Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.2.1-a+b-cos-^m-c+d-cos-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [932]. This is test number [89].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (932)	% 0.00 (0)
Mathematica	% 99.03 (923)	% 0.97 (9)
Maple	% 91.63 (854)	% 8.37 (78)
Maxima	% 31.22 (291)	% 68.78 (641)
Fricas	% 47.53 (443)	% 52.47 (489)
Sympy	% 10.19 (95)	% 89.81 (837)
Giac	% 28.86 (269)	% 71.14 (663)
Mupad	% 33.26 (310)	% 66.74 (622)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

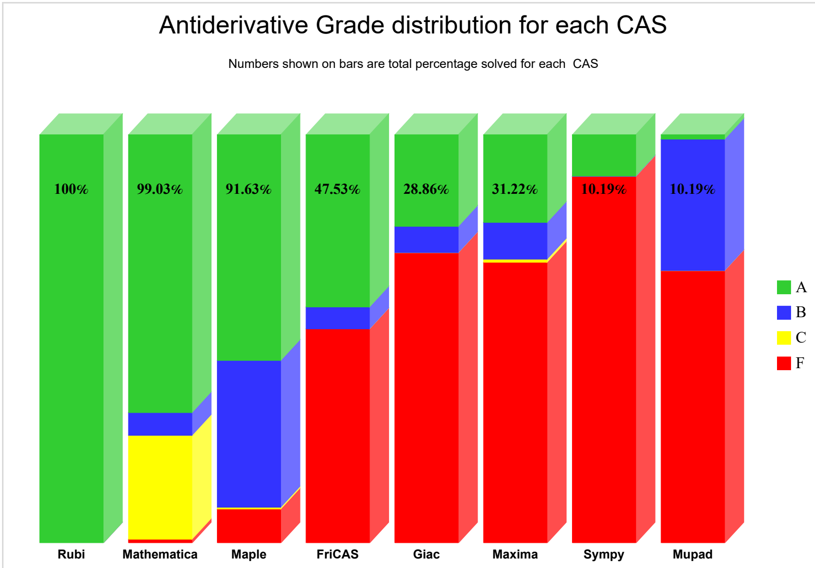
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

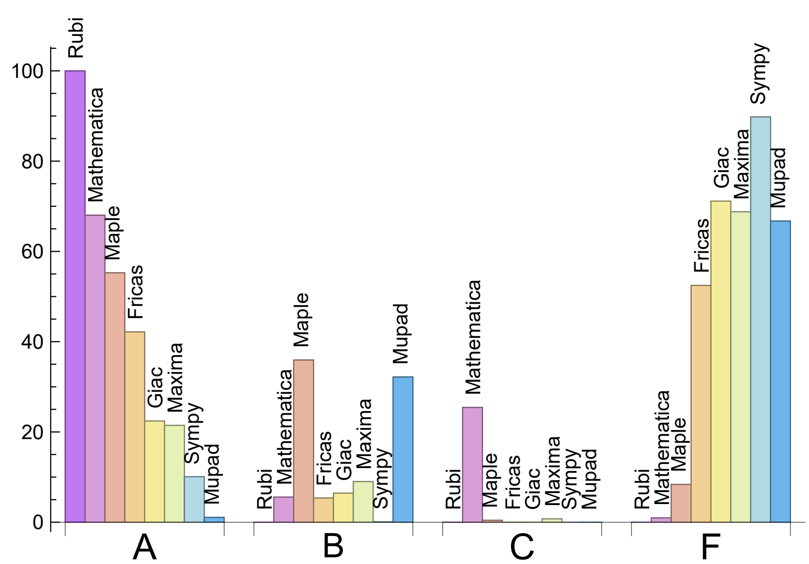
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	68.03	5.58	25.43	0.97
Maple	55.26	35.94	0.43	8.37
Maxima	21.46	9.01	0.75	68.78
Fricas	42.17	5.36	0.00	52.47
Sympy	10.09	0.11	0.00	89.81
Giac	22.42	6.44	0.00	71.14
Mupad	1.07	32.19	0.00	66.74

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	9	88.89 %	11.11 %	0.00 %
Maple	78	100.00 %	0.00 %	0.00 %
Maxima	641	83.46 %	6.40 %	10.14 %
Fricas	489	87.12 %	12.88 %	0.00 %
Sympy	837	44.56 %	55.44 %	0.00 %
Giac	663	90.35 %	8.14 %	1.51 %
Mupad	622	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

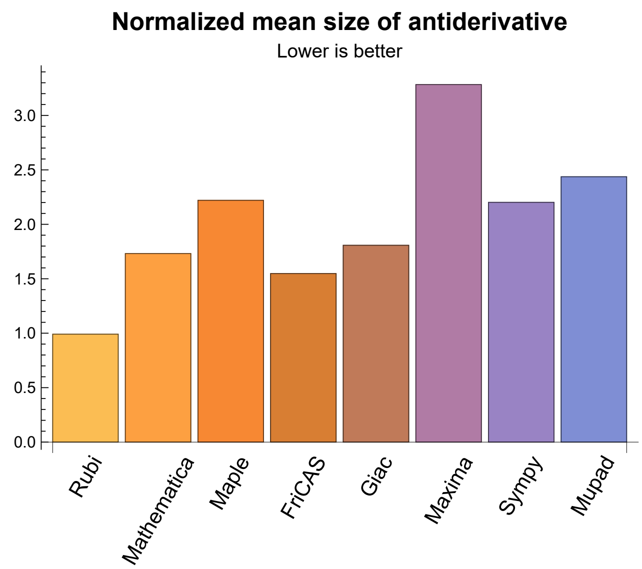
1.3 Performance

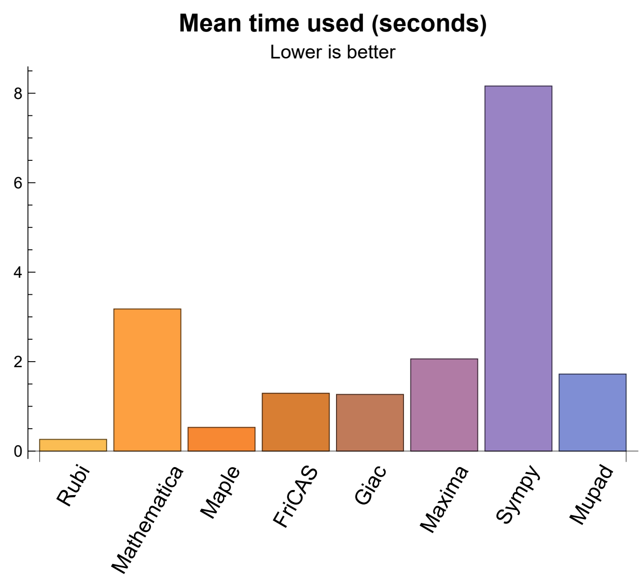
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	153.82	0.99	131.00	1.00
Mathematica	3.18	276.63	1.73	140.00	1.00
Maple	0.53	415.19	2.22	215.00	1.79
Maxima	2.06	373.74	3.28	118.00	1.26
Fricas	1.29	187.62	1.55	133.00	1.16
Sympy	8.16	201.08	2.20	124.00	1.83
Giac	1.27	175.53	1.81	106.00	1.04
Mupad	1.72	401.44	2.44	102.00	1.07

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{680, 681, 682, 683, 684, 685, 686, 687, 688, 689}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {22, 31, 117, 118, 119, 120, 128, 129, 130, 136, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 213, 214, 215, 216, 217, 228, 229, 230, 231, 232, 235, 236, 237, 242, 243, 244, 249, 250, 252, 253, 254, 256, 257, 260, 269, 286, 311, 356, 357, 358, 359, 360, 361, 362, 363, 367, 368, 369, 374, 379, 380, 382, 386, 387, 390, 392, 394, 400, 607, 610, 614, 617, 618, 619, 620, 621, 622, 625, 636, 638, 639, 643, 676, 677, 678, 679, 718, 720, 721, 724, 725, 726, 728, 730, 732, 734, 735, 736, 737, 743, 744, 745, 747, 748, 749, 750, 751, 754, 756, 757, 758, 761, 762, 763, 764, 765, 766, 768, 773, 774, 795, 796, 797, 798}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

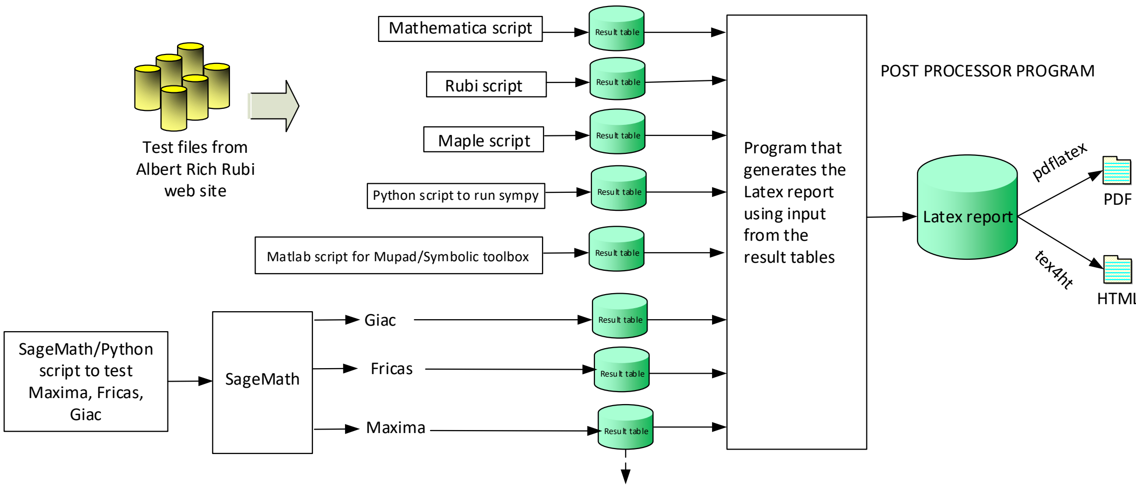
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 29, 33, 34, 35, 36, 39, 42, 43, 44, 45, 47, 48, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 137, 140, 141, 142, 143, 157, 171, 185, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 218, 219, 220, 221, 222, 227, 234, 240, 241, 246, 247, 248, 253, 254, 255, 256, 259, 260, 261, 262, 266, 267, 268, 272, 273, 274, 300, 313, 327, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 364, 370, 375, 376, 381, 382, 383, 388, 389, 390, 391, 394, 395, 396, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 493, 494, 495, 496, 497, 500, 501, 502, 503, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 538, 539, 540, 541, 542, 543, 546, 547, 548, 549, 550, 551, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 611, 612, 613, 614, 615, 619, 621, 622, 627, 628, 629, 630, 633, 634, 639, 650, 651, 656, 657, 661, 662, 663, 664, 665, 667, 668, 669, 670, 671, 672, 673, 675, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 727, 729, 730, 731, 732, 733, 734, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 749, 751, 752, 753, 754, 757, 758, 759, 760, 762, 763, 764, 765, 766, 767, 769, 770, 771, 772, 775, 776, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade: { 21, 22, 28, 30, 31, 32, 37, 38, 40, 41, 46, 49, 50, 51, 52, 59, 60, 61, 62, 69, 70, 71, 80, 81, 91, 92, 139, 586, 644, 645, 646, 647, 648, 649, 652, 653, 654, 655, 658, 659, 660, 676, 677, 678, 679, 720, 721, 726, 728, 748, 773, 774 }

C grade: { 117, 118, 119, 120, 128, 129, 130, 136, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 213, 214, 215, 216, 217, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 242, 243, 244, 245, 249, 250, 251, 252, 257, 258, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 371, 372, 374, 377, 378, 379, 380, 384, 385, 386, 387, 392, 393, 400, 459, 491, 492, 498, 499, 504, 505, 506, 507, 514, 515, 521, 522, 528, 529, 535, 536, 537, 544, 545, 552, 553, 559, 560, 603, 609, 610, 616, 617, 618, 620, 623, 624, 625, 626, 631, 632, 635, 636, 637, 638, 640, 641, 642, 643, 666, 674, 735, 750, 755, 756, 761, 768, 778, 788, 790 }

F grade: { 288, 289, 290, 373, 397, 398, 399, 401, 402 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 112, 113, 114, 115, 121, 122, 123, 124, 125, 131, 132, 133, 139, 140, 141, 142, 143, 146, 147, 149, 150, 153, 154, 155, 157, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 218, 219, 220, 224, 225, 226, 227, 231, 232, 238, 239, 240, 244, 246, 247, 253, 254, 255, 259, 260, 262, 263, 264, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 283, 284, 287, 293, 294, 295, 296, 297, 300, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 454, 455, 456, 459, 461, 463, 464, 466, 467, 490, 497, 504, 509, 510, 511, 513, 516, 517, 518, 520, 525, 527, 528, 533, 534, 535, 543, 546, 547, 548, 549, 554, 555, 556, 561, 564, 565, 572, 578, 583, 584, 605, 626, 627, 628, 650, 651, 656, 657, 658, 662, 663, 666, 667, 668, 669, 672, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 694, 695, 696, 700, 701, 702, 703, 704, 708, 709, 710, 711, 712, 715, 716, 733, 753, 754, 755, 779, 780, 783, 784, 785, 786, 793, 799, 800, 801, 802, 803, 806, 807, 808, 809, 810, 813, 814, 815, 816, 817, 820, 821, 822, 823, 824, 827, 828, 829, 830, 831, 834, 835, 836, 837, 838, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886 }

B grade: { 100, 101, 102, 103, 108, 109, 110, 111, 116, 117, 118, 119, 120, 127, 128, 129, 130, 134, 135, 136, 137, 138, 144, 145, 148, 151, 152, 156, 158, 159, 164, 165, 166, 171, 172, 173, 180, 185, 187, 188, 196, 201, 208, 209, 216, 217, 222, 223, 228, 229, 230, 233, 234, 235, 236, 237, 241, 242, 243, 245, 248, 249, 250, 251, 252, 256, 257, 258, 261, 265, 271, 281, 285, 286, 291, 292, 298, 299, 304, 305, 306, 311, 312, 313, 317, 327, 332, 343, 349, 356, 361, 362, 392, 449, 450, 451, 457, 458, 460, 462, 465, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 512, 514, 515, 519, 521, 522, 523, 524, 529, 530, 531, 532, 536, 537, 538, 539, 540, 541, 542, 544, 545, 551, 552, 553, 558, 559, 560, 562, 563, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 579, 580, 581, 582, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 652, 653, 654, 655, 659, 660, 661, 664, 665, 670, 671, 673, 674, 675, 690, 691, 697, 698, 699, 705, 706, 707, 713, 714, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 778, 781, 782, 791, 792, 794, 804, 805, 811, 812, 818, 819, 825, 826, 832, 833, 839, 840, 841 }

C grade: { 126, 526, 550, 557 }

F grade: { 221, 288, 289, 290, 396, 397, 398, 399, 400, 401, 402, 676, 677, 678, 679, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 795, 796, 797, 798, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 44, 45, 47, 48, 49, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 112, 113, 114, 115, 121, 210, 212, 218, 219, 220, 346, 348, 353, 354, 355, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 780, 793, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 854, 855, 858, 859, 860, 861, 862, 863, 866, 867, 868, 869, 873, 874, 875, 876, 880, 881, 882, 883 }

B grade: { 40, 42, 46, 50, 51, 52, 101, 102, 103, 109, 110, 118, 119, 126, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 221, 222, 223, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 339, 340, 341, 342, 343, 344, 345, 347, 349, 350, 351, 352, 356, 357, 358, 359, 360, 396, 781, 782, 783, 784, 848, 849, 856, 857, 864, 865, 870, 871, 872, 877, 878, 879, 884, 885, 886 }

C grade: { 278, 279, 280, 281, 285, 286, 287 }

F grade: { 100, 108, 111, 116, 117, 120, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 275, 276, 277, 282, 283, 284, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 138, 139, 140, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 403, 404, 405, 406, 407, 408, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 463, 464, 468, 469, 472, 480, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 778, 779, 780, 782, 784, 785, 791, 792, 793, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886 }

B grade: { 7, 8, 19, 101, 108, 109, 127, 128, 129, 134, 135, 136, 137, 141, 142, 143, 144, 145, 234, 235, 261, 283, 284, 285, 411, 412, 423, 456, 462, 465, 466, 467, 470, 471, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 781, 783, 786, 869 }

C grade: { }

F grade: { 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 397, 398, 399, 400, 401, 402, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, 24, 25, 26, 33, 34, 35, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 403, 404, 405, 406, 407, 408, 409, 410, 417, 418, 419, 420, 421, 428, 429, 430, 431, 438, 439, 440, 441, 452, 453, 454, 682, 683, 684, 685, 686, 687, 688, 780, 781, 782, 783, 791, 793, 844, 845, 868 }

B grade: { 411 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 49, 50, 51, 52, 59, 60, 61, 62, 69, 70, 71, 79, 80, 81, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 412, 413, 414, 415, 416, 422, 423, 424, 425, 426, 427, 432, 433, 434, 435, 436, 437, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 784, 785, 786, 787, 788, 789, 790, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 121, 122, 123, 124, 125, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 202, 203, 204, 205, 266, 267, 268, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 339, 340, 341, 342, 403, 404, 405, 406, 407, 408, 409, 410, 416, 417, 418, 419, 420, 421, 428, 429, 430, 431, 433, 434, 438, 439, 440, 441, 443, 444, 450, 451, 452, 454, 457, 459, 460, 462, 463, 464, 465, 467, 468, 470, 471, 472, 476, 478, 480, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 779, 780, 783, 785, 793 }

B grade: { 7, 8, 18, 19, 108, 116, 126, 127, 128, 129, 130, 136, 265, 271, 278, 411, 412, 413, 414, 415, 422, 423, 424, 425, 426, 427, 432, 435, 436, 437, 442, 445, 446, 447, 448, 449, 453, 455, 456, 458, 461, 466, 469, 473, 474, 475, 477, 479, 481, 482, 483, 484, 485, 778, 781, 782, 784, 786, 791, 792 }

C grade: { }

F grade: { 109, 110, 111, 117, 118, 119, 120, 137, 138, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 269, 270, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.8 Mupad

A grade: { 680, 681, 682, 683, 684, 685, 686, 687, 688, 689 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 124, 125, 126, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 202, 203, 204, 205, 210, 211, 212, 218, 219, 220, 221, 266, 267, 268, 272, 273, 274, 339, 340, 341, 342, 346, 347, 348, 353, 354, 355, 396, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 524, 525, 526, 548, 549, 550, 555, 556, 557, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 780, 781, 782, 783, 785, 786, 791, 792, 793, 794, 822, 823, 842, 843, 844, 845, 850, 851, 852, 853, 858, 859, 860, 861, 866, 867, 868, 873, 874, 875, 880, 881, 882 }

C grade: { }

F grade: { 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 206, 207, 208, 209, 213, 214, 215, 216, 217, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 551, 552, 553, 554, 558, 559, 560, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 784, 787, 788, 789, 790, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 846, 847, 848, 849, 854, 855, 856, 857, 862, 863, 864, 865, 869, 870, 871, 872, 876, 877, 878, 879, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	75	80	84	75	216	92	107
normalized size	1	1.00	0.66	0.70	0.74	0.66	1.89	0.81	0.94
time (sec)	N/A	0.070	0.138	0.055	0.321	1.054	3.160	0.462	2.976

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	65	70	69	64	168	77	93
normalized size	1	1.00	0.71	0.76	0.75	0.70	1.83	0.84	1.01
time (sec)	N/A	0.058	0.116	0.049	0.788	1.818	1.855	0.549	2.852

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	57	53	144	62	79
normalized size	1	1.00	0.96	0.79	0.75	0.70	1.89	0.82	1.04
time (sec)	N/A	0.052	0.087	0.045	0.297	0.931	0.912	0.494	3.602

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	46	42	92	47	55
normalized size	1	1.00	1.06	0.91	0.85	0.78	1.70	0.87	1.02
time (sec)	N/A	0.042	0.069	0.056	0.297	0.811	0.451	0.485	0.379

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	38	34	29	66	31	50
normalized size	1	1.00	0.84	1.00	0.89	0.76	1.74	0.82	1.32
time (sec)	N/A	0.014	0.050	0.042	0.295	2.334	0.200	0.292	0.745

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	15	15
normalized size	1	1.00	1.73	1.07	1.00	1.13	1.13	1.00	1.00
time (sec)	N/A	0.007	0.006	0.021	0.377	0.753	0.115	0.405	0.311

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	30	28	36	49	43	20
normalized size	1	1.00	1.00	1.88	1.75	2.25	3.06	2.69	1.25
time (sec)	N/A	0.020	0.008	0.071	0.299	1.676	4.922	0.476	0.343

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	38	60	0	63	47
normalized size	1	1.00	1.00	1.33	1.58	2.50	0.00	2.62	1.96
time (sec)	N/A	0.034	0.010	0.104	2.228	1.007	0.000	0.706	0.385

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	58	74	0	80	75
normalized size	1	1.00	1.00	1.09	1.23	1.57	0.00	1.70	1.60
time (sec)	N/A	0.048	0.014	0.104	0.503	1.633	0.000	0.610	0.712

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	70	88	0	96	102
normalized size	1	1.00	0.95	1.14	1.11	1.40	0.00	1.52	1.62
time (sec)	N/A	0.048	0.146	0.148	0.443	1.233	0.000	0.482	2.038

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	95	99	0	110	130
normalized size	1	1.00	0.89	1.08	1.12	1.16	0.00	1.29	1.53
time (sec)	N/A	0.062	0.151	0.149	0.973	0.871	0.000	0.523	3.339

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	65	112	107	110	0	124	158
normalized size	1	1.00	0.64	1.11	1.06	1.09	0.00	1.23	1.56
time (sec)	N/A	0.066	0.250	0.145	0.306	1.219	0.000	1.447	4.770

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	73	121	121	89	343	106	121
normalized size	1	1.00	0.57	0.94	0.94	0.69	2.66	0.82	0.94
time (sec)	N/A	0.130	0.196	0.061	0.302	1.644	3.530	0.803	2.864

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	61	96	95	76	221	89	105
normalized size	1	1.00	0.59	0.93	0.92	0.74	2.15	0.86	1.02
time (sec)	N/A	0.104	0.126	0.056	1.128	1.460	1.981	0.536	3.628

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	90	83	63	211	72	89
normalized size	1	1.00	0.61	1.03	0.95	0.72	2.43	0.83	1.02
time (sec)	N/A	0.097	0.129	0.055	0.485	1.563	1.022	0.810	3.495

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	69	41	64	61	49	107	54	61
normalized size	1	1.21	0.72	1.12	1.07	0.86	1.88	0.95	1.07
time (sec)	N/A	0.041	0.084	0.050	0.466	1.561	0.476	0.472	0.379

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	45	36	78	38	57
normalized size	1	1.00	0.76	1.16	1.00	0.80	1.73	0.84	1.27
time (sec)	N/A	0.014	0.047	0.047	0.297	1.173	0.249	0.456	0.724

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	47	51	43	53	0	79	33
normalized size	1	1.00	1.38	1.50	1.26	1.56	0.00	2.32	0.97
time (sec)	N/A	0.058	0.013	0.097	0.298	1.428	0.000	0.661	0.399

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	50	49	76	0	79	56
normalized size	1	1.00	0.82	1.47	1.44	2.24	0.00	2.32	1.65
time (sec)	N/A	0.058	0.015	0.110	0.989	1.613	0.000	0.576	0.390

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	58	88	83	0	90	83
normalized size	1	1.00	1.00	1.07	1.63	1.54	0.00	1.67	1.54
time (sec)	N/A	0.079	0.012	0.130	0.452	1.339	0.000	0.759	0.706

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	162	78	85	96	0	106	112
normalized size	1	1.00	2.45	1.18	1.29	1.45	0.00	1.61	1.70
time (sec)	N/A	0.088	5.785	0.127	0.348	1.635	0.000	0.642	2.008

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	797	102	145	111	0	122	141
normalized size	1	1.00	8.30	1.06	1.51	1.16	0.00	1.27	1.47
time (sec)	N/A	0.108	6.431	0.135	0.308	1.895	0.000	0.675	3.352

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	73	143	143	89	379	106	121
normalized size	1	1.00	0.57	1.11	1.11	0.69	2.94	0.82	0.94
time (sec)	N/A	0.147	0.185	0.056	1.975	0.879	3.653	0.623	2.869

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	63	121	117	76	272	88	105
normalized size	1	1.00	0.60	1.15	1.11	0.72	2.59	0.84	1.00
time (sec)	N/A	0.117	0.128	0.051	0.634	2.161	2.077	0.789	3.711

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	88	51	100	94	63	224	71	89
normalized size	1	1.04	0.60	1.18	1.11	0.74	2.64	0.84	1.05
time (sec)	N/A	0.078	0.121	0.045	0.838	1.528	1.028	0.397	3.486

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	70	50	121	55	63
normalized size	1	1.00	0.70	1.17	1.11	0.79	1.92	0.87	1.00
time (sec)	N/A	0.053	0.069	0.051	0.357	1.968	0.502	0.429	0.396

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	72	67	65	0	100	88
normalized size	1	1.00	1.37	1.22	1.14	1.10	0.00	1.69	1.49
time (sec)	N/A	0.062	0.075	0.087	1.050	1.639	0.000	0.638	0.437

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	211	65	64	91	0	80	57
normalized size	1	1.00	4.40	1.35	1.33	1.90	0.00	1.67	1.19
time (sec)	N/A	0.067	0.732	0.138	1.004	0.798	0.000	0.607	0.410

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	71	99	98	0	100	88
normalized size	1	1.00	0.85	1.20	1.68	1.66	0.00	1.69	1.49
time (sec)	N/A	0.083	0.028	0.119	1.111	1.054	0.000	0.849	0.440

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	154	80	111	98	0	106	112
normalized size	1	1.00	2.14	1.11	1.54	1.36	0.00	1.47	1.56
time (sec)	N/A	0.095	5.403	0.118	0.308	0.665	0.000	0.601	2.039

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	797	101	156	111	0	122	141
normalized size	1	1.00	8.57	1.09	1.68	1.19	0.00	1.31	1.52
time (sec)	N/A	0.117	6.387	0.132	1.100	1.026	0.000	0.831	3.293

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	487	124	179	124	0	138	170
normalized size	1	1.00	4.27	1.09	1.57	1.09	0.00	1.21	1.49
time (sec)	N/A	0.127	1.433	0.151	1.025	0.974	0.000	0.539	4.626

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	73	169	165	89	434	106	121
normalized size	1	1.00	0.57	1.33	1.30	0.70	3.42	0.83	0.95
time (sec)	N/A	0.157	0.197	0.053	1.754	0.908	3.867	0.558	2.855

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	114	63	133	128	76	280	89	105
normalized size	1	1.12	0.62	1.30	1.25	0.75	2.75	0.87	1.03
time (sec)	N/A	0.107	0.147	0.053	0.302	0.933	2.167	0.523	3.685

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	56	111	106	63	224	72	89
normalized size	1	1.00	0.64	1.28	1.22	0.72	2.57	0.83	1.02
time (sec)	N/A	0.081	0.111	0.053	0.739	1.027	1.075	0.708	3.557

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	91	94	89	80	0	116	93
normalized size	1	1.00	1.25	1.29	1.22	1.10	0.00	1.59	1.27
time (sec)	N/A	0.081	0.109	0.114	1.523	0.812	0.000	0.698	0.411

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	241	86	85	105	0	129	117
normalized size	1	1.00	3.30	1.18	1.16	1.44	0.00	1.77	1.60
time (sec)	N/A	0.083	1.246	0.121	0.673	0.885	0.000	0.574	0.596

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	272	86	110	111	0	129	115
normalized size	1	1.00	3.73	1.18	1.51	1.52	0.00	1.77	1.58
time (sec)	N/A	0.087	1.237	0.154	0.661	1.714	0.000	0.608	0.589

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	93	120	110	0	116	117
normalized size	1	1.00	0.84	1.27	1.64	1.51	0.00	1.59	1.60
time (sec)	N/A	0.096	0.037	0.140	1.855	0.962	0.000	0.669	0.621

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	797	102	182	111	0	122	141
normalized size	1	1.00	8.30	1.06	1.90	1.16	0.00	1.27	1.47
time (sec)	N/A	0.126	6.365	0.164	1.013	1.998	0.000	0.859	3.502

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	498	123	190	124	0	138	170
normalized size	1	1.00	4.49	1.11	1.71	1.12	0.00	1.24	1.53
time (sec)	N/A	0.144	1.431	0.135	1.957	0.759	0.000	0.803	4.612

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	211	146	270	137	0	154	199
normalized size	1	1.00	1.55	1.07	1.99	1.01	0.00	1.13	1.46
time (sec)	N/A	0.182	0.776	0.210	1.370	0.780	0.000	0.706	3.874

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	173	171	217	79	882	101	98
normalized size	1	1.00	1.47	1.45	1.84	0.67	7.47	0.86	0.83
time (sec)	N/A	0.107	0.321	0.074	1.240	1.054	6.506	0.437	1.943

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	143	136	176	70	570	88	70
normalized size	1	1.00	1.52	1.45	1.87	0.74	6.06	0.94	0.74
time (sec)	N/A	0.092	0.276	0.071	1.555	1.101	3.898	0.685	0.597

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	117	103	133	57	325	73	89
normalized size	1	1.00	1.54	1.36	1.75	0.75	4.28	0.96	1.17
time (sec)	N/A	0.061	0.231	0.078	1.334	1.169	2.320	0.364	0.409

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	89	68	92	46	129	58	66
normalized size	1	1.00	2.07	1.58	2.14	1.07	3.00	1.35	1.53
time (sec)	N/A	0.080	0.196	0.054	2.251	0.999	1.405	0.451	0.400

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	57	37	49	37	27	28	23
normalized size	1	1.00	1.97	1.28	1.69	1.28	0.93	0.97	0.79
time (sec)	N/A	0.034	0.072	0.050	1.223	0.895	0.751	0.502	0.331

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	23	22	20	16	16
normalized size	1	1.00	0.77	0.77	1.05	1.00	0.91	0.73	0.73
time (sec)	N/A	0.012	0.013	0.041	1.118	0.910	0.508	0.407	0.315

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	103	58	75	65	0	54	31
normalized size	1	1.00	2.71	1.53	1.97	1.71	0.00	1.42	0.82
time (sec)	N/A	0.048	0.150	0.077	1.119	0.927	0.000	0.499	0.354

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	188	99	119	97	0	84	67
normalized size	1	1.00	3.55	1.87	2.25	1.83	0.00	1.58	1.26
time (sec)	N/A	0.076	0.694	0.090	0.490	1.101	0.000	0.473	0.395

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	244	143	162	112	0	101	95
normalized size	1	1.00	2.94	1.72	1.95	1.35	0.00	1.22	1.14
time (sec)	N/A	0.093	1.298	0.108	0.313	1.062	0.000	0.806	0.454

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	368	183	205	124	0	114	96
normalized size	1	1.00	3.57	1.78	1.99	1.20	0.00	1.11	0.93
time (sec)	N/A	0.095	4.271	0.121	0.816	1.055	0.000	0.498	0.571

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	199	156	207	108	700	108	135
normalized size	1	1.00	1.60	1.26	1.67	0.87	5.65	0.87	1.09
time (sec)	N/A	0.185	0.433	0.070	1.357	1.075	9.744	0.879	0.499

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	177	122	164	99	413	95	113
normalized size	1	1.00	1.55	1.07	1.44	0.87	3.62	0.83	0.99
time (sec)	N/A	0.152	0.289	0.054	1.351	1.998	6.124	0.563	0.453

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	114	88	118	90	201	79	91
normalized size	1	1.00	1.42	1.10	1.48	1.12	2.51	0.99	1.14
time (sec)	N/A	0.170	0.348	0.062	1.502	1.345	3.638	0.460	0.422

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	105	56	72	80	56	50	35
normalized size	1	1.00	1.84	0.98	1.26	1.40	0.98	0.88	0.61
time (sec)	N/A	0.085	0.225	0.058	1.001	0.991	2.019	0.415	0.358

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	32	47	51	48	31	30
normalized size	1	1.00	1.09	0.58	0.85	0.93	0.87	0.56	0.55
time (sec)	N/A	0.038	0.114	0.050	0.310	1.527	1.291	0.451	0.334

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	32	46	49	44	31	30
normalized size	1	1.00	0.96	0.58	0.84	0.89	0.80	0.56	0.55
time (sec)	N/A	0.027	0.048	0.044	1.101	1.065	0.895	0.763	0.334

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	152	77	98	114	0	77	43
normalized size	1	1.00	2.30	1.17	1.48	1.73	0.00	1.17	0.65
time (sec)	N/A	0.112	0.289	0.088	0.993	0.896	0.000	0.436	0.373

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	239	120	145	146	0	106	92
normalized size	1	1.00	2.95	1.48	1.79	1.80	0.00	1.31	1.14
time (sec)	N/A	0.173	1.124	0.109	0.646	0.680	0.000	0.635	0.413

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	292	162	190	162	0	122	122
normalized size	1	1.00	2.45	1.36	1.60	1.36	0.00	1.03	1.03
time (sec)	N/A	0.190	1.812	0.140	0.947	1.098	0.000	0.598	0.428

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	343	204	234	172	0	135	153
normalized size	1	1.00	2.58	1.53	1.76	1.29	0.00	1.02	1.15
time (sec)	N/A	0.199	3.882	0.100	0.950	0.956	0.000	0.926	0.447

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	173	141	184	135	473	113	137
normalized size	1	1.00	1.13	0.92	1.20	0.88	3.09	0.74	0.90
time (sec)	N/A	0.265	0.572	0.064	1.256	1.016	13.647	0.378	0.472

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	161	107	137	126	240	96	113
normalized size	1	1.00	1.35	0.90	1.15	1.06	2.02	0.81	0.95
time (sec)	N/A	0.273	0.534	0.056	1.228	1.184	8.555	0.548	0.438

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	154	75	92	116	75	68	81
normalized size	1	1.00	1.60	0.78	0.96	1.21	0.78	0.71	0.84
time (sec)	N/A	0.184	0.237	0.056	0.689	1.746	5.154	0.472	0.421

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	86	45	67	75	68	46	45
normalized size	1	1.00	1.04	0.54	0.81	0.90	0.82	0.55	0.54
time (sec)	N/A	0.094	0.187	0.048	1.011	2.212	3.363	0.514	0.355

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	32	47	73	48	31	30
normalized size	1	1.00	0.86	0.39	0.57	0.88	0.58	0.37	0.36
time (sec)	N/A	0.058	0.137	0.045	0.482	0.764	2.279	0.414	0.338

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	45	67	75	63	46	45
normalized size	1	1.00	0.78	0.54	0.81	0.90	0.76	0.55	0.54
time (sec)	N/A	0.046	0.081	0.041	0.333	0.772	1.634	0.463	0.351

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	201	96	119	158	0	94	58
normalized size	1	1.00	2.07	0.99	1.23	1.63	0.00	0.97	0.60
time (sec)	N/A	0.201	0.483	0.082	1.119	1.206	0.000	0.586	0.405

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	286	139	165	190	0	122	111
normalized size	1	1.00	2.55	1.24	1.47	1.70	0.00	1.09	0.99
time (sec)	N/A	0.279	1.146	0.082	1.477	0.891	0.000	0.520	0.401

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	343	181	211	206	0	139	141
normalized size	1	1.00	2.20	1.16	1.35	1.32	0.00	0.89	0.90
time (sec)	N/A	0.305	3.852	0.115	0.959	1.149	0.000	0.722	0.393

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	289	160	204	171	530	128	159
normalized size	1	1.00	1.57	0.87	1.11	0.93	2.88	0.70	0.86
time (sec)	N/A	0.382	0.584	0.063	1.120	1.733	29.670	0.603	0.518

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	263	126	158	162	280	112	137
normalized size	1	1.00	1.75	0.84	1.05	1.08	1.87	0.75	0.91
time (sec)	N/A	0.373	0.416	0.058	0.812	0.655	19.067	0.492	0.472

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	224	94	112	152	95	83	102
normalized size	1	1.00	1.76	0.74	0.88	1.20	0.75	0.65	0.80
time (sec)	N/A	0.283	0.341	0.050	1.264	0.820	12.004	0.515	0.429

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	112	58	87	99	88	59	58
normalized size	1	1.00	0.98	0.51	0.76	0.87	0.77	0.52	0.51
time (sec)	N/A	0.199	0.271	0.053	0.864	0.594	8.349	0.448	0.387

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	58	87	99	87	59	58
normalized size	1	1.00	0.88	0.52	0.78	0.88	0.78	0.53	0.52
time (sec)	N/A	0.112	0.260	0.045	0.835	1.058	6.051	0.562	0.390

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	58	87	99	85	59	58
normalized size	1	1.00	0.78	0.52	0.78	0.88	0.76	0.53	0.52
time (sec)	N/A	0.078	0.235	0.044	1.195	0.915	4.464	0.559	0.389

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	77	56	87	99	83	59	58
normalized size	1	1.00	0.69	0.50	0.78	0.88	0.74	0.53	0.52
time (sec)	N/A	0.070	0.165	0.043	0.997	0.917	3.423	0.440	0.380

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	185	115	139	202	0	110	83
normalized size	1	1.00	1.54	0.96	1.16	1.68	0.00	0.92	0.69
time (sec)	N/A	0.287	0.860	0.088	1.478	1.039	0.000	0.693	0.373

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	341	158	186	234	0	139	130
normalized size	1	1.00	2.53	1.17	1.38	1.73	0.00	1.03	0.96
time (sec)	N/A	0.391	4.148	0.087	1.474	0.900	0.000	0.608	0.420

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	455	200	231	250	0	155	160
normalized size	1	1.00	2.46	1.08	1.25	1.35	0.00	0.84	0.86
time (sec)	N/A	0.432	6.266	0.111	1.104	1.087	0.000	0.878	0.466

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	345	179	224	207	588	145	181
normalized size	1	1.00	1.53	0.80	1.00	0.92	2.61	0.64	0.80
time (sec)	N/A	0.516	0.781	0.066	2.430	0.882	64.315	1.471	0.583

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	319	145	178	198	320	129	159
normalized size	1	1.00	1.67	0.76	0.93	1.04	1.68	0.68	0.83
time (sec)	N/A	0.490	0.729	0.058	0.774	1.188	42.517	0.743	0.513

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	280	113	132	188	116	100	125
normalized size	1	1.00	1.67	0.67	0.79	1.12	0.69	0.60	0.74
time (sec)	N/A	0.392	0.490	0.054	0.699	0.706	28.097	0.551	0.478

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	138	71	107	123	107	72	127
normalized size	1	1.00	0.89	0.46	0.69	0.79	0.69	0.46	0.82
time (sec)	N/A	0.300	0.277	0.047	0.992	1.022	19.963	1.415	0.432

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	58	87	123	87	59	58
normalized size	1	1.00	0.85	0.39	0.59	0.84	0.59	0.40	0.39
time (sec)	N/A	0.228	0.240	0.056	1.905	0.855	15.351	0.456	0.393

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	110	45	67	123	68	46	45
normalized size	1	1.00	0.79	0.32	0.48	0.88	0.49	0.33	0.32
time (sec)	N/A	0.144	0.237	0.057	0.678	0.726	11.579	0.501	0.357

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	97	58	87	123	85	59	58
normalized size	1	1.00	0.68	0.41	0.61	0.86	0.59	0.41	0.41
time (sec)	N/A	0.105	0.191	0.047	0.973	0.581	9.184	0.570	0.397

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	89	71	107	123	102	72	127
normalized size	1	1.00	0.62	0.50	0.75	0.86	0.71	0.50	0.89
time (sec)	N/A	0.091	0.162	0.039	1.013	1.080	7.603	0.377	0.417

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	211	134	159	246	0	126	99
normalized size	1	1.00	1.38	0.88	1.04	1.61	0.00	0.82	0.65
time (sec)	N/A	0.377	1.811	0.095	0.962	1.244	0.000	0.788	0.392

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	453	177	206	278	0	155	149
normalized size	1	1.00	2.70	1.05	1.23	1.65	0.00	0.92	0.89
time (sec)	N/A	0.533	6.367	0.094	1.058	1.045	0.000	0.553	0.448

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	507	219	251	294	0	171	179
normalized size	1	1.00	2.26	0.98	1.12	1.31	0.00	0.76	0.80
time (sec)	N/A	0.540	6.349	0.111	0.686	1.144	0.000	0.788	0.478

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	164	84	127	147	129	85	75
normalized size	1	1.00	0.89	0.46	0.69	0.80	0.70	0.46	0.41
time (sec)	N/A	0.410	0.363	0.055	0.673	0.801	39.013	0.474	0.882

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	151	84	127	147	124	85	151
normalized size	1	1.00	0.86	0.48	0.72	0.84	0.70	0.48	0.86
time (sec)	N/A	0.318	0.324	0.048	1.489	0.843	30.356	0.708	0.463

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	92	97	79	72	0	129	-1
normalized size	1	1.00	0.58	0.61	0.50	0.46	0.00	0.82	-0.01
time (sec)	N/A	0.241	0.277	0.178	1.132	0.722	0.000	0.654	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	80	84	65	62	0	105	-1
normalized size	1	1.00	0.66	0.69	0.53	0.51	0.00	0.86	-0.01
time (sec)	N/A	0.176	0.160	0.170	1.452	0.682	0.000	1.259	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	68	71	51	52	0	81	-1
normalized size	1	1.00	0.79	0.83	0.59	0.60	0.00	0.94	-0.01
time (sec)	N/A	0.115	0.102	0.166	1.396	0.816	0.000	0.528	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	58	36	40	0	56	-1
normalized size	1	1.00	0.96	1.04	0.64	0.71	0.00	1.00	-0.02
time (sec)	N/A	0.046	0.073	0.165	1.883	1.129	0.000	0.474	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	20	32	0	30	33
normalized size	1	1.00	1.12	1.65	0.77	1.23	0.00	1.15	1.27
time (sec)	N/A	0.013	0.030	0.000	1.637	1.044	0.000	0.393	0.464

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	50	180	0	146	0	58	-1
normalized size	1	1.00	1.35	4.86	0.00	3.95	0.00	1.57	-0.03
time (sec)	N/A	0.051	0.047	0.505	0.000	1.019	0.000	0.527	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	79	379	1170	140	0	104	-1
normalized size	1	1.00	1.27	6.11	18.87	2.26	0.00	1.68	-0.02
time (sec)	N/A	0.104	0.095	0.476	2.591	1.279	0.000	0.752	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	94	545	2642	155	0	131	-1
normalized size	1	1.00	0.92	5.34	25.90	1.52	0.00	1.28	-0.01
time (sec)	N/A	0.162	0.181	0.527	11.912	1.155	0.000	1.011	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	109	709	5115	165	0	154	-1
normalized size	1	1.00	0.79	5.14	37.07	1.20	0.00	1.12	-0.01
time (sec)	N/A	0.218	0.329	0.575	171.932	0.743	0.000	0.687	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	93	99	84	78	0	134	-1
normalized size	1	1.00	0.57	0.61	0.52	0.48	0.00	0.83	-0.01
time (sec)	N/A	0.247	0.263	0.147	1.053	1.545	0.000	0.668	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	81	86	69	67	0	109	-1
normalized size	1	1.00	0.70	0.74	0.59	0.58	0.00	0.94	-0.01
time (sec)	N/A	0.141	0.169	0.155	1.007	0.990	0.000	0.639	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	71	53	55	0	83	-1
normalized size	1	1.00	0.78	0.83	0.62	0.64	0.00	0.97	-0.01
time (sec)	N/A	0.065	0.100	0.162	1.127	0.589	0.000	0.518	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	38	44	0	58	-1
normalized size	1	1.00	0.93	0.98	0.64	0.75	0.00	0.98	-0.02
time (sec)	N/A	0.029	0.064	0.144	1.130	0.968	0.000	0.411	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	207	0	127	0	1884	-1
normalized size	1	1.00	0.98	3.14	0.00	1.92	0.00	28.55	-0.02
time (sec)	N/A	0.107	0.075	0.454	0.000	1.683	0.000	5.344	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	81	381	1314	146	0	0	-1
normalized size	1	1.00	1.25	5.86	20.22	2.25	0.00	0.00	-0.02
time (sec)	N/A	0.118	0.112	0.440	1.670	1.045	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	97	545	3216	162	0	0	-1
normalized size	1	1.00	0.92	5.14	30.34	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.232	0.570	3.697	1.892	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	110	710	0	173	0	0	-1
normalized size	1	1.00	0.76	4.93	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.348	0.584	0.000	0.883	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	107	112	111	101	0	171	-1
normalized size	1	1.00	0.53	0.55	0.55	0.50	0.00	0.84	-0.00
time (sec)	N/A	0.363	0.498	0.183	1.219	0.981	0.000	2.026	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	95	99	94	88	0	144	-1
normalized size	1	1.00	0.65	0.68	0.64	0.60	0.00	0.99	-0.01
time (sec)	N/A	0.160	0.288	0.184	1.177	0.732	0.000	1.329	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	84	86	77	75	0	117	-1
normalized size	1	1.00	0.72	0.74	0.66	0.65	0.00	1.01	-0.01
time (sec)	N/A	0.087	0.229	0.143	0.971	0.605	0.000	0.643	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	60	62	0	90	-1
normalized size	1	1.00	0.80	0.82	0.67	0.70	0.00	1.01	-0.01
time (sec)	N/A	0.050	0.104	0.000	0.992	0.920	0.000	0.723	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	89	244	0	147	0	5671	-1
normalized size	1	1.00	0.91	2.49	0.00	1.50	0.00	57.87	-0.01
time (sec)	N/A	0.202	0.510	0.564	0.000	1.223	0.000	56.699	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	1547	408	0	164	0	0	-1
normalized size	1	1.00	16.82	4.43	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.198	36.227	0.479	0.000	1.199	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	1693	545	3667	170	0	0	-1
normalized size	1	1.00	15.97	5.14	34.59	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.222	36.008	0.527	13.577	0.675	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	1825	709	6703	183	0	0	-1
normalized size	1	1.00	12.67	4.92	46.55	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.282	36.113	0.563	131.605	0.963	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	2069	872	0	196	0	0	-1
normalized size	1	1.00	11.37	4.79	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.345	35.840	0.598	0.000	1.475	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	83	86	77	75	0	117	-1
normalized size	1	1.00	0.70	0.72	0.65	0.63	0.00	0.98	-0.01
time (sec)	N/A	0.068	0.240	0.159	1.066	0.701	0.000	0.804	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	130	194	0	153	0	118	-1
normalized size	1	1.00	0.75	1.11	0.00	0.88	0.00	0.68	-0.01
time (sec)	N/A	0.372	0.199	0.460	0.000	1.293	0.000	1.249	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	118	183	0	143	0	116	-1
normalized size	1	1.00	0.84	1.31	0.00	1.02	0.00	0.83	-0.01
time (sec)	N/A	0.239	0.167	0.388	0.000	0.680	0.000	1.288	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	135	0	131	0	79	97
normalized size	1	1.00	1.00	1.30	0.00	1.26	0.00	0.76	0.93
time (sec)	N/A	0.125	0.112	0.288	0.000	1.145	0.000	1.675	0.381

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	53	120	0	122	0	74	60
normalized size	1	1.00	0.73	1.64	0.00	1.67	0.00	1.01	0.82
time (sec)	N/A	0.051	0.045	0.312	0.000	1.308	0.000	1.058	0.396

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	54	90	126	0	93	45
normalized size	1	1.00	0.87	1.17	1.96	2.74	0.00	2.02	0.98
time (sec)	N/A	0.022	0.012	0.064	1.309	0.883	0.000	0.615	0.357

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	224	0	164	0	162	-1
normalized size	1	1.00	0.76	2.64	0.00	1.93	0.00	1.91	-0.01
time (sec)	N/A	0.114	0.052	0.608	0.000	0.734	0.000	1.869	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	1540	466	0	236	0	290	-1
normalized size	1	1.00	14.26	4.31	0.00	2.19	0.00	2.69	-0.01
time (sec)	N/A	0.213	27.563	0.673	0.000	0.899	0.000	2.709	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	1791	671	0	251	0	371	-1
normalized size	1	1.00	12.18	4.56	0.00	1.71	0.00	2.52	-0.01
time (sec)	N/A	0.340	31.172	0.676	0.000	0.800	0.000	2.343	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	1921	875	0	263	0	451	-1
normalized size	1	1.00	10.61	4.83	0.00	1.45	0.00	2.49	-0.01
time (sec)	N/A	0.489	30.131	0.624	0.000	0.849	0.000	3.735	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	226	265	0	184	0	137	-1
normalized size	1	1.00	1.23	1.45	0.00	1.01	0.00	0.75	-0.01
time (sec)	N/A	0.400	1.355	0.317	0.000	0.979	0.000	1.271	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	196	234	0	174	0	115	-1
normalized size	1	1.00	1.35	1.61	0.00	1.20	0.00	0.79	-0.01
time (sec)	N/A	0.261	0.963	0.319	0.000	0.680	0.000	1.348	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	164	173	0	164	0	102	-1
normalized size	1	1.00	1.56	1.65	0.00	1.56	0.00	0.97	-0.01
time (sec)	N/A	0.134	0.464	0.310	0.000	0.705	0.000	2.046	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	140	0	154	0	81	-1
normalized size	1	1.00	0.70	1.82	0.00	2.00	0.00	1.05	-0.01
time (sec)	N/A	0.059	0.098	0.302	0.000	0.877	0.000	1.434	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	138	0	153	0	81	-1
normalized size	1	1.00	0.82	1.79	0.00	1.99	0.00	1.05	-0.01
time (sec)	N/A	0.039	0.068	0.000	0.000	1.783	0.000	1.244	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	1787	290	0	254	0	189	-1
normalized size	1	1.00	15.68	2.54	0.00	2.23	0.00	1.66	-0.01
time (sec)	N/A	0.221	23.749	0.667	0.000	1.537	0.000	2.764	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	103	567	0	286	0	0	-1
normalized size	1	1.00	0.72	3.94	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.374	0.484	0.628	0.000	1.212	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	1941	807	0	302	0	0	-1
normalized size	1	1.00	10.49	4.36	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.499	28.429	1.052	0.000	1.625	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	587	242	0	208	0	146	-1
normalized size	1	1.00	3.21	1.32	0.00	1.14	0.00	0.80	-0.01
time (sec)	N/A	0.412	6.350	0.321	0.000	0.834	0.000	3.961	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	216	208	0	198	0	124	-1
normalized size	1	1.00	1.49	1.43	0.00	1.37	0.00	0.86	-0.01
time (sec)	N/A	0.270	4.046	0.419	0.000	1.120	0.000	3.130	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	103	174	0	188	0	103	-1
normalized size	1	1.00	0.96	1.63	0.00	1.76	0.00	0.96	-0.01
time (sec)	N/A	0.137	1.132	0.356	0.000	1.044	0.000	1.860	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	65	174	0	188	0	103	-1
normalized size	1	1.00	0.61	1.63	0.00	1.76	0.00	0.96	-0.01
time (sec)	N/A	0.080	0.249	0.311	0.000	1.015	0.000	1.462	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	65	174	0	188	0	103	-1
normalized size	1	1.00	0.61	1.63	0.00	1.76	0.00	0.96	-0.01
time (sec)	N/A	0.063	0.145	0.302	0.000	1.196	0.000	2.360	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	1919	325	0	298	0	211	-1
normalized size	1	1.00	13.33	2.26	0.00	2.07	0.00	1.47	-0.01
time (sec)	N/A	0.335	24.138	0.609	0.000	1.062	0.000	3.591	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	2051	601	0	330	0	0	-1
normalized size	1	1.00	11.79	3.45	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.519	24.098	0.603	0.000	1.985	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	490	270	0	0	0	0	87
normalized size	1	1.00	4.41	2.43	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.078	6.169	0.421	0.000	0.852	0.000	0.000	0.764

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	232	219	0	0	0	0	80
normalized size	1	1.00	2.67	2.52	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.067	5.617	0.547	0.000	5.958	0.000	0.000	0.131

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	222	225	0	0	0	0	53
normalized size	1	1.00	3.64	3.69	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.051	5.006	0.561	0.000	0.984	0.000	0.000	0.121

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	0	0	0	27
normalized size	1	1.00	4.43	4.29	0.00	0.00	0.00	0.00	0.77
time (sec)	N/A	0.039	24.645	0.404	0.000	1.975	0.000	0.000	0.461

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	209	146	0	0	0	0	60
normalized size	1	1.00	3.67	2.56	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.050	9.674	0.451	0.000	1.101	0.000	0.000	0.645

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	444	369	0	0	0	0	87
normalized size	1	1.00	5.35	4.45	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.059	6.154	0.945	0.000	1.869	0.000	0.000	0.780

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	477	384	0	0	0	0	87
normalized size	1	1.00	4.30	3.46	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.074	6.173	0.834	0.000	1.166	0.000	0.000	0.871

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	532	260	0	0	0	0	136
normalized size	1	1.00	3.62	1.77	0.00	0.00	0.00	0.00	0.93
time (sec)	N/A	0.138	6.136	0.513	0.000	2.067	0.000	0.000	0.769

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	500	272	0	0	0	0	129
normalized size	1	1.00	4.13	2.25	0.00	0.00	0.00	0.00	1.07
time (sec)	N/A	0.119	6.128	0.457	0.000	1.662	0.000	0.000	0.649

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	235	250	0	0	0	0	104
normalized size	1	1.00	2.47	2.63	0.00	0.00	0.00	0.00	1.09
time (sec)	N/A	0.093	5.622	0.451	0.000	0.823	0.000	0.000	0.742

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	224	228	0	0	0	0	59
normalized size	1	1.00	3.34	3.40	0.00	0.00	0.00	0.00	0.88
time (sec)	N/A	0.081	5.115	0.484	0.000	2.032	0.000	0.000	0.680

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	104	0	0	0	0	82
normalized size	1	1.00	0.89	2.36	0.00	0.00	0.00	0.00	1.86
time (sec)	N/A	0.080	0.167	0.500	0.000	0.913	0.000	0.000	0.802

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	454	371	0	0	0	0	109
normalized size	1	1.00	4.99	4.08	0.00	0.00	0.00	0.00	1.20
time (sec)	N/A	0.093	6.156	0.816	0.000	0.876	0.000	0.000	0.867

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	487	386	0	0	0	0	114
normalized size	1	1.00	4.02	3.19	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.119	6.199	0.818	0.000	0.918	0.000	0.000	0.989

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	532	260	0	0	0	0	206
normalized size	1	1.00	3.62	1.77	0.00	0.00	0.00	0.00	1.40
time (sec)	N/A	0.153	6.137	0.730	0.000	1.277	0.000	0.000	0.782

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	500	272	0	0	0	0	143
normalized size	1	1.00	4.13	2.25	0.00	0.00	0.00	0.00	1.18
time (sec)	N/A	0.128	6.124	0.515	0.000	0.546	0.000	0.000	0.649

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	233	250	0	0	0	0	104
normalized size	1	1.00	2.56	2.75	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.109	5.711	0.915	0.000	1.407	0.000	0.000	0.618

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	240	172	0	0	0	0	104
normalized size	1	1.00	2.64	1.89	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.110	4.670	0.556	0.000	0.753	0.000	0.000	0.645

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	463	371	0	0	0	0	126
normalized size	1	1.00	5.09	4.08	0.00	0.00	0.00	0.00	1.38
time (sec)	N/A	0.105	6.196	0.934	0.000	1.924	0.000	0.000	1.018

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	485	386	0	0	0	0	154
normalized size	1	1.00	4.15	3.30	0.00	0.00	0.00	0.00	1.32
time (sec)	N/A	0.127	6.215	0.908	0.000	2.022	0.000	0.000	1.097

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	515	439	0	0	0	0	145
normalized size	1	1.00	3.50	2.99	0.00	0.00	0.00	0.00	0.99
time (sec)	N/A	0.148	6.246	0.948	0.000	1.097	0.000	0.000	1.210

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	271	273	0	0	0	0	221
normalized size	1	1.00	1.57	1.58	0.00	0.00	0.00	0.00	1.28
time (sec)	N/A	0.205	3.621	0.494	0.000	1.959	0.000	0.000	0.880

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	532	260	0	0	0	0	223
normalized size	1	1.00	3.62	1.77	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.164	6.151	0.543	0.000	0.656	0.000	0.000	0.807

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	500	272	0	0	0	0	146
normalized size	1	1.00	4.13	2.25	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.140	6.168	0.493	0.000	1.056	0.000	0.000	0.705

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	497	194	0	0	0	0	149
normalized size	1	1.00	4.18	1.63	0.00	0.00	0.00	0.00	1.25
time (sec)	N/A	0.122	6.201	0.523	0.000	0.901	0.000	0.000	0.794

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	70	292	0	0	0	0	145
normalized size	1	1.00	0.71	2.98	0.00	0.00	0.00	0.00	1.48
time (sec)	N/A	0.122	0.316	0.861	0.000	2.743	0.000	0.000	0.840

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	283	386	0	0	0	0	202
normalized size	1	1.00	2.34	3.19	0.00	0.00	0.00	0.00	1.67
time (sec)	N/A	0.146	4.340	0.903	0.000	1.764	0.000	0.000	1.301

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	298	439	0	0	0	0	199
normalized size	1	1.00	2.03	2.99	0.00	0.00	0.00	0.00	1.35
time (sec)	N/A	0.167	5.142	1.201	0.000	1.284	0.000	0.000	1.337

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	315	229	0	0	0	0	-1
normalized size	1	1.00	2.46	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	1.809	0.534	0.000	0.869	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	289	215	0	0	0	0	-1
normalized size	1	1.00	2.89	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	1.285	0.627	0.000	1.138	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	264	199	0	0	0	0	-1
normalized size	1	1.00	3.67	2.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	2.642	0.614	0.000	1.499	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	256	198	0	0	0	0	-1
normalized size	1	1.00	3.66	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	1.054	0.480	0.000	0.993	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	257	200	0	0	0	0	-1
normalized size	1	1.00	3.67	2.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.039	0.484	0.000	1.224	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	297	253	0	0	0	0	-1
normalized size	1	1.00	3.09	2.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	2.091	0.534	0.000	0.919	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	332	413	0	0	0	0	-1
normalized size	1	1.00	2.68	3.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	3.939	0.889	0.000	0.623	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	367	283	0	0	0	0	-1
normalized size	1	1.00	2.29	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	2.625	0.536	0.000	1.165	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	337	270	0	0	0	0	-1
normalized size	1	1.00	2.44	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	2.028	0.539	0.000	2.627	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	319	257	0	0	0	0	-1
normalized size	1	1.00	2.85	2.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	2.883	0.559	0.000	0.667	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	640	257	0	0	0	0	-1
normalized size	1	1.00	5.87	2.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	6.319	0.584	0.000	0.983	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	63	188	0	0	0	0	-1
normalized size	1	1.00	1.11	3.30	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.215	0.477	0.000	0.966	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	304	257	0	0	0	0	-1
normalized size	1	1.00	2.79	2.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	2.043	0.515	0.000	2.006	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	334	405	0	0	0	0	-1
normalized size	1	1.00	2.46	2.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	1.960	0.604	0.000	0.906	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	364	413	0	0	0	0	-1
normalized size	1	1.00	2.25	2.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	5.846	1.028	0.000	1.032	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	388	296	0	0	0	0	-1
normalized size	1	1.00	1.87	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	2.747	0.640	0.000	1.106	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	369	283	0	0	0	0	-1
normalized size	1	1.00	2.04	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.323	1.896	0.469	0.000	1.643	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	349	270	0	0	0	0	-1
normalized size	1	1.00	2.25	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.299	4.196	0.746	0.000	1.038	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	705	270	0	0	0	0	-1
normalized size	1	1.00	4.55	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.308	6.413	0.483	0.000	0.675	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	334	270	0	0	0	0	-1
normalized size	1	1.00	2.15	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	3.949	0.521	0.000	0.558	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	334	270	0	0	0	0	-1
normalized size	1	1.00	2.15	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.305	3.393	0.500	0.000	1.798	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	705	268	0	0	0	0	-1
normalized size	1	1.00	4.55	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.312	6.373	0.604	0.000	1.419	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	364	555	0	0	0	0	-1
normalized size	1	1.00	2.01	3.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.347	2.010	0.731	0.000	1.326	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	394	453	0	0	0	0	-1
normalized size	1	1.00	1.90	2.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	2.643	1.121	0.000	2.881	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	105	196	1921	108	0	0	-1
normalized size	1	1.00	0.68	1.27	12.47	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.347	0.282	2.014	0.982	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	91	161	1059	98	0	0	-1
normalized size	1	1.00	0.78	1.39	9.13	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.207	0.174	1.706	1.745	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	77	123	791	88	0	0	-1
normalized size	1	1.00	1.07	1.71	10.99	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.093	0.183	1.841	0.891	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	50	80	146	119	0	0	-1
normalized size	1	1.00	1.35	2.16	3.95	3.22	0.00	0.00	-0.03
time (sec)	N/A	0.058	0.048	0.137	1.729	1.967	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	42	98	49	0	58	41
normalized size	1	1.00	1.08	1.17	2.72	1.36	0.00	1.61	1.14
time (sec)	N/A	0.057	0.050	0.152	1.327	0.790	0.000	0.780	0.454

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	54	190	61	0	87	82
normalized size	1	1.00	0.66	0.70	2.47	0.79	0.00	1.13	1.06
time (sec)	N/A	0.109	0.089	0.176	1.436	0.899	0.000	0.958	1.270

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	66	64	237	71	0	116	132
normalized size	1	1.00	0.57	0.56	2.06	0.62	0.00	1.01	1.15
time (sec)	N/A	0.165	0.093	0.155	1.111	1.074	0.000	1.403	2.367

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	66	74	283	81	0	143	415
normalized size	1	1.00	0.43	0.48	1.85	0.53	0.00	0.93	2.71
time (sec)	N/A	0.228	0.131	0.163	1.484	0.994	0.000	1.637	5.642

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	106	197	1942	114	0	0	-1
normalized size	1	1.00	0.66	1.23	12.14	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.373	0.209	2.066	2.230	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	92	160	1080	103	0	0	-1
normalized size	1	1.00	0.77	1.33	9.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.229	0.173	1.913	1.045	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	168	803	90	0	0	-1
normalized size	1	1.00	1.05	2.24	10.71	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.111	0.164	1.965	1.122	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	85	249	997	109	0	0	-1
normalized size	1	1.00	1.12	3.28	13.12	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.149	0.174	1.655	1.411	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	52	55	125	62	0	0	89
normalized size	1	1.00	0.64	0.68	1.54	0.77	0.00	0.00	1.10
time (sec)	N/A	0.118	0.117	0.143	0.732	5.661	0.000	0.000	1.217

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	62	65	217	73	0	0	133
normalized size	1	1.00	0.51	0.54	1.79	0.60	0.00	0.00	1.10
time (sec)	N/A	0.173	0.149	0.137	1.135	2.164	0.000	0.000	2.103

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	72	75	263	86	0	0	157
normalized size	1	1.00	0.45	0.47	1.63	0.53	0.00	0.00	0.98
time (sec)	N/A	0.235	0.223	0.145	1.202	0.980	0.000	0.000	4.588

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	182	234	7450	137	0	0	-1
normalized size	1	1.00	0.91	1.17	37.25	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.358	4.403	0.237	3.271	2.616	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	182	197	1964	124	0	0	-1
normalized size	1	1.00	1.14	1.23	12.28	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.295	4.300	0.196	2.231	1.770	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	182	188	1106	111	0	0	-1
normalized size	1	1.00	1.52	1.57	9.22	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.232	4.173	0.196	1.812	2.009	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	182	269	973	127	0	0	-1
normalized size	1	1.00	1.60	2.36	8.54	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.225	4.244	0.178	1.816	1.033	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	356	333	1395	131	0	0	-1
normalized size	1	1.00	3.02	2.82	11.82	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.226	9.926	0.187	1.542	2.381	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	64	67	151	81	0	0	135
normalized size	1	1.00	0.53	0.55	1.25	0.67	0.00	0.00	1.12
time (sec)	N/A	0.225	0.180	0.167	1.366	1.013	0.000	0.000	2.140

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	74	77	243	94	0	0	163
normalized size	1	1.00	0.46	0.48	1.51	0.58	0.00	0.00	1.01
time (sec)	N/A	0.286	5.256	0.140	0.853	3.811	0.000	0.000	4.457

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	84	87	289	107	0	0	279
normalized size	1	1.00	0.42	0.43	1.44	0.53	0.00	0.00	1.39
time (sec)	N/A	0.351	5.352	0.158	1.155	1.937	0.000	0.000	6.415

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	121	50	0	0	42
normalized size	1	1.00	1.34	0.00	3.18	1.32	0.00	0.00	1.11
time (sec)	N/A	0.056	0.094	0.155	1.131	1.097	0.000	0.000	0.581

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	50	80	146	119	0	0	-1
normalized size	1	1.00	1.35	2.16	3.95	3.22	0.00	0.00	-0.03
time (sec)	N/A	0.060	0.065	0.105	1.225	1.141	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	188	91	420	164	0	0	-1
normalized size	1	1.00	4.95	2.39	11.05	4.32	0.00	0.00	-0.03
time (sec)	N/A	0.072	3.614	0.168	0.858	0.838	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	289	196	0	155	0	0	-1
normalized size	1	1.00	1.69	1.15	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.420	1.248	0.214	0.000	1.700	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	227	159	0	143	0	0	-1
normalized size	1	1.00	1.77	1.24	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.280	1.286	0.182	0.000	1.177	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	161	125	0	89	0	0	-1
normalized size	1	1.00	1.69	1.32	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.437	0.175	0.000	1.102	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	69	0	159	0	0	-1
normalized size	1	1.00	0.91	1.23	0.00	2.84	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.052	0.127	0.000	1.028	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	180	206	0	132	0	0	-1
normalized size	1	1.00	1.94	2.22	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.128	2.473	0.171	0.000	1.119	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	473	274	0	145	0	0	-1
normalized size	1	1.00	3.61	2.09	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.236	7.671	0.178	0.000	2.212	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	1540	341	0	157	0	0	-1
normalized size	1	1.00	9.11	2.02	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.365	10.018	0.273	0.000	2.008	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	286	187	0	135	0	0	-1
normalized size	1	1.00	2.27	1.48	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.901	0.165	0.000	1.986	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	224	151	0	125	0	0	-1
normalized size	1	1.00	2.64	1.78	0.00	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.863	0.137	0.000	1.474	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	135	124	0	70	0	0	-1
normalized size	1	1.00	2.50	2.30	0.00	1.30	0.00	0.00	-0.02
time (sec)	N/A	0.117	0.262	0.144	0.000	0.883	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	49	63	0	54	0	0	-1
normalized size	1	1.00	1.81	2.33	0.00	2.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	0.040	0.094	0.000	0.661	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	178	210	0	121	0	0	-1
normalized size	1	1.00	2.87	3.39	0.00	1.95	0.00	0.00	-0.02
time (sec)	N/A	0.085	1.795	0.146	0.000	2.126	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	471	278	0	134	0	0	-1
normalized size	1	1.00	4.81	2.84	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.165	6.634	0.145	0.000	3.608	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	1538	344	0	146	0	0	-1
normalized size	1	1.00	11.48	2.57	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.242	7.918	0.169	0.000	2.244	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	229	227	0	192	0	0	-1
normalized size	1	1.00	1.32	1.30	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.425	5.358	0.184	0.000	1.741	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	215	195	0	182	0	0	-1
normalized size	1	1.00	1.60	1.46	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.292	3.787	0.167	0.000	1.828	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	118	146	0	145	0	0	-1
normalized size	1	1.00	1.22	1.51	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.336	0.146	0.000	3.327	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	106	170	0	146	0	0	-1
normalized size	1	1.00	1.09	1.75	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.581	0.142	0.000	1.143	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	456	245	0	171	0	0	-1
normalized size	1	1.00	3.33	1.79	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.247	7.310	0.180	0.000	1.190	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	589	313	0	185	0	0	-1
normalized size	1	1.00	3.33	1.77	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.388	9.235	0.192	0.000	1.266	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	385	344	0	236	0	0	-1
normalized size	1	1.00	1.80	1.61	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.577	6.713	0.189	0.000	1.780	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	349	312	0	226	0	0	-1
normalized size	1	1.00	2.01	1.79	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.440	6.643	0.174	0.000	2.027	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	149	214	0	180	0	0	-1
normalized size	1	1.00	1.09	1.56	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.816	0.158	0.000	0.952	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	122	213	0	178	0	0	-1
normalized size	1	1.00	0.89	1.55	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.250	1.044	0.156	0.000	1.232	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	134	245	0	180	0	0	-1
normalized size	1	1.00	0.98	1.79	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.261	1.284	0.174	0.000	1.119	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	506	303	0	205	0	0	-1
normalized size	1	1.00	2.86	1.71	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.403	7.903	0.187	0.000	2.263	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	639	377	0	219	0	0	-1
normalized size	1	1.00	2.94	1.74	0.00	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.546	10.789	0.192	0.000	1.781	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	448	464	0	280	0	0	-1
normalized size	1	1.00	1.76	1.83	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.749	6.751	0.194	0.000	3.853	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	412	432	0	270	0	0	-1
normalized size	1	1.00	1.93	2.02	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.601	6.744	0.177	0.000	3.503	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	176	280	0	214	0	0	-1
normalized size	1	1.00	0.99	1.58	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.404	2.676	0.184	0.000	1.430	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	148	280	0	214	0	0	-1
normalized size	1	1.00	0.84	1.58	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.403	1.934	0.180	0.000	1.154	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	149	280	0	214	0	0	-1
normalized size	1	1.00	0.84	1.58	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.403	2.860	0.169	0.000	1.135	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	148	313	0	214	0	0	-1
normalized size	1	1.00	0.84	1.77	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.410	2.165	0.168	0.000	1.162	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	559	377	0	239	0	0	-1
normalized size	1	1.00	2.58	1.74	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.547	8.441	0.207	0.000	0.999	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	273	435	0	253	0	0	-1
normalized size	1	1.00	1.06	1.69	0.00	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.703	8.433	0.207	0.000	0.977	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	347	346	0	248	0	0	-1
normalized size	1	1.00	1.60	1.59	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.557	6.040	0.190	0.000	2.032	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	158	346	0	248	0	0	-1
normalized size	1	1.00	0.73	1.59	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.571	2.214	0.186	0.000	1.139	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	36	0	31	0	0	-1
normalized size	1	1.00	1.88	2.25	0.00	1.94	0.00	0.00	-0.06
time (sec)	N/A	0.043	0.023	0.054	0.000	0.806	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	32	42	0	105	0	0	-1
normalized size	1	1.00	0.78	1.02	0.00	2.56	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.020	0.063	0.000	0.968	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	289	165	1063	155	0	0	-1
normalized size	1	1.00	2.24	1.28	8.24	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.192	4.217	0.205	1.090	0.613	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	264	94	795	142	0	0	-1
normalized size	1	1.00	3.11	1.11	9.35	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.751	0.146	0.977	0.935	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	278	84	148	155	0	122	-1
normalized size	1	1.00	5.79	1.75	3.08	3.23	0.00	2.54	-0.02
time (sec)	N/A	0.066	0.539	0.118	1.052	0.951	0.000	1.046	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	40	46	82	42	0	62	42
normalized size	1	1.00	1.08	1.24	2.22	1.14	0.00	1.68	1.14
time (sec)	N/A	0.057	0.047	0.124	0.627	2.316	0.000	1.081	0.794

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	56	174	52	0	90	85
normalized size	1	1.00	0.66	0.71	2.20	0.66	0.00	1.14	1.08
time (sec)	N/A	0.116	0.119	0.132	0.854	2.979	0.000	0.963	1.385

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	62	66	221	64	0	120	158
normalized size	1	1.00	0.53	0.56	1.87	0.54	0.00	1.02	1.34
time (sec)	N/A	0.173	0.148	0.125	0.978	0.694	0.000	2.300	2.890

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	284	164	1305	124	0	0	-1
normalized size	1	1.00	2.49	1.44	11.45	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.479	0.148	0.755	0.865	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	252	94	966	111	0	0	-1
normalized size	1	1.00	3.50	1.31	13.42	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.733	0.128	1.028	0.792	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	277	83	221	64	0	119	-1
normalized size	1	1.00	7.49	2.24	5.97	1.73	0.00	3.22	-0.03
time (sec)	N/A	0.043	0.508	0.087	1.069	0.868	0.000	1.402	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	39	45	75	41	0	59	31
normalized size	1	1.00	1.11	1.29	2.14	1.17	0.00	1.69	0.89
time (sec)	N/A	0.042	0.040	0.101	0.783	0.740	0.000	0.642	0.872

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	55	164	51	0	87	84
normalized size	1	1.00	0.68	0.73	2.19	0.68	0.00	1.16	1.12
time (sec)	N/A	0.085	0.096	0.107	0.891	2.046	0.000	1.409	1.555

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	61	65	209	63	0	117	156
normalized size	1	1.00	0.54	0.58	1.87	0.56	0.00	1.04	1.39
time (sec)	N/A	0.132	0.115	0.109	0.655	1.186	0.000	0.822	2.004

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	256	197	0	226	0	157	-1
normalized size	1	1.00	1.38	1.06	0.00	1.22	0.00	0.85	-0.01
time (sec)	N/A	0.445	1.148	0.165	0.000	0.797	0.000	4.296	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	228	167	0	212	0	131	-1
normalized size	1	1.00	1.62	1.18	0.00	1.50	0.00	0.93	-0.01
time (sec)	N/A	0.295	0.891	0.151	0.000	0.774	0.000	2.601	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	161	118	0	162	0	86	-1
normalized size	1	1.00	1.50	1.10	0.00	1.51	0.00	0.80	-0.01
time (sec)	N/A	0.180	0.383	0.110	0.000	0.915	0.000	1.935	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	118	77	209	144	0	137	-1
normalized size	1	1.00	2.03	1.33	3.60	2.48	0.00	2.36	-0.02
time (sec)	N/A	0.067	0.327	0.136	1.359	1.325	0.000	0.764	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	157	160	351	152	0	68	-1
normalized size	1	1.00	1.65	1.68	3.69	1.60	0.00	0.72	-0.01
time (sec)	N/A	0.133	0.385	0.145	0.997	0.581	0.000	0.773	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	171	171	504	165	0	90	-1
normalized size	1	1.00	1.27	1.27	3.73	1.22	0.00	0.67	-0.01
time (sec)	N/A	0.251	0.321	0.168	1.187	1.025	0.000	0.730	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	C	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	218	305	692	177	0	136	-1
normalized size	1	1.00	1.26	1.76	4.00	1.02	0.00	0.79	-0.01
time (sec)	N/A	0.401	0.661	0.198	0.983	0.908	0.000	1.279	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	255	194	0	239	0	162	-1
normalized size	1	1.00	1.58	1.20	0.00	1.48	0.00	1.01	-0.01
time (sec)	N/A	0.302	0.195	0.147	0.000	0.932	0.000	1.809	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	227	166	0	225	0	141	-1
normalized size	1	1.00	1.92	1.41	0.00	1.91	0.00	1.19	-0.01
time (sec)	N/A	0.214	0.215	0.133	0.000	1.293	0.000	1.746	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	160	117	0	170	0	105	-1
normalized size	1	1.00	1.88	1.38	0.00	2.00	0.00	1.24	-0.01
time (sec)	N/A	0.129	0.119	0.097	0.000	1.203	0.000	1.796	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	C	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	110	84	248	84	0	79	-1
normalized size	1	1.00	2.34	1.79	5.28	1.79	0.00	1.68	-0.02
time (sec)	N/A	0.048	0.136	0.128	0.995	0.817	0.000	0.617	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	C	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	152	159	400	144	0	72	-1
normalized size	1	1.00	1.83	1.92	4.82	1.73	0.00	0.87	-0.01
time (sec)	N/A	0.093	0.172	0.133	1.021	0.841	0.000	0.639	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	170	170	563	157	0	89	-1
normalized size	1	1.00	1.39	1.39	4.61	1.29	0.00	0.73	-0.01
time (sec)	N/A	0.182	0.280	0.148	1.014	0.886	0.000	0.785	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	15.608	0.130	0.000	1.011	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	3.425	0.139	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	2.703	0.129	0.000	1.452	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	268	384	0	0	0	0	-1
normalized size	1	1.00	1.77	2.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	1.629	0.792	0.000	1.410	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	255	369	0	0	0	0	-1
normalized size	1	1.00	2.07	3.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	1.150	0.981	0.000	1.075	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	124	146	0	0	0	0	-1
normalized size	1	1.00	1.28	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	1.353	0.609	0.000	2.623	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	141	150	0	0	0	0	-1
normalized size	1	1.00	1.88	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	1.112	0.513	0.000	0.794	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	140	225	0	0	0	0	-1
normalized size	1	1.00	1.39	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	1.295	0.508	0.000	1.956	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	224	219	0	0	0	0	-1
normalized size	1	1.00	1.76	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.943	0.494	0.000	0.949	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	198	270	0	0	0	0	-1
normalized size	1	1.00	1.31	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	2.158	0.546	0.000	4.012	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	261	386	0	0	0	0	-1
normalized size	1	1.00	1.62	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	1.872	1.025	0.000	0.689	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	250	371	0	0	0	0	-1
normalized size	1	1.00	1.91	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	1.359	0.984	0.000	1.890	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	104	0	0	0	0	-1
normalized size	1	1.00	0.75	1.62	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.143	0.553	0.000	1.062	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	127	228	0	0	0	0	-1
normalized size	1	1.00	1.19	2.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	1.050	0.466	0.000	2.126	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	0	0	0	-1
normalized size	1	1.00	1.01	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	1.513	0.752	0.000	1.475	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	149	272	0	0	0	0	-1
normalized size	1	1.00	0.93	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	1.710	0.597	0.000	2.203	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	0	0	0	-1
normalized size	1	1.00	1.49	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	2.755	1.051	0.000	1.145	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	259	386	0	0	0	0	-1
normalized size	1	1.00	1.65	2.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	1.901	0.951	0.000	0.943	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	157	371	0	0	0	0	-1
normalized size	1	1.00	1.20	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.971	0.877	0.000	1.969	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	135	172	0	0	0	0	-1
normalized size	1	1.00	1.03	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	1.308	0.755	0.000	2.242	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	137	250	0	0	0	0	-1
normalized size	1	1.00	1.05	1.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	1.280	0.556	0.000	1.134	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	0	0	0	-1
normalized size	1	1.00	0.91	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	1.800	0.707	0.000	0.897	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	0	0	0	-1
normalized size	1	1.00	0.83	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	2.288	0.602	0.000	1.111	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	271	439	0	0	0	0	-1
normalized size	1	1.00	1.45	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	2.032	1.022	0.000	1.038	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	278	386	0	0	0	0	-1
normalized size	1	1.00	1.73	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	2.495	1.081	0.000	1.560	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	0	0	0	-1
normalized size	1	1.00	0.59	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.340	0.825	0.000	1.451	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	150	194	0	0	0	0	-1
normalized size	1	1.00	0.94	1.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	1.500	0.680	0.000	1.933	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	0	0	0	-1
normalized size	1	1.00	0.91	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	1.560	0.539	0.000	1.445	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	0	0	0	-1
normalized size	1	1.00	0.83	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	2.119	0.582	0.000	0.960	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	285	413	0	0	0	0	-1
normalized size	1	1.00	1.74	2.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	3.287	0.898	0.000	0.957	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	256	253	0	0	0	0	-1
normalized size	1	1.00	1.88	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	1.959	0.657	0.000	1.090	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	180	200	0	0	0	0	-1
normalized size	1	1.00	1.64	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	1.017	0.519	0.000	1.166	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	181	198	0	0	0	0	-1
normalized size	1	1.00	1.65	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.961	0.605	0.000	1.093	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	311	199	0	0	0	0	-1
normalized size	1	1.00	2.78	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	1.695	0.529	0.000	0.984	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	312	215	0	0	0	0	-1
normalized size	1	1.00	2.23	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	4.117	0.831	0.000	0.883	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	341	229	0	0	0	0	-1
normalized size	1	1.00	2.03	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	2.693	0.589	0.000	1.039	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	287	413	0	0	0	0	-1
normalized size	1	1.00	1.42	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	2.444	0.992	0.000	1.020	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	252	405	0	0	0	0	-1
normalized size	1	1.00	1.43	2.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	1.303	0.746	0.000	1.025	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	242	257	0	0	0	0	-1
normalized size	1	1.00	1.62	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	1.215	0.649	0.000	1.497	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	0	0	0	-1
normalized size	1	1.00	1.27	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.367	0.564	0.000	0.928	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	239	257	0	0	0	0	-1
normalized size	1	1.00	1.60	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	1.387	0.689	0.000	1.468	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	259	257	0	0	0	0	-1
normalized size	1	1.00	1.70	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	1.994	0.648	0.000	1.030	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	257	270	0	0	0	0	-1
normalized size	1	1.00	1.44	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	1.812	0.644	0.000	0.729	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	271	283	0	0	0	0	-1
normalized size	1	1.00	1.36	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	1.844	0.669	0.000	1.491	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	363	555	0	0	0	0	-1
normalized size	1	1.00	1.64	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	2.312	0.909	0.000	1.084	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	274	268	0	0	0	0	-1
normalized size	1	1.00	1.41	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.360	2.354	0.556	0.000	0.986	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	363	270	0	0	0	0	-1
normalized size	1	1.00	1.86	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.350	2.094	0.631	0.000	0.709	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	363	270	0	0	0	0	-1
normalized size	1	1.00	1.86	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.345	2.003	0.639	0.000	0.975	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	272	270	0	0	0	0	-1
normalized size	1	1.00	1.39	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.356	3.043	0.657	0.000	1.107	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	378	270	0	0	0	0	-1
normalized size	1	1.00	1.94	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.359	2.154	0.891	0.000	1.066	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	285	283	0	0	0	0	-1
normalized size	1	1.00	1.29	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	2.324	0.778	0.000	1.448	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	71	82	283	81	0	143	163
normalized size	1	1.00	0.46	0.54	1.85	0.53	0.00	0.93	1.07
time (sec)	N/A	0.282	0.211	0.225	1.016	0.984	0.000	1.163	5.191

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	61	72	237	71	0	116	134
normalized size	1	1.00	0.53	0.63	2.06	0.62	0.00	1.01	1.17
time (sec)	N/A	0.221	0.127	0.212	0.981	1.033	0.000	0.586	1.839

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	62	190	59	0	87	84
normalized size	1	1.00	0.66	0.81	2.47	0.77	0.00	1.13	1.09
time (sec)	N/A	0.159	0.102	0.197	1.123	0.791	0.000	0.620	0.785

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	50	98	40	0	58	43
normalized size	1	1.00	1.08	1.39	2.72	1.11	0.00	1.61	1.19
time (sec)	N/A	0.103	0.068	0.198	1.243	1.000	0.000	0.942	0.294

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	70	100	146	119	0	0	-1
normalized size	1	1.00	1.23	1.75	2.56	2.09	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.087	0.233	1.460	1.197	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	132	791	88	0	0	-1
normalized size	1	1.00	1.05	1.43	8.60	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.129	0.219	1.632	0.755	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	111	169	1059	108	0	0	-1
normalized size	1	1.00	0.82	1.24	7.79	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.263	0.242	1.641	1.225	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	72	83	263	86	0	0	221
normalized size	1	1.00	0.45	0.52	1.63	0.53	0.00	0.00	1.37
time (sec)	N/A	0.308	0.280	0.197	1.245	0.656	0.000	0.000	4.145

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	62	73	217	73	0	0	135
normalized size	1	1.00	0.51	0.60	1.79	0.60	0.00	0.00	1.12
time (sec)	N/A	0.239	0.189	0.195	0.962	0.692	0.000	0.000	1.626

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	52	63	125	60	0	0	91
normalized size	1	1.00	0.64	0.78	1.54	0.74	0.00	0.00	1.12
time (sec)	N/A	0.175	0.138	0.184	1.008	0.896	0.000	0.000	0.791

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	85	168	997	91	0	0	-1
normalized size	1	1.00	0.89	1.75	10.39	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.180	0.216	1.346	1.166	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	99	130	803	90	0	0	-1
normalized size	1	1.00	1.04	1.37	8.45	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.152	0.221	1.264	1.068	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	111	170	1080	112	0	0	-1
normalized size	1	1.00	0.79	1.21	7.71	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.250	0.237	1.567	1.141	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	126	205	1942	123	0	0	-1
normalized size	1	1.00	0.70	1.14	10.79	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.305	0.546	0.257	1.863	0.799	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	84	95	289	107	0	0	306
normalized size	1	1.00	0.42	0.47	1.44	0.53	0.00	0.00	1.52
time (sec)	N/A	0.414	5.383	0.207	1.021	0.830	0.000	0.000	4.848

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	74	85	243	94	0	0	227
normalized size	1	1.00	0.46	0.53	1.51	0.58	0.00	0.00	1.41
time (sec)	N/A	0.347	5.358	0.199	1.040	1.054	0.000	0.000	4.205

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	64	75	151	81	0	0	137
normalized size	1	1.00	0.53	0.62	1.25	0.67	0.00	0.00	1.13
time (sec)	N/A	0.285	0.283	0.187	0.823	1.025	0.000	0.000	1.623

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	404	268	1395	128	0	0	-1
normalized size	1	1.00	2.93	1.94	10.11	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.288	6.315	0.224	1.808	1.053	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	202	186	973	111	0	0	-1
normalized size	1	1.00	1.51	1.39	7.26	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.282	3.003	0.224	1.570	0.790	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	202	166	1106	120	0	0	-1
normalized size	1	1.00	1.44	1.19	7.90	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.292	3.014	0.241	1.737	1.001	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	202	207	1964	133	0	0	-1
normalized size	1	1.00	1.12	1.15	10.91	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.361	3.115	0.253	1.709	0.819	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	202	242	7450	146	0	0	-1
normalized size	1	1.00	0.92	1.10	33.86	0.66	0.00	0.00	-0.00
time (sec)	N/A	0.419	3.098	0.268	2.144	0.780	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	1540	294	0	130	0	0	-1
normalized size	1	1.00	10.00	1.91	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.284	7.813	0.224	0.000	1.250	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	473	228	0	114	0	0	-1
normalized size	1	1.00	4.01	1.93	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.201	6.606	0.216	0.000	1.097	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	178	144	0	86	0	0	-1
normalized size	1	1.00	2.17	1.76	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.953	0.180	0.000	1.692	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	68	82	0	39	0	0	-1
normalized size	1	1.00	1.45	1.74	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.079	0.107	0.186	0.000	1.044	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	171	134	0	70	0	0	-1
normalized size	1	1.00	1.82	1.43	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.599	0.207	0.000	1.094	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	257	159	0	125	0	0	-1
normalized size	1	1.00	2.06	1.27	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.877	0.220	0.000	0.933	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	1542	294	0	141	0	0	-1
normalized size	1	1.00	8.16	1.56	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.421	7.761	0.250	0.000	1.234	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	475	227	0	125	0	0	-1
normalized size	1	1.00	3.15	1.50	0.00	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.291	6.632	0.228	0.000	0.720	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	180	142	0	98	0	0	-1
normalized size	1	1.00	1.59	1.26	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.180	1.873	0.213	0.000	1.879	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	76	71	88	0	144	0	0	-1
normalized size	1	1.36	1.27	1.57	0.00	2.57	0.00	0.00	-0.02
time (sec)	N/A	0.115	0.095	0.209	0.000	1.177	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	135	173	134	0	89	0	0	-1
normalized size	1	1.29	1.65	1.28	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.254	0.218	0.000	1.155	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	259	167	0	143	0	0	-1
normalized size	1	1.00	1.54	0.99	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.386	0.450	0.224	0.000	0.937	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	0	258	0	161	0	0	-1
normalized size	1	1.00	0.00	1.31	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.511	0.000	0.215	0.000	1.072	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	458	184	0	136	0	0	-1
normalized size	1	1.00	2.92	1.17	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.353	6.508	0.202	0.000	1.117	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	99	151	0	126	0	0	-1
normalized size	1	1.00	0.85	1.29	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.523	0.204	0.000	1.002	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	140	156	0	125	0	0	-1
normalized size	1	1.00	1.20	1.33	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.456	0.207	0.000	1.342	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	316	203	0	182	0	0	-1
normalized size	1	1.00	1.82	1.17	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.396	6.541	0.209	0.000	1.418	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	316	235	0	201	0	0	-1
normalized size	1	1.00	1.48	1.10	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.548	6.570	0.218	0.000	1.214	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	641	316	0	195	0	0	-1
normalized size	1	1.00	2.70	1.33	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.640	8.110	0.236	0.000	0.982	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	508	258	0	170	0	0	-1
normalized size	1	1.00	2.58	1.31	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.495	6.815	0.224	0.000	0.886	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	131	222	0	169	0	0	-1
normalized size	1	1.00	0.83	1.41	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.352	0.939	0.209	0.000	1.096	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	122	221	0	167	0	0	-1
normalized size	1	1.00	0.78	1.41	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.350	0.689	0.227	0.000	1.197	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	164	222	0	169	0	0	-1
normalized size	1	1.00	1.04	1.41	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.352	0.695	0.222	0.000	1.042	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	373	320	0	235	0	0	-1
normalized size	1	1.00	1.74	1.50	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.528	2.248	0.224	0.000	1.535	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	412	352	0	245	0	0	-1
normalized size	1	1.00	1.62	1.39	0.00	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.666	3.233	0.236	0.000	1.798	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	696	390	0	229	0	0	-1
normalized size	1	1.00	2.51	1.41	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.780	8.396	0.253	0.000	1.247	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	561	326	0	204	0	0	-1
normalized size	1	1.00	2.37	1.38	0.00	0.86	0.00	0.00	-0.00
time (sec)	N/A	0.631	6.924	0.241	0.000	1.066	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	153	288	0	203	0	0	-1
normalized size	1	1.00	0.78	1.46	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.481	3.775	0.222	0.000	1.077	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	125	288	0	203	0	0	-1
normalized size	1	1.00	0.63	1.46	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.481	1.180	0.224	0.000	1.259	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	153	288	0	203	0	0	-1
normalized size	1	1.00	0.78	1.46	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.497	3.687	0.222	0.000	1.899	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	196	288	0	203	0	0	-1
normalized size	1	1.00	0.99	1.46	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.485	4.389	0.211	0.000	1.226	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	454	440	0	279	0	0	-1
normalized size	1	1.00	1.79	1.73	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.668	6.794	0.257	0.000	2.292	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	460	472	0	289	0	0	-1
normalized size	1	1.00	1.56	1.61	0.00	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.817	3.484	0.257	0.000	2.379	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	163	354	0	237	0	0	-1
normalized size	1	1.00	0.69	1.49	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.631	4.352	0.250	0.000	1.031	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	395	354	0	237	0	0	-1
normalized size	1	1.00	1.67	1.49	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.636	6.095	0.252	0.000	1.156	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	121	41	0	0	44
normalized size	1	1.00	1.34	0.00	3.18	1.08	0.00	0.00	1.16
time (sec)	N/A	0.118	0.117	0.226	1.010	1.089	0.000	0.000	0.736

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.531	3.058	3.820	0.000	1.133	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	1.271	2.712	0.000	1.086	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.525	3.568	0.000	0.757	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	208	0	0	0	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.021	1.041	0.000	1.409	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.866	0.817	0.000	1.056	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	1.113	0.399	0.000	1.053	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	135	100	105	97	286	122	175
normalized size	1	1.00	0.90	0.67	0.70	0.65	1.91	0.81	1.17
time (sec)	N/A	0.102	0.198	0.040	0.361	1.125	8.965	1.291	3.229

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	89	90	94	86	238	107	154
normalized size	1	1.00	0.70	0.70	0.73	0.67	1.86	0.84	1.20
time (sec)	N/A	0.086	0.186	0.037	0.465	0.913	5.396	0.597	3.245

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	78	80	84	75	216	92	115
normalized size	1	1.00	0.68	0.70	0.74	0.66	1.89	0.81	1.01
time (sec)	N/A	0.082	0.099	0.045	0.578	0.991	3.267	0.499	0.659

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	70	69	64	168	77	115
normalized size	1	1.00	0.97	0.76	0.75	0.70	1.83	0.84	1.25
time (sec)	N/A	0.064	0.130	0.040	0.467	0.764	1.863	0.475	4.255

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	57	53	144	62	75
normalized size	1	1.00	0.96	0.79	0.75	0.70	1.89	0.82	0.99
time (sec)	N/A	0.059	0.094	0.043	0.899	1.028	0.919	0.490	0.578

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	46	42	92	47	55
normalized size	1	1.00	1.06	0.91	0.85	0.78	1.70	0.87	1.02
time (sec)	N/A	0.047	0.064	0.039	0.504	1.194	0.462	0.444	0.580

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	38	34	29	66	31	31
normalized size	1	1.00	0.92	1.00	0.89	0.76	1.74	0.82	0.82
time (sec)	N/A	0.015	0.062	0.042	0.655	0.901	0.198	0.432	0.525

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	15	17
normalized size	1	1.00	1.73	1.07	1.00	1.13	1.13	1.00	1.13
time (sec)	N/A	0.009	0.006	0.015	0.674	0.761	0.118	0.331	0.466

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	30	28	36	49	43	57
normalized size	1	1.00	1.00	1.88	1.75	2.25	3.06	2.69	3.56
time (sec)	N/A	0.023	0.006	0.063	0.322	0.889	4.981	0.747	0.535

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	38	60	0	63	47
normalized size	1	1.00	1.00	1.33	1.58	2.50	0.00	2.62	1.96
time (sec)	N/A	0.037	0.008	0.073	0.702	1.005	0.000	0.533	0.514

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	58	74	0	105	81
normalized size	1	1.00	1.00	1.09	1.23	1.57	0.00	2.23	1.72
time (sec)	N/A	0.049	0.015	0.078	0.342	0.939	0.000	0.611	1.132

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	70	88	0	122	111
normalized size	1	1.00	0.95	1.14	1.11	1.40	0.00	1.94	1.76
time (sec)	N/A	0.053	0.169	0.082	0.532	0.787	0.000	0.517	2.337

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	95	99	0	164	150
normalized size	1	1.00	0.89	1.08	1.12	1.16	0.00	1.93	1.76
time (sec)	N/A	0.065	0.230	0.091	0.634	1.035	0.000	0.693	3.076

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	112	107	110	0	178	180
normalized size	1	1.00	0.87	1.11	1.06	1.09	0.00	1.76	1.78
time (sec)	N/A	0.070	0.334	0.086	0.692	1.001	0.000	0.533	3.121

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	123	120	120	110	343	127	143
normalized size	1	1.00	0.82	0.80	0.80	0.73	2.29	0.85	0.95
time (sec)	N/A	0.106	0.309	0.046	0.528	0.976	4.021	0.547	0.668

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	85	95	94	86	221	102	117
normalized size	1	1.00	0.77	0.86	0.85	0.77	1.99	0.92	1.05
time (sec)	N/A	0.108	0.139	0.042	0.343	0.955	2.021	0.549	0.609

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	89	82	77	211	82	93
normalized size	1	1.00	0.85	0.88	0.81	0.76	2.09	0.81	0.92
time (sec)	N/A	0.092	0.161	0.048	0.834	0.817	1.060	0.461	0.599

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	63	60	52	107	60	72
normalized size	1	1.00	0.83	0.89	0.85	0.73	1.51	0.85	1.01
time (sec)	N/A	0.050	0.152	0.039	0.689	0.721	0.483	0.679	0.552

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	44	40	78	43	42
normalized size	1	1.00	0.92	1.02	0.88	0.80	1.56	0.86	0.84
time (sec)	N/A	0.015	0.075	0.036	0.774	0.910	0.254	0.476	0.533

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	46	49	42	52	0	78	73
normalized size	1	1.00	1.39	1.48	1.27	1.58	0.00	2.36	2.21
time (sec)	N/A	0.062	0.013	0.079	0.601	1.139	0.000	0.545	0.553

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	49	48	74	0	77	181
normalized size	1	1.00	0.97	1.48	1.45	2.24	0.00	2.33	5.48
time (sec)	N/A	0.066	0.084	0.079	0.421	0.912	0.000	0.650	0.570

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	67	78	87	93	0	127	99
normalized size	1	1.00	1.14	1.32	1.47	1.58	0.00	2.15	1.68
time (sec)	N/A	0.078	0.014	0.096	0.570	0.889	0.000	0.625	1.166

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	89	84	100	0	178	141
normalized size	1	1.00	0.89	1.11	1.05	1.25	0.00	2.22	1.76
time (sec)	N/A	0.089	0.224	0.106	0.334	1.096	0.000	0.559	2.592

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	142	144	133	0	258	184
normalized size	1	1.00	0.75	1.29	1.31	1.21	0.00	2.35	1.67
time (sec)	N/A	0.095	0.278	0.099	0.672	0.667	0.000	0.626	3.103

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	118	157	132	136	0	272	221
normalized size	1	1.00	0.87	1.16	0.98	1.01	0.00	2.01	1.64
time (sec)	N/A	0.108	0.556	0.098	0.553	1.428	0.000	0.613	3.230

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	193	159	145	145	132	393	150	380
normalized size	1	1.14	0.94	0.85	0.85	0.78	2.31	0.88	2.24
time (sec)	N/A	0.215	0.325	0.044	0.614	1.035	3.932	0.593	2.104

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	130	123	119	110	284	124	319
normalized size	1	1.00	0.72	0.68	0.66	0.61	1.58	0.69	1.77
time (sec)	N/A	0.221	0.326	0.044	0.666	1.450	2.175	0.572	2.045

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	102	95	84	233	96	279
normalized size	1	1.00	0.83	0.84	0.79	0.69	1.93	0.79	2.31
time (sec)	N/A	0.116	0.269	0.043	0.364	1.378	1.078	0.510	1.952

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	90	80	76	72	66	128	72	77
normalized size	1	1.18	1.05	1.00	0.95	0.87	1.68	0.95	1.01
time (sec)	N/A	0.069	0.125	0.038	0.614	0.695	0.522	0.397	0.602

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	105	90	69	72	0	137	123
normalized size	1	1.00	1.44	1.23	0.95	0.99	0.00	1.88	1.68
time (sec)	N/A	0.115	0.137	0.077	0.345	0.854	0.000	0.514	0.718

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	88	68	66	94	0	129	97
normalized size	1	1.00	1.29	1.00	0.97	1.38	0.00	1.90	1.43
time (sec)	N/A	0.122	0.348	0.084	0.886	1.078	0.000	0.582	0.624

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	55	95	101	112	0	143	136
normalized size	1	1.00	0.70	1.20	1.28	1.42	0.00	1.81	1.72
time (sec)	N/A	0.134	0.177	0.096	0.625	1.125	0.000	0.704	0.670

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	70	118	113	126	0	205	157
normalized size	1	1.00	0.64	1.08	1.04	1.16	0.00	1.88	1.44
time (sec)	N/A	0.182	0.256	0.098	0.833	1.080	0.000	0.549	2.634

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	90	160	158	140	0	330	224
normalized size	1	1.00	0.68	1.20	1.19	1.05	0.00	2.48	1.68
time (sec)	N/A	0.203	0.420	0.112	0.803	1.044	0.000	0.686	4.221

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	120	206	181	170	0	367	260
normalized size	1	1.00	0.71	1.22	1.07	1.01	0.00	2.17	1.54
time (sec)	N/A	0.217	0.899	0.115	0.652	0.893	0.000	0.656	4.223

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	181	190	192	171	495	197	476
normalized size	1	1.00	0.73	0.77	0.78	0.69	2.00	0.80	1.93
time (sec)	N/A	0.401	0.411	0.046	0.664	0.891	6.451	0.653	2.074

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	156	174	170	150	459	168	214
normalized size	1	1.00	0.66	0.74	0.72	0.64	1.95	0.71	0.91
time (sec)	N/A	0.320	0.437	0.050	0.583	0.810	4.021	0.595	0.835

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	133	138	133	121	301	134	363
normalized size	1	1.00	0.78	0.81	0.78	0.71	1.77	0.79	2.14
time (sec)	N/A	0.204	0.486	0.040	0.674	1.044	2.250	0.642	2.037

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	104	116	111	96	240	107	123
normalized size	1	1.00	0.76	0.85	0.81	0.70	1.75	0.78	0.90
time (sec)	N/A	0.147	0.205	0.041	0.697	0.775	1.109	0.537	0.657

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	128	131	95	98	0	212	158
normalized size	1	1.00	1.20	1.22	0.89	0.92	0.00	1.98	1.48
time (sec)	N/A	0.228	0.156	0.082	0.681	1.149	0.000	0.591	0.824

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	119	109	90	116	0	170	150
normalized size	1	1.00	1.04	0.96	0.79	1.02	0.00	1.49	1.32
time (sec)	N/A	0.234	0.658	0.089	0.632	1.219	0.000	0.555	0.715

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	174	114	115	130	0	177	152
normalized size	1	1.00	1.61	1.06	1.06	1.20	0.00	1.64	1.41
time (sec)	N/A	0.252	2.441	0.101	0.858	0.751	0.000	0.861	0.720

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	77	135	125	138	0	221	185
normalized size	1	1.00	0.67	1.17	1.09	1.20	0.00	1.92	1.61
time (sec)	N/A	0.252	0.387	0.099	0.631	1.227	0.000	0.619	0.775

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	101	188	187	163	0	360	245
normalized size	1	1.00	0.66	1.22	1.21	1.06	0.00	2.34	1.59
time (sec)	N/A	0.336	0.487	0.110	0.978	0.915	0.000	0.696	4.293

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	125	225	195	182	0	461	304
normalized size	1	1.00	0.66	1.20	1.04	0.97	0.00	2.45	1.62
time (sec)	N/A	0.361	0.746	0.125	1.201	1.097	0.000	0.718	4.456

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	154	302	275	217	0	592	370
normalized size	1	1.00	0.69	1.36	1.24	0.98	0.00	2.67	1.67
time (sec)	N/A	0.380	1.008	0.115	0.762	1.099	0.000	0.783	4.308

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	153	672	0	479	0	393	474
normalized size	1	1.00	0.79	3.48	0.00	2.48	0.00	2.04	2.46
time (sec)	N/A	0.545	0.638	0.063	0.000	1.717	0.000	0.622	1.720

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	122	367	0	400	0	249	203
normalized size	1	1.00	0.82	2.48	0.00	2.70	0.00	1.68	1.37
time (sec)	N/A	0.327	0.331	0.058	0.000	1.175	0.000	0.478	1.178

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	222	0	334	0	177	168
normalized size	1	1.00	0.88	2.02	0.00	3.04	0.00	1.61	1.53
time (sec)	N/A	0.184	0.236	0.063	0.000	0.968	0.000	0.527	1.069

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	72	102	0	269	1744	126	190
normalized size	1	1.00	0.95	1.34	0.00	3.54	22.95	1.66	2.50
time (sec)	N/A	0.119	0.142	0.046	0.000	1.097	122.504	0.545	0.905

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	67	0	223	320	240	99
normalized size	1	1.00	0.98	1.14	0.00	3.78	5.42	4.07	1.68
time (sec)	N/A	0.056	0.091	0.049	0.000	1.227	24.697	0.547	0.776

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	175	172	78	43
normalized size	1	1.00	0.98	0.90	0.00	3.57	3.51	1.59	0.88
time (sec)	N/A	0.031	0.036	0.037	0.000	1.134	4.024	0.459	0.517

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	102	88	0	278	0	119	99
normalized size	1	1.00	1.50	1.29	0.00	4.09	0.00	1.75	1.46
time (sec)	N/A	0.072	0.082	0.070	0.000	1.077	0.000	0.897	0.836

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	115	134	0	382	0	153	324
normalized size	1	1.00	1.35	1.58	0.00	4.49	0.00	1.80	3.81
time (sec)	N/A	0.130	0.378	0.084	0.000	1.299	0.000	0.638	1.045

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	238	262	0	459	0	211	1087
normalized size	1	1.00	2.00	2.20	0.00	3.86	0.00	1.77	9.13
time (sec)	N/A	0.324	1.057	0.106	0.000	1.684	0.000	0.741	1.764

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	258	400	0	535	0	286	991
normalized size	1	1.00	1.64	2.55	0.00	3.41	0.00	1.82	6.31
time (sec)	N/A	0.522	2.463	0.110	0.000	1.349	0.000	0.665	2.676

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	176	504	0	747	0	333	3852
normalized size	1	1.00	0.66	1.89	0.00	2.81	0.00	1.25	14.48
time (sec)	N/A	0.723	0.902	0.071	0.000	1.298	0.000	0.932	7.205

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	213	144	358	0	651	0	262	3751
normalized size	1	1.28	0.87	2.16	0.00	3.92	0.00	1.58	22.60
time (sec)	N/A	0.429	0.783	0.063	0.000	1.054	0.000	0.565	7.039

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	113	238	0	554	0	847	3180
normalized size	1	1.00	0.73	1.54	0.00	3.57	0.00	5.46	20.52
time (sec)	N/A	0.256	0.734	0.063	0.000	1.739	0.000	1.116	6.124

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	200	0	470	0	175	2872
normalized size	1	1.00	0.95	1.85	0.00	4.35	0.00	1.62	26.59
time (sec)	N/A	0.143	0.406	0.056	0.000	1.410	0.000	0.577	6.208

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	116	0	321	0	135	99
normalized size	1	1.00	0.98	1.36	0.00	3.78	0.00	1.59	1.16
time (sec)	N/A	0.069	0.238	0.048	0.000	1.003	0.000	0.625	0.738

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	116	0	320	0	135	99
normalized size	1	1.00	0.98	1.35	0.00	3.72	0.00	1.57	1.15
time (sec)	N/A	0.055	0.188	0.041	0.000	1.352	0.000	0.496	0.707

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	146	221	0	592	0	198	2886
normalized size	1	1.00	1.24	1.87	0.00	5.02	0.00	1.68	24.46
time (sec)	N/A	0.219	0.371	0.088	0.000	1.857	0.000	0.703	5.977

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	163	271	0	750	0	332	3176
normalized size	1	1.00	1.05	1.75	0.00	4.84	0.00	2.14	20.49
time (sec)	N/A	0.407	0.966	0.094	0.000	2.113	0.000	0.703	5.920

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	285	401	0	899	0	293	3699
normalized size	1	1.00	1.31	1.85	0.00	4.14	0.00	1.35	17.05
time (sec)	N/A	0.681	5.522	0.128	0.000	3.271	0.000	0.963	6.927

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	499	535	0	1001	0	368	3843
normalized size	1	1.00	1.85	1.98	0.00	3.71	0.00	1.36	14.23
time (sec)	N/A	0.967	6.154	0.124	0.000	2.774	0.000	1.389	7.093

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	199	802	0	1161	0	1735	5962
normalized size	1	1.00	0.66	2.67	0.00	3.87	0.00	5.78	19.87
time (sec)	N/A	0.783	2.019	0.069	0.000	1.205	0.000	1.888	8.670

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	177	679	0	1029	0	354	5350
normalized size	1	1.00	0.80	3.07	0.00	4.66	0.00	1.60	24.21
time (sec)	N/A	0.488	1.484	0.071	0.000	1.321	0.000	1.135	8.232

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	149	639	0	913	0	319	5102
normalized size	1	1.00	0.83	3.57	0.00	5.10	0.00	1.78	28.50
time (sec)	N/A	0.304	1.125	0.063	0.000	1.092	0.000	1.043	8.801

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	115	400	0	587	0	250	203
normalized size	1	1.00	0.77	2.68	0.00	3.94	0.00	1.68	1.36
time (sec)	N/A	0.173	0.552	0.051	0.000	0.819	0.000	0.738	2.987

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	115	475	0	555	0	271	207
normalized size	1	1.00	0.86	3.54	0.00	4.14	0.00	2.02	1.54
time (sec)	N/A	0.123	0.384	0.051	0.000	0.862	0.000	0.632	3.151

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	400	0	585	0	251	203
normalized size	1	1.00	0.85	3.01	0.00	4.40	0.00	1.89	1.53
time (sec)	N/A	0.110	0.399	0.047	0.000	0.790	0.000	0.535	2.880

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	192	660	0	1142	0	344	5090
normalized size	1	1.00	1.05	3.63	0.00	6.27	0.00	1.89	27.97
time (sec)	N/A	0.456	1.140	0.096	0.000	2.757	0.000	1.272	9.058

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	205	712	0	1346	0	380	5347
normalized size	1	1.00	0.88	3.07	0.00	5.80	0.00	1.64	23.05
time (sec)	N/A	0.784	4.211	0.105	0.000	2.503	0.000	1.282	8.453

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	427	845	0	1524	0	801	5910
normalized size	1	1.00	1.40	2.77	0.00	5.00	0.00	2.63	19.38
time (sec)	N/A	1.078	6.183	0.136	0.000	6.036	0.000	1.523	9.160

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	240	1396	0	1593	0	563	7494
normalized size	1	1.00	0.78	4.55	0.00	5.19	0.00	1.83	24.41
time (sec)	N/A	0.900	5.781	0.075	0.000	1.268	0.000	1.325	9.895

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	227	1356	0	1445	0	531	7247
normalized size	1	1.00	0.91	5.42	0.00	5.78	0.00	2.12	28.99
time (sec)	N/A	0.571	2.733	0.075	0.000	1.294	0.000	2.111	12.371

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	158	776	0	893	0	399	378
normalized size	1	1.00	0.71	3.50	0.00	4.02	0.00	1.80	1.70
time (sec)	N/A	0.337	1.234	0.063	0.000	1.004	0.000	1.883	4.211

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	162	930	0	893	0	427	381
normalized size	1	1.00	0.79	4.51	0.00	4.33	0.00	2.07	1.85
time (sec)	N/A	0.281	1.195	0.054	0.000	0.918	0.000	1.122	4.149

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	164	931	0	891	0	427	382
normalized size	1	1.00	0.85	4.85	0.00	4.64	0.00	2.22	1.99
time (sec)	N/A	0.226	1.060	0.054	0.000	0.703	0.000	0.954	4.253

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	159	776	0	895	0	399	378
normalized size	1	1.00	0.86	4.22	0.00	4.86	0.00	2.17	2.05
time (sec)	N/A	0.217	0.917	0.054	0.000	1.022	0.000	0.749	4.123

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	274	1377	0	1815	0	554	7235
normalized size	1	1.00	1.09	5.49	0.00	7.23	0.00	2.21	28.82
time (sec)	N/A	0.788	3.073	0.102	0.000	5.129	0.000	1.663	12.506

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	416	1429	0	2048	0	587	7490
normalized size	1	1.00	1.35	4.64	0.00	6.65	0.00	1.91	24.32
time (sec)	N/A	1.270	6.226	0.121	0.000	6.321	0.000	1.630	9.946

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	214	827	0	0	0	0	-1
normalized size	1	1.00	0.81	3.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	1.141	1.088	0.000	1.464	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	180	665	0	0	0	0	-1
normalized size	1	1.00	0.87	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.861	0.825	0.000	0.838	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	137	452	0	0	0	0	-1
normalized size	1	1.00	0.85	2.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.555	0.740	0.000	0.871	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	170	0	0	0	0	-1
normalized size	1	1.00	1.00	2.98	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.059	0.000	0.000	0.750	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0	-1
normalized size	1	1.00	0.69	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	2.180	1.089	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	307	622	0	0	0	0	-1
normalized size	1	1.00	1.56	3.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.498	7.328	0.826	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	515	977	0	0	0	0	-1
normalized size	1	1.00	1.97	3.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	6.495	0.908	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	262	995	0	0	0	0	-1
normalized size	1	1.00	0.83	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.518	1.357	0.875	0.000	1.251	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	214	827	0	0	0	0	-1
normalized size	1	1.00	0.83	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.391	1.154	0.701	0.000	1.332	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	174	663	0	0	0	0	-1
normalized size	1	1.00	0.87	3.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.782	0.756	0.000	0.868	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	134	450	0	0	0	0	-1
normalized size	1	1.00	0.85	2.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.546	0.000	0.000	0.783	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	107	249	0	0	0	0	-1
normalized size	1	1.00	0.60	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.314	2.299	0.716	0.000	3.558	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	363	740	0	0	0	0	-1
normalized size	1	1.00	1.74	3.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	11.177	0.776	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	508	980	0	0	0	0	-1
normalized size	1	1.00	1.99	3.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.763	6.453	0.867	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	268	1140	0	0	0	0	-1
normalized size	1	1.00	0.72	3.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	1.206	0.993	0.000	1.384	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	263	995	0	0	0	0	-1
normalized size	1	1.00	0.85	3.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	1.378	0.946	0.000	1.292	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	214	827	0	0	0	0	-1
normalized size	1	1.00	0.86	3.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.883	0.853	0.000	1.187	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	177	662	0	0	0	0	-1
normalized size	1	1.00	0.90	3.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.261	0.774	0.856	0.000	1.425	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	379	528	0	0	0	0	-1
normalized size	1	1.00	1.71	2.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.586	1.761	0.768	0.000	2.711	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	390	960	0	0	0	0	-1
normalized size	1	1.00	1.76	4.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.592	2.194	0.845	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	395	1134	0	0	0	0	-1
normalized size	1	1.00	1.46	4.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.877	2.646	0.962	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	434	1742	0	0	0	0	-1
normalized size	1	1.00	1.34	5.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.169	4.123	1.110	0.000	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	211	824	0	0	0	0	-1
normalized size	1	1.00	0.86	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	1.098	0.915	0.000	1.421	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	92	275	0	0	0	0	-1
normalized size	1	1.00	0.67	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.245	0.648	0.000	0.808	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	81	253	0	0	0	0	-1
normalized size	1	1.00	0.77	2.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.140	0.657	0.000	1.297	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	231	0	0	0	0	-1
normalized size	1	1.00	0.88	2.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.063	0.617	0.000	1.236	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	137	0	0	0	0	-1
normalized size	1	1.00	1.00	5.96	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.024	0.415	0.000	1.158	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	158	0	0	0	0	-1
normalized size	1	1.00	0.85	3.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.087	0.051	0.549	0.000	2.655	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	157	350	0	0	0	0	-1
normalized size	1	1.00	1.65	3.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	1.121	0.795	0.000	1.586	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	194	408	0	0	0	0	-1
normalized size	1	1.00	1.44	3.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.364	1.268	0.954	0.000	1.838	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	114	276	0	0	0	0	-1
normalized size	1	1.00	0.81	1.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.208	0.754	0.000	1.843	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	104	253	0	0	0	0	-1
normalized size	1	1.00	0.97	2.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.207	0.724	0.000	0.978	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	94	231	0	0	0	0	-1
normalized size	1	1.00	1.18	2.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.097	0.739	0.000	1.093	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	44	138	0	0	0	0	-1
normalized size	1	1.00	1.83	5.75	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.031	0.460	0.000	1.767	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	61	159	0	0	0	0	-1
normalized size	1	1.00	1.22	3.18	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	0.062	0.796	0.000	2.531	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	178	351	0	0	0	0	-1
normalized size	1	1.00	1.82	3.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	1.467	0.877	0.000	1.771	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	237	408	0	0	0	0	-1
normalized size	1	1.00	1.72	2.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.374	1.884	1.043	0.000	1.764	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	182	665	0	0	0	0	-1
normalized size	1	1.00	0.85	3.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.914	0.825	0.000	1.538	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	137	453	0	0	0	0	116
normalized size	1	1.00	0.83	2.75	0.00	0.00	0.00	0.00	0.70
time (sec)	N/A	0.188	0.616	0.944	0.000	0.944	0.000	0.000	0.564

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	86	220	0	0	0	0	80
normalized size	1	1.00	0.70	1.80	0.00	0.00	0.00	0.00	0.66
time (sec)	N/A	0.108	2.416	0.773	0.000	0.718	0.000	0.000	0.660

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	75	0	0	0	0	52
normalized size	1	1.00	1.00	1.32	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.037	0.045	0.102	0.000	1.077	0.000	0.000	0.599

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	166	0	0	0	0	-1
normalized size	1	1.00	1.00	2.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.126	0.087	0.596	0.000	0.000	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	310	532	0	0	0	0	-1
normalized size	1	1.00	1.50	2.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	8.500	1.160	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	518	710	0	0	0	0	-1
normalized size	1	1.00	1.93	2.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	6.397	0.989	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	242	1285	0	0	0	0	-1
normalized size	1	1.00	0.74	3.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.513	1.313	1.132	0.000	0.980	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	197	984	0	0	0	0	-1
normalized size	1	1.00	0.77	3.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.881	1.003	0.000	1.417	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	159	530	0	0	0	0	-1
normalized size	1	1.00	0.85	2.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.700	0.939	0.000	1.600	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	137	373	0	0	0	0	-1
normalized size	1	1.00	0.81	2.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.528	0.982	0.000	1.292	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	83	217	0	0	0	0	-1
normalized size	1	1.00	0.78	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.206	0.793	0.000	1.193	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	402	376	0	0	0	0	-1
normalized size	1	1.00	2.28	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.394	5.191	0.922	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	441	894	0	0	0	0	-1
normalized size	1	1.00	1.59	3.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.780	4.147	2.025	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	597	1542	0	0	0	0	-1
normalized size	1	1.00	1.73	4.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.082	6.483	2.392	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	272	1684	0	0	0	0	-1
normalized size	1	1.00	0.62	3.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	1.991	4.373	0.000	2.460	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	237	1291	0	0	0	0	-1
normalized size	1	1.00	0.69	3.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.570	1.642	3.383	0.000	1.639	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	188	907	0	0	0	0	-1
normalized size	1	1.00	0.67	3.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.379	1.273	2.878	0.000	1.605	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	175	846	0	0	0	0	-1
normalized size	1	1.00	0.67	3.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	1.150	2.853	0.000	1.773	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	154	742	0	0	0	0	-1
normalized size	1	1.00	0.63	3.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	1.003	2.727	0.000	1.359	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	158	489	0	0	0	0	-1
normalized size	1	1.00	0.71	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.934	1.655	0.000	1.528	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	464	845	0	0	0	0	-1
normalized size	1	1.00	1.45	2.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.871	4.801	2.718	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	638	1320	0	0	0	0	-1
normalized size	1	1.00	1.68	3.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.100	6.579	3.606	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	189	616	0	0	0	0	-1
normalized size	1	1.00	0.67	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	1.418	2.355	0.000	2.823	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	81	231	0	0	0	0	-1
normalized size	1	1.00	0.73	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.184	0.757	0.000	1.913	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	231	0	0	0	0	78
normalized size	1	1.00	0.90	2.96	0.00	0.00	0.00	0.00	1.00
time (sec)	N/A	0.102	0.083	0.594	0.000	1.506	0.000	0.000	0.090

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	155	0	0	0	0	54
normalized size	1	1.00	0.84	3.04	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.052	0.060	0.609	0.000	1.461	0.000	0.000	0.628

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	0	0	0	0	39
normalized size	1	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.70
time (sec)	N/A	0.012	0.030	0.017	0.000	1.101	0.000	0.000	0.573

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	138	0	0	0	0	-1
normalized size	1	1.00	1.00	5.75	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.053	0.483	0.000	2.124	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	158	350	0	0	0	0	-1
normalized size	1	1.00	1.56	3.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.256	1.176	0.839	0.000	2.266	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	195	408	0	0	0	0	-1
normalized size	1	1.00	1.42	2.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.371	1.323	0.839	0.000	2.276	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	254	0	0	0	0	-1
normalized size	1	1.00	0.90	2.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.170	0.633	0.000	1.162	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	94	232	0	0	0	0	78
normalized size	1	1.00	1.18	2.90	0.00	0.00	0.00	0.00	0.98
time (sec)	N/A	0.102	0.125	0.677	0.000	1.300	0.000	0.000	0.089

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	60	158	0	0	0	0	52
normalized size	1	1.00	1.13	2.98	0.00	0.00	0.00	0.00	0.98
time (sec)	N/A	0.050	0.074	0.729	0.000	0.742	0.000	0.000	0.140

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	44	54	0	0	0	0	39
normalized size	1	1.00	1.83	2.25	0.00	0.00	0.00	0.00	1.62
time (sec)	N/A	0.012	0.038	0.066	0.000	1.205	0.000	0.000	0.570

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	45	139	0	0	0	0	-1
normalized size	1	1.00	1.80	5.56	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.065	0.593	0.000	2.840	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	179	351	0	0	0	0	-1
normalized size	1	1.00	1.72	3.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	1.517	0.828	0.000	1.643	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	236	408	0	0	0	0	-1
normalized size	1	1.00	1.69	2.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.366	1.901	1.058	0.000	1.628	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	77	290	0	0	0	0	87
normalized size	1	1.00	0.69	2.61	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.075	0.528	0.713	0.000	2.929	0.000	0.000	0.962

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	66	262	0	0	0	0	80
normalized size	1	1.00	0.76	3.01	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.060	0.245	0.738	0.000	1.564	0.000	0.000	0.740

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	229	0	0	0	0	53
normalized size	1	1.00	0.87	3.75	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.051	0.114	0.641	0.000	2.042	0.000	0.000	0.653

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	152	0	0	0	0	33
normalized size	1	1.00	1.00	4.34	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.039	0.068	0.689	0.000	1.893	0.000	0.000	0.260

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	148	0	0	0	0	60
normalized size	1	1.00	0.89	2.60	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.048	0.145	0.735	0.000	0.979	0.000	0.000	0.973

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	397	0	0	0	0	87
normalized size	1	1.00	0.78	4.78	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.059	0.424	1.594	0.000	1.305	0.000	0.000	1.231

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	502	0	0	0	0	87
normalized size	1	1.00	0.86	4.52	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.074	0.318	1.830	0.000	1.256	0.000	0.000	1.461

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	113	398	0	0	0	0	135
normalized size	1	1.00	0.71	2.49	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.120	0.803	0.797	0.000	2.058	0.000	0.000	1.044

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	98	362	0	0	0	0	128
normalized size	1	1.00	0.73	2.68	0.00	0.00	0.00	0.00	0.95
time (sec)	N/A	0.104	0.633	0.746	0.000	1.538	0.000	0.000	0.890

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	79	321	0	0	0	0	102
normalized size	1	1.00	0.78	3.18	0.00	0.00	0.00	0.00	1.01
time (sec)	N/A	0.090	0.322	0.694	0.000	0.854	0.000	0.000	1.006

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	283	0	0	0	0	76
normalized size	1	1.00	0.89	3.93	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.084	0.165	0.797	0.000	1.157	0.000	0.000	0.941

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	202	0	0	0	0	81
normalized size	1	1.00	0.91	2.97	0.00	0.00	0.00	0.00	1.19
time (sec)	N/A	0.086	0.299	0.692	0.000	0.747	0.000	0.000	1.134

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	514	0	0	0	0	108
normalized size	1	1.00	0.77	5.41	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.095	0.620	1.580	0.000	1.036	0.000	0.000	1.240

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	660	0	0	0	0	113
normalized size	1	1.00	0.92	4.89	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.106	0.397	2.086	0.000	1.091	0.000	0.000	1.385

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	137	470	0	0	0	0	178
normalized size	1	1.00	0.71	2.42	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.220	1.030	0.749	0.000	1.582	0.000	0.000	1.046

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	110	421	0	0	0	0	146
normalized size	1	1.00	0.69	2.65	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.202	0.792	0.744	0.000	1.151	0.000	0.000	0.938

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	84	376	0	0	0	0	125
normalized size	1	1.00	0.72	3.24	0.00	0.00	0.00	0.00	1.08
time (sec)	N/A	0.177	0.401	0.685	0.000	1.827	0.000	0.000	0.876

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	86	303	0	0	0	0	124
normalized size	1	1.00	0.69	2.44	0.00	0.00	0.00	0.00	1.00
time (sec)	N/A	0.188	0.560	0.800	0.000	1.300	0.000	0.000	0.930

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	85	631	0	0	0	0	128
normalized size	1	1.00	0.71	5.26	0.00	0.00	0.00	0.00	1.07
time (sec)	N/A	0.190	1.329	1.642	0.000	1.175	0.000	0.000	1.638

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	125	738	0	0	0	0	156
normalized size	1	1.00	0.84	4.95	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.211	0.958	2.068	0.000	0.851	0.000	0.000	1.738

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	177	847	0	0	0	0	147
normalized size	1	1.00	0.91	4.37	0.00	0.00	0.00	0.00	0.76
time (sec)	N/A	0.233	0.813	2.599	0.000	0.872	0.000	0.000	2.001

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	158	516	0	0	0	0	-1
normalized size	1	1.00	1.41	4.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.391	2.044	0.866	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	81	227	0	0	0	0	-1
normalized size	1	1.00	1.08	3.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.304	0.790	0.000	152.496	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	188	0	0	0	0	-1
normalized size	1	1.00	0.91	3.55	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.101	0.076	0.675	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0	-1
normalized size	1	1.00	1.00	5.17	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.045	0.080	0.631	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	195	354	0	0	0	0	-1
normalized size	1	1.00	2.53	4.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	3.111	0.944	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	210	452	0	0	0	0	-1
normalized size	1	1.00	1.64	3.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.546	4.585	1.959	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	266	1070	0	0	0	0	-1
normalized size	1	1.00	1.09	4.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.705	1.957	2.591	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	251	815	0	0	0	0	-1
normalized size	1	1.00	1.36	4.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.459	1.849	2.239	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	194	794	0	0	0	0	-1
normalized size	1	1.00	1.19	4.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.384	3.402	1.808	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	229	713	0	0	0	0	-1
normalized size	1	1.00	1.55	4.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.397	3.700	1.912	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	238	612	0	0	0	0	-1
normalized size	1	1.00	1.52	3.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.436	3.512	1.361	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	278	874	0	0	0	0	-1
normalized size	1	1.00	1.28	4.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.683	3.135	2.492	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	294	1008	0	0	0	0	-1
normalized size	1	1.00	1.05	3.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.995	3.572	3.361	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	354	2194	0	0	0	0	-1
normalized size	1	1.00	1.02	6.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.039	3.230	3.687	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	309	1935	0	0	0	0	-1
normalized size	1	1.00	1.10	6.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.774	3.007	3.520	0.000	160.504	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	284	1914	0	0	0	0	-1
normalized size	1	1.00	1.08	7.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.777	2.171	3.079	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	272	1836	0	0	0	0	-1
normalized size	1	1.00	1.11	7.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.657	2.069	3.046	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	291	1736	0	0	0	0	-1
normalized size	1	1.00	1.16	6.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.682	3.067	2.955	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	301	1176	0	0	0	0	-1
normalized size	1	1.00	1.15	4.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.775	2.999	1.915	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	334	1992	0	0	0	0	-1
normalized size	1	1.00	1.02	6.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.071	3.621	3.708	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	349	2128	0	0	0	0	-1
normalized size	1	1.00	0.88	5.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.350	5.957	5.452	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	1152	1233	0	0	0	0	-1
normalized size	1	1.00	2.63	2.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.922	17.595	0.274	0.000	88.706	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	314	801	0	0	0	0	-1
normalized size	1	1.00	0.85	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.575	7.065	0.269	0.000	3.067	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	137	197	0	0	0	0	-1
normalized size	1	1.00	1.01	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	1.275	0.219	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	203	789	0	0	0	0	-1
normalized size	1	1.00	0.89	3.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	3.456	0.248	0.000	1.409	0.000	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	247	880	0	0	0	0	-1
normalized size	1	1.00	0.91	3.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.402	7.868	0.298	0.000	1.903	0.000	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	453	1555	0	0	0	0	-1
normalized size	1	1.00	1.38	4.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.646	13.293	0.265	0.000	2.515	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	1304	1826	0	0	0	0	-1
normalized size	1	1.00	3.35	4.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.924	6.233	0.309	0.000	1.586	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	1189	1683	0	0	0	0	-1
normalized size	1	1.00	2.34	3.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.257	19.590	0.259	0.000	3.292	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	437	1421	0	0	0	0	-1
normalized size	1	1.00	1.01	3.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.170	12.274	0.186	0.000	3.992	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	339	1003	0	0	0	0	-1
normalized size	1	1.00	0.90	2.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	7.575	0.270	0.000	1.294	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	357	1183	0	0	0	0	-1
normalized size	1	1.00	1.06	3.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	13.127	0.225	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	256	1075	0	0	0	0	-1
normalized size	1	1.00	0.92	3.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	4.980	0.264	0.000	1.408	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	443	1539	0	0	0	0	-1
normalized size	1	1.00	1.36	4.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.661	13.522	0.217	0.000	1.032	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	1302	1827	0	0	0	0	-1
normalized size	1	1.00	3.36	4.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.949	6.261	0.272	0.000	0.683	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	1368	2503	0	0	0	0	-1
normalized size	1	1.00	3.01	5.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.318	6.296	0.415	0.000	0.762	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	1203	1866	0	0	0	0	-1
normalized size	1	1.00	2.38	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.356	19.063	0.247	0.000	2.227	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	329	1629	0	0	0	0	-1
normalized size	1	1.00	0.74	3.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.003	6.617	0.209	0.000	3.671	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	1185	1626	0	0	0	0	-1
normalized size	1	1.00	2.66	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.010	18.189	0.195	0.000	56.373	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	328	1485	0	0	0	0	-1
normalized size	1	1.00	0.84	3.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.739	7.044	0.283	0.000	61.289	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	427	1750	0	0	0	0	-1
normalized size	1	1.00	1.26	5.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.765	11.796	0.223	0.000	1.111	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	1302	1827	0	0	0	0	-1
normalized size	1	1.00	3.36	4.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.044	6.297	0.275	0.000	0.845	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	1368	2504	0	0	0	0	-1
normalized size	1	1.00	3.01	5.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.409	6.308	0.375	0.000	0.937	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	1431	2789	0	0	0	0	-1
normalized size	1	1.00	2.74	5.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.775	6.347	0.594	0.000	0.738	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	414	479	622	0	0	0	0	-1
normalized size	1	1.09	1.26	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.736	4.456	0.252	0.000	1.371	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	130	159	0	0	0	0	-1
normalized size	1	1.00	1.12	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.924	0.246	0.000	65.069	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	170	123	0	0	0	0	-1
normalized size	1	1.00	1.56	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	1.403	0.214	0.000	0.964	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	211	612	0	0	0	0	-1
normalized size	1	1.00	0.94	2.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	4.957	0.235	0.000	2.339	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	371	883	0	0	0	0	-1
normalized size	1	1.00	1.35	3.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.404	13.968	0.325	0.000	1.840	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	1201	1661	0	0	0	0	-1
normalized size	1	1.00	2.58	3.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.986	6.223	0.220	0.000	3.017	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	985	1206	0	0	0	0	-1
normalized size	1	1.00	2.55	3.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.503	17.734	0.249	0.000	81.385	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	196	809	0	0	0	0	-1
normalized size	1	1.00	0.74	3.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	4.817	0.211	0.000	1.995	0.000	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	202	830	0	0	0	0	-1
normalized size	1	1.00	0.76	3.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	5.793	0.230	0.000	1.218	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	1233	1452	0	0	0	0	-1
normalized size	1	1.00	4.33	5.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	6.284	0.250	0.000	0.984	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	1269	1781	0	0	0	0	-1
normalized size	1	1.00	3.55	4.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.724	6.359	0.222	0.000	0.840	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	1314	2478	0	0	0	0	-1
normalized size	1	1.00	3.03	5.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.057	6.384	0.267	0.000	1.349	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	1282	3911	0	0	0	0	-1
normalized size	1	1.00	2.58	7.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.064	6.305	0.271	0.000	62.141	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	277	1782	0	0	0	0	-1
normalized size	1	1.00	0.81	5.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	6.379	0.297	0.000	1.047	0.000	0.000	0.000

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	1273	2417	0	0	0	0	-1
normalized size	1	1.00	3.55	6.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	6.253	0.212	0.000	2.170	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	1296	2743	0	0	0	0	-1
normalized size	1	1.00	3.40	7.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.732	6.279	0.289	0.000	1.126	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	1321	3693	0	0	0	0	-1
normalized size	1	1.00	3.32	9.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.813	6.398	0.272	0.000	2.247	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1351	4189	0	0	0	0	-1
normalized size	1	1.00	2.86	8.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.173	6.490	0.319	0.000	1.303	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	131	115	0	0	0	0	-1
normalized size	1	1.00	4.09	3.59	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	3.660	0.202	0.000	0.719	0.000	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	156	107	0	0	0	0	-1
normalized size	1	1.00	6.24	4.28	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.935	0.187	0.000	1.878	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	143	119	0	0	0	0	-1
normalized size	1	1.00	2.55	2.12	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.117	1.102	0.184	0.000	1.422	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	153	132	0	0	0	0	-1
normalized size	1	1.00	3.12	2.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.110	1.521	0.192	0.000	1.947	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	140	116	0	0	0	0	-1
normalized size	1	1.00	2.41	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	1.051	0.215	0.000	1.612	0.000	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	144	125	0	0	0	0	-1
normalized size	1	1.00	2.40	2.08	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	1.043	0.194	0.000	0.885	0.000	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	144	123	0	0	0	0	-1
normalized size	1	1.00	1.71	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	1.256	0.198	0.000	1.636	0.000	0.000	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	153	137	0	0	0	0	-1
normalized size	1	1.00	1.87	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	1.058	0.201	0.000	1.308	0.000	0.000	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	150	122	0	0	0	0	-1
normalized size	1	1.00	2.78	2.26	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.101	0.614	0.187	0.000	0.765	0.000	0.000	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	158	109	0	0	0	0	-1
normalized size	1	1.00	3.36	2.32	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.401	0.190	0.000	0.999	0.000	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	145	121	0	0	0	0	-1
normalized size	1	1.00	4.26	3.56	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	0.553	0.165	0.000	0.808	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	155	129	0	0	0	0	-1
normalized size	1	1.00	5.74	4.78	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	0.494	0.181	0.000	0.735	0.000	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	154	127	0	0	0	0	-1
normalized size	1	1.00	1.92	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.615	0.160	0.000	0.657	0.000	0.000	0.000

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	146	107	0	0	0	0	-1
normalized size	1	1.00	1.78	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.483	0.126	0.000	0.864	0.000	0.000	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	160	98	0	0	0	0	-1
normalized size	1	1.00	2.58	1.58	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.689	0.120	0.000	1.510	0.000	0.000	0.000

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	155	128	0	0	0	0	-1
normalized size	1	1.00	2.58	2.13	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.461	0.168	0.000	1.423	0.000	0.000	0.000

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	175	142	0	0	0	0	-1
normalized size	1	1.00	2.27	1.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	2.841	0.209	0.000	1.950	0.000	0.000	0.000

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	140	132	0	0	0	0	-1
normalized size	1	1.00	1.87	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.692	0.182	0.000	2.193	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	145	144	0	0	0	0	-1
normalized size	1	1.00	1.46	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	1.767	0.192	0.000	1.205	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	155	161	0	0	0	0	-1
normalized size	1	1.00	1.53	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	2.051	0.185	0.000	1.563	0.000	0.000	0.000

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	115	144	0	0	0	0	-1
normalized size	1	1.00	1.58	1.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	1.439	0.178	0.000	1.474	0.000	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	117	153	0	0	0	0	-1
normalized size	1	1.00	1.56	2.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.841	0.175	0.000	0.857	0.000	0.000	0.000

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	135	158	0	0	0	0	-1
normalized size	1	1.00	1.36	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.985	0.179	0.000	1.062	0.000	0.000	0.000

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	113	168	0	0	0	0	-1
normalized size	1	1.00	1.16	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.600	0.175	0.000	1.326	0.000	0.000	0.000

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	194	159	0	0	0	0	-1
normalized size	1	1.00	1.96	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.766	0.171	0.000	1.695	0.000	0.000	0.000

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	142	142	0	0	0	0	-1
normalized size	1	1.00	1.46	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.224	0.187	0.000	1.382	0.000	0.000	0.000

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	147	154	0	0	0	0	-1
normalized size	1	1.00	1.91	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.509	0.172	0.000	1.295	0.000	0.000	0.000

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	156	164	0	0	0	0	-1
normalized size	1	1.00	1.97	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.509	0.184	0.000	1.403	0.000	0.000	0.000

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	117	168	0	0	0	0	-1
normalized size	1	1.00	1.23	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.503	0.175	0.000	1.070	0.000	0.000	0.000

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	119	180	0	0	0	0	-1
normalized size	1	1.00	1.23	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.223	0.171	0.000	1.545	0.000	0.000	0.000

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	140	152	0	0	0	0	-1
normalized size	1	1.00	1.82	1.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.159	0.171	0.000	2.048	0.000	0.000	0.000

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	115	164	0	0	0	0	-1
normalized size	1	1.00	1.53	2.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.305	0.175	0.000	3.856	0.000	0.000	0.000

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	4614	0	0	0	0	0	-1
normalized size	1	1.00	26.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	21.701	0.087	0.000	0.000	0.000	0.000	0.000

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	4613	0	0	0	0	0	-1
normalized size	1	1.00	26.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	21.328	0.088	0.000	0.000	0.000	0.000	0.000

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	4605	0	0	0	0	0	-1
normalized size	1	1.00	26.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	21.337	0.147	0.000	0.000	0.000	0.000	0.000

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	4608	0	0	0	0	0	-1
normalized size	1	1.00	26.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	21.203	0.161	0.000	0.000	0.000	0.000	0.000

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	33.561	0.204	0.000	5.659	0.000	0.000	0.000

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	80.140	0.199	0.000	5.216	0.000	0.000	0.000

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	19.413	0.166	0.000	3.567	0.000	0.000	0.000

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	9.177	0.156	0.000	3.796	0.000	0.000	0.000

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	2.743	0.167	0.000	1.966	0.000	0.000	0.000

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	1.931	0.157	0.000	3.121	0.000	0.000	0.000

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.534	0.161	0.000	1.107	0.000	0.000	0.000

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	82.317	0.151	0.000	0.963	0.000	0.000	0.000

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	29.301	0.153	0.000	1.502	0.000	0.000	0.000

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	86.229	0.146	0.000	1.761	0.000	0.000	0.000

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	97	502	0	0	0	0	-1
normalized size	1	1.00	0.64	3.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.347	2.184	0.000	1.408	0.000	0.000	0.000

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	85	397	0	0	0	0	-1
normalized size	1	1.00	0.69	3.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.239	1.912	0.000	1.483	0.000	0.000	0.000

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	148	0	0	0	0	-1
normalized size	1	1.00	0.73	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.111	0.744	0.000	1.360	0.000	0.000	0.000

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	0	0	0	-1
normalized size	1	1.00	0.69	2.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.074	0.774	0.000	1.051	0.000	0.000	0.000

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	229	0	0	0	0	-1
normalized size	1	1.00	0.75	2.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.139	0.738	0.000	1.009	0.000	0.000	0.000

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	88	262	0	0	0	0	-1
normalized size	1	1.00	0.69	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.323	0.862	0.000	1.012	0.000	0.000	0.000

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	99	290	0	0	0	0	-1
normalized size	1	1.00	0.66	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.565	0.740	0.000	1.716	0.000	0.000	0.000

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	139	689	0	0	0	0	-1
normalized size	1	1.00	0.70	3.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.872	2.776	0.000	1.229	0.000	0.000	0.000

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	126	660	0	0	0	0	-1
normalized size	1	1.00	0.72	3.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	1.313	2.471	0.000	0.842	0.000	0.000	0.000

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	93	514	0	0	0	0	-1
normalized size	1	1.00	0.69	3.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.336	1.935	0.000	0.624	0.000	0.000	0.000

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	83	202	0	0	0	0	-1
normalized size	1	1.00	0.77	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.198	0.909	0.000	1.647	0.000	0.000	0.000

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	0	0	0	-1
normalized size	1	1.00	0.78	2.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.187	0.806	0.000	1.318	0.000	0.000	0.000

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	100	321	0	0	0	0	-1
normalized size	1	1.00	0.71	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.455	0.784	0.000	2.375	0.000	0.000	0.000

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	120	362	0	0	0	0	-1
normalized size	1	1.00	0.69	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.682	0.895	0.000	2.518	0.000	0.000	0.000

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	135	398	0	0	0	0	-1
normalized size	1	1.00	0.68	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	1.109	1.176	0.000	2.339	0.000	0.000	0.000

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	191	847	0	0	0	0	-1
normalized size	1	1.00	0.82	3.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	1.198	3.344	0.000	0.959	0.000	0.000	0.000

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	134	738	0	0	0	0	-1
normalized size	1	1.00	0.71	3.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	1.673	2.566	0.000	2.336	0.000	0.000	0.000

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	106	631	0	0	0	0	-1
normalized size	1	1.00	0.66	3.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.506	2.233	0.000	1.147	0.000	0.000	0.000

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	108	303	0	0	0	0	-1
normalized size	1	1.00	0.65	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.631	0.955	0.000	0.625	0.000	0.000	0.000

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	106	376	0	0	0	0	-1
normalized size	1	1.00	0.68	2.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.467	1.094	0.000	1.148	0.000	0.000	0.000

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	132	421	0	0	0	0	-1
normalized size	1	1.00	0.66	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.956	0.928	0.000	1.153	0.000	0.000	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	159	470	0	0	0	0	-1
normalized size	1	1.00	0.68	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	1.238	1.018	0.000	1.083	0.000	0.000	0.000

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	165	452	0	0	0	0	-1
normalized size	1	1.00	0.88	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.547	3.020	2.224	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	83	354	0	0	0	0	-1
normalized size	1	1.00	0.71	3.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	5.160	1.051	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0	-1
normalized size	1	1.00	1.29	3.06	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.131	0.336	0.828	0.000	0.000	0.000	0.000	0.000

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	47	188	0	0	0	0	-1
normalized size	1	1.00	0.51	2.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.244	0.902	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	176	227	0	0	0	0	-1
normalized size	1	1.00	1.30	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	6.519	1.203	0.000	80.170	0.000	0.000	0.000

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	196	516	0	0	0	0	-1
normalized size	1	1.00	1.14	3.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.391	6.787	1.026	0.000	0.000	0.000	0.000	0.000

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	655	1008	0	0	0	0	-1
normalized size	1	1.00	1.92	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.966	6.798	3.864	0.000	0.000	0.000	0.000	0.000

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	351	874	0	0	0	0	-1
normalized size	1	1.00	1.27	3.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	4.626	2.563	0.000	0.000	0.000	0.000	0.000

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	584	612	0	0	0	0	-1
normalized size	1	1.00	2.69	2.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	6.670	1.681	0.000	0.000	0.000	0.000	0.000

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	574	713	0	0	0	0	-1
normalized size	1	1.00	2.76	3.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	6.673	2.306	0.000	0.000	0.000	0.000	0.000

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	250	794	0	0	0	0	-1
normalized size	1	1.00	1.12	3.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	5.644	1.972	0.000	0.000	0.000	0.000	0.000

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	319	815	0	0	0	0	-1
normalized size	1	1.00	1.30	3.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.474	6.429	2.402	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	747	2128	0	0	0	0	-1
normalized size	1	1.00	1.64	4.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.451	6.834	5.951	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	723	1992	0	0	0	0	-1
normalized size	1	1.00	1.86	5.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.032	6.897	4.537	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	694	1176	0	0	0	0	-1
normalized size	1	1.00	2.16	3.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	6.768	2.151	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	395	1736	0	0	0	0	-1
normalized size	1	1.00	1.25	5.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.752	5.983	3.828	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	665	1836	0	0	0	0	-1
normalized size	1	1.00	2.20	6.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	6.750	3.508	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	280	1914	0	0	0	0	-1
normalized size	1	1.00	0.88	6.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.685	5.657	3.459	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	353	1563	0	0	0	0	-1
normalized size	1	1.00	0.96	4.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.753	13.078	0.246	0.000	1.345	0.000	0.000	0.000

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	301	888	0	0	0	0	-1
normalized size	1	1.00	0.97	2.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.502	10.993	0.286	0.000	1.156	0.000	0.000	0.000

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	215	797	0	0	0	0	-1
normalized size	1	1.00	0.80	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	6.049	0.254	0.000	1.261	0.000	0.000	0.000

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	146	199	0	0	0	0	-1
normalized size	1	1.00	0.94	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	1.364	0.238	0.000	0.000	0.000	0.000	0.000

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	565	803	0	0	0	0	-1
normalized size	1	1.00	1.31	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.650	13.852	0.345	0.000	2.105	0.000	0.000	0.000

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	1113	1241	0	0	0	0	-1
normalized size	1	1.00	2.23	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.991	18.157	0.274	0.000	59.854	0.000	0.000	0.000

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	441	1835	0	0	0	0	-1
normalized size	1	1.00	1.03	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.050	13.881	0.293	0.000	0.873	0.000	0.000	0.000

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	345	1547	0	0	0	0	-1
normalized size	1	1.00	0.95	4.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.759	11.870	0.264	0.000	0.832	0.000	0.000	0.000

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	291	1085	0	0	0	0	-1
normalized size	1	1.00	0.92	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.526	7.216	0.315	0.000	0.880	0.000	0.000	0.000

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	639	1191	0	0	0	0	-1
normalized size	1	1.00	1.61	3.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.556	17.488	0.277	0.000	48.678	0.000	0.000	0.000

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	322	1005	0	0	0	0	-1
normalized size	1	1.00	0.74	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.715	11.669	0.333	0.000	53.097	0.000	0.000	0.000

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	845	1423	0	0	0	0	-1
normalized size	1	1.00	1.71	2.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.270	17.804	0.278	0.000	2.388	0.000	0.000	0.000

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	961	1691	0	0	0	0	-1
normalized size	1	1.00	1.69	2.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.356	17.623	0.307	0.000	2.603	0.000	0.000	0.000

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	521	2512	0	0	0	0	-1
normalized size	1	1.00	1.05	5.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.518	16.292	0.373	0.000	1.263	0.000	0.000	0.000

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	443	1835	0	0	0	0	-1
normalized size	1	1.00	1.04	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.167	13.831	0.277	0.000	0.830	0.000	0.000	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	376	1758	0	0	0	0	-1
normalized size	1	1.00	0.99	4.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	15.169	0.290	0.000	0.863	0.000	0.000	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	399	1493	0	0	0	0	-1
normalized size	1	1.00	0.88	3.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.824	11.995	0.352	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	736	1631	0	0	0	0	-1
normalized size	1	1.00	1.46	3.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.107	16.679	0.219	0.000	67.251	0.000	0.000	0.000

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	3679	1631	0	0	0	0	-1
normalized size	1	1.00	7.31	3.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.101	22.320	0.239	0.000	59.196	0.000	0.000	0.000

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	970	1868	0	0	0	0	-1
normalized size	1	1.00	1.71	3.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.451	17.705	0.299	0.000	1.786	0.000	0.000	0.000

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	1642	2327	0	0	0	0	-1
normalized size	1	1.00	2.57	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.883	16.608	0.437	0.000	2.919	0.000	0.000	0.000

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	322	891	0	0	0	0	-1
normalized size	1	1.00	1.03	2.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	14.044	0.326	0.000	1.268	0.000	0.000	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	296	620	0	0	0	0	-1
normalized size	1	1.00	1.12	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	12.595	0.235	0.000	0.633	0.000	0.000	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	103	125	0	0	0	0	-1
normalized size	1	1.00	0.80	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.873	0.292	0.000	0.514	0.000	0.000	0.000

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	146	143	0	0	0	0	-1
normalized size	1	1.00	1.07	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.793	0.221	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	507	630	0	0	0	0	-1
normalized size	1	1.00	1.07	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.796	12.101	0.282	0.000	2.130	0.000	0.000	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	1153	1248	0	0	0	0	-1
normalized size	1	1.00	2.28	2.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.954	18.920	0.285	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	440	1789	0	0	0	0	-1
normalized size	1	1.00	1.11	4.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.823	14.443	0.198	0.000	1.365	0.000	0.000	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	369	1457	0	0	0	0	-1
normalized size	1	1.00	1.14	4.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	12.131	0.273	0.000	1.334	0.000	0.000	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	237	832	0	0	0	0	-1
normalized size	1	1.00	0.77	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	8.457	0.314	0.000	2.248	0.000	0.000	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	235	811	0	0	0	0	-1
normalized size	1	1.00	0.77	2.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	4.018	0.289	0.000	1.517	0.000	0.000	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	1175	1214	0	0	0	0	-1
normalized size	1	1.00	2.63	2.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	18.054	0.367	0.000	61.423	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	1025	1675	0	0	0	0	-1
normalized size	1	1.00	1.95	3.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.091	15.194	0.249	0.000	40.816	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	546	4197	0	0	0	0	-1
normalized size	1	1.00	1.06	8.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.302	17.547	0.270	0.000	1.045	0.000	0.000	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	525	3701	0	0	0	0	-1
normalized size	1	1.00	1.20	8.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.925	17.266	0.291	0.000	0.841	0.000	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	471	2745	0	0	0	0	-1
normalized size	1	1.00	1.12	6.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.844	14.454	0.306	0.000	0.824	0.000	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	455	2419	0	0	0	0	-1
normalized size	1	1.00	1.14	6.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.758	13.505	0.274	0.000	0.690	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	359	1790	0	0	0	0	-1
normalized size	1	1.00	0.94	4.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.729	8.405	0.341	0.000	0.560	0.000	0.000	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	1716	3920	0	0	0	0	-1
normalized size	1	1.00	3.08	7.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.213	14.454	0.281	0.000	45.658	0.000	0.000	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	242	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	1.779	1.542	0.000	1.004	0.000	0.000	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	197	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	0.905	1.296	0.000	0.890	0.000	0.000	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	168	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.357	1.120	0.000	1.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	112	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.170	1.063	0.000	0.992	0.000	0.000	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	6703	0	0	0	0	0	-1
normalized size	1	1.00	35.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	24.616	0.770	0.000	0.896	0.000	0.000	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	7214	0	0	0	0	0	-1
normalized size	1	1.00	24.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	26.460	0.426	0.000	0.836	0.000	0.000	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	222	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	0.882	1.509	0.000	0.975	0.000	0.000	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	159	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.343	1.296	0.000	1.894	0.000	0.000	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	107	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.200	1.096	0.000	0.751	0.000	0.000	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	47	67	0	30	0	71	-1
normalized size	1	1.00	1.81	2.58	0.00	1.15	0.00	2.73	-0.04
time (sec)	N/A	0.078	0.080	0.193	0.000	1.848	0.000	0.534	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	64	67	0	32	0	46	-1
normalized size	1	1.00	0.98	1.03	0.00	0.49	0.00	0.71	-0.02
time (sec)	N/A	0.100	0.072	0.141	0.000	1.468	0.000	0.863	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	51	45	24	87	30	25
normalized size	1	1.00	0.78	1.38	1.22	0.65	2.35	0.81	0.68
time (sec)	N/A	0.020	0.060	0.053	0.318	0.744	0.277	0.496	0.630

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	31	150	144	70	333	88	29
normalized size	1	1.00	1.19	5.77	5.54	2.69	12.81	3.38	1.12
time (sec)	N/A	0.031	0.214	0.056	0.345	0.820	2.459	0.400	0.677

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	74	143	27	114	1370	28
normalized size	1	1.00	1.00	2.64	5.11	0.96	4.07	48.93	1.00
time (sec)	N/A	0.045	0.204	0.374	0.633	1.440	4.992	55.249	0.892

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	27	48	115	56	80	47	33
normalized size	1	1.00	1.04	1.85	4.42	2.15	3.08	1.81	1.27
time (sec)	N/A	0.033	0.128	0.072	0.337	0.752	2.375	0.357	0.640

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	45	48	92	36	0	86	-1
normalized size	1	1.00	1.61	1.71	3.29	1.29	0.00	3.07	-0.04
time (sec)	N/A	0.042	0.115	0.301	0.535	1.032	0.000	0.455	0.000

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	33	43	0	36	0	35	37
normalized size	1	1.00	1.27	1.65	0.00	1.38	0.00	1.35	1.42
time (sec)	N/A	0.016	0.048	0.227	0.000	0.970	0.000	0.960	0.742

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	48	0	68	0	59	85
normalized size	1	1.00	1.00	1.71	0.00	2.43	0.00	2.11	3.04
time (sec)	N/A	0.042	0.076	0.202	0.000	0.956	0.000	1.827	5.331

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	164	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.658	0.226	0.000	0.905	0.000	0.000	0.000

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	213	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	3.531	0.200	0.000	1.095	0.000	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	133	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.272	0.245	0.000	1.086	0.000	0.000	0.000

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	197	0	0	0	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	1.420	0.171	0.000	1.048	0.000	0.000	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	64	117	0	194	235	281	93
normalized size	1	1.00	1.02	1.86	0.00	3.08	3.73	4.46	1.48
time (sec)	N/A	0.093	0.134	0.090	0.000	3.912	32.916	0.741	0.937

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	51	0	22	0	50	37
normalized size	1	1.00	1.00	2.32	0.00	1.00	0.00	2.27	1.68
time (sec)	N/A	0.031	0.105	0.062	0.000	0.824	0.000	1.091	0.845

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	39	52	43	56	72	74
normalized size	1	1.00	0.66	0.83	1.11	0.91	1.19	1.53	1.57
time (sec)	N/A	0.067	0.064	0.096	0.429	1.880	2.424	0.329	0.667

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	171	0	0	0	0	56
normalized size	1	1.00	1.00	2.95	0.00	0.00	0.00	0.00	0.97
time (sec)	N/A	0.045	0.075	0.983	0.000	0.914	0.000	0.000	1.035

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	259	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	2.110	0.186	0.000	1.199	0.000	0.000	0.000

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	253	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	1.916	0.176	0.000	0.995	0.000	0.000	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	189	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	0.463	0.249	0.000	1.069	0.000	0.000	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	188	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.476	0.204	0.000	0.892	0.000	0.000	0.000

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	100	299	0	0	0	0	-1
normalized size	1	1.00	0.60	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.584	0.875	0.000	0.946	0.000	0.000	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	91	271	0	0	0	0	-1
normalized size	1	1.00	0.65	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.309	0.894	0.000	2.358	0.000	0.000	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	75	238	0	0	0	0	-1
normalized size	1	1.00	0.69	2.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.111	0.984	0.000	2.176	0.000	0.000	0.000

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	55	161	0	0	0	0	-1
normalized size	1	1.00	0.69	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.086	0.936	0.000	0.934	0.000	0.000	0.000

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	73	213	0	0	0	0	-1
normalized size	1	1.00	0.70	2.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.190	0.986	0.000	1.341	0.000	0.000	0.000

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	85	453	0	0	0	0	-1
normalized size	1	1.00	0.62	3.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.265	1.091	0.000	1.043	0.000	0.000	0.000

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	107	575	0	0	0	0	-1
normalized size	1	1.00	0.63	3.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.490	2.368	0.000	0.730	0.000	0.000	0.000

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	103	301	0	0	0	0	-1
normalized size	1	1.00	0.61	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.302	0.794	0.000	0.762	0.000	0.000	0.000

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	88	273	0	0	0	0	-1
normalized size	1	1.00	0.63	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.053	0.874	0.000	0.954	0.000	0.000	0.000

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	76	240	0	0	0	0	-1
normalized size	1	1.00	0.68	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.043	0.902	0.000	1.076	0.000	0.000	0.000

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	57	163	0	0	0	0	-1
normalized size	1	1.00	0.69	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.067	0.849	0.000	2.298	0.000	0.000	0.000

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	73	215	0	0	0	0	-1
normalized size	1	1.00	0.66	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.150	1.017	0.000	3.150	0.000	0.000	0.000

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	87	455	0	0	0	0	-1
normalized size	1	1.00	0.62	3.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.160	0.968	0.000	0.925	0.000	0.000	0.000

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	107	576	0	0	0	0	-1
normalized size	1	1.00	0.61	3.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.285	2.370	0.000	0.858	0.000	0.000	0.000

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	100	301	0	0	0	0	-1
normalized size	1	1.00	0.58	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.064	0.898	0.000	2.221	0.000	0.000	0.000

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	89	273	0	0	0	0	-1
normalized size	1	1.00	0.61	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.065	0.960	0.000	1.164	0.000	0.000	0.000

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	78	240	0	0	0	0	-1
normalized size	1	1.00	0.67	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.046	0.879	0.000	1.174	0.000	0.000	0.000

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	54	163	0	0	0	0	-1
normalized size	1	1.00	0.64	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.104	0.808	0.000	2.036	0.000	0.000	0.000

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	73	215	0	0	0	0	-1
normalized size	1	1.00	0.65	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.214	0.987	0.000	1.094	0.000	0.000	0.000

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	87	455	0	0	0	0	-1
normalized size	1	1.00	0.61	3.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.211	1.135	0.000	1.040	0.000	0.000	0.000

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	102	578	0	0	0	0	-1
normalized size	1	1.00	0.58	3.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.286	2.435	0.000	0.812	0.000	0.000	0.000

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	101	298	0	0	0	0	-1
normalized size	1	1.00	0.58	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.453	0.956	0.000	1.229	0.000	0.000	0.000

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	88	270	0	0	0	0	-1
normalized size	1	1.00	0.61	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.206	0.958	0.000	1.091	0.000	0.000	0.000

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	78	237	0	0	0	0	94
normalized size	1	1.00	0.69	2.10	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.093	0.058	1.010	0.000	0.666	0.000	0.000	0.282

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	160	0	0	0	0	48
normalized size	1	1.00	0.66	1.95	0.00	0.00	0.00	0.00	0.59
time (sec)	N/A	0.068	0.054	0.901	0.000	1.195	0.000	0.000	0.342

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	73	212	0	0	0	0	-1
normalized size	1	1.00	0.69	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.103	1.158	0.000	0.880	0.000	0.000	0.000

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	84	405	0	0	0	0	-1
normalized size	1	1.00	0.62	3.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.160	2.308	0.000	0.929	0.000	0.000	0.000

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	101	578	0	0	0	0	-1
normalized size	1	1.00	0.60	3.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.256	2.598	0.000	1.147	0.000	0.000	0.000

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	104	301	0	0	0	0	-1
normalized size	1	1.00	0.59	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.169	1.083	0.000	1.081	0.000	0.000	0.000

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	88	273	0	0	0	0	-1
normalized size	1	1.00	0.60	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.146	0.990	0.000	1.805	0.000	0.000	0.000

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	75	240	0	0	0	0	-1
normalized size	1	1.00	0.65	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.098	0.940	0.000	0.988	0.000	0.000	0.000

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	163	0	0	0	0	-1
normalized size	1	1.00	0.67	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.053	0.853	0.000	0.869	0.000	0.000	0.000

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	76	215	0	0	0	0	-1
normalized size	1	1.00	0.68	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.075	1.000	0.000	0.990	0.000	0.000	0.000

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	87	455	0	0	0	0	-1
normalized size	1	1.00	0.62	3.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.076	1.057	0.000	1.051	0.000	0.000	0.000

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	104	578	0	0	0	0	-1
normalized size	1	1.00	0.61	3.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.112	2.731	0.000	1.135	0.000	0.000	0.000

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	104	301	0	0	0	0	-1
normalized size	1	1.00	0.59	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.095	0.931	0.000	0.949	0.000	0.000	0.000

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	91	273	0	0	0	0	-1
normalized size	1	1.00	0.62	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.089	0.921	0.000	0.916	0.000	0.000	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	78	240	0	0	0	0	-1
normalized size	1	1.00	0.67	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.082	1.052	0.000	0.934	0.000	0.000	0.000

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	163	0	0	0	0	-1
normalized size	1	1.00	0.67	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.053	0.995	0.000	1.058	0.000	0.000	0.000

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	76	215	0	0	0	0	-1
normalized size	1	1.00	0.68	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.068	1.108	0.000	0.916	0.000	0.000	0.000

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	87	455	0	0	0	0	-1
normalized size	1	1.00	0.61	3.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.068	1.056	0.000	0.962	0.000	0.000	0.000

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	104	578	0	0	0	0	-1
normalized size	1	1.00	0.60	3.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.079	2.531	0.000	0.569	0.000	0.000	0.000

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	104	578	0	0	0	0	-1
normalized size	1	1.00	0.59	3.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.076	2.507	0.000	0.833	0.000	0.000	0.000

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	81	91	93	252	0	0	105
normalized size	1	1.00	0.47	0.53	0.54	1.47	0.00	0.00	0.61
time (sec)	N/A	0.069	0.174	0.325	0.714	0.844	0.000	0.000	2.297

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	69	74	68	230	0	0	92
normalized size	1	1.00	0.51	0.54	0.50	1.69	0.00	0.00	0.68
time (sec)	N/A	0.055	0.129	0.261	0.728	1.316	0.000	0.000	1.245

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	57	55	40	204	99	0	79
normalized size	1	1.00	0.58	0.56	0.41	2.08	1.01	0.00	0.81
time (sec)	N/A	0.022	0.116	0.211	0.652	0.682	40.283	0.000	0.822

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	39	40	181	46	0	35
normalized size	1	1.00	0.71	0.66	0.68	3.07	0.78	0.00	0.59
time (sec)	N/A	0.012	0.053	0.180	0.602	0.941	12.093	0.000	0.291

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	40	54	92	210	0	0	-1
normalized size	1	1.00	0.67	0.90	1.53	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.035	0.161	0.587	1.859	0.000	0.000	0.000

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	59	120	205	0	0	-1
normalized size	1	1.00	0.74	0.87	1.76	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.051	0.171	0.651	0.963	0.000	0.000	0.000

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	65	120	716	225	0	0	-1
normalized size	1	1.00	0.61	1.12	6.69	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.113	0.227	0.709	1.018	0.000	0.000	0.000

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	76	139	957	253	0	0	-1
normalized size	1	1.00	0.52	0.96	6.60	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.318	0.236	0.716	1.069	0.000	0.000	0.000

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	81	91	100	261	0	0	106
normalized size	1	1.00	0.46	0.51	0.56	1.47	0.00	0.00	0.60
time (sec)	N/A	0.069	0.163	0.263	0.748	1.134	0.000	0.000	1.910

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	69	74	74	237	0	0	93
normalized size	1	1.00	0.49	0.53	0.53	1.69	0.00	0.00	0.66
time (sec)	N/A	0.056	0.136	0.233	1.071	1.020	0.000	0.000	1.226

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	58	55	43	209	0	0	50
normalized size	1	1.00	0.57	0.54	0.43	2.07	0.00	0.00	0.50
time (sec)	N/A	0.023	0.045	0.190	1.014	1.987	0.000	0.000	0.517

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	39	40	184	0	0	36
normalized size	1	1.00	0.69	0.64	0.66	3.02	0.00	0.00	0.59
time (sec)	N/A	0.013	0.069	0.183	1.144	0.820	0.000	0.000	0.854

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	40	54	95	212	0	0	-1
normalized size	1	1.00	0.65	0.87	1.53	3.42	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.045	0.150	1.254	1.096	0.000	0.000	0.000

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	50	59	123	208	0	0	-1
normalized size	1	1.00	0.71	0.84	1.76	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.056	0.155	1.130	1.216	0.000	0.000	0.000

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	65	120	747	232	0	0	-1
normalized size	1	1.00	0.59	1.09	6.79	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.095	0.171	1.271	1.095	0.000	0.000	0.000

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	77	139	992	260	0	0	-1
normalized size	1	1.00	0.52	0.93	6.66	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.045	0.203	1.132	1.036	0.000	0.000	0.000

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	81	91	110	279	0	0	108
normalized size	1	1.00	0.43	0.49	0.59	1.49	0.00	0.00	0.58
time (sec)	N/A	0.072	0.198	0.255	0.655	1.025	0.000	0.000	2.192

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	69	74	82	251	0	0	64
normalized size	1	1.00	0.47	0.50	0.55	1.70	0.00	0.00	0.43
time (sec)	N/A	0.056	0.157	0.218	0.664	1.009	0.000	0.000	0.713

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	57	55	47	219	0	0	52
normalized size	1	1.00	0.53	0.51	0.44	2.05	0.00	0.00	0.49
time (sec)	N/A	0.024	0.147	0.166	0.646	2.472	0.000	0.000	1.045

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	39	40	190	0	0	38
normalized size	1	1.00	0.65	0.60	0.62	2.92	0.00	0.00	0.58
time (sec)	N/A	0.014	0.083	0.141	0.583	1.969	0.000	0.000	1.041

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	40	54	99	216	0	0	-1
normalized size	1	1.00	0.61	0.82	1.50	3.27	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.065	0.188	0.589	1.694	0.000	0.000	0.000

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	59	127	214	0	0	-1
normalized size	1	1.00	0.68	0.80	1.72	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.092	0.160	0.647	1.428	0.000	0.000	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	65	121	803	242	0	0	-1
normalized size	1	1.00	0.56	1.04	6.92	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.153	0.181	0.696	0.939	0.000	0.000	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	76	139	1060	274	0	0	-1
normalized size	1	1.00	0.48	0.89	6.75	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.195	0.211	0.701	1.034	0.000	0.000	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	69	74	68	236	0	0	95
normalized size	1	1.00	0.51	0.54	0.50	1.74	0.00	0.00	0.70
time (sec)	N/A	0.056	0.110	0.264	0.669	2.003	0.000	0.000	1.822

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	57	55	40	210	0	0	82
normalized size	1	1.00	0.58	0.56	0.41	2.14	0.00	0.00	0.84
time (sec)	N/A	0.024	0.092	0.227	0.664	0.945	0.000	0.000	1.369

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	39	40	187	46	0	61
normalized size	1	1.00	0.71	0.66	0.68	3.17	0.78	0.00	1.03
time (sec)	N/A	0.013	0.046	0.184	0.594	0.933	12.676	0.000	0.537

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	40	54	92	215	0	0	-1
normalized size	1	1.00	0.67	0.90	1.53	3.58	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.037	0.183	0.592	1.107	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	59	125	211	0	0	-1
normalized size	1	1.00	0.74	0.87	1.84	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.051	0.173	0.672	1.058	0.000	0.000	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	65	120	722	231	0	0	-1
normalized size	1	1.00	0.61	1.12	6.75	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.074	0.196	0.710	0.878	0.000	0.000	0.000

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	76	139	957	259	0	0	-1
normalized size	1	1.00	0.52	0.96	6.60	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.090	0.215	0.705	0.851	0.000	0.000	0.000

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	69	74	68	236	0	0	95
normalized size	1	1.00	0.47	0.50	0.46	1.59	0.00	0.00	0.64
time (sec)	N/A	0.056	0.086	0.216	0.694	1.177	0.000	0.000	1.566

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	57	55	40	210	0	0	82
normalized size	1	1.00	0.53	0.51	0.37	1.96	0.00	0.00	0.77
time (sec)	N/A	0.023	0.103	0.174	0.678	1.161	0.000	0.000	0.688

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	39	40	187	0	0	61
normalized size	1	1.00	0.65	0.60	0.62	2.88	0.00	0.00	0.94
time (sec)	N/A	0.014	0.048	0.166	0.598	1.101	0.000	0.000	1.021

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	40	54	92	215	0	0	-1
normalized size	1	1.00	0.61	0.82	1.39	3.26	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.047	0.158	0.601	1.189	0.000	0.000	0.000

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	59	133	211	0	0	-1
normalized size	1	1.00	0.68	0.80	1.80	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.063	0.190	0.674	0.732	0.000	0.000	0.000

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	65	121	739	231	0	0	-1
normalized size	1	1.00	0.56	1.04	6.37	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.072	0.175	0.703	0.885	0.000	0.000	0.000

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	76	139	983	259	0	0	-1
normalized size	1	1.00	0.48	0.89	6.26	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.106	0.199	0.716	0.899	0.000	0.000	0.000

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	72	74	68	236	0	0	95
normalized size	1	1.00	0.49	0.50	0.46	1.59	0.00	0.00	0.64
time (sec)	N/A	0.057	0.074	0.214	0.669	0.810	0.000	0.000	1.559

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	60	55	40	210	0	0	82
normalized size	1	1.00	0.56	0.51	0.37	1.96	0.00	0.00	0.77
time (sec)	N/A	0.024	0.060	0.176	0.676	1.025	0.000	0.000	0.700

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	39	40	187	0	0	61
normalized size	1	1.00	0.69	0.60	0.62	2.88	0.00	0.00	0.94
time (sec)	N/A	0.014	0.051	0.147	0.598	1.534	0.000	0.000	0.481

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	43	54	92	215	0	0	-1
normalized size	1	1.00	0.65	0.82	1.39	3.26	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.037	0.144	0.604	0.872	0.000	0.000	0.000

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	59	133	211	0	0	-1
normalized size	1	1.00	0.68	0.80	1.80	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.054	0.167	0.657	0.925	0.000	0.000	0.000

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	65	120	757	231	0	0	-1
normalized size	1	1.00	0.56	1.03	6.53	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.078	0.199	0.693	0.887	0.000	0.000	0.000

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	76	139	1033	259	0	0	-1
normalized size	1	1.00	0.48	0.89	6.58	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.106	0.201	0.705	0.944	0.000	0.000	0.000

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.180	0.395	0.000	0.990	0.000	0.000	0.000

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	89	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.147	0.291	0.000	0.616	0.000	0.000	0.000

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	86	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.087	0.158	0.000	0.557	0.000	0.000	0.000

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	86	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.102	0.275	0.000	0.701	0.000	0.000	0.000

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	86	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.131	0.260	0.000	0.644	0.000	0.000	0.000

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	94	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.094	0.255	0.000	1.672	0.000	0.000	0.000

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.214	0.362	0.000	0.955	0.000	0.000	0.000

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	89	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.195	0.299	0.000	0.629	0.000	0.000	0.000

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	86	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.049	0.143	0.000	0.779	0.000	0.000	0.000

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	87	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.014	0.229	0.000	0.809	0.000	0.000	0.000

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.026	0.243	0.000	0.674	0.000	0.000	0.000

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.115	0.271	0.000	0.770	0.000	0.000	0.000

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.182	0.330	0.000	0.908	0.000	0.000	0.000

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	89	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.023	0.252	0.000	0.915	0.000	0.000	0.000

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.016	0.121	0.000	0.811	0.000	0.000	0.000

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	85	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.072	0.200	0.000	0.771	0.000	0.000	0.000

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	89	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.156	0.234	0.000	0.704	0.000	0.000	0.000

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	89	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.150	0.282	0.000	1.093	0.000	0.000	0.000

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.183	0.326	0.000	0.651	0.000	0.000	0.000

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	89	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.092	0.321	0.000	0.616	0.000	0.000	0.000

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.126	0.107	0.000	0.776	0.000	0.000	0.000

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	86	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.153	0.190	0.000	0.820	0.000	0.000	0.000

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	89	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.203	0.224	0.000	0.576	0.000	0.000	0.000

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	89	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.196	0.251	0.000	0.652	0.000	0.000	0.000

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	130	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.268	1.685	0.000	0.658	0.000	0.000	0.000

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.314	1.636	0.000	0.691	0.000	0.000	0.000

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.252	1.178	0.000	0.766	0.000	0.000	0.000

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.177	1.163	0.000	0.646	0.000	0.000	0.000

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	109	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.199	0.894	0.000	0.787	0.000	0.000	0.000

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	109	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.192	1.085	0.000	0.936	0.000	0.000	0.000

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	118	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.180	1.370	0.000	0.748	0.000	0.000	0.000

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.179	0.817	0.000	0.772	0.000	0.000	0.000

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.452	0.335	0.000	0.903	0.000	0.000	0.000

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.358	0.279	0.000	0.846	0.000	0.000	0.000

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.259	0.257	0.000	0.786	0.000	0.000	0.000

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.236	0.211	0.000	0.670	0.000	0.000	0.000

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	133	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.267	0.233	0.000	0.981	0.000	0.000	0.000

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.232	0.205	0.000	0.670	0.000	0.000	0.000

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.237	0.199	0.000	0.986	0.000	0.000	0.000

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.236	0.214	0.000	0.502	0.000	0.000	0.000

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	140	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.515	0.243	0.000	0.747	0.000	0.000	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.321	0.243	0.000	0.902	0.000	0.000	0.000

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.307	0.220	0.000	0.825	0.000	0.000	0.000

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.320	0.188	0.000	0.875	0.000	0.000	0.000

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.310	0.186	0.000	0.689	0.000	0.000	0.000

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	140	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.344	0.198	0.000	0.871	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [503] had the largest ratio of [.5000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	4	1.00	19	0.210
2	A	6	4	1.00	19	0.210
3	A	6	4	1.00	19	0.210
4	A	5	4	1.00	19	0.210
5	A	1	1	1.00	17	0.059
6	A	2	1	1.00	10	0.100
7	A	2	2	1.00	17	0.118
8	A	4	4	1.00	19	0.210
9	A	5	5	1.00	19	0.263
10	A	5	4	1.00	19	0.210
11	A	6	4	1.00	19	0.210
12	A	6	4	1.00	19	0.210
13	A	11	4	1.00	21	0.190
14	A	9	4	1.00	21	0.190
15	A	9	4	1.00	21	0.190
16	A	2	2	1.21	19	0.105
17	A	1	1	1.00	12	0.083
18	A	3	3	1.00	19	0.158
19	A	5	4	1.00	21	0.190
20	A	7	5	1.00	21	0.238
21	A	8	5	1.00	21	0.238
22	A	9	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	13	4	1.00	21	0.190
24	A	11	4	1.00	21	0.190
25	A	8	6	1.04	19	0.316
26	A	7	5	1.00	12	0.417
27	A	6	5	1.00	19	0.263
28	A	6	5	1.00	21	0.238
29	A	7	5	1.00	21	0.238
30	A	9	5	1.00	21	0.238
31	A	11	5	1.00	21	0.238
32	A	11	4	1.00	21	0.190
33	A	15	4	1.00	21	0.190
34	A	11	6	1.12	19	0.316
35	A	10	5	1.00	12	0.417
36	A	8	6	1.00	19	0.316
37	A	8	6	1.00	21	0.286
38	A	8	6	1.00	21	0.286
39	A	9	5	1.00	21	0.238
40	A	12	5	1.00	21	0.238
41	A	13	5	1.00	21	0.238
42	A	15	4	1.00	21	0.190
43	A	7	5	1.00	21	0.238
44	A	6	5	1.00	21	0.238
45	A	2	2	1.00	21	0.095
46	A	4	4	1.00	21	0.190
47	A	2	2	1.00	19	0.105
48	A	1	1	1.00	12	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
49	A	3	3	1.00	19	0.158
50	A	5	5	1.00	21	0.238
51	A	6	6	1.00	21	0.286
52	A	6	5	1.00	21	0.238
53	A	7	6	1.00	21	0.286
54	A	3	3	1.00	21	0.143
55	A	6	6	1.00	21	0.286
56	A	3	3	1.00	21	0.143
57	A	2	2	1.00	19	0.105
58	A	2	2	1.00	12	0.167
59	A	4	4	1.00	19	0.210
60	A	6	6	1.00	21	0.286
61	A	7	7	1.00	21	0.333
62	A	7	6	1.00	21	0.286
63	A	4	3	1.00	21	0.143
64	A	7	7	1.00	21	0.333
65	A	5	5	1.00	21	0.238
66	A	3	3	1.00	21	0.143
67	A	3	3	1.00	19	0.158
68	A	3	2	1.00	12	0.167
69	A	5	4	1.00	19	0.210
70	A	7	6	1.00	21	0.286
71	A	8	7	1.00	21	0.333
72	A	5	3	1.00	21	0.143
73	A	8	7	1.00	21	0.333
74	A	6	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
75	A	5	5	1.00	21	0.238
76	A	4	4	1.00	21	0.190
77	A	4	3	1.00	19	0.158
78	A	4	2	1.00	12	0.167
79	A	6	4	1.00	19	0.210
80	A	8	6	1.00	21	0.286
81	A	9	7	1.00	21	0.333
82	A	6	3	1.00	21	0.143
83	A	9	7	1.00	21	0.333
84	A	7	6	1.00	21	0.286
85	A	6	6	1.00	21	0.286
86	A	6	6	1.00	21	0.286
87	A	5	4	1.00	21	0.190
88	A	5	3	1.00	19	0.158
89	A	5	2	1.00	12	0.167
90	A	7	4	1.00	19	0.210
91	A	9	6	1.00	21	0.286
92	A	10	7	1.00	21	0.333
93	A	7	6	1.00	21	0.286
94	A	7	7	1.00	21	0.333
95	A	5	4	1.00	23	0.174
96	A	4	4	1.00	23	0.174
97	A	3	3	1.00	23	0.130
98	A	2	2	1.00	21	0.095
99	A	1	1	1.00	14	0.071
100	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
101	A	3	3	1.00	23	0.130
102	A	4	3	1.00	23	0.130
103	A	5	3	1.00	23	0.130
104	A	6	6	1.00	23	0.261
105	A	4	4	1.00	23	0.174
106	A	3	3	1.00	21	0.143
107	A	2	2	1.00	14	0.143
108	A	4	4	1.00	21	0.190
109	A	4	4	1.00	23	0.174
110	A	5	5	1.00	23	0.217
111	A	6	5	1.00	23	0.217
112	A	6	6	1.00	23	0.261
113	A	5	4	1.00	23	0.174
114	A	4	3	1.00	21	0.143
115	A	3	2	1.00	14	0.143
116	A	4	4	1.00	21	0.190
117	A	4	4	1.00	23	0.174
118	A	4	4	1.00	23	0.174
119	A	5	5	1.00	23	0.217
120	A	6	5	1.00	23	0.217
121	A	4	2	1.00	14	0.143
122	A	7	7	1.00	23	0.304
123	A	6	6	1.00	23	0.261
124	A	4	4	1.00	23	0.174
125	A	3	3	1.00	21	0.143
126	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	A	5	4	1.00	21	0.190
128	A	6	5	1.00	23	0.217
129	A	7	6	1.00	23	0.261
130	A	8	6	1.00	23	0.261
131	A	7	7	1.00	23	0.304
132	A	6	6	1.00	23	0.261
133	A	4	4	1.00	23	0.174
134	A	3	3	1.00	21	0.143
135	A	3	3	1.00	14	0.214
136	A	6	5	1.00	21	0.238
137	A	7	6	1.00	23	0.261
138	A	8	6	1.00	23	0.261
139	A	7	7	1.00	23	0.304
140	A	6	6	1.00	23	0.261
141	A	4	4	1.00	23	0.174
142	A	4	4	1.00	21	0.190
143	A	4	3	1.00	14	0.214
144	A	7	6	1.00	21	0.286
145	A	8	7	1.00	23	0.304
146	A	6	4	1.00	21	0.190
147	A	5	4	1.00	21	0.190
148	A	4	4	1.00	21	0.190
149	A	3	3	1.00	21	0.143
150	A	4	4	1.00	21	0.190
151	A	5	4	1.00	21	0.190
152	A	6	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	10	4	1.00	23	0.174
154	A	9	4	1.00	23	0.174
155	A	7	4	1.00	23	0.174
156	A	6	4	1.00	23	0.174
157	A	6	4	1.00	23	0.174
158	A	7	4	1.00	23	0.174
159	A	9	4	1.00	23	0.174
160	A	12	4	1.00	23	0.174
161	A	10	4	1.00	23	0.174
162	A	8	4	1.00	23	0.174
163	A	8	5	1.00	23	0.217
164	A	8	4	1.00	23	0.174
165	A	10	4	1.00	23	0.174
166	A	12	4	1.00	23	0.174
167	A	16	4	1.00	23	0.174
168	A	13	4	1.00	23	0.174
169	A	11	4	1.00	23	0.174
170	A	10	5	1.00	23	0.217
171	A	10	5	1.00	23	0.217
172	A	11	4	1.00	23	0.174
173	A	13	4	1.00	23	0.174
174	A	6	5	1.00	23	0.217
175	A	5	5	1.00	23	0.217
176	A	4	4	1.00	23	0.174
177	A	4	4	1.00	23	0.174
178	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
179	A	5	5	1.00	23	0.217
180	A	6	5	1.00	23	0.217
181	A	7	6	1.00	23	0.261
182	A	6	6	1.00	23	0.261
183	A	5	5	1.00	23	0.217
184	A	5	5	1.00	23	0.217
185	A	3	3	1.00	23	0.130
186	A	5	5	1.00	23	0.217
187	A	6	6	1.00	23	0.261
188	A	7	6	1.00	23	0.261
189	A	8	6	1.00	23	0.261
190	A	7	6	1.00	23	0.261
191	A	6	5	1.00	23	0.217
192	A	6	6	1.00	23	0.261
193	A	6	5	1.00	23	0.217
194	A	6	5	1.00	23	0.217
195	A	6	5	1.00	23	0.217
196	A	7	6	1.00	23	0.261
197	A	8	6	1.00	23	0.261
198	A	5	3	1.00	25	0.120
199	A	4	3	1.00	25	0.120
200	A	3	3	1.00	25	0.120
201	A	2	2	1.00	25	0.080
202	A	1	1	1.00	25	0.040
203	A	2	2	1.00	25	0.080
204	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
205	A	4	2	1.00	25	0.080
206	A	6	5	1.00	25	0.200
207	A	5	5	1.00	25	0.200
208	A	4	4	1.00	25	0.160
209	A	4	4	1.00	25	0.160
210	A	3	3	1.00	25	0.120
211	A	4	4	1.00	25	0.160
212	A	5	4	1.00	25	0.160
213	A	6	5	1.00	25	0.200
214	A	5	5	1.00	25	0.200
215	A	4	4	1.00	25	0.160
216	A	4	4	1.00	25	0.160
217	A	4	4	1.00	25	0.160
218	A	3	3	1.00	25	0.120
219	A	4	4	1.00	25	0.160
220	A	5	4	1.00	25	0.160
221	A	2	2	1.00	25	0.080
222	A	2	2	1.00	25	0.080
223	A	2	2	1.00	28	0.071
224	A	7	7	1.00	25	0.280
225	A	6	6	1.00	25	0.240
226	A	5	5	1.00	25	0.200
227	A	2	2	1.00	25	0.080
228	A	4	4	1.00	25	0.160
229	A	5	5	1.00	25	0.200
230	A	6	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	7	6	1.00	23	0.261
232	A	6	5	1.00	23	0.217
233	A	5	4	1.00	23	0.174
234	A	2	2	1.00	23	0.087
235	A	3	3	1.00	23	0.130
236	A	5	5	1.00	23	0.217
237	A	6	5	1.00	23	0.217
238	A	7	7	1.00	25	0.280
239	A	6	6	1.00	25	0.240
240	A	4	4	1.00	25	0.160
241	A	4	4	1.00	25	0.160
242	A	5	5	1.00	25	0.200
243	A	6	5	1.00	25	0.200
244	A	8	8	1.00	25	0.320
245	A	7	7	1.00	25	0.280
246	A	5	5	1.00	25	0.200
247	A	5	5	1.00	25	0.200
248	A	5	5	1.00	25	0.200
249	A	6	6	1.00	25	0.240
250	A	7	6	1.00	25	0.240
251	A	9	8	1.00	25	0.320
252	A	8	7	1.00	25	0.280
253	A	6	6	1.00	25	0.240
254	A	6	5	1.00	25	0.200
255	A	6	5	1.00	25	0.200
256	A	6	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
257	A	7	6	1.00	25	0.240
258	A	8	6	1.00	25	0.240
259	A	7	6	1.00	25	0.240
260	A	7	6	1.00	25	0.240
261	A	2	2	1.00	15	0.133
262	A	2	2	1.00	17	0.118
263	A	4	3	1.00	26	0.115
264	A	3	3	1.00	26	0.115
265	A	2	2	1.00	26	0.077
266	A	1	1	1.00	26	0.038
267	A	2	2	1.00	26	0.077
268	A	3	2	1.00	26	0.077
269	A	4	3	1.00	25	0.120
270	A	3	3	1.00	25	0.120
271	A	2	2	1.00	25	0.080
272	A	1	1	1.00	25	0.040
273	A	2	2	1.00	25	0.080
274	A	3	2	1.00	25	0.080
275	A	7	7	1.00	26	0.269
276	A	6	6	1.00	26	0.231
277	A	5	5	1.00	26	0.192
278	A	2	2	1.00	26	0.077
279	A	4	4	1.00	26	0.154
280	A	5	5	1.00	26	0.192
281	A	6	5	1.00	26	0.192
282	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
283	A	6	6	1.00	25	0.240
284	A	5	5	1.00	25	0.200
285	A	2	2	1.00	25	0.080
286	A	3	3	1.00	25	0.120
287	A	5	5	1.00	25	0.200
288	A	3	3	1.00	25	0.120
289	A	3	3	1.00	25	0.120
290	A	3	3	1.00	25	0.120
291	A	9	6	1.00	21	0.286
292	A	8	6	1.00	21	0.286
293	A	7	6	1.00	21	0.286
294	A	6	5	1.00	21	0.238
295	A	7	6	1.00	21	0.286
296	A	8	6	1.00	21	0.286
297	A	9	6	1.00	21	0.286
298	A	9	7	1.00	23	0.304
299	A	8	7	1.00	23	0.304
300	A	5	5	1.00	23	0.217
301	A	7	6	1.00	23	0.261
302	A	8	7	1.00	23	0.304
303	A	9	7	1.00	23	0.304
304	A	17	6	1.00	23	0.261
305	A	15	6	1.00	23	0.261
306	A	13	6	1.00	23	0.261
307	A	13	7	1.00	23	0.304
308	A	13	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	15	6	1.00	23	0.261
310	A	17	6	1.00	23	0.261
311	A	19	6	1.00	23	0.261
312	A	17	6	1.00	23	0.261
313	A	16	7	1.00	23	0.304
314	A	16	7	1.00	23	0.304
315	A	17	6	1.00	23	0.261
316	A	19	6	1.00	23	0.261
317	A	9	7	1.00	23	0.304
318	A	8	7	1.00	23	0.304
319	A	7	6	1.00	23	0.261
320	A	7	6	1.00	23	0.261
321	A	7	6	1.00	23	0.261
322	A	8	7	1.00	23	0.304
323	A	9	7	1.00	23	0.304
324	A	10	8	1.00	23	0.348
325	A	9	8	1.00	23	0.348
326	A	8	7	1.00	23	0.304
327	A	5	5	1.00	23	0.217
328	A	8	7	1.00	23	0.304
329	A	8	7	1.00	23	0.304
330	A	9	8	1.00	23	0.348
331	A	10	8	1.00	23	0.348
332	A	10	8	1.00	23	0.348
333	A	9	7	1.00	23	0.304
334	A	9	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
335	A	9	8	1.00	23	0.348
336	A	9	8	1.00	23	0.348
337	A	9	7	1.00	23	0.304
338	A	10	8	1.00	23	0.348
339	A	5	3	1.00	25	0.120
340	A	4	3	1.00	25	0.120
341	A	3	3	1.00	25	0.120
342	A	2	2	1.00	25	0.080
343	A	3	3	1.00	25	0.120
344	A	4	4	1.00	25	0.160
345	A	5	4	1.00	25	0.160
346	A	6	5	1.00	25	0.200
347	A	5	5	1.00	25	0.200
348	A	4	4	1.00	25	0.160
349	A	5	5	1.00	25	0.200
350	A	5	5	1.00	25	0.200
351	A	6	6	1.00	25	0.240
352	A	7	6	1.00	25	0.240
353	A	6	5	1.00	25	0.200
354	A	5	5	1.00	25	0.200
355	A	4	4	1.00	25	0.160
356	A	5	5	1.00	25	0.200
357	A	5	5	1.00	25	0.200
358	A	5	5	1.00	25	0.200
359	A	6	6	1.00	25	0.240
360	A	7	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	7	6	1.00	23	0.261
362	A	6	6	1.00	23	0.261
363	A	4	4	1.00	23	0.174
364	A	3	3	1.00	23	0.130
365	A	6	5	1.00	23	0.217
366	A	7	6	1.00	23	0.261
367	A	7	6	1.00	25	0.240
368	A	6	6	1.00	25	0.240
369	A	5	5	1.00	25	0.200
370	A	3	3	1.36	25	0.120
371	A	6	6	1.29	25	0.240
372	A	7	7	1.00	25	0.280
373	A	7	6	1.00	25	0.240
374	A	6	6	1.00	25	0.240
375	A	5	5	1.00	25	0.200
376	A	5	5	1.00	25	0.200
377	A	7	7	1.00	25	0.280
378	A	8	8	1.00	25	0.320
379	A	8	7	1.00	25	0.280
380	A	7	7	1.00	25	0.280
381	A	6	6	1.00	25	0.240
382	A	6	6	1.00	25	0.240
383	A	6	6	1.00	25	0.240
384	A	8	8	1.00	25	0.320
385	A	9	9	1.00	25	0.360
386	A	9	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
387	A	8	7	1.00	25	0.280
388	A	7	6	1.00	25	0.240
389	A	7	6	1.00	25	0.240
390	A	7	6	1.00	25	0.240
391	A	7	7	1.00	25	0.280
392	A	9	8	1.00	25	0.320
393	A	10	9	1.00	25	0.360
394	A	8	7	1.00	25	0.280
395	A	8	7	1.00	25	0.280
396	A	3	3	1.00	25	0.120
397	A	7	6	1.00	21	0.286
398	A	6	5	1.00	21	0.238
399	A	4	3	1.00	21	0.143
400	A	3	2	1.00	19	0.105
401	A	4	3	1.00	21	0.143
402	A	5	4	1.00	21	0.190
403	A	8	4	1.00	19	0.210
404	A	7	4	1.00	19	0.210
405	A	7	4	1.00	19	0.210
406	A	6	4	1.00	19	0.210
407	A	6	4	1.00	19	0.210
408	A	5	4	1.00	19	0.210
409	A	1	1	1.00	17	0.059
410	A	2	1	1.00	10	0.100
411	A	2	2	1.00	17	0.118
412	A	4	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
413	A	5	5	1.00	19	0.263
414	A	5	4	1.00	19	0.210
415	A	6	4	1.00	19	0.210
416	A	6	4	1.00	19	0.210
417	A	7	5	1.00	21	0.238
418	A	7	5	1.00	21	0.238
419	A	6	5	1.00	21	0.238
420	A	2	2	1.00	19	0.105
421	A	1	1	1.00	12	0.083
422	A	3	3	1.00	19	0.158
423	A	4	4	1.00	21	0.190
424	A	5	5	1.00	21	0.238
425	A	6	6	1.00	21	0.286
426	A	6	5	1.00	21	0.238
427	A	7	5	1.00	21	0.238
428	A	8	6	1.14	21	0.286
429	A	4	3	1.00	21	0.143
430	A	3	2	1.00	19	0.105
431	A	2	2	1.18	12	0.167
432	A	4	4	1.00	19	0.210
433	A	4	4	1.00	21	0.190
434	A	4	4	1.00	21	0.190
435	A	6	6	1.00	21	0.286
436	A	7	7	1.00	21	0.333
437	A	7	6	1.00	21	0.286
438	A	9	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
439	A	5	3	1.00	21	0.143
440	A	4	2	1.00	19	0.105
441	A	3	3	1.00	12	0.250
442	A	5	5	1.00	19	0.263
443	A	5	5	1.00	21	0.238
444	A	5	5	1.00	21	0.238
445	A	5	5	1.00	21	0.238
446	A	7	7	1.00	21	0.333
447	A	8	8	1.00	21	0.381
448	A	8	7	1.00	21	0.333
449	A	7	6	1.00	21	0.286
450	A	6	6	1.00	21	0.286
451	A	5	5	1.00	21	0.238
452	A	5	5	1.00	21	0.238
453	A	3	3	1.00	19	0.158
454	A	2	2	1.00	12	0.167
455	A	4	4	1.00	19	0.210
456	A	6	6	1.00	21	0.286
457	A	6	6	1.00	21	0.286
458	A	7	6	1.00	21	0.286
459	A	7	6	1.00	21	0.286
460	A	6	6	1.28	21	0.286
461	A	5	5	1.00	21	0.238
462	A	4	4	1.00	21	0.190
463	A	4	4	1.00	19	0.210
464	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
465	A	5	5	1.00	19	0.263
466	A	6	6	1.00	21	0.286
467	A	7	6	1.00	21	0.286
468	A	8	6	1.00	21	0.286
469	A	7	7	1.00	21	0.333
470	A	6	6	1.00	21	0.286
471	A	5	5	1.00	21	0.238
472	A	5	5	1.00	21	0.238
473	A	5	4	1.00	19	0.210
474	A	5	5	1.00	12	0.417
475	A	6	6	1.00	19	0.316
476	A	7	6	1.00	21	0.286
477	A	8	6	1.00	21	0.286
478	A	7	7	1.00	21	0.333
479	A	6	6	1.00	21	0.286
480	A	6	6	1.00	21	0.286
481	A	6	5	1.00	21	0.238
482	A	6	4	1.00	19	0.210
483	A	6	5	1.00	12	0.417
484	A	7	6	1.00	19	0.316
485	A	8	6	1.00	21	0.286
486	A	8	8	1.00	23	0.348
487	A	7	7	1.00	23	0.304
488	A	6	6	1.00	21	0.286
489	A	2	2	1.00	14	0.143
490	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
491	A	9	9	1.00	23	0.391
492	A	10	10	1.00	23	0.435
493	A	9	8	1.00	23	0.348
494	A	8	7	1.00	23	0.304
495	A	7	6	1.00	21	0.286
496	A	6	6	1.00	14	0.429
497	A	8	8	1.00	21	0.381
498	A	9	9	1.00	23	0.391
499	A	10	10	1.00	23	0.435
500	A	10	8	1.00	23	0.348
501	A	9	7	1.00	23	0.304
502	A	8	6	1.00	21	0.286
503	A	7	7	1.00	14	0.500
504	A	9	9	1.00	21	0.429
505	A	9	9	1.00	23	0.391
506	A	10	10	1.00	23	0.435
507	A	11	10	1.00	23	0.435
508	A	8	7	1.00	14	0.500
509	A	6	6	1.00	23	0.261
510	A	5	5	1.00	23	0.217
511	A	4	4	1.00	21	0.190
512	A	1	1	1.00	14	0.071
513	A	3	3	1.00	21	0.143
514	A	6	6	1.00	23	0.261
515	A	7	7	1.00	23	0.304
516	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
517	A	5	5	1.00	23	0.217
518	A	4	4	1.00	21	0.190
519	A	1	1	1.00	14	0.071
520	A	3	3	1.00	21	0.143
521	A	6	6	1.00	23	0.261
522	A	7	7	1.00	23	0.304
523	A	7	7	1.00	23	0.304
524	A	6	6	1.00	23	0.261
525	A	5	5	1.00	21	0.238
526	A	2	2	1.00	14	0.143
527	A	2	2	1.00	21	0.095
528	A	9	9	1.00	23	0.391
529	A	10	10	1.00	23	0.435
530	A	8	8	1.00	23	0.348
531	A	7	7	1.00	23	0.304
532	A	6	6	1.00	23	0.261
533	A	6	6	1.00	21	0.286
534	A	4	4	1.00	14	0.286
535	A	7	7	1.00	21	0.333
536	A	10	10	1.00	23	0.435
537	A	11	10	1.00	23	0.435
538	A	9	9	1.00	23	0.391
539	A	8	8	1.00	23	0.348
540	A	7	7	1.00	23	0.304
541	A	7	7	1.00	23	0.304
542	A	7	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
543	A	7	7	1.00	14	0.500
544	A	10	10	1.00	21	0.476
545	A	11	11	1.00	23	0.478
546	A	8	7	1.00	14	0.500
547	A	5	5	1.00	23	0.217
548	A	4	4	1.00	23	0.174
549	A	3	3	1.00	21	0.143
550	A	1	1	1.00	14	0.071
551	A	1	1	1.00	21	0.048
552	A	6	6	1.00	23	0.261
553	A	7	7	1.00	23	0.304
554	A	5	5	1.00	23	0.217
555	A	4	4	1.00	23	0.174
556	A	3	3	1.00	21	0.143
557	A	1	1	1.00	14	0.071
558	A	1	1	1.00	21	0.048
559	A	6	6	1.00	23	0.261
560	A	7	7	1.00	23	0.304
561	A	6	4	1.00	21	0.190
562	A	5	4	1.00	21	0.190
563	A	4	4	1.00	21	0.190
564	A	3	3	1.00	21	0.143
565	A	4	4	1.00	21	0.190
566	A	5	4	1.00	21	0.190
567	A	6	4	1.00	21	0.190
568	A	7	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
569	A	6	5	1.00	23	0.217
570	A	5	5	1.00	23	0.217
571	A	4	4	1.00	23	0.174
572	A	4	4	1.00	23	0.174
573	A	5	5	1.00	23	0.217
574	A	6	5	1.00	23	0.217
575	A	7	6	1.00	23	0.261
576	A	6	6	1.00	23	0.261
577	A	5	5	1.00	23	0.217
578	A	5	5	1.00	23	0.217
579	A	5	5	1.00	23	0.217
580	A	6	6	1.00	23	0.261
581	A	7	6	1.00	23	0.261
582	A	6	6	1.00	23	0.261
583	A	5	5	1.00	23	0.217
584	A	3	3	1.00	23	0.130
585	A	1	1	1.00	23	0.043
586	A	5	5	1.00	23	0.217
587	A	7	7	1.00	23	0.304
588	A	7	7	1.00	23	0.304
589	A	6	6	1.00	23	0.261
590	A	6	6	1.00	23	0.261
591	A	6	6	1.00	23	0.261
592	A	6	6	1.00	23	0.261
593	A	7	7	1.00	23	0.304
594	A	8	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
595	A	8	8	1.00	23	0.348
596	A	7	7	1.00	23	0.304
597	A	7	7	1.00	23	0.304
598	A	7	7	1.00	23	0.304
599	A	7	7	1.00	23	0.304
600	A	7	7	1.00	23	0.304
601	A	8	7	1.00	23	0.304
602	A	9	7	1.00	23	0.304
603	A	7	7	1.00	25	0.280
604	A	7	7	1.00	25	0.280
605	A	1	1	1.00	25	0.040
606	A	3	3	1.00	25	0.120
607	A	4	4	1.00	25	0.160
608	A	5	5	1.00	25	0.200
609	A	6	5	1.00	25	0.200
610	A	8	8	1.00	25	0.320
611	A	8	8	1.00	25	0.320
612	A	6	6	1.00	25	0.240
613	A	5	5	1.00	25	0.200
614	A	4	4	1.00	25	0.160
615	A	5	5	1.00	25	0.200
616	A	6	5	1.00	25	0.200
617	A	7	5	1.00	25	0.200
618	A	8	8	1.00	25	0.320
619	A	7	7	1.00	25	0.280
620	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
621	A	6	6	1.00	25	0.240
622	A	5	5	1.00	25	0.200
623	A	6	5	1.00	25	0.200
624	A	7	5	1.00	25	0.200
625	A	8	5	1.00	25	0.200
626	A	8	8	1.09	25	0.320
627	A	1	1	1.00	25	0.040
628	A	1	1	1.00	25	0.040
629	A	3	3	1.00	25	0.120
630	A	4	4	1.00	25	0.160
631	A	7	7	1.00	25	0.280
632	A	6	6	1.00	25	0.240
633	A	4	4	1.00	25	0.160
634	A	4	4	1.00	25	0.160
635	A	4	4	1.00	25	0.160
636	A	5	5	1.00	25	0.200
637	A	6	5	1.00	25	0.200
638	A	7	7	1.00	25	0.280
639	A	5	5	1.00	25	0.200
640	A	5	5	1.00	25	0.200
641	A	5	5	1.00	25	0.200
642	A	5	5	1.00	25	0.200
643	A	6	5	1.00	25	0.200
644	A	1	1	1.00	25	0.040
645	A	1	1	1.00	25	0.040
646	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
647	A	2	2	1.00	25	0.080
648	A	1	1	1.00	25	0.040
649	A	1	1	1.00	25	0.040
650	A	2	2	1.00	25	0.080
651	A	2	2	1.00	25	0.080
652	A	2	2	1.00	27	0.074
653	A	2	2	1.00	27	0.074
654	A	1	1	1.00	27	0.037
655	A	1	1	1.00	27	0.037
656	A	2	2	1.00	27	0.074
657	A	2	2	1.00	27	0.074
658	A	1	1	1.00	27	0.037
659	A	1	1	1.00	27	0.037
660	A	1	1	1.00	25	0.040
661	A	1	1	1.00	25	0.040
662	A	2	2	1.00	25	0.080
663	A	2	2	1.00	25	0.080
664	A	1	1	1.00	25	0.040
665	A	1	1	1.00	25	0.040
666	A	2	2	1.00	25	0.080
667	A	2	2	1.00	25	0.080
668	A	2	2	1.00	27	0.074
669	A	2	2	1.00	27	0.074
670	A	1	1	1.00	27	0.037
671	A	1	1	1.00	27	0.037
672	A	2	2	1.00	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
673	A	2	2	1.00	27	0.074
674	A	1	1	1.00	27	0.037
675	A	1	1	1.00	27	0.037
676	A	5	3	1.00	23	0.130
677	A	5	3	1.00	23	0.130
678	A	5	3	1.00	23	0.130
679	A	5	3	1.00	23	0.130
680	A	0	0	0.00	0	0.000
681	A	0	0	0.00	0	0.000
682	A	0	0	0.00	0	0.000
683	A	0	0	0.00	0	0.000
684	A	0	0	0.00	0	0.000
685	A	0	0	0.00	0	0.000
686	A	0	0	0.00	0	0.000
687	A	0	0	0.00	0	0.000
688	A	0	0	0.00	0	0.000
689	A	0	0	0.00	0	0.000
690	A	9	6	1.00	21	0.286
691	A	8	6	1.00	21	0.286
692	A	7	6	1.00	21	0.286
693	A	6	5	1.00	21	0.238
694	A	7	6	1.00	21	0.286
695	A	8	6	1.00	21	0.286
696	A	9	6	1.00	21	0.286
697	A	10	7	1.00	23	0.304
698	A	9	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
699	A	8	7	1.00	23	0.304
700	A	7	6	1.00	23	0.261
701	A	7	6	1.00	23	0.261
702	A	8	7	1.00	23	0.304
703	A	9	7	1.00	23	0.304
704	A	10	7	1.00	23	0.304
705	A	10	8	1.00	23	0.348
706	A	9	8	1.00	23	0.348
707	A	8	7	1.00	23	0.304
708	A	8	7	1.00	23	0.304
709	A	8	7	1.00	23	0.304
710	A	9	8	1.00	23	0.348
711	A	10	8	1.00	23	0.348
712	A	11	10	1.00	23	0.435
713	A	7	7	1.00	23	0.304
714	A	3	3	1.00	23	0.130
715	A	5	5	1.00	23	0.217
716	A	9	8	1.00	23	0.348
717	A	10	9	1.00	23	0.391
718	A	12	10	1.00	23	0.435
719	A	11	10	1.00	23	0.435
720	A	10	9	1.00	23	0.391
721	A	10	9	1.00	23	0.391
722	A	10	9	1.00	23	0.391
723	A	10	9	1.00	23	0.391
724	A	13	11	1.00	23	0.478

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
725	A	12	11	1.00	23	0.478
726	A	11	10	1.00	23	0.435
727	A	11	10	1.00	23	0.435
728	A	11	10	1.00	23	0.435
729	A	11	10	1.00	23	0.435
730	A	6	6	1.00	25	0.240
731	A	5	5	1.00	25	0.200
732	A	4	4	1.00	25	0.160
733	A	2	2	1.00	25	0.080
734	A	8	8	1.00	25	0.320
735	A	8	8	1.00	25	0.320
736	A	7	6	1.00	25	0.240
737	A	6	6	1.00	25	0.240
738	A	5	5	1.00	25	0.200
739	A	6	6	1.00	25	0.240
740	A	7	7	1.00	25	0.280
741	A	9	9	1.00	25	0.360
742	A	9	9	1.00	25	0.360
743	A	8	6	1.00	25	0.240
744	A	7	6	1.00	25	0.240
745	A	6	6	1.00	25	0.240
746	A	7	7	1.00	25	0.280
747	A	8	8	1.00	25	0.320
748	A	8	8	1.00	25	0.320
749	A	9	9	1.00	25	0.360
750	A	10	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
751	A	5	5	1.00	25	0.200
752	A	4	4	1.00	25	0.160
753	A	2	2	1.00	25	0.080
754	A	2	2	1.00	25	0.080
755	A	9	9	1.00	25	0.360
756	A	8	8	1.00	25	0.320
757	A	6	6	1.00	25	0.240
758	A	5	5	1.00	25	0.200
759	A	5	5	1.00	25	0.200
760	A	5	5	1.00	25	0.200
761	A	7	7	1.00	25	0.280
762	A	8	8	1.00	25	0.320
763	A	7	6	1.00	25	0.240
764	A	6	6	1.00	25	0.240
765	A	6	6	1.00	25	0.240
766	A	6	6	1.00	25	0.240
767	A	6	6	1.00	25	0.240
768	A	8	8	1.00	25	0.320
769	A	6	5	1.00	21	0.238
770	A	5	4	1.00	21	0.190
771	A	4	3	1.00	21	0.143
772	A	3	2	1.00	19	0.105
773	A	5	3	1.00	21	0.143
774	A	8	3	1.00	21	0.143
775	A	8	6	1.00	21	0.286
776	A	7	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
777	A	6	4	1.00	19	0.210
778	A	2	2	1.00	21	0.095
779	A	3	3	1.00	19	0.158
780	A	1	1	1.00	25	0.040
781	A	1	1	1.00	27	0.037
782	A	1	1	1.00	31	0.032
783	A	1	1	1.00	27	0.037
784	A	1	1	1.00	29	0.034
785	A	2	2	1.00	25	0.080
786	A	1	1	1.00	29	0.034
787	A	3	3	1.00	25	0.120
788	A	3	3	1.00	25	0.120
789	A	3	3	1.00	25	0.120
790	A	3	3	1.00	25	0.120
791	A	3	3	1.00	28	0.107
792	A	2	2	1.00	23	0.087
793	A	2	2	1.00	21	0.095
794	A	3	3	1.00	28	0.107
795	A	7	4	1.00	25	0.160
796	A	7	4	1.00	25	0.160
797	A	7	4	1.00	25	0.160
798	A	7	4	1.00	25	0.160
799	A	9	7	1.00	31	0.226
800	A	8	7	1.00	29	0.241
801	A	6	6	1.00	23	0.261
802	A	6	6	1.00	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
803	A	7	7	1.00	31	0.226
804	A	8	7	1.00	31	0.226
805	A	9	7	1.00	31	0.226
806	A	9	7	1.00	29	0.241
807	A	7	6	1.00	23	0.261
808	A	7	7	1.00	29	0.241
809	A	6	6	1.00	31	0.194
810	A	7	7	1.00	31	0.226
811	A	8	7	1.00	31	0.226
812	A	9	7	1.00	31	0.226
813	A	8	6	1.00	23	0.261
814	A	8	7	1.00	29	0.241
815	A	7	7	1.00	31	0.226
816	A	6	6	1.00	31	0.194
817	A	7	7	1.00	31	0.226
818	A	8	7	1.00	31	0.226
819	A	9	7	1.00	31	0.226
820	A	9	7	1.00	31	0.226
821	A	8	7	1.00	31	0.226
822	A	7	7	1.00	29	0.241
823	A	5	5	1.00	23	0.217
824	A	7	7	1.00	29	0.241
825	A	8	7	1.00	31	0.226
826	A	9	7	1.00	31	0.226
827	A	9	7	1.00	31	0.226
828	A	8	7	1.00	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
829	A	7	7	1.00	31	0.226
830	A	6	6	1.00	29	0.207
831	A	6	6	1.00	23	0.261
832	A	8	7	1.00	29	0.241
833	A	9	7	1.00	31	0.226
834	A	9	7	1.00	31	0.226
835	A	8	7	1.00	31	0.226
836	A	7	7	1.00	31	0.226
837	A	6	6	1.00	31	0.194
838	A	7	7	1.00	29	0.241
839	A	7	6	1.00	23	0.261
840	A	9	7	1.00	29	0.241
841	A	8	6	1.00	23	0.261
842	A	7	5	1.00	33	0.152
843	A	6	5	1.00	33	0.152
844	A	2	2	1.00	33	0.061
845	A	3	2	1.00	33	0.061
846	A	3	3	1.00	33	0.091
847	A	5	5	1.00	33	0.152
848	A	6	6	1.00	33	0.182
849	A	6	5	1.00	33	0.152
850	A	7	5	1.00	33	0.152
851	A	6	5	1.00	33	0.152
852	A	2	2	1.00	33	0.061
853	A	3	2	1.00	33	0.061
854	A	3	3	1.00	33	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
855	A	5	5	1.00	33	0.152
856	A	6	6	1.00	33	0.182
857	A	6	5	1.00	33	0.152
858	A	7	5	1.00	33	0.152
859	A	6	5	1.00	33	0.152
860	A	2	2	1.00	33	0.061
861	A	3	2	1.00	33	0.061
862	A	3	3	1.00	33	0.091
863	A	5	5	1.00	33	0.152
864	A	6	6	1.00	33	0.182
865	A	6	5	1.00	33	0.152
866	A	6	5	1.00	33	0.152
867	A	2	2	1.00	33	0.061
868	A	3	2	1.00	33	0.061
869	A	3	3	1.00	33	0.091
870	A	5	5	1.00	33	0.152
871	A	6	6	1.00	33	0.182
872	A	6	5	1.00	33	0.152
873	A	6	5	1.00	33	0.152
874	A	2	2	1.00	33	0.061
875	A	3	2	1.00	33	0.061
876	A	3	3	1.00	33	0.091
877	A	5	5	1.00	33	0.152
878	A	6	6	1.00	33	0.182
879	A	6	5	1.00	33	0.152
880	A	6	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
881	A	2	2	1.00	33	0.061
882	A	3	2	1.00	33	0.061
883	A	3	3	1.00	33	0.091
884	A	5	5	1.00	33	0.152
885	A	6	6	1.00	33	0.182
886	A	6	5	1.00	33	0.152
887	A	4	3	1.00	31	0.097
888	A	4	3	1.00	29	0.103
889	A	3	2	1.00	23	0.087
890	A	4	3	1.00	29	0.103
891	A	4	3	1.00	31	0.097
892	A	4	3	1.00	31	0.097
893	A	4	3	1.00	31	0.097
894	A	4	3	1.00	29	0.103
895	A	3	2	1.00	23	0.087
896	A	4	3	1.00	29	0.103
897	A	4	3	1.00	31	0.097
898	A	4	3	1.00	31	0.097
899	A	4	3	1.00	31	0.097
900	A	4	3	1.00	29	0.103
901	A	3	2	1.00	23	0.087
902	A	4	3	1.00	29	0.103
903	A	4	3	1.00	31	0.097
904	A	4	3	1.00	31	0.097
905	A	4	3	1.00	31	0.097
906	A	4	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
907	A	3	2	1.00	23	0.087
908	A	4	3	1.00	29	0.103
909	A	4	3	1.00	31	0.097
910	A	4	3	1.00	31	0.097
911	A	4	3	1.00	29	0.103
912	A	4	3	1.00	29	0.103
913	A	4	3	1.00	27	0.111
914	A	3	2	1.00	21	0.095
915	A	4	3	1.00	27	0.111
916	A	4	3	1.00	29	0.103
917	A	4	3	1.00	29	0.103
918	A	4	3	1.00	29	0.103
919	A	4	3	1.00	31	0.097
920	A	4	3	1.00	31	0.097
921	A	4	3	1.00	31	0.097
922	A	4	3	1.00	31	0.097
923	A	4	3	1.00	31	0.097
924	A	4	3	1.00	31	0.097
925	A	4	3	1.00	31	0.097
926	A	4	3	1.00	31	0.097
927	A	4	3	1.00	31	0.097
928	A	4	3	1.00	31	0.097
929	A	4	3	1.00	31	0.097
930	A	4	3	1.00	31	0.097
931	A	4	3	1.00	31	0.097
932	A	4	3	1.00	31	0.097

Chapter 3

Listing of integrals

3.1 $\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=114

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin}{}$$

```
[Out] 5/16*a*x+a*sin(d*x+c)/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3*
sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin
(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2633, 2635, 8}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Cos[c + d*x]),x]
```

```
[Out] (5*a*x)/16 + (a*Sin[c + d*x])/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx &= a\int \cos^5(c+dx)\,dx + a\int \cos^6(c+dx)\,dx \\
&= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1-2\right.\right.}{} \\
&= \frac{a\sin(c+dx)}{d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} \\
&= \frac{a\sin(c+dx)}{d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} \\
&= \frac{5ax}{16} + \frac{a\sin(c+dx)}{d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c}{24d}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 75, normalized size = 0.66

$$
\frac{a\left(192\sin^5(c+dx) - 640\sin^3(c+dx) + 960\sin(c+dx) + 5(45\sin(2(c+dx)) + 9\sin(4(c+dx)) + \sin(6(c+dx)\right.}{960d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(960*Sin[c + d*x] - 640*Sin[c + d*x]^3 + 192*Sin[c + d*x]^5 + 5*(60*c +
60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])))/(96
0*d)
```

fricas [A] time = 1.05, size = 75, normalized size = 0.66

$$\frac{75\,adx + \left(40\,a\cos\left(dx+c\right)^{5} + 48\,a\cos\left(dx+c\right)^{4} + 50\,a\cos\left(dx+c\right)^{3} + 64\,a\cos\left(dx+c\right)^{2} + 75\,a\cos\left(dx+c\right) + 1}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + (40*a*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 + 50*a*cos(d*x
 + c)^3 + 64*a*cos(d*x + c)^2 + 75*a*cos(d*x + c) + 128*a)*sin(d*x + c))/d
```

giac [A] time = 0.46, size = 92, normalized size = 0.81

$$\frac{5}{16}\,ax+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{15\,a\sin\left(2\,dx+2\,c\right)}{64\,d}+\frac{5\,a\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x + 1/192*a*sin(6*d*x + 6*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 3/64*a*
sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 15/64*a*sin(2*d*x + 2*c)/d
 + 5/8*a*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 80, normalized size = 0.70

$$\frac{a\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*cos(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 84, normalized size = 0.74

$$\frac{64\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a-5\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a - 5*(4
*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2
*c))*a)/d
```

mupad [B] time = 2.98, size = 107, normalized size = 0.94

$$\frac{5\,a\,x}{16}+\frac{\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{39\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}+\frac{133\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{283\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{107\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{27\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*cos(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + ((27*a*tan(c/2 + (d*x)/2))/8 + (107*a*tan(c/2 + (d*x)/2)^3)/24
 + (283*a*tan(c/2 + (d*x)/2)^5)/20 + (133*a*tan(c/2 + (d*x)/2)^7)/20 + (39*
a*tan(c/2 + (d*x)/2)^9)/8 + (5*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 + (d
*x)/2)^2 + 1)^6)
```

sympy [A] time = 3.16, size = 216, normalized size = 1.89

$$\begin{cases} \frac{5ax\sin^{6}(c+dx)}{16}+\frac{15ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{15ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{5ax\cos^{6}(c+dx)}{16}+\frac{5a\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{8a\sin^{5}(c+dx)}{15d} \\ x\left(a\cos(c)+a\right)\cos^{5}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8*a*sin(c + d*x)**5/(15*d) + 5
*a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)
**2/(3*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + a*sin(c + d*x)*cos(c
 + d*x)**4/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**5, True))
```

3.2 $\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=92

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] 3/8*a*x+a*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin
(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2635, 8, 2633}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c
 + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + a\int \cos^5(c+dx)\,dx$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1-2\right.\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} -$$
$$= \frac{3ax}{8} + \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c}{4d}$$

Mathematica [A] time = 0.12, size = 65, normalized size = 0.71

$$\frac{a\left(96\sin^5(c+dx)-320\sin^3(c+dx)+480\sin(c+dx)+15(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Cos[c + d*x]),x]
[Out] (a*(480*Sin[c + d*x] - 320*Sin[c + d*x]^3 + 96*Sin[c + d*x]^5 + 15*(12*(c +
 d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])))/(480*d)
```

fricas [A] time = 1.82, size = 64, normalized size = 0.70

$$\frac{45\,adx + \left(24\,a\cos\left(dx+c\right)^4 + 30\,a\cos\left(dx+c\right)^3 + 32\,a\cos\left(dx+c\right)^2 + 45\,a\cos\left(dx+c\right) + 64\,a\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="fricas")
[Out] 1/120*(45*a*d*x + (24*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^3 + 32*a*cos(d*x
 + c)^2 + 45*a*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

giac [A] time = 0.55, size = 77, normalized size = 0.84

$$\frac{3}{8}\,ax + \frac{a\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{a\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{a\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x + 1/80*a*sin(5*d*x + 5*c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 5/48*a*si
n(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 5/8*a*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 70, normalized size = 0.76

$$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.79, size = 69, normalized size = 0.75

$$\frac{32\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right.\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 2.85, size = 93, normalized size = 1.01

$$\frac{3\,a\,x}{8}+\frac{\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{116\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{19\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (19*a*tan(c/2 + (d*x)/2)^3)/6 +
(116*a*tan(c/2 + (d*x)/2)^5)/15 + (13*a*tan(c/2 + (d*x)/2)^7)/6 + (3*a*tan(
c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 1.86, size = 168, normalized size = 1.83

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{8a\sin^5(c+dx)}{15d} + \frac{4a\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \\ x(a\cos(c)+a)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)
**3*cos(c + d*x)**2/(3*d) + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + a*sin(
c + d*x)*cos(c + d*x)**4/d + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d,
0)), (x*(a*cos(c) + a)*cos(c)**4, True))
```

3.3 $\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] 3/8*a*x+a*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin
(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2633, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^3(c+dx)\,dx + a\int \cos^4(c+dx)\,dx$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1-\right.\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} -$$
$$= \frac{3ax}{8} + \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c}{4d}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 0.96

$$\frac{3a(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*
Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.93, size = 53, normalized size = 0.70

$$\frac{9\,adx + \left(6\,a\cos\left(dx+c\right)^3 + 8\,a\cos\left(dx+c\right)^2 + 9\,a\cos\left(dx+c\right) + 16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c)
 + 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.49, size = 62, normalized size = 0.82

$$\frac{3}{8}\,ax + \frac{a\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{a\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{3\,a\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x + 1/32*a*sin(4*d*x + 4*c)/d + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*a*sin
(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 60, normalized size = 0.79

$$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.30, size = 57, normalized size = 0.75

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)a-3\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 3.60, size = 79, normalized size = 1.04

$$\frac{3\,a\,x}{8}+\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (31*a*tan(c/2 + (d*x)/2)^3)/12 +
 (49*a*tan(c/2 + (d*x)/2)^5)/12 + (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c/2
 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 0.91, size = 144, normalized size = 1.89

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2a\sin^3(c+dx)}{3d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} + \\ x(a\cos(c)+a)\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a*
sin(c + d*x)**3/(3*d) + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a*sin(c +
d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**3, True))
```

3.4 $\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] 1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2635, 8, 2633}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^2(c+dx)\,dx + a\int \cos^3(c+dx)\,dx$$
$$= \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx - \frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(\right.}{d}$$
$$= \frac{ax}{2} + \frac{a\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 1.06

$$\frac{a(c+dx)}{2d} - \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} + \frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*Si
n[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.81, size = 42, normalized size = 0.78

$$\frac{3\,adx + \left(2\,a\cos\left(dx+c\right)^2 + 3\,a\cos\left(dx+c\right) + 4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))/
d
```

giac [A] time = 0.48, size = 47, normalized size = 0.87

$$\frac{1}{2}\,ax + \frac{a\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{a\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{3\,a\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*a*sin(
d*x + c)/d
```

maple [A] time = 0.06, size = 49, normalized size = 0.91

$$\frac{\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

maxima [A] time = 0.30, size = 46, normalized size = 0.85

$$-\frac{4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a)/d
```

mupad [B] time = 0.38, size = 55, normalized size = 1.02

$$\frac{a\,x}{2}+\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x)),x)
```

```
[Out] (a*x)/2 + (2*a*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) +
(a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

sympy [A] time = 0.45, size = 92, normalized size = 1.70

$$\begin{cases}\frac{ax\sin^2(c+dx)}{2}+\frac{ax\cos^2(c+dx)}{2}+\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d\neq 0\\ x\left(a\cos\left(c\right)+a\right)\cos^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + 2*a*sin(c + d*x)
**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d + a*sin(c + d*x)*cos(c + d*x)/
(2*d), Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**2, True))
```

3.5 $\int \cos(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] 1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2734}

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))\,dx=\frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 0.84

$$\frac{a(2(c+dx)+4\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 2.33, size = 29, normalized size = 0.76

$$\frac{adx + (a\cos(dx + c) + 2\,a)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.29, size = 31, normalized size = 0.82

$$\frac{1}{2}\,ax + \frac{a\sin(2\,dx + 2\,c)}{4\,d} + \frac{a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="giac")
[Out] 1/2*a*x + 1/4*a*sin(2*d*x + 2*c)/d + a*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + \sin(dx + c)\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c)),x)
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+sin(d*x+c)*a)
```

maxima [A] time = 0.29, size = 34, normalized size = 0.89

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))a + 4\,a\sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="maxima")
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d
```

mupad [B] time = 0.75, size = 50, normalized size = 1.32

$$\frac{a\,x}{2} + \frac{a\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 + 3\,a\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*cos(c + d*x)),x)
```

```
[Out] (a*x)/2 + (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (
d*x)/2)^2 + 1)^2)
```

sympy [A] time = 0.20, size = 66, normalized size = 1.74

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} + \frac{a\sin(c+dx)\cos(c+dx)}{2d} + \frac{a\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\cos(c)+a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) + a*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c),
True))
```

3.6 $\int (a+a\cos(c+dx))\,dx$

Optimal. Leaf size=15

$$\frac{a\sin(c+dx)}{d}+ax$$

```
[Out] a*x+a*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2637}

$$\frac{a\sin(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Cos[c + d*x],x]
```

```
[Out] a*x + (a*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))\,dx &= ax+a\int \cos(c+dx)\,dx\\ &= ax+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.73

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Cos[c + d*x],x]
```

```
[Out] a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

fricas [A] time = 0.75, size = 17, normalized size = 1.13

$$\frac{adx+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cos(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + a*sin(d*x + c))/d
```

giac [A] time = 0.40, size = 15, normalized size = 1.00

$$ax + \frac{a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cos(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + a*sin(d*x + c)/d
```

maple [A] time = 0.02, size = 16, normalized size = 1.07

$$ax + \frac{a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*cos(d*x+c),x)
```

```
[Out] a*x+a*sin(d*x+c)/d
```

maxima [A] time = 0.38, size = 15, normalized size = 1.00

$$ax + \frac{a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cos(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + a*sin(d*x + c)/d
```

mupad [B] time = 0.31, size = 15, normalized size = 1.00

$$a\,x + \frac{a\,\sin(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + a*cos(c + d*x),x)
```

```
[Out] a*x + (a*sin(c + d*x))/d
```

sympy [A] time = 0.11, size = 17, normalized size = 1.13

$$ax + a\left(\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d \neq 0\\ x\cos(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cos(d*x+c),x)
```

```
[Out] a*x + a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))
```

3.7 $\int (a + a\cos(c + dx))\sec(c + dx)\,dx$

Optimal. Leaf size=16

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

```
[Out] a*x+a*arctanh(sin(d*x+c))/d
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2735, 3770}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))\sec(c+dx)\,dx &= ax+a\int \sec(c+dx)\,dx\\ &= ax+\frac{a\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

fricas [B] time = 1.68, size = 36, normalized size = 2.25

$$\frac{2\,adx + a\log\left(\sin\left(dx + c\right) + 1\right) - a\log\left(-\sin\left(dx + c\right) + 1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

giac [B] time = 0.48, size = 43, normalized size = 2.69

$$\frac{(dx + c)a + a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)))/d
```

maple [A] time = 0.07, size = 30, normalized size = 1.88

$$ax + \frac{a\ln\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d} + \frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*x+1/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*c*a
```

maxima [A] time = 0.30, size = 28, normalized size = 1.75

$$\frac{(dx + c)a + a\log\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a + a*log(sec(d*x + c) + tan(d*x + c)))/d
```

mupad [B] time = 0.34, size = 20, normalized size = 1.25

$$a\,x+\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x),x)
[Out] a*x + (2*a*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [A] time = 4.92, size = 49, normalized size = 3.06

$$ax+a\left(\begin{cases}\frac{x\tan(c)\sec(c)}{\tan(c)+\sec(c)}+\frac{x\sec^2(c)}{\tan(c)+\sec(c)} & \text{for } d=0\\ \frac{\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x)
[Out] a*x + a*Piecewise((x*tan(c)*sec(c)/(tan(c) + sec(c)) + x*sec(c)**2/(tan(c)
+ sec(c)), Eq(d, 0)), (log(tan(c + d*x) + sec(c + d*x))/d, True))
```

3.8 $\int (a + a\cos(c + dx))\sec^2(c + dx)\,dx$

Optimal. Leaf size=24

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3767, 8, 3770}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx = a\int \sec(c+dx)\,dx + a\int \sec^2(c+dx)\,dx$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{a\tan(c+dx)}{d}$$

Mathematica [**A**] time = 0.01, size = 24, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

fricas [**B**] time = 1.01, size = 60, normalized size = 2.50

$$\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,a\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

giac [**B**] time = 0.71, size = 63, normalized size = 2.62

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.10, size = 32, normalized size = 1.33

$$\frac{a\tan(dx+c)}{d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a*tan(d*x+c)/d+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 2.23, size = 38, normalized size = 1.58

$$\frac{a\left(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)+2\,a\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*a*tan(d*x + c))/
d
```

mupad [B] time = 0.39, size = 47, normalized size = 1.96

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^2,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
(d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos(c+dx)\sec^2(c+dx)\,dx+\int\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x)
)
```

3.9 $\int (a + a\cos(c + dx))\sec^3(c + dx)\,dx$

Optimal. Leaf size=47

$$\frac{a\tan(c + dx)}{d} + \frac{a\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a\tan(c + dx)\sec(c + dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2748, 3768, 3770, 3767, 8}

$$\frac{a\tan(c + dx)}{d} + \frac{a\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a\tan(c + dx)\sec(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx &= a\int \sec^2(c+dx)\,dx+a\int \sec^3(c+dx)\,dx\\ &= \frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a\int \sec(c+dx)\,dx-\frac{a\,\text{Subst}(\int 1\,dx,x,-\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

fricas [A] time = 1.63, size = 74, normalized size = 1.57

$$\frac{a\cos(dx+c)^2\log(\sin(dx+c)+1)-a\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(2\,a\cos(dx+c)+a)\sin(dx+}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.61, size = 80, normalized size = 1.70

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.10, size = 51, normalized size = 1.09

$$\frac{a\tan\left(dx+c\right)}{d}+\frac{a\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+
c))
```

maxima [A] time = 0.50, size = 58, normalized size = 1.23

$$-\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,a\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

mupad [B] time = 0.71, size = 75, normalized size = 1.60

$$\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
- 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos(c+dx)\sec^3(c+dx)\,dx+\int \sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x)
)
```

3.10 $\int (a + a\cos(c + dx))\sec^4(c + dx)\,dx$

Optimal. Leaf size=63

$$\frac{a\tan^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/
3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3767, 3768, 3770}

$$\frac{a\tan^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))\sec^4(c + dx)\,dx = a\int \sec^3(c + dx)\,dx + a\int \sec^4(c + dx)\,dx$$

$$= \frac{a\sec(c + dx)\tan(c + dx)}{2d} + \frac{1}{2}a\int \sec(c + dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1 + x^2\right)\right.}{}$$

$$= \frac{a\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a\tan(c + dx)}{d} + \frac{a\sec(c + dx)\tan(c + dx)}{2d} + \frac{a}{}$$

Mathematica [A] time = 0.15, size = 60, normalized size = 0.95

$$\frac{a\left(\frac{1}{3}\tan^3(c + dx) + \tan(c + dx)\right)}{d} + \frac{a\tanh^{-1}(\sin(c + dx))}{2d} + \frac{a\tan(c + dx)\sec(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 1.23, size = 88, normalized size = 1.40

$$\frac{3\,a\cos(dx + c)^3\log(\sin(dx + c) + 1) - 3\,a\cos(dx + c)^3\log(-\sin(dx + c) + 1) + 2\left(4\,a\cos(dx + c)^2 + 3\,a\cos\right.}{12\,d\cos(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.48, size = 96, normalized size = 1.52

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 + 9*
a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.15, size = 72, normalized size = 1.14

$$\frac{a\sec(dx+c)\tan(dx+c)}{2d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2a\tan(dx+c)}{3d}+\frac{a\tan(dx+c)\left(\sec^2(dx+c)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*tan(d
*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.44, size = 70, normalized size = 1.11

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a-3a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.04, size = 102, normalized size = 1.62

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2) - (4*a*tan(c/2 +
(d*x)/2)^3)/3 + a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(
c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos(c+dx)\sec^4(c+dx)\,dx + \int \sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x)
)
```

3.11 $\int (a+a\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1/
4*a*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3768, 3770, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^5(c+dx)\,dx = a\int \sec^4(c+dx)\,dx + a\int \sec^5(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+\right.\right.}{}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} +$$

Mathematica [A] time = 0.15, size = 76, normalized size = 0.89

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.87, size = 99, normalized size = 1.16

$$\frac{9\,a\cos(dx+c)^4\log(\sin(dx+c)+1)-9\,a\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(16\,a\cos(dx+c)^3+9\,a\cos\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.52, size = 110, normalized size = 1.29

$$\frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5 +
31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^4)/d
```

maple [A] time = 0.15, size = 92, normalized size = 1.08

$$\frac{2a\tan(dx+c)}{3d}+\frac{a\tan(dx+c)\left(\sec^{2}(dx+c)\right)}{3d}+\frac{a\left(\sec^{3}(dx+c)\right)\tan(dx+c)}{4d}+\frac{3a\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*sec(d*x+c)^3*tan(d
*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.97, size = 95, normalized size = 1.12

$$\frac{16\left(\tan(dx+c)^{3}+3\,\tan(dx+c)\right)a-3\,a\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.34, size = 130, normalized size = 1.53

$$\frac{-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{12}-\frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{12}+\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}+\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] ((13*a*tan(c/2 + (d*x)/2))/4 - (31*a*tan(c/2 + (d*x)/2)^3)/12 + (49*a*tan(c
/2 + (d*x)/2)^5)/12 - (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)/2
)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^
8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos\left(c+dx\right)\sec^5\left(c+dx\right)dx+\int \sec^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5, x)
)
```

3.12 $\int (a + a\cos(c + dx))\sec^6(c + dx)\,dx$

Optimal. Leaf size=101

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1/
4*a*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3767, 3768, 3770}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*a*Tan[c +
d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx = a\int \sec^5(c+dx)\,dx + a\int \sec^6(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+\right.\right.}{}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} +$$

Mathematica [A] time = 0.25, size = 65, normalized size = 0.64

$$\frac{a\left(45\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(24\tan^4(c+dx) + 80\tan^2(c+dx) + 30\sec^3(c+dx) + 45\sec(c+dx) +\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a*(45*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(120 + 45*Sec[c + d*x] + 30*Sec
[c + d*x]^3 + 80*Tan[c + d*x]^2 + 24*Tan[c + d*x]^4)))/(120*d)
```

fricas [A] time = 1.22, size = 110, normalized size = 1.09

$$\frac{45\,a\cos(dx+c)^5\log(\sin(dx+c)+1) - 45\,a\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\left(64\,a\cos(dx+c)^4 + 45\right.}{240\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(45*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 2*(64*a*cos(d*x + c)^4 + 45*a*cos(d*x + c)^3 + 32*a*co
s(d*x + c)^2 + 30*a*cos(d*x + c) + 24*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [A] time = 1.45, size = 124, normalized size = 1.23

$$\frac{45\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 45\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 130\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 464\,a\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(45*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(45*a*tan(1/2*d*x + 1/2*c)^9 - 130*a*tan(1/2*d*x + 1/2*c)
^7 + 464*a*tan(1/2*d*x + 1/2*c)^5 - 190*a*tan(1/2*d*x + 1/2*c)^3 + 195*a*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.14, size = 112, normalized size = 1.11

$$\frac{a\left(\sec^{3}\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3a\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{8a\tan\left(dx+c\right)}{15d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 1/4*a*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+8/15*a*tan(d*x+c)/d+1/5/d*a*tan(d*x+c)*sec(d*x+c)^4+4/
15/d*a*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.31, size = 107, normalized size = 1.06

$$\frac{16\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)a-15a\left(\frac{2\left(3\sin\left(dx+c\right)^{3}-5\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\sin\left(dx+c\right)^{2}+1}-3\log\left(\sin\left(dx+c\right)+1\cdots\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 15*a
*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
+ 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.77, size = 158, normalized size = 1.56

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}-\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6}+\frac{116\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{15}-\frac{19\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{6}+\frac{13\,a\,\tan\left(\frac{c}{2}+\cdots\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+5\tan\left(\frac{c}{2}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((13*a*tan(c/2 + (d*x)/2))/4 - (19*
a*tan(c/2 + (d*x)/2)^3)/6 + (116*a*tan(c/2 + (d*x)/2)^5)/15 - (13*a*tan(c/2
 + (d*x)/2)^7)/6 + (3*a*tan(c/2 + (d*x)/2)^9)/4)/(d*(5*tan(c/2 + (d*x)/2)^2
 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)
^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\sec^{6}\left(c+dx\right)dx+\int\sec^{6}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**6, x)
)
```

3.13 $\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{2a^2\sin^5(c+dx)}{5d}-\frac{4a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{24d}+$$

```
[Out] 11/16*a^2*x+2*a^2*sin(d*x+c)/d+11/16*a^2*cos(d*x+c)*sin(d*x+c)/d+11/24*a^2*
cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-4/3*a^2*sin(d*x
+c)^3/d+2/5*a^2*sin(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2635, 8, 2633}

$$\frac{2a^2\sin^5(c+dx)}{5d}-\frac{4a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{24d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (11*a^2*x)/16 + (2*a^2*Sin[c + d*x])/d + (11*a^2*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (11*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) + (2*a^2*Sin[c + d*x]^5
)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)+2a^2\cos^5(c+dx)+a^2\cos^6(c+dx)\right)\,dx$$

$$= a^2\int \cos^4(c+dx)\,dx + a^2\int \cos^6(c+dx)\,dx + \left(2a^2\right)\int \cos^5(c+dx)\,d$$

$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{4}\left(3a^2\right)\int$$

$$= \frac{2a^2\sin(c+dx)}{d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{11a^2\cos^3(c+dx)\sin(}{24d}$$

$$= \frac{3a^2x}{8} + \frac{2a^2\sin(c+dx)}{d} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{16d} + \frac{11a^2\cos^3(c}{}$$

$$= \frac{11a^2x}{16} + \frac{2a^2\sin(c+dx)}{d} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{16d} + \frac{11a^2\cos^3(c}{}$$

Mathematica [**A**] time = 0.20, size = 73, normalized size = 0.57

$$\frac{a^2(1200\sin(c+dx)+465\sin(2(c+dx))+200\sin(3(c+dx))+75\sin(4(c+dx))+24\sin(5(c+dx))+5\sin(6}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(660*d*x + 1200*Sin[c + d*x] + 465*Sin[2*(c + d*x)] + 200*Sin[3*(c + d
*x)] + 75*Sin[4*(c + d*x)] + 24*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(96
0*d)
```

fricas [**A**] time = 1.64, size = 89, normalized size = 0.69

$$\frac{165\,a^2dx+\left(40\,a^2\cos\left(dx+c\right)^5+96\,a^2\cos\left(dx+c\right)^4+110\,a^2\cos\left(dx+c\right)^3+128\,a^2\cos\left(dx+c\right)^2+165\,a^2\cos\left(\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(165*a^2*d*x + (40*a^2*cos(d*x + c)^5 + 96*a^2*cos(d*x + c)^4 + 110*a
^2*cos(d*x + c)^3 + 128*a^2*cos(d*x + c)^2 + 165*a^2*cos(d*x + c) + 256*a^2
)*sin(d*x + c))/d
```

giac [A] time = 0.80, size = 106, normalized size = 0.82

$$\frac{11}{16}\,a^2x+\frac{a^2\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{a^2\sin\left(5\,dx+5\,c\right)}{40\,d}+\frac{5\,a^2\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{5\,a^2\sin\left(3\,dx+3\,c\right)}{24\,d}+\frac{31\,a^2\sin\left(2\,dx+2\,c\right)}{64\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 11/16*a^2*x + 1/192*a^2*sin(6*d*x + 6*c)/d + 1/40*a^2*sin(5*d*x + 5*c)/d +
5/64*a^2*sin(4*d*x + 4*c)/d + 5/24*a^2*sin(3*d*x + 3*c)/d + 31/64*a^2*sin(2
*d*x + 2*c)/d + 5/4*a^2*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 121, normalized size = 0.94

$$\frac{a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(d}{2}\right.}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+2/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*(
1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.30, size = 121, normalized size = 0.94

$$\frac{128\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)a^2-5\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+\right.\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/960*(128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^
2)/d
```

mupad [B] time = 2.86, size = 121, normalized size = 0.94

$$\frac{11\,a^2\,x}{16}+\frac{\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{187\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+\frac{331\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{20}+\frac{501\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}+\frac{87\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8}+\frac{53\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (11*a^2*x)/16 + ((87*a^2*tan(c/2 + (d*x)/2)^3)/8 + (501*a^2*tan(c/2 + (d*x)
/2)^5)/20 + (331*a^2*tan(c/2 + (d*x)/2)^7)/20 + (187*a^2*tan(c/2 + (d*x)/2)
^9)/24 + (11*a^2*tan(c/2 + (d*x)/2)^11)/8 + (53*a^2*tan(c/2 + (d*x)/2))/8)/
(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 3.53, size = 343, normalized size = 2.66

$$\begin{cases}\frac{5a^2x\sin^6(c+dx)}{16}+\frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^2x\sin^4(c+dx)}{8}+\frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{5a^2x}{} \\ x\,(a\cos(c)+a)^2\cos^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 3*a**2*x*sin(c + d*x)**4/8 + 15*a**2*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*a**2*x*cos(c +
 d*x)**6/16 + 3*a**2*x*cos(c + d*x)**4/8 + 5*a**2*sin(c + d*x)**5*cos(c + d
*x)/(16*d) + 16*a**2*sin(c + d*x)**5/(15*d) + 5*a**2*sin(c + d*x)**3*cos(c
+ d*x)**3/(6*d) + 8*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a**2*sin
(c + d*x)**3*cos(c + d*x)/(8*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*
d) + 2*a**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*a**2*sin(c + d*x)*cos(c + d*
x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**4, True))
```

3.14 $\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2}{4}$$

```
[Out] 3/4*a^2*x+2*a^2*sin(d*x+c)/d+3/4*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*cos(d*
x+c)^3*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2633, 2635, 8}

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/4 + (2*a^2*Sin[c + d*x])/d + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - (a^2*Sin[c + d*x]^3)/d + (a
^2*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^3(c+dx)+2a^2\cos^4(c+dx)+a^2\cos^5(c+dx)\right)dx$$
$$= a^2\int \cos^3(c+dx)\,dx + a^2\int \cos^5(c+dx)\,dx + \left(2a^2\right)\int \cos^4(c+dx)\,d$$
$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(3a^2\right)\int \cos^2(c+dx)\,dx - \frac{a^2\,\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{2a^2\sin(c+dx)}{d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^3(c+dx)\sin(c}{2d}$$
$$= \frac{3a^2x}{4} + \frac{2a^2\sin(c+dx)}{d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^3(c+dx}{2}$$

Mathematica [A] time = 0.13, size = 61, normalized size = 0.59

$$\frac{a^2(110\sin(c+dx)+40\sin(2(c+dx))+15\sin(3(c+dx))+5\sin(4(c+dx))+\sin(5(c+dx))+60dx)}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(60*d*x + 110*Sin[c + d*x] + 40*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)]
 + 5*Sin[4*(c + d*x)] + Sin[5*(c + d*x)]))/(80*d)
```

fricas [A] time = 1.46, size = 76, normalized size = 0.74

$$\frac{15\,a^2dx+\left(4\,a^2\cos\left(dx+c\right)^4+10\,a^2\cos\left(dx+c\right)^3+12\,a^2\cos\left(dx+c\right)^2+15\,a^2\cos\left(dx+c\right)+24\,a^2\right)\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*c
os(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.54, size = 89, normalized size = 0.86

$$\frac{3}{4}\,a^2x+\frac{a^2\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{a^2\sin\left(4\,dx+4\,c\right)}{16\,d}+\frac{3\,a^2\sin\left(3\,dx+3\,c\right)}{16\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d}+\frac{11\,a^2\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*x + 1/80*a^2*sin(5*d*x + 5*c)/d + 1/16*a^2*sin(4*d*x + 4*c)/d + 3/1
6*a^2*sin(3*d*x + 3*c)/d + 1/2*a^2*sin(2*d*x + 2*c)/d + 11/8*a^2*sin(d*x +
c)/d
```

maple [A] time = 0.06, size = 96, normalized size = 0.93

$$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5}+2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*(2+cos(d*x+c)^2
)*sin(d*x+c))
```

maxima [A] time = 1.13, size = 95, normalized size = 0.92

$$\frac{16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2-80\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2+15(12dx+12}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 80
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

mupad [B] time = 3.63, size = 105, normalized size = 1.02

$$\frac{3a^2x}{4}+\frac{\frac{3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2}+7a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{72a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5}+9a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\frac{13a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/4 + (9*a^2*tan(c/2 + (d*x)/2)^3 + (72*a^2*tan(c/2 + (d*x)/2)^5)/5
 + 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/2 + (d*x)/2)^9)/2 + (13*a^2*ta
n(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 1.98, size = 221, normalized size = 2.15

$$\begin{cases} \frac{3a^2x\sin^4(c+dx)}{4} + \frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{3a^2x\cos^4(c+dx)}{4} + \frac{8a^2\sin^5(c+dx)}{15d} + \frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{3a^2\sin^3(c+dx)\cos(c+dx)}{4d} \\ x\left(a\cos(c)+a\right)^2\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/4 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/2 + 3*a**2*x*cos(c + d*x)**4/4 + 8*a**2*sin(c + d*x)**5/(15*d) + 4*a*
*2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a**2*sin(c + d*x)**3*cos(c + d
*x)/(4*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**
4/d + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a**2*sin(c + d*x)*cos(c +
 d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**3, True))
```

3.15 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=87

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

```
[Out] 7/8*a^2*x+2*a^2*sin(d*x+c)/d+7/8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*
x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2635, 8, 2633}

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (7*a^2*x)/8 + (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a^2*Sin[c + d*x]^3)/(3*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^2(c+dx)+2a^2\cos^3(c+dx)+a^2\cos^4(c+dx)\right)dx$$
$$= a^2\int \cos^2(c+dx)\,dx + a^2\int \cos^4(c+dx)\,dx + \left(2a^2\right)\int \cos^3(c+dx)\,d$$
$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{2}a^2\int 1\,dx$$
$$= \frac{a^2x}{2} + \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)}{4d}$$
$$= \frac{7a^2x}{8} + \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+d}{4}$$

Mathematica [A] time = 0.13, size = 53, normalized size = 0.61

$$\frac{a^2(144\sin(c+dx)+48\sin(2(c+dx))+16\sin(3(c+dx))+3\sin(4(c+dx))+84dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)]
 + 3*Sin[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 1.56, size = 63, normalized size = 0.72

$$\frac{21\,a^2dx + \left(6\,a^2\cos\left(dx+c\right)^3 + 16\,a^2\cos\left(dx+c\right)^2 + 21\,a^2\cos\left(dx+c\right) + 32\,a^2\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*c
os(d*x + c) + 32*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.81, size = 72, normalized size = 0.83

$$\frac{7}{8}a^2x + \frac{a^2\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a^2\sin\left(3\,dx+3\,c\right)}{6\,d} + \frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d} + \frac{3\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 7/8*a^2*x + 1/32*a^2*sin(4*d*x + 4*c)/d + 1/6*a^2*sin(3*d*x + 3*c)/d + 1/2*
a^2*sin(2*d*x + 2*c)/d + 3/2*a^2*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 90, normalized size = 1.03

$$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

maxima [A] time = 0.48, size = 83, normalized size = 0.95

$$-\frac{64\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2-3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^2-24(2dx+2c+}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
a^2)/d
```

mupad [B] time = 3.50, size = 89, normalized size = 1.02

$$\frac{7a^2x}{8}+\frac{\frac{7a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}+\frac{77a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{12}+\frac{83a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12}+\frac{25a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (7*a^2*x)/8 + ((83*a^2*tan(c/2 + (d*x)/2)^3)/12 + (77*a^2*tan(c/2 + (d*x)/2
)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 + (25*a^2*tan(c/2 + (d*x)/2))/4)/(
d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 1.02, size = 211, normalized size = 2.43

$$\begin{cases} \frac{3a^2x\sin^4(c+dx)}{8} + \frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^2x\sin^2(c+dx)}{2} + \frac{3a^2x\cos^4(c+dx)}{8} + \frac{a^2x\cos^2(c+dx)}{2} + \frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d} + \\ x\left(a\cos(c)+a\right)^2\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + a**2*x*sin(c + d*x)**2/2 + 3*a**2*x*cos(c + d*x)**4/8 + a**2*x*co
s(c + d*x)**2/2 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*a**2*sin(c
+ d*x)**3/(3*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*a**2*sin(c
+ d*x)*cos(c + d*x)**2/d + a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a*cos(c) + a)**2*cos(c)**2, True))
```

3.16 $\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=57

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+a^2x$$

```
[Out] a^2*x+2*a^2*sin(d*x+c)/d+a^2*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2751, 2644}

$$\frac{4a^2\sin(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{3d}+a^2x+\frac{\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] a^2*x + (4*a^2*Sin[c + d*x])/(3*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(3*d)
+ ((a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx=\frac{(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{2}{3}\int (a+a\cos(c+dx))^2\,dx$$
$$=a^2x+\frac{4a^2\sin(c+dx)}{3d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{3d}+\frac{(a+a\cos(c+dx))^2}{3d}$$

Mathematica [A] time = 0.08, size = 41, normalized size = 0.72

$$\frac{a^2(21\sin(c+dx)+6\sin(2(c+dx))+\sin(3(c+dx))+12dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(1
2*d)
```

fricas [A] time = 1.56, size = 49, normalized size = 0.86

$$\frac{3\,a^2dx+\left(a^2\cos\left(dx+c\right)^2+3\,a^2\cos\left(dx+c\right)+5\,a^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x + (a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x
+ c))/d
```

giac [A] time = 0.47, size = 54, normalized size = 0.95

$$a^2x+\frac{a^2\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d}+\frac{7\,a^2\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a^2*x + 1/12*a^2*sin(3*d*x + 3*c)/d + 1/2*a^2*sin(2*d*x + 2*c)/d + 7/4*a^2*
sin(d*x + c)/d
```

maple [A] time = 0.05, size = 64, normalized size = 1.12

$$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+a^2*sin(d*x+c))
```

maxima [A] time = 0.47, size = 61, normalized size = 1.07

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}-6\,a^{2}\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^2 - 6*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.38, size = 61, normalized size = 1.07

$$a^{2}\,x+\frac{5\,a^{2}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^{2}\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^{2}\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (5*a^2*sin(c + d*x))/(3*d) + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*d
) + (a^2*cos(c + d*x)*sin(c + d*x))/d
```

sympy [A] time = 0.48, size = 107, normalized size = 1.88

$$\begin{cases} a^{2}x\sin^{2}\left(c+dx\right)+a^{2}x\cos^{2}\left(c+dx\right)+\frac{2a^{2}\sin^{3}\left(c+dx\right)}{3d}+\frac{a^{2}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}+\frac{a^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{a^{2}\sin\left(c+dx\right)}{d} & \text{fo} \\ x\left(a\cos\left(c\right)+a\right)^{2}\cos\left(c\right) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2 + a**2*x*cos(c + d*x)**2 + 2*a**2*sin(c +
 d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2/d + a**2*sin(c + d*x)*co
s(c + d*x)/d + a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c),
 True))
```

3.17 $\int (a + a\cos(c + dx))^2 \, dx$

Optimal. Leaf size=45

$$\frac{2a^2\sin(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{3a^2x}{2}$$

```
[Out] 3/2*a^2*x+2*a^2*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{2a^2\sin(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2,x]
[Out] (3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\,dx = \frac{3a^2x}{2} + \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 0.76

$$\frac{a^2(6(c+dx)+8\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2,x]
[Out] (a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 1.17, size = 36, normalized size = 0.80

$$\frac{3\,a^2dx + \left(a^2\cos\left(dx+c\right) + 4\,a^2\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.46, size = 38, normalized size = 0.84

$$\frac{3}{2}\,a^2x + \frac{a^2\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{2\,a^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*x + 1/4*a^2*sin(2*d*x + 2*c)/d + 2*a^2*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 52, normalized size = 1.16

$$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 2a^2\sin\left(dx+c\right) + a^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*sin(d*x+c)+a^2*(d*
x+c))
```

maxima [A] time = 0.30, size = 45, normalized size = 1.00

$$a^2x + \frac{\left(2\,dx + 2\,c + \sin\left(2\,dx+2\,c\right)\right)a^2}{4\,d} + \frac{2\,a^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 2*a^2*sin(d*x + c)/d
```

mupad [B] time = 0.72, size = 57, normalized size = 1.27

$$\frac{3\,a^2\,x}{2}+\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/2 + (3*a^2*tan(c/2 + (d*x)/2)^3 + 5*a^2*tan(c/2 + (d*x)/2))/(d*(t
an(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [A] time = 0.25, size = 78, normalized size = 1.73

$$\begin{cases} \frac{a^2 x \sin^2{(c+dx)}}{2}+\frac{a^2 x \cos^2{(c+dx)}}{2}+a^2 x+\frac{a^2 \sin{(c+dx)}\cos{(c+dx)}}{2d}+\frac{2a^2 \sin{(c+dx)}}{d} & \text{for}\: d \neq 0 \\ x\left(a\cos{(c)}+a\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x + a
**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*a**2*sin(c + d*x)/d, Ne(d, 0)), (x*
(a*cos(c) + a)**2, True))
```

3.18 $\int (a + a\cos(c + dx))^2 \sec(c + dx)\, dx$

Optimal. Leaf size=34

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

```
[Out] 2*a^2*x+a^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2746, 2735, 3770}

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x],x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx = \frac{a^2\sin(c+dx)}{d} + \int \left(a^2+2a^2\cos(c+dx)\right)\sec(c+dx)\,dx$$
$$= 2a^2x + \frac{a^2\sin(c+dx)}{d} + a^2\int \sec(c+dx)\,dx$$
$$= 2a^2x + \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\sin(c+dx)}{d}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.38

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\sin(c)\cos(dx)}{d} + \frac{a^2\cos(c)\sin(dx)}{d} + 2a^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x],x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Co
s[c]*Sin[d*x])/d
```

fricas [A] time = 1.43, size = 53, normalized size = 1.56

$$\frac{4\,a^2dx + a^2\log\left(\sin\left(dx+c\right)+1\right) - a^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) + 2
*a^2*sin(d*x + c))/d
```

giac [B] time = 0.66, size = 79, normalized size = 2.32

$$\frac{2\,(dx+c)a^2 + a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

maple [A] time = 0.10, size = 51, normalized size = 1.50

$$2a^2x + \frac{a^2\sin(dx+c)}{d} + \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{2a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c),x)
```

```
[Out] 2*a^2*x+a^2*sin(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*c
```

maxima [A] time = 0.30, size = 43, normalized size = 1.26

$$\frac{2(dx+c)a^2 + a^2\log(\sec(dx+c)+\tan(dx+c)) + a^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(d*x + c)*a^2 + a^2*log(sec(d*x + c) + tan(d*x + c)) + a^2*sin(d*x + c))
/d
```

mupad [B] time = 0.40, size = 33, normalized size = 0.97

$$2\,a^2\,x + \frac{a^2\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] 2*a^2*x + (a^2*(2*atanh(tan(c/2 + (d*x)/2)) + sin(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cos(c+dx)\sec(c+dx)\,dx + \int \cos^2(c+dx)\sec(c+dx)\,dx + \int \sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)**2*s
ec(c + d*x), x) + Integral(sec(c + d*x), x))
```

3.19 $\int (a + a\cos(c + dx))^2 \sec^2(c + dx)\, dx$

Optimal. Leaf size=34

$$\frac{a^2\tan(c+dx)}{d} + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} + a^2x$$

```
[Out] a^2*x+2*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 3770, 3767, 8}

$$\frac{a^2\tan(c+dx)}{d} + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} + a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx &= \int \left(a^2+2a^2\sec(c+dx)+a^2\sec^2(c+dx)\right)\,dx \\
&= a^2x+a^2\int \sec^2(c+dx)\,dx+\left(2a^2\right)\int \sec(c+dx)\,dx \\
&= a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\,\mathrm{Subst}(\int 1\,dx,x,-\tan(c+dx))}{d} \\
&= a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 28, normalized size = 0.82

$$
a^2\left(\frac{\tan(c+dx)}{d}+\frac{2\tanh^{-1}(\sin(c+dx))}{d}+x\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] a^2*(x + (2*ArcTanh[Sin[c + d*x]])/d + Tan[c + d*x]/d)
```

fricas [**B**] time = 1.61, size = 76, normalized size = 2.24

$$
\frac{a^2dx\cos(dx+c)+a^2\cos(dx+c)\log(\sin(dx+c)+1)-a^2\cos(dx+c)\log(-\sin(dx+c)+1)+a^2\sin(dx+c}{d\cos(dx+c)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d*
x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

giac [**B**] time = 0.58, size = 79, normalized size = 2.32

$$
\frac{(dx+c)a^2+2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

maple [A] time = 0.11, size = 50, normalized size = 1.47

$$a^2x + \frac{2a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^2\tan(dx+c)}{d} + \frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x)
```

```
[Out] a^2*x+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*tan(d*x+c)/d+1/d*a^2*c
```

maxima [A] time = 0.99, size = 49, normalized size = 1.44

$$\frac{(dx+c)a^2 + a^2\Big(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\Big) + a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + a^2*
tan(d*x + c))/d
```

mupad [B] time = 0.39, size = 56, normalized size = 1.65

$$a^2\,x + \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] a^2*x + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2))/(d
*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cos(c+dx)\sec^2(c+dx)\,dx + \int \cos^2(c+dx)\sec^2(c+dx)\,dx + \int \sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

3.20 $\int (a + a\cos(c + dx))^2 \sec^3(c + dx)\, dx$

Optimal. Leaf size=54

$$\frac{2a^2\tan(c+dx)}{d} + \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 3/2*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*tan(d*x
+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{2a^2\tan(c+dx)}{d} + \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 \sec^3(c+dx)\,dx = \int \left(a^2\sec(c+dx) + 2a^2\sec^2(c+dx) + a^2\sec^3(c+dx)\right)\,dx$$
$$= a^2\int \sec(c+dx)\,dx + a^2\int \sec^3(c+dx)\,dx + \left(2a^2\right)\int \sec^2(c+dx)\,dx$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a^2\int \sec(c+d$$
$$= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{2a^2\tan(c+dx)}{d} + \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

fricas [A] time = 1.34, size = 83, normalized size = 1.54

$$\frac{3\,a^2\cos(dx+c)^2\log(\sin(dx+c)+1) - 3\,a^2\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\left(4\,a^2\cos(dx+c)+a^2\right)\sin}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log(
-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x
+ c)^2)
```

giac [A] time = 0.76, size = 90, normalized size = 1.67

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.13, size = 58, normalized size = 1.07

$$\frac{3a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{2a^2\tan\left(dx+c\right)}{d}+\frac{a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x)
```

```
[Out] 3/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*t
an(d*x+c)/d
```

maxima [A] time = 0.45, size = 88, normalized size = 1.63

$$-\frac{a^2\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,a^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 8*a^2*tan(d*x + c))/d
```

mupad [B] time = 0.71, size = 83, normalized size = 1.54

$$\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (3*a^2*tan(c/2 + (d*x)/2)^3 - 5*a^2*t
an(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cos(c+dx)\sec^3(c+dx)\,dx + \int \cos^2(c+dx)\sec^3(c+dx)\,dx + \int \sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(cos(c + d*x)**
2*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x))
```

3.21 $\int (a + a\cos(c + dx))^2 \sec^4(c + dx)\, dx$

Optimal. Leaf size=66

$$\frac{a^2\tan^3(c+dx)}{3d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

```
[Out] a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d+1/
3*a^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3767, 8, 3768, 3770}

$$\frac{a^2\tan^3(c+dx)}{3d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*
Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sec^4(c+dx)\,dx = \int \left(a^2\sec^2(c+dx)+2a^2\sec^3(c+dx)+a^2\sec^4(c+dx)\right)dx$$
$$= a^2\int \sec^2(c+dx)\,dx + a^2\int \sec^4(c+dx)\,dx + \left(2a^2\right)\int \sec^3(c+dx)\,dx$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + a^2\int \sec(c+dx)\,dx - \frac{a^2\,\mathrm{Subst}(\int 1\,dx, x, -$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d}$$

Mathematica [B] time = 5.78, size = 162, normalized size = 2.45

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-2\tan(c)\cos(c+dx)-\sec(c)(-4\sin(2c+dx)+3\sin(c+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

```
[Out] -1/48*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3*(12*Cos[c
 + d*x]^3*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) - Sec[c]*(13*Sin[d*x] - 4*Sin[2*c + d*x] + 3*Sin[c + 2
*d*x] + 3*Sin[3*c + 2*d*x] + 5*Sin[2*c + 3*d*x]) - 2*Cos[c + d*x]*Tan[c]))/
d
```

fricas [A] time = 1.63, size = 96, normalized size = 1.45

$$\frac{3\,a^2\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,a^2\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(5\,a^2\cos(dx+c)^2+3\,a^2\,\mathrm{c}\right.}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*si
n(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.64, size = 106, normalized size = 1.61

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2*c
)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.13, size = 78, normalized size = 1.18

$$\frac{5a^2\tan(dx+c)}{3d}+\frac{a^2\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\left(\sec^2(dx+c)\right)\tan(dx+c}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x)
```

```
[Out] 5/3*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(
d*x+c))+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

maxima [A] time = 0.35, size = 85, normalized size = 1.29

$$\frac{2\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^2-3\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6\,a^2\,\mathrm{ta}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan
(d*x + c))/d
```

mupad [B] time = 2.01, size = 112, normalized size = 1.70

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{16\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+6\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^5 - (16*a^2
*tan(c/2 + (d*x)/2)^3)/3 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/
2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\cos\left(c+dx\right)\sec^{4}\left(c+dx\right)\,dx+\int\cos^{2}\left(c+dx\right)\sec^{4}\left(c+dx\right)\,dx+\int\sec^{4}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**4,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**
2*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

3.22 $\int (a + a\cos(c + dx))^2 \sec^5(c + dx)\, dx$

Optimal. Leaf size=96

$$\frac{2a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{7a^2\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] 7/8*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+7/8*a^2*sec(d*x+c)*tan(d*x
+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 3768, 3770, 3767}

$$\frac{2a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{7a^2\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*Tan[c + d*x])/d + (7*a^2*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*
a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx &= \int \left(a^2\sec^3(c+dx)+2a^2\sec^4(c+dx)+a^2\sec^5(c+dx)\right)\,dx\\
&= a^2\int \sec^3(c+dx)\,dx+a^2\int \sec^5(c+dx)\,dx+\left(2a^2\right)\int \sec^4(c+dx)\,dx\\
&= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{2}a^2\int \sec(c\\
&= \frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\sec(c+dx)\tan(c+dx)}{8d}\\
&= \frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\sec(c+dx)\tan(c+dx}{8d}
\end{aligned}$$

Mathematica [B] time = 6.43, size = 797, normalized size = 8.30

$$-\frac{7(\cos(c+dx)a+a)^2\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d}+\frac{7(\cos(c+dx)a+a)^2\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\right.}{32d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

```
[Out] (-7*(a + a*Cos[c + d*x])^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec
[c/2 + (d*x)/2]^4)/(32*d) + (7*(a + a*Cos[c + d*x])^2*Log[Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4)/(32*d) + ((a + a*Cos[c + d*x]
)^2*Sec[c/2 + (d*x)/2]^4)/(64*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4
) + ((a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(12*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[
c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a + a*Cos[
c + d*x])^2*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(3*d*(Cos[c/2] - Sin[c/2])*(
Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - ((a + a*Cos[c + d*x])^2*Sec[c/2
 + (d*x)/2]^4)/(64*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + ((a + a
*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c
/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^2
*Sec[c/2 + (d*x)/2]^4*(-29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c/2] + Sin[
c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^
```

```
2*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(3*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 1.89, size = 111, normalized size = 1.16

$$\frac{21\,a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-21\,a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(32\,a^2\cos\left(dx+c\right)^3+2\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(21*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 21*a^2*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(32*a^2*cos(d*x + c)^3 + 21*a^2*cos(d*x + c)^2 +
16*a^2*cos(d*x + c) + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.68, size = 122, normalized size = 1.27

$$\frac{21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+83\,a^2\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(21*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 21*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*a^2*tan(1/2*d*x +
1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.14, size = 102, normalized size = 1.06

$$\frac{7a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{7a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{4a^2\tan\left(dx+c\right)}{3d}+\frac{2a^2\left(\sec^2\left(dx+c\right)\right)\tan\left(dx\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x)
```

```
[Out] 7/8*a^2*sec(d*x+c)*tan(d*x+c)/d+7/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+4/3*a^2
*tan(d*x+c)/d+2/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*
x+c)/d
```

maxima [A] time = 0.31, size = 145, normalized size = 1.51

$$\frac{32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{2}-3\,a^{2}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.35, size = 141, normalized size = 1.47

$$\frac{7\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{77\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{83\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{25\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] (7*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((83*a^2*tan(c/2 + (d*x)/2)^3)/12
 - (77*a^2*tan(c/2 + (d*x)/2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 - (25*
a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.23 $\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=129

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx}{24d}$$

```
[Out] 23/16*a^3*x+4*a^3*sin(d*x+c)/d+23/16*a^3*cos(d*x+c)*sin(d*x+c)/d+23/24*a^3*
cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-7/3*a^3*sin(d*x
+c)^3/d+3/5*a^3*sin(d*x+c)^5/d
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2633, 2635, 8}

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^5
)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx = \int \left(a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+3a^3\cos^5(c+dx)+a^3\cos^6(c+\right.$$
$$= a^3\int \cos^3(c+dx)\,dx + a^3\int \cos^6(c+dx)\,dx + \left(3a^3\right)\int \cos^4(c+dx)\,dx$$
$$= \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}\left(5a^3\right)\int$$
$$= \frac{4a^3\sin(c+dx)}{d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{23a^3\cos^3(c+dx)\sin(c}{24d}$$
$$= \frac{9a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{23a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{23a^3\cos^3(c+}{2}$$
$$= \frac{23a^3x}{16} + \frac{4a^3\sin(c+dx)}{d} + \frac{23a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{23a^3\cos^3(c}{}$$

Mathematica [A] time = 0.18, size = 73, normalized size = 0.57

$$\frac{a^3(2520\sin(c+dx)+945\sin(2(c+dx))+380\sin(3(c+dx))+135\sin(4(c+dx))+36\sin(5(c+dx))+5\sin(6(}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c +
d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(
960*d)
```

fricas [A] time = 0.88, size = 89, normalized size = 0.69

$$\frac{345\,a^3dx + \left(40\,a^3\cos\left(dx+c\right)^5 + 144\,a^3\cos\left(dx+c\right)^4 + 230\,a^3\cos\left(dx+c\right)^3 + 272\,a^3\cos\left(dx+c\right)^2 + 345\,a^3\cos\left(\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^3*cos(d*x + c)^4 + 230*
a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*a^
3)*sin(d*x + c))/d
```

giac [A] time = 0.62, size = 106, normalized size = 0.82

$$\frac{23}{16}a^3x+\frac{a^3\sin(6dx+6c)}{192d}+\frac{3a^3\sin(5dx+5c)}{80d}+\frac{9a^3\sin(4dx+4c)}{64d}+\frac{19a^3\sin(3dx+3c)}{48d}+\frac{63a^3\sin(2dx+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 23/16*a^3*x + 1/192*a^3*sin(6*d*x + 6*c)/d + 3/80*a^3*sin(5*d*x + 5*c)/d +
9/64*a^3*sin(4*d*x + 4*c)/d + 19/48*a^3*sin(3*d*x + 3*c)/d + 63/64*a^3*sin(
2*d*x + 2*c)/d + 21/8*a^3*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 143, normalized size = 1.11

$$\frac{a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3c}{}\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^3*(2+co
s(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 1.97, size = 143, normalized size = 1.11

$$\frac{192\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

mupad [B] time = 2.87, size = 121, normalized size = 0.94

$$\frac{23\,a^3\,x}{16}+\frac{\frac{23\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{391\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{759\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{969\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{211\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8}+\frac{105\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (23*a^3*x)/16 + ((211*a^3*tan(c/2 + (d*x)/2)^3)/8 + (969*a^3*tan(c/2 + (d*x
)/2)^5)/20 + (759*a^3*tan(c/2 + (d*x)/2)^7)/20 + (391*a^3*tan(c/2 + (d*x)/2
)^9)/24 + (23*a^3*tan(c/2 + (d*x)/2)^11)/8 + (105*a^3*tan(c/2 + (d*x)/2))/8
)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 3.65, size = 379, normalized size = 2.94

$$\begin{cases} \frac{5a^3x\sin^6(c+dx)}{16}+\frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{9a^3x\sin^4(c+dx)}{8}+\frac{15a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{9a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{5a^3x\,c \\ x\left(a\cos(c)+a\right)^3\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(c + d*x)**6/16 + 15*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 9*a**3*x*sin(c + d*x)**4/8 + 15*a**3*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*a**3*x*cos(c +
 d*x)**6/16 + 9*a**3*x*cos(c + d*x)**4/8 + 5*a**3*sin(c + d*x)**5*cos(c + d
*x)/(16*d) + 8*a**3*sin(c + d*x)**5/(5*d) + 5*a**3*sin(c + d*x)**3*cos(c +
d*x)**3/(6*d) + 4*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*a**3*sin(c + d
*x)**3*cos(c + d*x)/(8*d) + 2*a**3*sin(c + d*x)**3/(3*d) + 11*a**3*sin(c +
d*x)*cos(c + d*x)**5/(16*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*a*
*3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d
, Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c)**3, True))
```

3.24 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=105

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+$$

```
[Out] 13/8*a^3*x+4*a^3*sin(d*x+c)/d+13/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/4*a^3*cos(
d*x+c)^3*sin(d*x+c)/d-5/3*a^3*sin(d*x+c)^3/d+1/5*a^3*sin(d*x+c)^5/d
```

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2635, 8, 2633}

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3)/
(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx = \int \left(a^3\cos^2(c+dx)+3a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+a^3\cos^5(c+\right.$$

$$= a^3\int \cos^2(c+dx)\,dx + a^3\int \cos^5(c+dx)\,dx + \left(3a^3\right)\int \cos^3(c+dx)\,dx$$

$$= \frac{a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{2}a^3\int 1\,dx$$

$$= \frac{a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+dx}{4d}$$

$$= \frac{13a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+}{}$$

Mathematica [A] time = 0.13, size = 63, normalized size = 0.60

$$\frac{a^3(1380\sin(c+dx)+480\sin(2(c+dx))+170\sin(3(c+dx))+45\sin(4(c+dx))+6\sin(5(c+dx))+780dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c + d
*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

fricas [A] time = 2.16, size = 76, normalized size = 0.72

$$\frac{195\,a^3dx + \left(24\,a^3\cos\left(dx+c\right)^4 + 90\,a^3\cos\left(dx+c\right)^3 + 152\,a^3\cos\left(dx+c\right)^2 + 195\,a^3\cos\left(dx+c\right) + 304\,a^3\right)\sin\left(dx\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*a
^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.79, size = 88, normalized size = 0.84

$$\frac{13}{8}a^3x + \frac{a^3\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{3\,a^3\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{17\,a^3\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{a^3\sin\left(2\,dx+2\,c\right)}{d} + \frac{23\,a^3\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 13/8*a^3*x + 1/80*a^3*sin(5*d*x + 5*c)/d + 3/32*a^3*sin(4*d*x + 4*c)/d + 17
/48*a^3*sin(3*d*x + 3*c)/d + a^3*sin(2*d*x + 2*c)/d + 23/8*a^3*sin(d*x + c)
/d
```

maple [A] time = 0.05, size = 121, normalized size = 1.15

$$\frac{\frac{a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5} + 3a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + a^3\left(2+\cos^2(dx+c)\right)\sin(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(2+cos(d*x+c)^2)*si
n(d*x+c)+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.63, size = 117, normalized size = 1.11

$$\frac{32\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a^3 - 480\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a^3 + 45\,(12\,dx +}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 48
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3)/d
```

mupad [B] time = 3.71, size = 105, normalized size = 1.00

$$\frac{13\,a^3\,x}{8}+\frac{\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{91\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{133\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (13*a^3*x)/8 + ((133*a^3*tan(c/2 + (d*x)/2)^3)/6 + (416*a^3*tan(c/2 + (d*x)
/2)^5)/15 + (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (13*a^3*tan(c/2 + (d*x)/2)^9)
/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 2.08, size = 272, normalized size = 2.59

$$\begin{cases} \frac{9a^3x\sin^4(c+dx)}{8} + \frac{9a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^3x\sin^2(c+dx)}{2} + \frac{9a^3x\cos^4(c+dx)}{8} + \frac{a^3x\cos^2(c+dx)}{2} + \frac{8a^3\sin^5(c+dx)}{15d} + \frac{4a^3\sin^3(c+dx)}{3d} \\ x(a\cos(c)+a)^3\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + a**3*x*sin(c + d*x)**2/2 + 9*a**3*x*cos(c + d*x)**4/8 + a**3*x*co
s(c + d*x)**2/2 + 8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*co
s(c + d*x)**2/(3*d) + 9*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3*si
n(c + d*x)**3/d + a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**3*sin(c + d*x
)*cos(c + d*x)**3/(8*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**2/d + a**3*sin(
c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c)**2, Tru
e))
```

3.25 $\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=85

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

```
[Out] 15/8*a^3*x+4*a^3*sin(d*x+c)/d+15/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos(
d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3\sin^3(c+dx)}{4d}+\frac{3a^3\sin(c+dx)}{d}+\frac{9a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}+\frac{\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (15*a^3*x)/8 + (3*a^3*Sin[c + d*x])/d + (9*a^3*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + ((a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/(
4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx &= \frac{(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{3}{4}\int(a+a\cos(c+dx))^3\,dx\\
&= \frac{(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{3}{4}\int\left(a^3+3a^3\cos(c+dx)+3a^3\cos^2(\right.\\
&= \frac{3a^3x}{4}+\frac{(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\left(3a^3\right)\int\cos^3(c+dx)\,dx+\frac{1}{4}\\
&= \frac{3a^3x}{4}+\frac{9a^3\sin(c+dx)}{4d}+\frac{9a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{(a+a\cos(c+d}{4d}\\
&= \frac{15a^3x}{8}+\frac{3a^3\sin(c+dx)}{d}+\frac{9a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{(a+a\cos(c+}{4}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 51, normalized size = 0.60

$$
\frac{a^3(104\sin(c+dx)+32\sin(2(c+dx))+8\sin(3(c+dx))+\sin(4(c+dx))+60dx)}{32d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]
+ Sin[4*(c + d*x)]))/(32*d)
```

fricas [A] time = 1.53, size = 63, normalized size = 0.74

$$
\frac{15\,a^3dx+\left(2\,a^3\cos\left(dx+c\right)^3+8\,a^3\cos\left(dx+c\right)^2+15\,a^3\cos\left(dx+c\right)+24\,a^3\right)\sin\left(dx+c\right)}{8\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(15*a^3*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 15*a^3*cos
(d*x + c) + 24*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.40, size = 71, normalized size = 0.84

$$\frac{15}{8}\,a^3x + \frac{a^3\sin\left(4\,dx + 4\,c\right)}{32\,d} + \frac{a^3\sin\left(3\,dx + 3\,c\right)}{4\,d} + \frac{a^3\sin\left(2\,dx + 2\,c\right)}{d} + \frac{13\,a^3\sin\left(dx + c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 15/8*a^3*x + 1/32*a^3*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(3*d*x + 3*c)/d + a^3
*sin(2*d*x + 2*c)/d + 13/4*a^3*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 100, normalized size = 1.18

$$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + a^3\left(2+\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right) + 3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(
2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
a^3*sin(d*x+c))
```

maxima [A] time = 0.84, size = 94, normalized size = 1.11

$$-\frac{32\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^3 - \left(12\,dx + 12\,c + \sin\left(4\,dx + 4\,c\right) + 8\,\sin\left(2\,dx + 2\,c\right)\right)a^3 - 24\left(2\,dx + 2\,c + \cdots\right.}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^
3 - 32*a^3*sin(d*x + c))/d
```

mupad [B] time = 3.49, size = 89, normalized size = 1.05

$$\frac{15\,a^3\,x}{8}+\frac{\frac{15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{55\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{49\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (15*a^3*x)/8 + ((73*a^3*tan(c/2 + (d*x)/2)^3)/4 + (55*a^3*tan(c/2 + (d*x)/2
)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 + (49*a^3*tan(c/2 + (d*x)/2))/4)/(
d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 1.03, size = 224, normalized size = 2.64

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8}+\frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^3x\sin^2(c+dx)}{2}+\frac{3a^3x\cos^4(c+dx)}{8}+\frac{3a^3x\cos^2(c+dx)}{2}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d}+ \frac{2 \\ x\left(a\cos(c)+a\right)^3\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*
x*cos(c + d*x)**2/2 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3*si
n(c + d*x)**3/d + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*a**3*sin(c
+ d*x)*cos(c + d*x)**2/d + 3*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + a**3*si
n(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c), True))
```

3.26 $\int (a + a\cos(c + dx))^3 \, dx$

Optimal. Leaf size=63

$$-\frac{a^3 \sin^3(c+dx)}{3d} + \frac{4a^3 \sin(c+dx)}{d} + \frac{3a^3 \sin(c+dx)\cos(c+dx)}{2d} + \frac{5a^3 x}{2}$$

```
[Out] 5/2*a^3*x+4*a^3*sin(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d-1/3*a^3*sin(d*
x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2637, 2635, 8, 2633}

$$-\frac{a^3 \sin^3(c+dx)}{3d} + \frac{4a^3 \sin(c+dx)}{d} + \frac{3a^3 \sin(c+dx)\cos(c+dx)}{2d} + \frac{5a^3 x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2
*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^3\,dx &= \int \left(a^3+3a^3\cos(c+dx)+3a^3\cos^2(c+dx)+a^3\cos^3(c+dx)\right)\,dx\\ &= a^3x+a^3\int \cos^3(c+dx)\,dx+\left(3a^3\right)\int \cos(c+dx)\,dx+\left(3a^3\right)\int \cos^2(c+dx)\,dx\\ &= a^3x+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int 1\,dx-\frac{a^3\,\mathrm{Subst}\left(\int\right.}{}\\ &= \frac{5a^3x}{2}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 44, normalized size = 0.70

$$\frac{a^3(45\sin(c+dx)+9\sin(2(c+dx))+\sin(3(c+dx))+30c+30dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(30*c + 30*d*x + 45*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + Sin[3*(c + d*x
)]))/(12*d)
```

fricas [A] time = 1.97, size = 50, normalized size = 0.79

$$\frac{15\,a^3dx+\left(2\,a^3\cos\left(dx+c\right)^2+9\,a^3\cos\left(dx+c\right)+22\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin(
d*x + c))/d
```

giac [A] time = 0.43, size = 55, normalized size = 0.87

$$\frac{5}{2}\,a^3x+\frac{a^3\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{3\,a^3\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{15\,a^3\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*x + 1/12*a^3*sin(3*d*x + 3*c)/d + 3/4*a^3*sin(2*d*x + 2*c)/d + 15/4
*a^3*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 74, normalized size = 1.17

$$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3\sin(dx+c)+a^3(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+3*a^3*sin(d*x+c)+a^3*(d*x+c))
```

maxima [A] time = 0.36, size = 70, normalized size = 1.11

$$a^3x-\frac{\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3}{3d}+\frac{3(2dx+2c+\sin(2dx+2c))a^3}{4d}+\frac{3a^3\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c + si
n(2*d*x + 2*c))*a^3/d + 3*a^3*sin(d*x + c)/d
```

mupad [B] time = 0.40, size = 63, normalized size = 1.00

$$\frac{5a^3x}{2}+\frac{11a^3\sin(c+dx)}{3d}+\frac{a^3\cos(c+dx)^2\sin(c+dx)}{3d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3,x)
```

```
[Out] (5*a^3*x)/2 + (11*a^3*sin(c + d*x))/(3*d) + (a^3*cos(c + d*x)^2*sin(c + d*x
))/(3*d) + (3*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

sympy [A] time = 0.50, size = 121, normalized size = 1.92

$$\begin{cases}\frac{3a^3x\sin^2(c+dx)}{2}+\frac{3a^3x\cos^2(c+dx)}{2}+a^3x+\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d} \\ x(a\cos(c)+a)^3\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*x
 + 2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*a
**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*a**3*sin(c + d*x)/d, Ne(d, 0)), (x*
(a*cos(c) + a)**3, True))
```

3.27 $\int (a + a\cos(c + dx))^3 \sec(c + dx)\, dx$

Optimal. Leaf size=59

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

```
[Out] 7/2*a^3*x+a^3*arctanh(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*cos(d*x+c)*s
in(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2757, 2637, 2635, 8, 3770}

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x],x]
```

```
[Out] (7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
```

```
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^3 \sec(c + dx)\, dx = \int \left(3a^3 + 3a^3\cos(c + dx) + a^3\cos^2(c + dx) + a^3\sec(c + dx)\right) dx$$
$$= 3a^3x + a^3 \int \cos^2(c + dx)\, dx + a^3 \int \sec(c + dx)\, dx + \left(3a^3\right) \int \cos(c + dx$$
$$= 3a^3x + \frac{a^3\tanh^{-1}(\sin(c + dx))}{d} + \frac{3a^3\sin(c + dx)}{d} + \frac{a^3\cos(c + dx)\sin(c +}{2d}$$
$$= \frac{7a^3x}{2} + \frac{a^3\tanh^{-1}(\sin(c + dx))}{d} + \frac{3a^3\sin(c + dx)}{d} + \frac{a^3\cos(c + dx)\sin(c +}{2d}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 1.37

$$\frac{a^3\left(12\sin(c + dx) + \sin(2(c + dx)) - 4\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) + 4\log\left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(\right.\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x],x]
```

```
[Out] (a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 1.64, size = 65, normalized size = 1.10

$$\frac{7\,a^3dx + a^3\log\left(\sin\left(dx + c\right) + 1\right) - a^3\log\left(-\sin\left(dx + c\right) + 1\right) + \left(a^3\cos\left(dx + c\right) + 6\,a^3\right)\sin\left(dx + c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(7*a^3*d*x + a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (
a^3*cos(d*x + c) + 6*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.64, size = 100, normalized size = 1.69

$$\frac{7\,(dx+c)a^3 + 2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(7*(d*x + c)*a^3 + 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.09, size = 72, normalized size = 1.22

$$\frac{a^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{7a^3x}{2} + \frac{7a^3c}{2d} + \frac{3a^3\sin(dx+c)}{d} + \frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c),x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+7/2*a^3*x+7/2/d*a^3*c+3*a^3*sin(d*x+c)/d+1/
d*a^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 1.05, size = 67, normalized size = 1.14

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3 + 12\,(dx+c)a^3 + 4\,a^3\log(\sec(dx+c)+\tan(dx+c)) + 12\,a^3\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 4*a^3*log(se
c(d*x + c) + tan(d*x + c)) + 12*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.44, size = 88, normalized size = 1.49

$$\frac{7\,a^3\,x}{2} + \frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (7*a^3*x)/2 + (2*a^3*atanh(tan(c/2 + (d*x)/2)))/d + (5*a^3*tan(c/2 + (d*x)/
2)^3 + 7*a^3*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*
x)/2)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int \cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int \sec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)**2
*sec(c + d*x), x) + Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(se
c(c + d*x), x))
```

3.28 $\int (a+a\cos(c+dx))^3 \sec^2(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+3a^3x$$

```
[Out] 3*a^3*x+3*a^3*arctanh(sin(d*x+c))/d+a^3*sin(d*x+c)/d+a^3*tan(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 2637, 3770, 3767, 8}

$$\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Tan
[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx = \int \left(3a^3+a^3\cos(c+dx)+3a^3\sec(c+dx)+a^3\sec^2(c+dx)\right)dx$$
$$= 3a^3x+a^3\int \cos(c+dx)\,dx+a^3\int \sec^2(c+dx)\,dx+\left(3a^3\right)\int \sec(c+d$$
$$= 3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\,\text{Subst}(\int 1\,dx,x,-}{d}$$
$$= 3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}$$

Mathematica [B] time = 0.73, size = 211, normalized size = 4.40

$$\frac{1}{8}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(c)\cos(dx)}{d}+\frac{\cos(c)\sin(dx)}{d}+\frac{\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(3*x - (3*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]])/d + (3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d +
(Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d + Sin[(d*x)/2]/(d*(Cos[c/2] - Sin
[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(d*x)/2]/(d*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/8
```

fricas [A] time = 0.80, size = 91, normalized size = 1.90

$$\frac{6\,a^3dx\cos(dx+c)+3\,a^3\cos(dx+c)\log(\sin(dx+c)+1)-3\,a^3\cos(dx+c)\log(-\sin(dx+c)+1)+2\left(a^3\cos\right.}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*
a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(d*
x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.61, size = 80, normalized size = 1.67

$$\frac{3\,(dx+c)a^3+3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^4 - 1))/d
```

maple [A] time = 0.14, size = 65, normalized size = 1.35

$$3a^3x+\frac{3a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a^3\sin\left(dx+c\right)}{d}+\frac{a^3\tan\left(dx+c\right)}{d}+\frac{3a^3c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x)
```

```
[Out] 3*a^3*x+3/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*sin(d*x+c)/d+a^3*tan(d*x+c)/d
+3/d*a^3*c
```

maxima [A] time = 1.00, size = 64, normalized size = 1.33

$$\frac{6\,(dx+c)a^3+3\,a^3\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+2\,a^3\sin\left(dx+c\right)+2\,a^3\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

mupad [B] time = 0.41, size = 57, normalized size = 1.19

$$3\,a^3\,x+\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] 3*a^3*x + (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (4*a^3*tan(c/2 + (d*x)/2))/
(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int 3\cos^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int \cos^{3}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(3*cos(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**2, x) + In
tegral(sec(c + d*x)**2, x))
```

3.29 $\int (a + a\cos(c+dx))^3 \sec^3(c+dx)\, dx$

Optimal. Leaf size=59

$$\frac{3a^3\tan(c+dx)}{d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}+a^3x$$

```
[Out] a^3*x+7/2*a^3*arctanh(sin(d*x+c))/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*t
an(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{3a^3\tan(c+dx)}{d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

```
[Out] a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^3
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 \sec^3(c+dx)\,dx = \int \left(a^3 + 3a^3\sec(c+dx) + 3a^3\sec^2(c+dx) + a^3\sec^3(c+dx)\right) dx$$
$$= a^3x + a^3\int \sec^3(c+dx)\,dx + \left(3a^3\right)\int \sec(c+dx)\,dx + \left(3a^3\right)\int \sec^2(c +$$
$$= a^3x + \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a^3\int \text{se}$$
$$= a^3x + \frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c}{2d}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.85

$$a^3\left(\frac{3\tan(c+dx)}{d} + \frac{7\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\tan(c+dx)\sec(c+dx)}{2d} + x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

```
[Out] a^3*(x + (7*ArcTanh[Sin[c + d*x]])/(2*d) + (3*Tan[c + d*x])/d + (Sec[c + d*
x]*Tan[c + d*x])/(2*d))
```

fricas [A] time = 1.05, size = 98, normalized size = 1.66

$$\frac{4\,a^3dx\cos(dx+c)^2 + 7\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1) - 7\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\left(6\,a\right.}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
- 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a^3
)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.85, size = 100, normalized size = 1.69

$$\frac{2\,(dx+c)a^3 + 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 7\,a^3\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.12, size = 71, normalized size = 1.20

$$a^3x + \frac{a^3c}{d} + \frac{7a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d} + \frac{3a^3\tan\left(dx+c\right)}{d} + \frac{a^3\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x)
```

```
[Out] a^3*x+1/d*a^3*c+7/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)/d+1/2*
a^3*sec(d*x+c)*tan(d*x+c)/d
```

maxima [A] time = 1.11, size = 99, normalized size = 1.68

$$\frac{4\,(dx+c)a^3 - a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 6\,a^3\left(\log\left(\sin\left(dx+c\right)+1\right) - \log\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*a^3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 12*a^3*tan(d*x + c))/d
```

mupad [B] time = 0.44, size = 88, normalized size = 1.49

$$a^3\,x + \frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] a^3*x + (7*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^3 -
 7*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^
2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cos(c+dx)\sec^3(c+dx)\,dx+\int 3\cos^2(c+dx)\sec^3(c+dx)\,dx+\int \cos^3(c+dx)\sec^3(c+dx)\,dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**3,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(3*cos(c + d*x)
**2*sec(c + d*x)**3, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(sec(c + d*x)**3, x))
```

3.30 $\int (a + a\cos(c+dx))^3 \sec^4(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 5/2*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c)*tan(d*x
+c)/d+1/3*a^3*tan(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^3 \sec^4(c + dx)\,dx = \int \left(a^3 \sec(c + dx) + 3a^3 \sec^2(c + dx) + 3a^3 \sec^3(c + dx) + a^3 \sec^4(c + d\right.$$

$$= a^3 \int \sec(c + dx)\,dx + a^3 \int \sec^4(c + dx)\,dx + \left(3a^3\right) \int \sec^2(c + dx)\,dx +$$

$$= \frac{a^3 \tanh^{-1}(\sin(c + dx))}{d} + \frac{3a^3 \sec(c + dx)\tan(c + dx)}{2d} + \frac{1}{2}\left(3a^3\right) \int \sec($$

$$= \frac{5a^3 \tanh^{-1}(\sin(c + dx))}{2d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{3a^3 \sec(c + dx)\tan(c + dx}{2d}$$

Mathematica [B] time = 5.40, size = 154, normalized size = 2.14

$$-\frac{a^3 \sec^6\left(\frac{1}{2}(c + dx)\right)(\sec(c + dx) + 1)^3 \left(-4\tan(c)\cos(c + dx) - \sec(c)(-20\sin(2c + dx) + 9\sin(c + 2dx) + 9\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

```
[Out] -1/192*(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(60*Cos[c + d*x]^3*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) - Sec[c]*(50*Sin[d*x] - 20*Sin[2*c + d*x] + 9*Sin[c + 2*d*x] + 9*Sin
[3*c + 2*d*x] + 22*Sin[2*c + 3*d*x]) - 4*Cos[c + d*x]*Tan[c]))/d
```

fricas [A] time = 0.66, size = 98, normalized size = 1.36

$$\frac{15\,a^3 \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 15\,a^3 \cos(dx + c)^3 \log(-\sin(dx + c) + 1) + 2\left(22\,a^3 \cos(dx + c)^2 + 9\right.}{12\,d \cos(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*l
og(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2*a
^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.60, size = 106, normalized size = 1.47

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x + 1
/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.12, size = 80, normalized size = 1.11

$$\frac{5a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{11a^3\tan\left(dx+c\right)}{3d}+\frac{3a^3\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a^3\tan\left(dx+c\right)\left(\sec^2\left(dx+\right.\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x)
```

```
[Out] 5/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c
)*tan(d*x+c)/d+1/3/d*a^3*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.31, size = 111, normalized size = 1.54

$$\frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-9\,a^3\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,a^3\left(\mathrm{lo}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*a^3*tan(d*x + c))/d
```

mupad [B] time = 2.04, size = 112, normalized size = 1.56

$$\frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{40\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^5 - (40*a^3
*tan(c/2 + (d*x)/2)^3)/3 + 11*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)
/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.31 $\int (a + a\cos(c + dx))^3 \sec^5(c + dx)\,dx$

Optimal. Leaf size=93

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\sec}{8d}$$

```
[Out] 15/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(d
*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3767, 8, 3768, 3770}

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
a^3*Tan[c + d*x]^3)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx = \int \left(a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+a^3\sec^5(c+
$$
$$= a^3\int \sec^2(c+dx)\,dx + a^3\int \sec^5(c+dx)\,dx + \left(3a^3\right)\int \sec^3(c+dx)\,dx
$$
$$= \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(3a^3\right)\int
$$
$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+d
$$
$$= \frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+
$$

Mathematica [B] time = 6.39, size = 797, normalized size = 8.57

$$-\frac{15(\cos(c+dx)a+a)^3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d}+\frac{15(\cos(c+dx)a+a)^3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{64d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

```
[Out] (-15*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Se
c[c/2 + (d*x)/2]^6)/(64*d) + (15*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(64*d) + ((a + a*Cos[c + d*
x])^3*Sec[c/2 + (d*x)/2]^6)/(128*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)^4) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(16*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*C
os[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (3*(a + a
*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(8*d*(Cos[c/2] - Sin[c/
2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - ((a + a*Cos[c + d*x])^3*Se
```

```
c[c/2 + (d*x)/2]^6)/(128*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + (
(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(16*d*(Cos[c/2] +
 Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d
*x])^3*Sec[c/2 + (d*x)/2]^6*(-19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Cos[c/2]
+ Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (3*(a + a*Cos[c
+ d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(8*d*(Cos[c/2] + Sin[c/2])*(Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 1.03, size = 111, normalized size = 1.19

$$\frac{15\,a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-15\,a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,a^3\cos\left(dx+c\right)^3+\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2 +
8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.83, size = 122, normalized size = 1.31

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a^3\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x + 1
/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.13, size = 101, normalized size = 1.09

$$\frac{3a^3\tan\left(dx+c\right)}{d}+\frac{15a^3\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{15a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{a^3\tan\left(dx+c\right)\left(\sec^2\left(dx\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x)
```

```
[Out] 3*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(d*x+c)/d+15/8/d*a^3*ln(sec(d*x+c
)+tan(d*x+c))+1/d*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4*a^3*sec(d*x+c)^3*tan(d*x+
c)/d
```

maxima [A] time = 1.10, size = 156, normalized size = 1.68

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{3}-a^{3}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)\right.\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x + c
))/d
```

mupad [B] time = 3.29, size = 141, normalized size = 1.52

$$\frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{55\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{49\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] (15*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((73*a^3*tan(c/2 + (d*x)/2)^3)/4
 - (55*a^3*tan(c/2 + (d*x)/2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 - (49*
a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.32 $\int (a + a\cos(c + dx))^3 \sec^6(c + dx)\, dx$

Optimal. Leaf size=114

$$\frac{a^3 \tan^5(c+dx)}{5d}+\frac{5a^3 \tan^3(c+dx)}{3d}+\frac{4a^3 \tan(c+dx)}{d}+\frac{13a^3 \tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3 \tan(c+dx)\sec^3(c+dx)}{4d}+$$

```
[Out] 13/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+13/8*a^3*sec(d*x+c)*tan(d
*x+c)/d+3/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/3*a^3*tan(d*x+c)^3/d+1/5*a^3*ta
n(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 3768, 3770, 3767}

$$\frac{a^3 \tan^5(c+dx)}{5d}+\frac{5a^3 \tan^3(c+dx)}{3d}+\frac{4a^3 \tan(c+dx)}{d}+\frac{13a^3 \tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3 \tan(c+dx)\sec^3(c+dx)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^6(c+dx)\,dx = \int \Big(a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+3a^3\sec^5(c+dx)+a^3\sec^6(c+a$$

$$= a^3\int \sec^3(c+dx)\,dx + a^3\int \sec^6(c+dx)\,dx + \left(3a^3\right)\int \sec^4(c+dx)\,dx$$

$$= \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{2}a^3\int \sec($$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+dx}{8d}$$

$$= \frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+}{8d}$$

Mathematica [B] time = 1.43, size = 487, normalized size = 4.27

$$-\frac{a^3\sec(c)\sec^5(c+dx)\Big(1440\sin(2c+dx)-1500\sin(c+2dx)-1500\sin(3c+2dx)-3040\sin(2c+3dx)-390}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

```
[Out] -1/3840*(a^3*Sec[c]*Sec[c + d*x]^5*(975*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 975*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 195*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1950*Cos
[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) + 1950*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 975*Cos[2*c + 3*d*x]*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 975*Cos[4*c + 3*d*x]*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] - 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] - 195*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 4640*Sin[d*x] + 1440*Sin[2*c + d*x] - 1500*Sin[c + 2*d*x] - 1500*Si
n[3*c + 2*d*x] - 3040*Sin[2*c + 3*d*x] - 390*Sin[3*c + 4*d*x] - 390*Sin[5*c
 + 4*d*x] - 608*Sin[4*c + 5*d*x]))/d
```

fricas [A] time = 0.97, size = 124, normalized size = 1.09

$$\frac{195\,a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-195\,a^3\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(304\,a^3\cos(dx+c)^4\right.}{240\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(195*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 195*a^3*cos(d*x + c)^
5*log(-sin(d*x + c) + 1) + 2*(304*a^3*cos(d*x + c)^4 + 195*a^3*cos(d*x + c)
^3 + 152*a^3*cos(d*x + c)^2 + 90*a^3*cos(d*x + c) + 24*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

giac [A] time = 0.54, size = 138, normalized size = 1.21

$$\frac{195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-910\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2*d
*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x + 1/
2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.15, size = 124, normalized size = 1.09

$$\frac{13a^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{13a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{38a^3\tan(dx+c)}{15d}+\frac{19a^3\tan(dx+c)\left(\sec^2\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x)
```

```
[Out] 13/8*a^3*sec(d*x+c)*tan(d*x+c)/d+13/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+38/15
*a^3*tan(d*x+c)/d+19/15/d*a^3*tan(d*x+c)*sec(d*x+c)^2+3/4*a^3*sec(d*x+c)^3*
tan(d*x+c)/d+1/5/d*a^3*tan(d*x+c)*sec(d*x+c)^4
```

maxima [A] time = 1.03, size = 179, normalized size = 1.57

$$16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3-45\,a^3\left(\frac{2(3}{\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.63, size = 170, normalized size = 1.49

$$\frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}-\frac{91\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{133\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\,\tan\cdots}{\cdots}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((416*a^3*tan(c/2 + (d*x)/2)^5)/
15 - (133*a^3*tan(c/2 + (d*x)/2)^3)/6 - (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (
13*a^3*tan(c/2 + (d*x)/2)^9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(5*tan(c
/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan
(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.33 $\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=127

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}$$

```
[Out] 49/16*a^4*x+8*a^4*sin(d*x+c)/d+49/16*a^4*cos(d*x+c)*sin(d*x+c)/d+41/24*a^4*
cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c)/d-4*a^4*sin(d*x+c
)^3/d+4/5*a^4*sin(d*x+c)^5/d
```

Rubi [A] time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2635, 8, 2633}

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)/(5
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx = \int \left(a^4\cos^2(c+dx) + 4a^4\cos^3(c+dx) + 6a^4\cos^4(c+dx) + 4a^4\cos^5(c+\right.$$
$$= a^4\int \cos^2(c+dx)\,dx + a^4\int \cos^6(c+dx)\,dx + \left(4a^4\right)\int \cos^3(c+dx)\,dx$$
$$= \frac{a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^4\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^5(c+}{}$$
$$= \frac{a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{41a^4\cos^3(c+d}{24}$$
$$= \frac{11a^4x}{4} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\cos^3(c+}{}$$
$$= \frac{49a^4x}{16} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\cos^3(c-}{}$$

Mathematica [A] time = 0.20, size = 73, normalized size = 0.57

$$\frac{a^4(5280\sin(c+dx) + 1905\sin(2(c+dx)) + 720\sin(3(c+dx)) + 225\sin(4(c+dx)) + 48\sin(5(c+dx)) + 5\sin(6}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c +
 d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/
(960*d)
```

fricas [A] time = 0.91, size = 89, normalized size = 0.70

$$\frac{735\,a^4dx + \left(40\,a^4\cos(dx+c)^5 + 192\,a^4\cos(dx+c)^4 + 410\,a^4\cos(dx+c)^3 + 576\,a^4\cos(dx+c)^2 + 735\,a^4\cos(}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

[Out] 1/240*(735*a^4*d*x + (40*a^4*cos(d*x + c)^5 + 192*a^4*cos(d*x + c)^4 + 410*
a^4*cos(d*x + c)^3 + 576*a^4*cos(d*x + c)^2 + 735*a^4*cos(d*x + c) + 1152*a
^4)*sin(d*x + c))/d

giac [A] time = 0.56, size = 106, normalized size = 0.83

$$\frac{49}{16}\,a^4x+\frac{a^4\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{a^4\sin\left(5\,dx+5\,c\right)}{20\,d}+\frac{15\,a^4\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{3\,a^4\sin\left(3\,dx+3\,c\right)}{4\,d}+\frac{127\,a^4\sin\left(2\,dx+2\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

[Out] 49/16*a^4*x + 1/192*a^4*sin(6*d*x + 6*c)/d + 1/20*a^4*sin(5*d*x + 5*c)/d +
15/64*a^4*sin(4*d*x + 4*c)/d + 3/4*a^4*sin(3*d*x + 3*c)/d + 127/64*a^4*sin(
2*d*x + 2*c)/d + 11/2*a^4*sin(d*x + c)/d

maple [A] time = 0.05, size = 169, normalized size = 1.33

$$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+6a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\,\mathrm{c}}{}\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x)
```

[Out] 1/d*(a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a^4
(1/4(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a^4*(2+co
s(d*x+c)^2)*sin(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

maxima [A] time = 1.75, size = 165, normalized size = 1.30

$$\frac{256\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^4-5\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + sin(2*
d*x + 2*c))*a^4)/d

mupad [B] time = 2.85, size = 121, normalized size = 0.95

$$\frac{49\,a^4\,x}{16}+\frac{\frac{49\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{833\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{1617\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{1967\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{1471\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{207\,a^4\,\tan\left(\frac{c}{2}+\right.}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (49*a^4*x)/16 + ((1471*a^4*tan(c/2 + (d*x)/2)^3)/24 + (1967*a^4*tan(c/2 + (
d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/20 + (833*a^4*tan(c/2 + (d*
x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 + (207*a^4*tan(c/2 + (d*x)/2
))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [A] time = 3.87, size = 434, normalized size = 3.42

$$\begin{cases} \frac{5a^4x\sin^6(c+dx)}{16}+\frac{15a^4x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{9a^4x\sin^4(c+dx)}{4}+\frac{15a^4x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{9a^4x\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{a^4x\sin}{} \\ x\left(a\cos(c)+a\right)^4\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((5*a**4*x*sin(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 9*a**4*x*sin(c + d*x)**4/4 + 15*a**4*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + 9*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + a**4*x*sin(c + d
*x)**2/2 + 5*a**4*x*cos(c + d*x)**6/16 + 9*a**4*x*cos(c + d*x)**4/4 + a**4*
x*cos(c + d*x)**2/2 + 5*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 32*a**4*
sin(c + d*x)**5/(15*d) + 5*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 16*
a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*a**4*sin(c + d*x)**3*cos(c +
 d*x)/(4*d) + 8*a**4*sin(c + d*x)**3/(3*d) + 11*a**4*sin(c + d*x)*cos(c + d
*x)**5/(16*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**4*sin(c + d*x
)*cos(c + d*x)**3/(4*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**2/d + a**4*sin(
c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**4*cos(c)**2, Tru
e))
```

3.34 $\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=102

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+$$

```
[Out] 7/2*a^4*x+8*a^4*sin(d*x+c)/d+7/2*a^4*cos(d*x+c)*sin(d*x+c)/d+a^4*cos(d*x+c)
^3*sin(d*x+c)/d-8/3*a^4*sin(d*x+c)^3/d+1/5*a^4*sin(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{16a^4\sin^3(c+dx)}{15d}+\frac{32a^4\sin(c+dx)}{5d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{5d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{10d}+\frac{7a^4x}{2}+\frac{\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (7*a^4*x)/2 + (32*a^4*Sin[c + d*x])/(5*d) + (27*a^4*Cos[c + d*x]*Sin[c + d*
x])/(10*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(5*d) + ((a + a*Cos[c + d*x]
)^4*Sin[c + d*x])/(5*d) - (16*a^4*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx = \frac{(a+a\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{4}{5}\int (a+a\cos(c+dx))^4\,dx$$

$$= \frac{(a+a\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{4}{5}\int \left(a^4+4a^4\cos(c+dx)+6a^4\cos^2(\right.$$

$$= \frac{4a^4x}{5} + \frac{(a+a\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}\left(4a^4\right)\int \cos^4(c+dx)\,dx + \frac{1}{5}$$

$$= \frac{4a^4x}{5} + \frac{16a^4\sin(c+dx)}{5d} + \frac{12a^4\cos(c+dx)\sin(c+dx)}{5d} + \frac{a^4\cos^3(c+dx}{5d}$$

$$= \frac{16a^4x}{5} + \frac{32a^4\sin(c+dx)}{5d} + \frac{27a^4\cos(c+dx)\sin(c+dx)}{10d} + \frac{a^4\cos^3(c+d}{5}$$

$$= \frac{7a^4x}{2} + \frac{32a^4\sin(c+dx)}{5d} + \frac{27a^4\cos(c+dx)\sin(c+dx)}{10d} + \frac{a^4\cos^3(c+dx}{5d}$$

Mathematica [A] time = 0.15, size = 63, normalized size = 0.62

$$\frac{a^4(1470\sin(c+dx)+480\sin(2(c+dx))+145\sin(3(c+dx))+30\sin(4(c+dx))+3\sin(5(c+dx))+840dx)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c + d
*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)
```

fricas [A] time = 0.93, size = 76, normalized size = 0.75

$$\frac{105\,a^4dx + \left(6\,a^4\cos\left(dx+c\right)^4 + 30\,a^4\cos\left(dx+c\right)^3 + 68\,a^4\cos\left(dx+c\right)^2 + 105\,a^4\cos\left(dx+c\right) + 166\,a^4\right)\sin\left(dx\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4*
cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.52, size = 89, normalized size = 0.87

$$\frac{7}{2}\,a^4x + \frac{a^4\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{a^4\sin\left(4\,dx+4\,c\right)}{8\,d} + \frac{29\,a^4\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{2\,a^4\sin\left(2\,dx+2\,c\right)}{d} + \frac{49\,a^4\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 7/2*a^4*x + 1/80*a^4*sin(5*d*x + 5*c)/d + 1/8*a^4*sin(4*d*x + 4*c)/d + 29/4
8*a^4*sin(3*d*x + 3*c)/d + 2*a^4*sin(2*d*x + 2*c)/d + 49/8*a^4*sin(d*x + c)
/d
```

maple [A] time = 0.05, size = 133, normalized size = 1.30

$$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5} + 4a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + 2a^4\left(2+\cos^2\left(dx+c\right)\right)\sin\left(d\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^4*(2+cos(d*x+c)^2)*
sin(d*x+c)+4*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*sin(d*x+c))
```

maxima [A] time = 0.30, size = 128, normalized size = 1.25

$$\frac{8\left(3\sin\left(dx+c\right)^5 - 10\sin\left(dx+c\right)^3 + 15\sin\left(dx+c\right)\right)a^4 - 240\left(\sin\left(dx+c\right)^3 - 3\sin\left(dx+c\right)\right)a^4 + 15\left(12\,dx+\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 240
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 1
20*a^4*sin(d*x + c))/d
```

mupad [B] time = 3.69, size = 105, normalized size = 1.03

$$\frac{7\,a^4\,x}{2}+\frac{7\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{98\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{896\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{158\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (7*a^4*x)/2 + ((158*a^4*tan(c/2 + (d*x)/2)^3)/3 + (896*a^4*tan(c/2 + (d*x)/
2)^5)/15 + (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*tan(c/2 + (d*x)/2)^9 + 2
5*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 2.17, size = 280, normalized size = 2.75

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{2} + 3a^4x\sin^2(c+dx)\cos^2(c+dx) + 2a^4x\sin^2(c+dx) + \frac{3a^4x\cos^4(c+dx)}{2} + 2a^4x\cos^2(c+dx) + \frac{8a^4\sin \\ x\,(a\cos(c)+a)^4\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/2 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2 + 2*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/2 + 2*a**4*x*co
s(c + d*x)**2 + 8*a**4*sin(c + d*x)**5/(15*d) + 4*a**4*sin(c + d*x)**3*cos(
c + d*x)**2/(3*d) + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*a**4*sin(
c + d*x)**3/d + a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5*a**4*sin(c + d*x)*c
os(c + d*x)**3/(2*d) + 6*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*a**4*sin(c
 + d*x)*cos(c + d*x)/d + a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)*
*4*cos(c), True))
```

3.35 $\int (a + a\cos(c + dx))^4 \, dx$

Optimal. Leaf size=87

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

```
[Out] 35/8*a^4*x+8*a^4*sin(d*x+c)/d+27/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(
d*x+c)^3*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2637, 2635, 8, 2633}

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int (a + a\cos(c + dx))^4 \, dx = \int \left(a^4 + 4a^4\cos(c + dx) + 6a^4\cos^2(c + dx) + 4a^4\cos^3(c + dx) + a^4\cos^4(c + dx)\right) dx$$
$$= a^4x + a^4\int \cos^4(c + dx)\, dx + \left(4a^4\right)\int \cos(c + dx)\, dx + \left(4a^4\right)\int \cos^3(c + dx)\, dx + (6$$
$$= a^4x + \frac{4a^4\sin(c + dx)}{d} + \frac{3a^4\cos(c + dx)\sin(c + dx)}{d} + \frac{a^4\cos^3(c + dx)\sin(c + dx)}{4d} +$$
$$= 4a^4x + \frac{8a^4\sin(c + dx)}{d} + \frac{27a^4\cos(c + dx)\sin(c + dx)}{8d} + \frac{a^4\cos^3(c + dx)\sin(c + dx)}{4d}$$
$$= \frac{35a^4x}{8} + \frac{8a^4\sin(c + dx)}{d} + \frac{27a^4\cos(c + dx)\sin(c + dx)}{8d} + \frac{a^4\cos^3(c + dx)\sin(c + dx)}{4d}$$

Mathematica [A] time = 0.11, size = 56, normalized size = 0.64

$$\frac{a^4(672\sin(c + dx) + 168\sin(2(c + dx)) + 32\sin(3(c + dx)) + 3\sin(4(c + dx)) + 420c + 420dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3*
(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 1.03, size = 63, normalized size = 0.72

$$\frac{105\, a^4 dx + \left(6\, a^4 \cos\left(dx + c\right)^3 + 32\, a^4 \cos\left(dx + c\right)^2 + 81\, a^4 \cos\left(dx + c\right) + 160\, a^4\right)\sin\left(dx + c\right)}{24\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4*
cos(d*x + c) + 160*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.71, size = 72, normalized size = 0.83

$$\frac{35}{8}\,a^4x + \frac{a^4 \sin\left(4\,dx + 4\,c\right)}{32\,d} + \frac{a^4 \sin\left(3\,dx + 3\,c\right)}{3\,d} + \frac{7\,a^4 \sin\left(2\,dx + 2\,c\right)}{4\,d} + \frac{7\,a^4 \sin\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 35/8*a^4*x + 1/32*a^4*sin(4*d*x + 4*c)/d + 1/3*a^4*sin(3*d*x + 3*c)/d + 7/4
*a^4*sin(2*d*x + 2*c)/d + 7*a^4*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 111, normalized size = 1.28

$$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a
^4*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+4*a^4*sin(d*x+c)+a^4*(d*x+c))
```

maxima [A] time = 0.74, size = 106, normalized size = 1.22

$$a^4x - \frac{4\left(\sin\left(dx + c\right)^3 - 3\,\sin\left(dx + c\right)\right)a^4}{3\,d} + \frac{\left(12\,dx + 12\,c + \sin\left(4\,dx + 4\,c\right) + 8\,\sin\left(2\,dx + 2\,c\right)\right)a^4}{32\,d} + \frac{3\left(2\,dx + 2\,c +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 4/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c +
 sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c + sin(2*d*
x + 2*c))*a^4/d + 4*a^4*sin(d*x + c)/d
```

mupad [B] time = 3.56, size = 89, normalized size = 1.02

$$\frac{35\,a^4\,x}{8} + \frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{385\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{511\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{93\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4,x)
```

```
[Out] (35*a^4*x)/8 + ((511*a^4*tan(c/2 + (d*x)/2)^3)/12 + (385*a^4*tan(c/2 + (d*x
)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 + (93*a^4*tan(c/2 + (d*x)/2))/
4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 1.08, size = 224, normalized size = 2.57

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + 3a^4x\sin^2(c+dx) + \frac{3a^4x\cos^4(c+dx)}{8} + 3a^4x\cos^2(c+dx) + a^4x + \frac{3a^4\sin^3(c \\ x\,(a\cos(c)+a)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*x*
cos(c + d*x)**2 + a**4*x + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 8*a*
*4*sin(c + d*x)**3/(3*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*a*
*4*sin(c + d*x)*cos(c + d*x)**2/d + 3*a**4*sin(c + d*x)*cos(c + d*x)/d + 4*
a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**4, True))
```

3.36 $\int (a+a\cos(c+dx))^4\sec(c+dx)\,dx$

Optimal. Leaf size=73

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

```
[Out] 6*a^4*x+a^4*arctanh(sin(d*x+c))/d+7*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*sin(d
*x+c)/d-1/3*a^4*sin(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2757, 2637, 2635, 8, 2633, 3770}

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x],x]
```

```
[Out] 6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*C
os[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^4 \sec(c+dx)\,dx = \int \left(4a^4 + 6a^4\cos(c+dx) + 4a^4\cos^2(c+dx) + a^4\cos^3(c+dx) + a^4\sec(c\right.$$

$$= 4a^4x + a^4\int \cos^3(c+dx)\,dx + a^4\int \sec(c+dx)\,dx + \left(4a^4\right)\int \cos^2(c+d$$

$$= 4a^4x + \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{6a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\sin(c}{d}$$

$$= 6a^4x + \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{7a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\sin(c}{d}$$

Mathematica [A] time = 0.11, size = 91, normalized size = 1.25

$$\frac{a^4\left(81\sin(c+dx) + 12\sin(2(c+dx)) + \sin(3(c+dx)) - 12\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 12\log\left(\sin\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x],x]
```

```
[Out] (a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + Sin
[3*(c + d*x)]))/(12*d)
```

fricas [A] time = 0.81, size = 80, normalized size = 1.10

$$\frac{36\,a^4dx + 3\,a^4\log\left(\sin\left(dx+c\right)+1\right) - 3\,a^4\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(a^4\cos\left(dx+c\right)^2 + 6\,a^4\cos\left(dx+c\right) + 20\,a\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1
) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.70, size = 116, normalized size = 1.59

$$\frac{18\,(dx+c)a^4 + 3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+38\,a^4\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*(18*(d*x + c)*a^4 + 3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 + 38*a^
4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^3)/d
```

maple [A] time = 0.11, size = 94, normalized size = 1.29

$$\frac{\sin(dx+c)\left(\cos^2(dx+c)\right)a^4}{3d}+\frac{20a^4\sin(dx+c)}{3d}+\frac{2a^4\cos(dx+c)\sin(dx+c)}{d}+6a^4x+\frac{6a^4c}{d}+\frac{a^4\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c),x)
```

```
[Out] 1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*si
n(d*x+c)/d+6*a^4*x+6/d*a^4*c+1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 1.52, size = 89, normalized size = 1.22

$$-\frac{\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^4-3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4-12\,(dx+c)a^4-3\,a^4\log(\sec(dx+c)+}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 3*(2*d*x + 2*c + sin(2*d*x +
2*c))*a^4 - 12*(d*x + c)*a^4 - 3*a^4*log(sec(d*x + c) + tan(d*x + c)) - 18*
a^4*sin(d*x + c))/d
```

mupad [B] time = 0.41, size = 93, normalized size = 1.27

$$6\,a^4\,x+\frac{20\,a^4\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x),x)
```

```
[Out] 6*a^4*x + (20*a^4*sin(c + d*x))/(3*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d + (a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (2*a^4*cos(
c + d*x)*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 6\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int 4\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c),x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sec(c + d*x), x) + Integral(6*cos(c + d*x)**2
*sec(c + d*x), x) + Integral(4*cos(c + d*x)**3*sec(c + d*x), x) + Integral(
cos(c + d*x)**4*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

3.37 $\int (a+a\cos(c+dx))^4\sec^2(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

```
[Out] 13/2*a^4*x+4*a^4*arctanh(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+1/2*a^4*cos(d*x+c
)*sin(d*x+c)/d+a^4*tan(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2757, 2637, 2635, 8, 3770, 3767}

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

```
[Out] (13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (
a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
```

```
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^4 \sec^2(c+dx)\,dx = \int \left(6a^4 + 4a^4\cos(c+dx) + a^4\cos^2(c+dx) + 4a^4\sec(c+dx) + a^4\sec^2(\right.$$

$$= 6a^4x + a^4\int \cos^2(c+dx)\,dx + a^4\int \sec^2(c+dx)\,dx + \left(4a^4\right)\int \cos(c+$$

$$= 6a^4x + \frac{4a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{4a^4\sin(c+dx)}{d} + \frac{a^4\cos(c+dx)\sin(}{2d}$$

$$= \frac{13a^4x}{2} + \frac{4a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{4a^4\sin(c+dx)}{d} + \frac{a^4\cos(c+dx)\sin}{2d}$$

Mathematica [B] time = 1.25, size = 241, normalized size = 3.30

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{16\sin(c)\cos(dx)}{d} + \frac{\sin(2c)\cos(2dx)}{d} + \frac{16\cos(c)\sin(dx)}{d} + \frac{\cos(2c)\sin(2}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x])/
d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

fricas [A] time = 0.89, size = 105, normalized size = 1.44

$$\frac{13\,a^4dx\cos\left(dx+c\right)+4\,a^4\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-4\,a^4\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(a^4\cos\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 4
*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*cos(
d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.57, size = 129, normalized size = 1.77

$$\frac{13\,(dx+c)a^4+8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(7\,a^4\tan\left(\frac{1}{2}\right.\right.}{\left(\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
+ 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.12, size = 86, normalized size = 1.18

$$\frac{a^4\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{13a^4x}{2}+\frac{13a^4c}{2d}+\frac{4a^4\sin\left(dx+c\right)}{d}+\frac{4a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a^4\tan\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x)
```

```
[Out] 1/2*a^4*cos(d*x+c)*sin(d*x+c)/d+13/2*a^4*x+13/2/d*a^4*c+4*a^4*sin(d*x+c)/d+
4/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*tan(d*x+c)/d
```

maxima [A] time = 0.67, size = 85, normalized size = 1.16

$$\frac{\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4+24\,(dx+c)a^4+8\,a^4\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+16\,a^4\sin}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*tan
(d*x + c))/d
```

mupad [B] time = 0.60, size = 117, normalized size = 1.60

$$\frac{13\,a^4\,x}{2}+\frac{8\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+11\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^2,x)
```

```
[Out] (13*a^4*x)/2 + (8*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)
/2)^3 - 5*a^4*tan(c/2 + (d*x)/2)^5 + 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2
 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\cos(c+dx)\sec^2(c+dx)\,dx+\int 6\cos^2(c+dx)\sec^2(c+dx)\,dx+\int 4\cos^3(c+dx)\sec^2(c+dx)\,dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**2,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(6*cos(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**2, x) +
Integral(cos(c + d*x)**4*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x)
)
```

3.38 $\int (a + a\cos(c + dx))^4 \sec^3(c + dx)\, dx$

Optimal. Leaf size=73

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

```
[Out] 4*a^4*x+13/2*a^4*arctanh(sin(d*x+c))/d+a^4*sin(d*x+c)/d+4*a^4*tan(d*x+c)/d+
1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2757, 2637, 3770, 3767, 8, 3768}

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

```
[Out] 4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4*
a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^4 \sec^3(c+dx)\,dx = \int \left(4a^4 + a^4\cos(c+dx) + 6a^4\sec(c+dx) + 4a^4\sec^2(c+dx) + a^4\sec^3(\right.$$

$$= 4a^4x + a^4\int \cos(c+dx)\,dx + a^4\int \sec^3(c+dx)\,dx + \left(4a^4\right)\int \sec^2(c+$$

$$= 4a^4x + \frac{6a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^4\sin(c+dx)}{d} + \frac{a^4\sec(c+dx)\tan(c}{2d}$$

$$= 4a^4x + \frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^4\sin(c+dx)}{d} + \frac{4a^4\tan(c+dx)}{d} +$$

Mathematica [B] time = 1.24, size = 272, normalized size = 3.73

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin(c)\cos(dx)}{d} + \frac{4\cos(c)\sin(dx)}{d} + \frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))))/64
```

fricas [A] time = 1.71, size = 111, normalized size = 1.52

$$\frac{16\,a^4dx\cos\left(dx+c\right)^2+13\,a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-13\,a^4\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) + 1
) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^2
+ 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.61, size = 129, normalized size = 1.77

$$\frac{8\,(dx+c)a^4+13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{2\left(7\,a^4\tan\left(\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.15, size = 86, normalized size = 1.18

$$\frac{a^4\sin\left(dx+c\right)}{d}+4a^4x+\frac{4a^4c}{d}+\frac{13a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{4a^4\tan\left(dx+c\right)}{d}+\frac{a^4\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x)
```

```
[Out] a^4*sin(d*x+c)/d+4*a^4*x+4/d*a^4*c+13/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a
^4*tan(d*x+c)/d+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

maxima [A] time = 0.66, size = 110, normalized size = 1.51

$$\frac{16\,(dx+c)a^4-a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^4\left(\log\left(\sin\left(dx+c\right)+1\right)-1\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

mupad [B] time = 0.59, size = 115, normalized size = 1.58

$$4\,a^4\,x+\frac{13\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{5\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+2\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-11\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^3,x)
```

```
[Out] 4*a^4*x + (13*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)^
3 + 5*a^4*tan(c/2 + (d*x)/2)^5 - 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (
d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.39 $\int (a+a\cos(c+dx))^4 \sec^4(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{a^4\tan^3(c+dx)}{3d}+\frac{7a^4\tan(c+dx)}{d}+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\tan(c+dx)\sec(c+dx)}{d}+a^4x$$

```
[Out] a^4*x+6*a^4*arctanh(sin(d*x+c))/d+7*a^4*tan(d*x+c)/d+2*a^4*sec(d*x+c)*tan(d
*x+c)/d+1/3*a^4*tan(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{a^4\tan^3(c+dx)}{3d}+\frac{7a^4\tan(c+dx)}{d}+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\tan(c+dx)\sec(c+dx)}{d}+a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

```
[Out] a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Tan[c + d*x])/d + (2*a^4*S
ec[c + d*x]*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx = \int \left(a^4+4a^4\sec(c+dx)+6a^4\sec^2(c+dx)+4a^4\sec^3(c+dx)+a^4\sec^4$$
$$= a^4x+a^4\int \sec^4(c+dx)\,dx+\left(4a^4\right)\int \sec(c+dx)\,dx+\left(4a^4\right)\int \sec^3(c+$$
$$= a^4x+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sec(c+dx)\tan(c+dx)}{d}+\left(2a^4\right)\int$$
$$= a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{7a^4\tan(c+dx)}{d}+\frac{2a^4\sec(c+dx)\tan(}{d}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.84

$$a^4\left(\frac{\tan^3(c+dx)}{3d}+\frac{7\tan(c+dx)}{d}+\frac{6\tanh^{-1}(\sin(c+dx))}{d}+\frac{2\tan(c+dx)\sec(c+dx)}{d}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

```
[Out] a^4*(x + (6*ArcTanh[Sin[c + d*x]])/d + (7*Tan[c + d*x])/d + (2*Sec[c + d*x]
*Tan[c + d*x])/d + Tan[c + d*x]^3/(3*d))
```

fricas [A] time = 0.96, size = 110, normalized size = 1.51

$$\frac{3\,a^4dx\cos\left(dx+c\right)^3+9\,a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-9\,a^4\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(20\,a^4\right.}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1)
- 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 + 6*
a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.67, size = 116, normalized size = 1.59

$$\frac{3\,(dx+c)a^4 + 18\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 18\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(15\,a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 - 38\,a^4 \tan}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 38*a
^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^3)/d
```

maple [A] time = 0.14, size = 93, normalized size = 1.27

$$a^4x + \frac{a^4c}{d} + \frac{6a^4 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{20a^4 \tan(dx+c)}{3d} + \frac{2a^4 \sec(dx+c)\tan(dx+c)}{d} + \frac{a^4 \tan(dx+c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x)
```

```
[Out] a^4*x+1/d*a^4*c+6/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3*a^4*tan(d*x+c)/d+2*a
^4*sec(d*x+c)*tan(d*x+c)/d+1/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 1.86, size = 120, normalized size = 1.64

$$\frac{\left(\tan(dx+c)^3 + 3\,\tan(dx+c)\right)a^4 + 3\,(dx+c)a^4 - 3\,a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - \right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 3*(d*x + c)*a^4 - 3*a^4*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) + 6*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*a^4*tan(d
*x + c))/d
```

mupad [B] time = 0.62, size = 117, normalized size = 1.60

$$a^4\,x + \frac{12\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d} - \frac{10\,a^4\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 - \frac{76\,a^4\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3}{3} + 18\,a^4\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 - 3\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 + 3\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^4,x)
```

```
[Out] a^4*x + (12*a^4*atanh(tan(c/2 + (d*x)/2)))/d - (10*a^4*tan(c/2 + (d*x)/2)^5
 - (76*a^4*tan(c/2 + (d*x)/2)^3)/3 + 18*a^4*tan(c/2 + (d*x)/2))/(d*(3*tan(c
/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.40 $\int (a+a\cos(c+dx))^4 \sec^5(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\mathrm{se}}{8d}$$

```
[Out] 35/8*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x+c)*tan(d
*x+c)/d+1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+4/3*a^4*tan(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\mathrm{se}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
4*a^4*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^4 \sec^5(c+dx)\,dx = \int \left(a^4 \sec(c+dx) + 4a^4 \sec^2(c+dx) + 6a^4 \sec^3(c+dx) + 4a^4 \sec^4(c+a$$

$$= a^4 \int \sec(c+dx)\,dx + a^4 \int \sec^5(c+dx)\,dx + \left(4a^4\right) \int \sec^2(c+dx)\,dx +$$

$$= \frac{a^4 \tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^4 \sec(c+dx)\tan(c+dx)}{d} + \frac{a^4 \sec^3(c+dx)}{4d}$$

$$= \frac{4a^4 \tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4 \tan(c+dx)}{d} + \frac{27a^4 \sec(c+dx)\tan(c+d}{8d}$$

$$= \frac{35a^4 \tanh^{-1}(\sin(c+dx))}{8d} + \frac{8a^4 \tan(c+dx)}{d} + \frac{27a^4 \sec(c+dx)\tan(c+}{8d}$$

Mathematica [B] time = 6.37, size = 797, normalized size = 8.30

$$-\frac{35(\cos(c+dx)a+a)^4 \log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d}+\frac{35(\cos(c+dx)a+a)^4 \log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

```
[Out] (-35*(a + a*Cos[c + d*x])^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Se
c[c/2 + (d*x)/2]^8)/(128*d) + (35*(a + a*Cos[c + d*x])^4*Log[Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8)/(128*d) + ((a + a*Cos[c +
d*x])^4*Sec[c/2 + (d*x)/2]^8)/(256*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])^4) + ((a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(24*d*(
Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a
*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(
Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (5*(a +
 a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(12*d*(Cos[c/2] - Sin
[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - ((a + a*Cos[c + d*x])^4
```

```
*Sec[c/2 + (d*x)/2]^8)/(256*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4)
+ ((a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(24*d*(Cos[c/2
] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c
+ d*x])^4*Sec[c/2 + (d*x)/2]^8*(-97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(Cos[c/
2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (5*(a + a*Cos
[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])
*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 2.00, size = 111, normalized size = 1.16

$$\frac{105\,a^4\cos(dx+c)^4\log(\sin(dx+c)+1)-105\,a^4\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(160\,a^4\cos(dx+c)^3\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^4
*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)^2
 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.86, size = 122, normalized size = 1.27

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*d*
x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.16, size = 102, normalized size = 1.06

$$\frac{35a^4\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{20a^4\tan(dx+c)}{3d}+\frac{27a^4\sec(dx+c)\tan(dx+c)}{8d}+\frac{4a^4\tan(dx+c)\left(\sec^2(\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x)
```

```
[Out] 35/8/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x
+c)*tan(d*x+c)/d+4/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2+1/4*a^4*sec(d*x+c)^3*tan
(d*x+c)/d
```

maxima [B] time = 1.01, size = 182, normalized size = 1.90

$$\frac{64\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{4}-3\,a^{4}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*a^4*(log(sin(
d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*a^4*tan(d*x + c))/d
```

mupad [B] time = 3.50, size = 141, normalized size = 1.47

$$\frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{385\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{511\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{93\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^5,x)
```

```
[Out] (35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((511*a^4*tan(c/2 + (d*x)/2)^3)/
12 - (385*a^4*tan(c/2 + (d*x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 -
(93*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*
x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.41 $\int (a + a\cos(c+dx))^4 \sec^6(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+7a^4$$

```
[Out] 7/2*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*x
+c)/d+a^4*sec(d*x+c)^3*tan(d*x+c)/d+8/3*a^4*tan(d*x+c)^3/d+1/5*a^4*tan(d*x+
c)^5/d
```

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = $\{2757, 3767, 8, 3768, 3770\}$

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+7a^4$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

```
[Out] (7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*
Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^6(c+dx)\,dx = \int \left(a^4\sec^2(c+dx)+4a^4\sec^3(c+dx)+6a^4\sec^4(c+dx)+4a^4\sec^5(c+\right.$$
$$= a^4\int \sec^2(c+dx)\,dx + a^4\int \sec^6(c+dx)\,dx + \left(4a^4\right)\int \sec^3(c+dx)\,dx$$
$$= \frac{2a^4\sec(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{d} + \left(2a^4\right)\int \sec$$
$$= \frac{2a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx}{2d}$$
$$= \frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx}{2d}$$

Mathematica [B] time = 1.43, size = 498, normalized size = 4.49

$$-\frac{a^4\sec(c)\sec^5(c+dx)\left(960\sin(2c+dx)-660\sin(c+2dx)-660\sin(3c+2dx)-1600\sin(2c+3dx)+60\sin(4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

```
[Out] -1/960*(a^4*Sec[c]*Sec[c + d*x]^5*(525*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ 105*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1050*Cos[
d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]) + 1050*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 525*Cos[2*c + 3*d*x]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] - 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Si
```

```
n[(c + d*x)/2]] - 105*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] - 2360*Sin[d*x] + 960*Sin[2*c + d*x] - 660*Sin[c + 2*d*x] - 660*Sin[3*
c + 2*d*x] - 1600*Sin[2*c + 3*d*x] + 60*Sin[4*c + 3*d*x] - 210*Sin[3*c + 4*
d*x] - 210*Sin[5*c + 4*d*x] - 332*Sin[4*c + 5*d*x]))/d
```

fricas [A] time = 0.76, size = 124, normalized size = 1.12

$$\frac{105\,a^4\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-105\,a^4\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(166\,a^4\cos\left(dx+c\right)^4\right.}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c)^
3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

giac [A] time = 0.80, size = 138, normalized size = 1.24

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\right.}{\left(\right.}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*d*
x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/2*c
)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.14, size = 123, normalized size = 1.11

$$\frac{83a^4\tan\left(dx+c\right)}{15d}+\frac{7a^4\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{7a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{34a^4\tan\left(dx+c\right)\left(\sec^2\left(d\right.\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x)
```

```
[Out] 83/15*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*x+c)/d+7/2/d*a^4*ln(sec(d*x
+c)+tan(d*x+c))+34/15/d*a^4*tan(d*x+c)*sec(d*x+c)^2+a^4*sec(d*x+c)^3*tan(d*
x+c)/d+1/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4
```

maxima [A] time = 1.96, size = 190, normalized size = 1.71

$$\frac{4\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)a^{4}+120\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)a^{4}-15a^{4}\left(\frac{2\left(3\sin\right.}{\sin(dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c))/d
```

mupad [B] time = 4.61, size = 170, normalized size = 1.53

$$\frac{7a^4\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{7a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\frac{98a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}+\frac{896a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15}-\frac{158a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+25a^4\tan}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^6,x)
```

```
[Out] (7*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((896*a^4*tan(c/2 + (d*x)/2)^5)/15 -
(158*a^4*tan(c/2 + (d*x)/2)^3)/3 - (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*
tan(c/2 + (d*x)/2)^9 + 25*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^
2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2
)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.42 $\int (a + a\cos(c + dx))^4 \sec^7(c + dx)\,dx$

Optimal. Leaf size=136

$$\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+$$

```
[Out] 49/16*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+49/16*a^4*sec(d*x+c)*tan
(d*x+c)/d+41/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*x+
c)/d+4*a^4*tan(d*x+c)^3/d+4/5*a^4*tan(d*x+c)^5/d
```

Rubi [A] time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2757, 3768, 3770, 3767}

$$\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

```
[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Se
c[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*
d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (
4*a^4*Tan[c + d*x]^5)/(5*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^7(c+dx)\,dx = \int \left(a^4\sec^3(c+dx)+4a^4\sec^4(c+dx)+6a^4\sec^5(c+dx)+4a^4\sec^6(c+\right.$$

$$= a^4\int \sec^3(c+dx)\,dx + a^4\int \sec^7(c+dx)\,dx + \left(4a^4\right)\int \sec^4(c+dx)\,dx$$

$$= \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^4\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^4\sec^5(c+}{}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{11a^4\sec(c+dx)\tan(c+dx}{4d}$$

$$= \frac{11a^4\tanh^{-1}(\sin(c+dx))}{4d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+}{16d}$$

$$= \frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+}{16d}$$

Mathematica [A] time = 0.78, size = 211, normalized size = 1.55

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(23520\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

```
[Out] -1/122880*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(2352
0*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-11520*Sin[c] + 3750*Sin[d*x] + 3750*
Sin[2*c + d*x] + 15360*Sin[c + 2*d*x] - 1920*Sin[3*c + 2*d*x] + 3845*Sin[2*
c + 3*d*x] + 3845*Sin[4*c + 3*d*x] + 6912*Sin[3*c + 4*d*x] + 735*Sin[4*c +
5*d*x] + 735*Sin[6*c + 5*d*x] + 1152*Sin[5*c + 6*d*x])))/d
```

fricas [A] time = 0.78, size = 137, normalized size = 1.01

$$\frac{735\,a^4\cos(dx+c)^6\log(\sin(dx+c)+1) - 735\,a^4\cos(dx+c)^6\log(-\sin(dx+c)+1) + 2\left(1152\,a^4\cos(dx+c)^5\right.}{480\,d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)^
6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x + c
)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x + c
) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 0.71, size = 154, normalized size = 1.13

$$\frac{735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-4165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1/2
*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*x +
 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 0.21, size = 146, normalized size = 1.07

$$\frac{49a^4\sec(dx+c)\tan(dx+c)}{16d}+\frac{49a^4\ln(\sec(dx+c)+\tan(dx+c))}{16d}+\frac{24a^4\tan(dx+c)}{5d}+\frac{12a^4\tan(dx+c)\left(\sec^2\right.}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x)
```

```
[Out] 49/16*a^4*sec(d*x+c)*tan(d*x+c)/d+49/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+24/
5*a^4*tan(d*x+c)/d+12/5/d*a^4*tan(d*x+c)*sec(d*x+c)^2+41/24*a^4*sec(d*x+c)^
3*tan(d*x+c)/d+4/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4+1/6*a^4*sec(d*x+c)^5*tan(d
*x+c)/d
```

maxima [B] time = 1.37, size = 270, normalized size = 1.99

$$\frac{128\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)a^4+640\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^4-5\,a^4\left(\frac{2(1}{\mathrm{s}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 6
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 - 40
*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*s
in(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) -
 180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*a^4
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)))/d
```

mupad [B] time = 3.87, size = 199, normalized size = 1.46

$$\frac{49\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\frac{49\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{833\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{1617\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}-\frac{1967\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{1471}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^7,x)
```

```
[Out] (49*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((1471*a^4*tan(c/2 + (d*x)/2)^3)
/24 - (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/
20 - (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 -
 (207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 +
(d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}$$

```
[Out] 15/8*x/a-4*sin(d*x+c)/a/d+15/8*cos(d*x+c)*sin(d*x+c)/a/d+5/4*cos(d*x+c)^3*s
in(d*x+c)/a/d-cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))+4/3*sin(d*x+c)^3/a
/d
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2767, 2748, 2633, 2635, 8}

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x]),x]
```

```
[Out] (15*x)/(8*a) - (4*Sin[c + d*x])/(a*d) + (15*Cos[c + d*x]*Sin[c + d*x])/(8*a
*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (Cos[c + d*x]^4*Sin[c + d*x
])/(d*(a + a*Cos[c + d*x])) + (4*Sin[c + d*x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \cos^3(c+dx)(4a-5a\cos(c+dx))\,dx}{a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{4\int \cos^3(c+dx)\,dx}{a} + \frac{5\int \cos^4(c+dx)\,dx}{a}$$

$$= \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{15\int \cos^2(c+dx)\,dx}{4a} + \frac{4\,\mathrm{Subs}\ldots}{}$$

$$= -\frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^4(c+d\ldots}{d(a+a\,c\ldots}$$

$$= \frac{15x}{8a} - \frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^4(\ldots}{d(a\ldots}$$

Mathematica [A] time = 0.32, size = 173, normalized size = 1.47

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-168\sin\left(c+\frac{dx}{2}\right)-120\sin\left(c+\frac{3dx}{2}\right)-120\sin\left(2c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{5dx}{2}\right)+40\sin\left(3\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2]
 - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120*S
in[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2] - 5
```

```
*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2] + 3
*Sin[5*c + (9*d*x)/2]))/(384*a*d)
```

fricas [A] time = 1.05, size = 79, normalized size = 0.67

$$\frac{45\,dx\cos\left(dx+c\right)+45\,dx+\left(6\,\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}+13\,\cos\left(dx+c\right)^{2}-19\,\cos\left(dx+c\right)-64\right)\sin\left(d\right.}{24\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(45*d*x*cos(d*x + c) + 45*d*x + (6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 +
 13*cos(d*x + c)^2 - 19*cos(d*x + c) - 64)*sin(d*x + c))/(a*d*cos(d*x + c)
+ a*d)
```

giac [A] time = 0.44, size = 101, normalized size = 0.86

$$\frac{\frac{45\,(dx+c)}{a}-\frac{24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+115\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+109\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2*
c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

maple [A] time = 0.07, size = 171, normalized size = 1.45

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{25\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{115\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{109\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{7\tan\left(\frac{c}{}\right.}{4ad\left(1+\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-25/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^7-115/12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-109/12/a
/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-7/4/a/d/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)+15/4/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.24, size = 217, normalized size = 1.84

$$-\frac{\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 1.94, size = 98, normalized size = 0.83

$$\frac{15\,x}{8\,a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}-\frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{115\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x)),x)
```

```
[Out] (15*x)/(8*a) - tan(c/2 + (d*x)/2)/(a*d) - ((7*tan(c/2 + (d*x)/2))/4 + (109*
tan(c/2 + (d*x)/2)^3)/12 + (115*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 + (d
*x)/2)^7)/4)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 6.51, size = 882, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((45*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) + 180*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 9
6*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 +
d*x/2)**2 + 24*a*d) + 270*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)*
```

```
*8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(
c/2 + d*x/2)**2 + 24*a*d) + 180*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d
*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*
d*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 24*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 + 96*
a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*
x/2)**2 + 24*a*d) - 246*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 9
6*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 +
d*x/2)**2 + 24*a*d) - 374*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 +
 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2
+ d*x/2)**2 + 24*a*d) - 314*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8
 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) - 66*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 +
 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2
+ d*x/2)**2 + 24*a*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a), True))
```

3.44 $\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{3x}{2a}$$

```
[Out] -3/2*x/a+4*sin(d*x+c)/a/d-3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)^3*sin(d*
x+c)/d/(a+a*cos(d*x+c))-4/3*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2767, 2748, 2635, 8, 2633}

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sin[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*
d) - (Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - (4*Sin[c + d*
x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \cos^2(c+dx)(3a-4a\cos(c+dx))\,dx}{a^2}$$
$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int \cos^2(c+dx)\,dx}{a} + \frac{4\int \cos^3(c+dx)\,dx}{a}$$
$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int 1\,dx}{2a} - \frac{4\,\mathrm{Subst}\left(\int \left(1-x\right.\right.}{}$$
$$= -\frac{3x}{2a} + \frac{4\sin(c+dx)}{ad} - \frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{4\sin^3(}{3}$$

Mathematica [A] time = 0.28, size = 143, normalized size = 1.52

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(21\sin\left(c+\frac{dx}{2}\right)+18\sin\left(c+\frac{3dx}{2}\right)+18\sin\left(2c+\frac{3dx}{2}\right)-2\sin\left(2c+\frac{5dx}{2}\right)-2\sin\left(3c+\frac{5a}{2}\right.\right.}{48ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
+ 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[2*
c + (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3*c
+ (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

fricas [A] time = 1.10, size = 70, normalized size = 0.74

$$-\frac{9\,dx\cos\left(dx+c\right)+9\,dx-\left(2\,\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}+7\,\cos\left(dx+c\right)+16\right)\sin\left(dx+c\right)}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*c
os(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.69, size = 88, normalized size = 0.94

$$-\frac{\frac{9\,(dx+c)}{a}-\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c)
^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/
2*c)^2 + 1)^3*a))/d
```

maple [A] time = 0.07, size = 136, normalized size = 1.45

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{5\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{16\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{3\arctan\left(\tan\left(\frac{dx}{2}+\right.\right.}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)+5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c
)^5+16/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+3/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-3/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.55, size = 176, normalized size = 1.87

$$\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a+\frac{3\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{9\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3
*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.60, size = 70, normalized size = 0.74

$$\frac{\frac{15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}+\frac{3\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{4}-\frac{\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{12}+\frac{\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{24}}{a\,d\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{3x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x)),x)
```

```
[Out] ((15*sin(c/2 + (d*x)/2))/8 + (3*sin((3*c)/2 + (3*d*x)/2))/4 - sin((5*c)/2 +
 (5*d*x)/2)/12 + sin((7*c)/2 + (7*d*x)/2)/24)/(a*d*cos(c/2 + (d*x)/2)) - (3
*x)/(2*a)
```

sympy [A] time = 3.90, size = 570, normalized size = 6.06

$$\begin{cases} -\frac{9dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}-\frac{27dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}-\frac{}{6ad\tan^6\left(\frac{c}{2}+\right.} \\ \frac{x\cos^4(c)}{a\cos(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-9*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*t
an(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*d*x*tan(c/2 +
 d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d
*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 +
 d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*
d) - 9*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d
*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/
2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) +
48*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)
```

```
**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*tan(c/2 + d*x/2)**3/(6*a*d*t
an(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**
2 + 6*a*d) + 24*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/
2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*cos(c)**
4/(a*cos(c) + a), True))
```

3.45 $\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3x}{2a}$$

```
[Out] 3/2*x/a-2*sin(d*x+c)/a/d+3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)^2*sin(d*x
+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2767, 2734}

$$-\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sin[c + d*x])/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) - (Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \cos(c+dx)(2a-3a\cos(c+dx))\,dx}{a^2}$$
$$= \frac{3x}{2a} - \frac{2\sin(c+dx)}{ad} + \frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [**A**] time = 0.23, size = 117, normalized size = 1.54

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-4\sin\left(c+\frac{dx}{2}\right)-3\sin\left(c+\frac{3dx}{2}\right)-3\sin\left(2c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{5dx}{2}\right)+12\right.}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2] -
 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c +
(3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2]))/(16*a*d)
```

fricas [**A**] time = 1.17, size = 57, normalized size = 0.75

$$\frac{3\,dx\cos\left(dx+c\right)+3\,dx+\left(\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*d*x*cos(d*x + c) + 3*d*x + (cos(d*x + c)^2 - cos(d*x + c) - 4)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [**A**] time = 0.36, size = 73, normalized size = 0.96

$$\frac{\frac{3\,(dx+c)}{a}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

maple [A] time = 0.08, size = 103, normalized size = 1.36

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*
c)^3-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+3/a/d*arctan(tan(1
/2*d*x+1/2*c))
```

maxima [A] time = 1.33, size = 133, normalized size = 1.75

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a+\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.41, size = 89, normalized size = 1.17

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(c+d\,x)}{2}+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x)),x)
```

```
[Out] -(sin(c/2 + (d*x)/2) - (3*cos(c/2 + (d*x)/2)*(c + d*x))/2 + 3*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2) - 2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(a*
d*cos(c/2 + (d*x)/2))
```

sympy [A] time = 2.32, size = 325, normalized size = 4.28

$$\begin{cases} \frac{3dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} + \frac{6dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} + \frac{3dx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} - \frac{}{2ad\tan^4\left(\frac{c}{2}+\right.} \\ \frac{x\cos^3(c)}{a\cos(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan
(c/2 + d*x/2)**2 + 2*a*d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/
2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*d*x/(2*a*d*tan(c/2 + d*x/2)*
*4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*tan(c/2 + d*x/2)**5/(2*a*d*tan(
c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*tan(c/2 + d*x/2)*
*3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*tan(
c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d
), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a), True))
```

3.46 $\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=43

$$\frac{\sin(c+dx)}{ad}+\frac{\sin(c+dx)}{ad(\cos(c+dx)+1)}-\frac{x}{a}$$

```
[Out] -x/a+sin(d*x+c)/a/d+sin(d*x+c)/a/d/(1+cos(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2746, 12, 2735, 2648}

$$\frac{\sin(c+dx)}{ad}+\frac{\sin(c+dx)}{ad(\cos(c+dx)+1)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(x/a) + Sin[c + d*x]/(a*d) + Sin[c + d*x]/(a*d*(1 + Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{\sin(c+dx)}{ad}-\frac{\int \frac{a\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}\\ &= \frac{\sin(c+dx)}{ad}-\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx\\ &= -\frac{x}{a}+\frac{\sin(c+dx)}{ad}+\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= -\frac{x}{a}+\frac{\sin(c+dx)}{ad}+\frac{\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.20, size = 89, normalized size = 2.07

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{3dx}{2}\right)-2dx\cos\left(c+\frac{dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)-2dx\cos\left(\right.\right.}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x)/
2]))/(4*a*d)
```

fricas [A] time = 1.00, size = 46, normalized size = 1.07

$$-\frac{dx\cos\left(dx+c\right)+dx-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

giac [A] time = 0.45, size = 58, normalized size = 1.35

$$-\frac{\frac{dx+c}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d
*x + 1/2*c)^2 + 1)*a))/d
```

maple [A] time = 0.05, size = 68, normalized size = 1.58

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)+2/a/d*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-
2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 2.25, size = 92, normalized size = 2.14

$$-\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos
(d*x + c) + 1)))/d
```

mupad [B] time = 0.40, size = 66, normalized size = 1.53

$$\frac{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+(-c-d\,x)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x)),x)
```

```
[Out] (sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)*(c + d*x) + 2*cos(c/2 + (d*x)/2)^2
*sin(c/2 + (d*x)/2))/(a*d*cos(c/2 + (d*x)/2))
```

sympy [A] time = 1.41, size = 129, normalized size = 3.00

$$\begin{cases} -\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - d*x/(
a*d*tan(c/2 + d*x/2)**2 + a*d) + tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)*
*2 + a*d) + 3*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)),
(x*cos(c)**2/(a*cos(c) + a), True))
```

3.47 $\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] x/a-sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2735, 2648}

$$\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x]),x]
```

```
[Out] x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= \frac{x}{a}-\frac{\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 1.97

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(dx\cos\left(\frac{1}{2}(c+dx)\right)-\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(d*x*Cos[(c + d*x)/2] - Sec[c/2]*Sin[(d*x)/2]))/(a*d*(1
 + Cos[c + d*x]))
```

fricas [A] time = 0.89, size = 37, normalized size = 1.28

$$\frac{dx\cos\left(dx+c\right)+dx-\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.50, size = 28, normalized size = 0.97

$$\frac{\frac{dx+c}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d
```

maple [A] time = 0.05, size = 37, normalized size = 1.28

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.22, size = 49, normalized size = 1.69

$$\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c
) + 1)))/d
```

mupad [B] time = 0.33, size = 23, normalized size = 0.79

$$\frac{x}{a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x)),x)
```

```
[Out] x/a - tan(c/2 + (d*x)/2)/(a*d)
```

sympy [A] time = 0.75, size = 27, normalized size = 0.93

$$\begin{cases} \frac{x}{a}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for } d\neq 0 \\ \frac{x\cos\left(c\right)}{a\cos\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((x/a - tan(c/2 + d*x/2)/(a*d), Ne(d, 0)), (x*cos(c)/(a*cos(c) + a
), True))
```

3.48 $\int \frac{1}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2648}

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-1),x]
```

```
[Out] Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\cos(c+dx)}\,dx = \frac{\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-1),x]
```

```
[Out] Tan[(c + d*x)/2]/(a*d)
```

fricas [A] time = 0.91, size = 22, normalized size = 1.00

$$\frac{\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="fricas")
[Out] sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.41, size = 16, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="giac")
[Out] tan(1/2*d*x + 1/2*c)/(a*d)
```

maple [A] time = 0.04, size = 17, normalized size = 0.77

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c)),x)
[Out] 1/a/d*tan(1/2*d*x+1/2*c)
```

maxima [A] time = 1.12, size = 23, normalized size = 1.05

$$\frac{\sin(dx+c)}{ad(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="maxima")
[Out] sin(d*x + c)/(a*d*(cos(d*x + c) + 1))
```

mupad [B] time = 0.31, size = 16, normalized size = 0.73

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(a*d)
```

sympy [A] time = 0.51, size = 20, normalized size = 0.91

$$\begin{cases} \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for } d \neq 0 \\ \frac{x}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)/(a*d), Ne(d, 0)), (x/(a*cos(c) + a), True))
```

3.49 $\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] arctanh(sin(d*x+c))/a/d-sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2747, 3770, 2648}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\int \sec(c+dx)\,dx}{a} - \int \frac{1}{a+a\cos(c+dx)}\,dx$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [B] time = 0.15, size = 103, normalized size = 2.71

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sin[(d*x)/2]))
/(a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.93, size = 65, normalized size = 1.71

$$\frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1)-(\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.50, size = 54, normalized size = 1.42

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/
a - tan(1/2*d*x + 1/2*c)/a)/d
```

maple [A] time = 0.08, size = 58, normalized size = 1.53

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d
*x+1/2*c)+1)
```

maxima [A] time = 1.12, size = 75, normalized size = 1.97

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.35, size = 31, normalized size = 0.82

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)) - tan(c/2 + (d*x)/2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(cos(c + d*x) + 1), x)/a
```

3.50 $\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{2\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] -arctanh(sin(d*x+c))/a/d+2*tan(d*x+c)/a/d-tan(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2768, 2748, 3767, 8, 3770}

$$\frac{2\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a*d)) + (2*Tan[c + d*x])/(a*d) - Tan[c + d*x]/(d*(
a + a*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{\int(-2a+a\cos(c+dx))\sec^2(c+dx)\,dx}{a^2}\\ &= -\frac{\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{\int\sec(c+dx)\,dx}{a}+\frac{2\int\sec^2(c+dx)\,dx}{a}\\ &= -\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{2\,\mathrm{Subst}(\int 1\,dx,x,-\tan(c+dx))}{ad}\\ &= -\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{2\tan(c+dx)}{ad}-\frac{\tan(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.69, size = 188, normalized size = 3.55

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+\right.\right.}\right.\right.}{ad(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + S
in[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(a*d*(1 + Cos[c +
 d*x]))
```

fricas [A] time = 1.10, size = 97, normalized size = 1.83

$$-\frac{\left(\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left(\cos(dx+c)^2+\cos(dx+c)\right)\log(-\sin(dx+c)+1)-}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c)
^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*x
+ c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

giac [A] time = 0.47, size = 84, normalized size = 1.58

$$-\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a))/d
```

maple [A] time = 0.09, size = 99, normalized size = 1.87

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}-\frac{1}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)-1/a/d/(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+
1/2*c)-1)-1/a/d/(tan(1/2*d*x+1/2*c)+1)-1/a/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.49, size = 119, normalized size = 2.25

$$-\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\,\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.40, size = 67, normalized size = 1.26

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2
+ (d*x)/2)))/(a*d) + tan(c/2 + (d*x)/2)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(cos(c + d*x) + 1), x)/a
```

3.51 $\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=83

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] 3/2*arctanh(sin(d*x+c))/a/d-2*tan(d*x+c)/a/d+3/2*sec(d*x+c)*tan(d*x+c)/a/d-
sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*Tan[c + d*x])/(a*d) + (3*Sec[c + d*x
]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x
]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{\int(-3a+2a\cos(c+dx))\sec^3(c+dx)\,dx}{a^2}\\ &= -\frac{\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{2\int\sec^2(c+dx)\,dx}{a}+\frac{3\int\sec^3(c+dx)\,dx}{a}\\ &= \frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{3\int\sec(c+dx)\,dx}{2a}+\frac{2\,\mathrm{Subst}(}{}\\ &= \frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{2\tan(c+dx)}{ad}+\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec(c+dx)\tan(}{d(a+a\cos(c+}\end{aligned}$$

Mathematica [B] time = 1.30, size = 244, normalized size = 2.94

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(-4*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(-6*Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^(-2) - (4*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])))))/(2*a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 1.06, size = 112, normalized size = 1.35

$$\frac{3\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)}{4\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x
 + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c)^2 + co
s(d*x + c) - 1)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

giac [A] time = 0.81, size = 101, normalized size = 1.22

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/a - 2*tan(1/2*d*x + 1/2*c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

maple [A] time = 0.11, size = 143, normalized size = 1.72

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}-\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2+3/2/a/d/(tan(1/2
*d*x+1/2*c)-1)-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)-1/2/a/d/(tan(1/2*d*x+1/2*c)
+1)^2+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.31, size = 162, normalized size = 1.95

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) + 1)
))/d
```

mupad [B] time = 0.45, size = 95, normalized size = 1.14

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - tan(c/2 + (d*x)/2)/(a*d) - (tan(c/2 +
 (d*x)/2) - 3*tan(c/2 + (d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*ta
n(c/2 + (d*x)/2)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^3(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x) + 1), x)/a
```

3.52 $\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=103

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] -3/2*arctanh(sin(d*x+c))/a/d+4*tan(d*x+c)/a/d-3/2*sec(d*x+c)*tan(d*x+c)/a/d
-sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))+4/3*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2768, 2748, 3767, 3768, 3770}

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(2*a*d) + (4*Tan[c + d*x])/(a*d) - (3*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c +
d*x])) + (4*Tan[c + d*x]^3)/(3*a*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int(-4a+3a\cos(c+dx))\sec^4(c+dx)\,dx}{a^2}$$
$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int\sec^3(c+dx)\,dx}{a} + \frac{4\int\sec^4(c+dx)\,dx}{a}$$
$$= -\frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int\sec(c+dx)\,dx}{2a} - \frac{4\,\mathrm{Subs}}{}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{4\tan(c+dx)}{ad} - \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\,\mathrm{ta}}{d(a+a\cos(}$$

Mathematica [B] time = 4.27, size = 368, normalized size = 3.57

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(6\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{8}\sec(c)\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-12\sin(2c+dx)-6\sin(c+2dx)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(6*Sec[c/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2]*Sec[c]*Sec[c
 + d*x]^3*(9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*
Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 27*Cos[d*x]*(Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]) + 27*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*
```

```
x)/2] + Sin[(c + d*x)/2]] - 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 48*Sin[d*x] - 12*Sin[2*c + d*x] - 6*Sin[c + 2*d*x] - 6*Sin[3
*c + 2*d*x] + 20*Sin[2*c + 3*d*x]))/8))/(3*a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 1.05, size = 124, normalized size = 1.20

$$-\frac{9\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx+c\right)+1\right)-9\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\log\left(-\sin\left(dx+c\right)+1\right.}{12\left(ad\cos\left(dx+c\right)^{4}+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(9*(cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 9*(cos(d
*x + c)^4 + cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*cos(d*x + c)^3 +
 7*cos(d*x + c)^2 - cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a
*d*cos(d*x + c)^3)
```

giac [A] time = 0.50, size = 114, normalized size = 1.11

$$-\frac{\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{3}a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a - 6*tan(1/2*d*x + 1/2*c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^5 - 16*t
an(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 -
1)^3*a))/d
```

maple [A] time = 0.12, size = 183, normalized size = 1.78

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{3ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{5}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)-1/3/a/d/(tan(1/2*d*x+1/2*c)-1)^3-1/a/d/(tan(1/2*d*
x+1/2*c)-1)^2-5/2/a/d/(tan(1/2*d*x+1/2*c)-1)+3/2/a/d*ln(tan(1/2*d*x+1/2*c)-
```

```
1)-1/3/a/d/(tan(1/2*d*x+1/2*c)+1)^3+1/a/d/(tan(1/2*d*x+1/2*c)+1)^2-5/2/a/d/
(tan(1/2*d*x+1/2*c)+1)-3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.82, size = 205, normalized size = 1.99

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
+ 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(d*x
 + c) + 1)))/d
```

mupad [B] time = 0.57, size = 96, normalized size = 0.93

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(a*d) - (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (3*tan(c/2
 + (d*x)/2) - (16*tan(c/2 + (d*x)/2)^3)/3 + 5*tan(c/2 + (d*x)/2)^5)/(a*d*(t
an(c/2 + (d*x)/2)^2 - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^4(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(cos(c + d*x) + 1), x)/a
```

3.53 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -5*x/a^2+12*sin(d*x+c)/a^2/d-5*cos(d*x+c)*sin(d*x+c)/a^2/d-10/3*cos(d*x+c)^
3*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*
x+c))^2-4*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2748, 2635, 8, 2633}

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-5*x)/a^2 + (12*Sin[c + d*x])/(a^2*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(a^2
*d) - (10*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (Cos[
c + d*x]^4*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) - (4*Sin[c + d*x]^3)/
(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^3(c+dx)(4a-6a\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \cos^2(c+dx)\left(30a^2-\right.}{3a^4}$$

$$= -\frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{10\int \cos^2(c+dx)\,dx}{a^2} +$$

$$= -\frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{5x}{a^2} + \frac{12\sin(c+dx)}{a^2d} - \frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} -$$

Mathematica [A] time = 0.43, size = 199, normalized size = 1.60

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-156\sin\left(c+\frac{dx}{2}\right)+342\sin\left(c+\frac{3dx}{2}\right)+118\sin\left(2c+\frac{3dx}{2}\right)+30\sin\left(2c+\frac{5dx}{2}\right)+30\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-360*d*x*Cos[(d*x)/2] - 360*d*x*Cos[c + (d*x)
/2] - 120*d*x*Cos[c + (3*d*x)/2] - 120*d*x*Cos[2*c + (3*d*x)/2] + 516*Sin[(
d*x)/2] - 156*Sin[c + (d*x)/2] + 342*Sin[c + (3*d*x)/2] + 118*Sin[2*c + (3*
d*x)/2] + 30*Sin[2*c + (5*d*x)/2] + 30*Sin[3*c + (5*d*x)/2] - 3*Sin[3*c + (
7*d*x)/2] - 3*Sin[4*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2] + Sin[5*c + (9*d*
x)/2]))/(192*a^2*d)
```

fricas [A] time = 1.08, size = 108, normalized size = 0.87

$$-\frac{15\,dx\cos\left(dx+c\right)^2+30\,dx\cos\left(dx+c\right)+15\,dx-\left(\cos\left(dx+c\right)^4-\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)^2+33\,\cos\left(dx\right.\right.}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(15*d*x*cos(d*x + c)^2 + 30*d*x*cos(d*x + c) + 15*d*x - (cos(d*x + c)^
4 - cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 33*cos(d*x + c) + 24)*sin(d*x + c))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.88, size = 108, normalized size = 0.87

$$-\frac{\frac{30\,(dx+c)}{a^2}-\frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(30*(d*x + c)/a^2 - 4*(15*tan(1/2*d*x + 1/2*c)^5 + 20*tan(1/2*d*x + 1/
2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (a^
4*tan(1/2*d*x + 1/2*c)^3 - 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.07, size = 156, normalized size = 1.26

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d\,a^2}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{10\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{40\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2*tan(1/2*d*x+1/2*c)+10/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+40/3/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)^3+6/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)-10/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.36, size = 207, normalized size = 1.67

$$\frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1) -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^2)/d
```

mupad [B] time = 0.50, size = 135, normalized size = 1.09

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-60\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^2,x)
```

```
[Out] -(sin(c/2 + (d*x)/2) - 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 60*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*
```

```
x)/2) - 16*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 30*cos(c/2 + (d*x)/2)^
3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

sympy [A] time = 9.74, size = 700, normalized size = 5.65

$$\begin{cases} -\frac{30dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{90dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} - \frac{}{6a^2d\,\mathrm{t}} \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-30*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a
**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*
d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 +
 d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*d*x*tan(c/2 + d
*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18
*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*d*x/(6*a**2*d*tan(c/2 + d*x/2)
**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) - tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/
2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*tan(c/2 + d*
x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*
a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*tan(c/2 + d*x/2)**5/(6*a**2*d*
tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d
*x/2)**2 + 6*a**2*d) + 160*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**
6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*
d) + 63*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2
+ d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(
c)**5/(a*cos(c) + a)**2, True))
```

3.54 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$-\frac{16\sin(c+dx)}{3a^2d}-\frac{8\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] 7/2*x/a^2-16/3*sin(d*x+c)/a^2/d+7/2*cos(d*x+c)*sin(d*x+c)/a^2/d-8/3*cos(d*x
+c)^2*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*co
s(d*x+c))^2
```

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2765, 2977, 2734}

$$-\frac{16\sin(c+dx)}{3a^2d}-\frac{8\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (7*x)/(2*a^2) - (16*Sin[c + d*x])/(3*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*d) - (8*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) -
 (Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(3a-5a\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{8\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \cos(c+dx)\left(16a^2-21a\right.}{3a^4}$$
$$= \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos}{3a}$$

Mathematica [A] time = 0.29, size = 177, normalized size = 1.55

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(147\sin\left(c+\frac{dx}{2}\right)-239\sin\left(c+\frac{3dx}{2}\right)-63\sin\left(2c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{5dx}{2}\right)-15\sin\left(3c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(252*d*x*Cos[(d*x)/2] + 252*d*x*Cos[c + (d*x)/
2] + 84*d*x*Cos[c + (3*d*x)/2] + 84*d*x*Cos[2*c + (3*d*x)/2] - 381*Sin[(d*x
)/2] + 147*Sin[c + (d*x)/2] - 239*Sin[c + (3*d*x)/2] - 63*Sin[2*c + (3*d*x)
/2] - 15*Sin[2*c + (5*d*x)/2] - 15*Sin[3*c + (5*d*x)/2] + 3*Sin[3*c + (7*d*
x)/2] + 3*Sin[4*c + (7*d*x)/2]))/(192*a^2*d)
```

fricas [A] time = 2.00, size = 99, normalized size = 0.87

$$\frac{21\,dx\cos(dx+c)^2 + 42\,dx\cos(dx+c) + 21\,dx + \left(3\cos(dx+c)^3 - 6\cos(dx+c)^2 - 43\cos(dx+c) - 32\right)\sin(}{6\left(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c)
^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.56, size = 95, normalized size = 0.83

$$\frac{\frac{21\,(dx+c)}{a^2} - \frac{6\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 21*
a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.05, size = 122, normalized size = 1.07

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d\,a^2}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2
*tan(1/2*d*x+1/2*c)+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.35, size = 164, normalized size = 1.44

$$-\frac{\frac{6\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{42\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^2)/d
```

mupad [B] time = 0.45, size = 113, normalized size = 0.99

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-22\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-30\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (sin(c/2 + (d*x)/2) - 22*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 30*cos(c
/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x
)/2) + 21*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

sympy [A] time = 6.12, size = 413, normalized size = 3.62

$$\begin{cases} \frac{21dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}+\frac{42dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}+\frac{21dx}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}+ \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((21*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a*
*2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d
*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*d*x/(
6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) +
tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x
/2)**2 + 6*a**2*d) - 19*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**4 +
 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 71*tan(c/2 + d*x/2)**3/(6*a**2
*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 39*tan
(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**2, True))
```

3.55 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=80

$$\frac{4\sin(c+dx)}{3a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{2x}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -2*x/a^2+4/3*sin(d*x+c)/a^2/d+2*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x
+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.17, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2968, 3023, 12, 2735, 2648}

$$\frac{4\sin(c+dx)}{3a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{2x}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 + (4*Sin[c + d*x])/(3*a^2*d) + (2*Sin[c + d*x])/(a^2*d*(1 + Cos[
c + d*x])) - (Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{\int \frac{\cos(c+dx)(2a-4a\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}\\
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{\int \frac{2a\cos(c+dx)-4a\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\
&= \frac{4\sin(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{\int \frac{6a^2\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^3}\\
&= \frac{4\sin(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{2\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}\\
&= -\frac{2x}{a^2}+\frac{4\sin(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{2\int \frac{1}{a+a\cos(c+dx)}\,dx}{a}\\
&= -\frac{2x}{a^2}+\frac{4\sin(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{2\sin(c+dx)}{d\left(a^2+a^2\cos(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 114, normalized size = 1.42

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-6(\sin(c+dx)-2dx)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-16\,\mathrm{se}\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] - 16*Cos[(c + d*x)/2]^2*Sec[c/2
]*Sin[(d*x)/2] - 6*Cos[(c + d*x)/2]^3*(-2*d*x + Sin[c + d*x]) + Cos[(c + d*
x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 1.34, size = 90, normalized size = 1.12

$$-\frac{6\,dx\cos(dx+c)^2+12\,dx\cos(dx+c)+6\,dx-\left(3\,\cos(dx+c)^2+14\,\cos(dx+c)+10\right)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*d*x*cos(d*x + c)^2 + 12*d*x*cos(d*x + c) + 6*d*x - (3*cos(d*x + c)^
2 + 14*cos(d*x + c) + 10)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d)
```

giac [A] time = 0.46, size = 79, normalized size = 0.99

$$-\frac{\frac{12\,(dx+c)}{a^2}-\frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 +
 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/
d
```

maple [A] time = 0.06, size = 88, normalized size = 1.10

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d\,a^2}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2*tan(1/
2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.50, size = 118, normalized size = 1.48

$$\frac{\frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12 \sin(dx+c)}{\left(a^2+\frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x + c)
/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.42, size = 91, normalized size = 1.14

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(c+d\,x)}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^2,x)
```

```
[Out] -(sin(c/2 + (d*x)/2) - 16*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 12*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6
*a^2*d*cos(c/2 + (d*x)/2)^3)
```

sympy [A] time = 3.64, size = 201, normalized size = 2.51

$$\begin{cases} -\frac{12dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}-\frac{12dx}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}-\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}+\frac{14\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}+\frac{27\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} & \text{for} \\ \frac{x\cos^3(c)}{(a\cos(c)+a)^2} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) - 12*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - tan(c/2 + d*x/2)
**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 14*tan(c/2 + d*x/2)**3/(6*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*tan(c/2 + d*x/2)/(6*a**2*d*tan(c
/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a)**2, True
))
```

3.56 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$-\frac{5\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{x}{a^2}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] x/a^2-5/3*sin(d*x+c)/a^2/d/(1+cos(d*x+c))+1/3*sin(d*x+c)/d/(a+a*cos(d*x+c))
^2
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2758, 2735, 2648}

$$-\frac{5\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{x}{a^2}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] x/a^2 - (5*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(3*d*(
a + a*Cos[c + d*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{-2a+3a\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= \frac{x}{a^2} + \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{5\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}$$
$$= \frac{x}{a^2} + \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{5\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [A] time = 0.22, size = 105, normalized size = 1.84

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6dx\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-10\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos^2\left(\right.\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(6*d*x*Cos[(c + d*x)/2]^3 + Sec[c/2]*Sin[(d*x)/2] - 10*
Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*Tan[c/2]))/(3*a
^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.99, size = 80, normalized size = 1.40

$$\frac{3\,dx\cos\left(dx+c\right)^2+6\,dx\cos\left(dx+c\right)+3\,dx-\left(5\,\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) +
4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.41, size = 50, normalized size = 0.88

$$\frac{\frac{6\,(dx+c)}{a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/a^6)/d
```

maple [A] time = 0.06, size = 56, normalized size = 0.98

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d\,a^2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(
tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.00, size = 72, normalized size = 1.26

$$-\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 0.36, size = 35, normalized size = 0.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,d\,x}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2) + 6*d*x)/(6*a^2*d)
```

sympy [A] time = 2.02, size = 56, normalized size = 0.98

$$\begin{cases} \frac{x}{a^2}+\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x\cos^2(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/a**2 + tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*tan(c/2 + d*x/2)/(2*
a**2*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**2, True))
```

3.57 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{2\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -1/3*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+2/3*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2750, 2648}

$$\frac{2\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(3*d*(a^2 + a
^2*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{2\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}$$
$$= -\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{2\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [A] time = 0.11, size = 60, normalized size = 1.09

$$\frac{\sec\left(\frac{c}{2}\right)\left(-3\sin\left(c+\frac{dx}{2}\right)+2\sin\left(c+\frac{3dx}{2}\right)+3\sin\left(\frac{dx}{2}\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(3*Sin[(d*x)/2] - 3*Sin[c + (d*x)/2] + 2*Sin[c
 + (3*d*x)/2]))/(12*a^2*d)
```

fricas [A] time = 1.53, size = 51, normalized size = 0.93

$$\frac{(2\cos(dx+c)+1)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)
```

giac [A] time = 0.45, size = 31, normalized size = 0.56

$$-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

maple [A] time = 0.05, size = 32, normalized size = 0.58

$$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.31, size = 47, normalized size = 0.85

$$\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

mupad [B] time = 0.33, size = 30, normalized size = 0.55

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^2,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 3))/(6*a^2*d)
```

sympy [A] time = 1.29, size = 48, normalized size = 0.87

$$\begin{cases} -\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**3/(6*a**2*d) + tan(c/2 + d*x/2)/(2*a**2*d), N
e(d, 0)), (x*cos(c)/(a*cos(c) + a)**2, True))
```

3.58 $\int \frac{1}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] 1/3*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$\frac{\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*d*(a^2 + a^2*Co
s[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^2}\,dx &= \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}\\ &= \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.96

$$\frac{\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-2),x]
```

```
[Out] (Cos[(c + d*x)/2]*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*a^2*d*(1
+ Cos[c + d*x])^2)
```

fricas [A] time = 1.06, size = 49, normalized size = 0.89

$$\frac{(\cos(dx+c)+2)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)
```

giac [A] time = 0.76, size = 31, normalized size = 0.56

$$\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

maple [A] time = 0.04, size = 32, normalized size = 0.58

$$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.10, size = 46, normalized size = 0.84

$$\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

mupad [B] time = 0.33, size = 30, normalized size = 0.55

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+3\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 3))/(6*a^2*d)
```

sympy [A] time = 0.90, size = 44, normalized size = 0.80

$$\begin{cases} \frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d \neq 0 \\ \frac{x}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**3/(6*a**2*d) + tan(c/2 + d*x/2)/(2*a**2*d), Ne
(d, 0)), (x/(a*cos(c) + a)**2, True))
```

3.59 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{4\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] arctanh(sin(d*x+c))/a^2/d-4/3*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*sin(d*x+c
)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2766, 2978, 12, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{4\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^2*d) - (4*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x]
)) - Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(3a-a\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\ &= -\frac{4\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int 3a^2\sec(c+dx)\,dx}{3a^4}\\ &= -\frac{4\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \sec(c+dx)\,dx}{a^2}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{4\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}\end{aligned}$$

Mathematica [B] time = 0.29, size = 152, normalized size = 2.30

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.\right.}{3a^2d(\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sin[(d*x)/
2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*Tan[c/2]
))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.90, size = 114, normalized size = 1.73

$$\frac{3\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log(-\sin(d}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c
) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.44, size = 77, normalized size = 1.17

$$\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

maple [A] time = 0.09, size = 77, normalized size = 1.17

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d\,a^2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2*tan(1/2*d*x+1/2*c)-1/d/a^2*ln(tan
(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.99, size = 98, normalized size = 1.48

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

mupad [B] time = 0.37, size = 43, normalized size = 0.65

$$-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] -(9*tan(c/2 + (d*x)/2) - 12*atanh(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)^
3)/(6*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.60 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=81

$$\frac{10\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -2*arctanh(sin(d*x+c))/a^2/d+10/3*tan(d*x+c)/a^2/d-2*tan(d*x+c)/a^2/d/(1+co
s(d*x+c))-1/3*tan(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.17, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{10\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*ArcTanh[Sin[c + d*x]])/(a^2*d) + (10*Tan[c + d*x])/(3*a^2*d) - (2*Tan[c
 + d*x])/(a^2*d*(1 + Cos[c + d*x])) - Tan[c + d*x]/(3*d*(a + a*Cos[c + d*x]
)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
```

```
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(4a-2a\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(10a^2-6a^2\cos(c+dx)\right)\sec^2}{3a^4}$$

$$= -\frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{2\int\sec(c+dx)\,dx}{a^2} + \frac{10\int\sec^2(}{3a}$$

$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{10\,\mathrm{Sub}}{}$$

$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{10\tan(c+dx)}{3a^2d} - \frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+d}{3d(a+a\cos(c}$$

Mathematica [B] time = 1.12, size = 239, normalized size = 2.95

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\right.\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]
*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*(2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] -
Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(
1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.68, size = 146, normalized size = 1.80

$$-\frac{3\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c)^3+2\cos(dx+c)^2+\text{co}\right.}{3\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c)
 + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x +
 c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

giac [A] time = 0.63, size = 106, normalized size = 1.31

$$-\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^
2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.11, size = 120, normalized size = 1.48

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d\,a^2}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{1}{d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}-\frac{1}{d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*tan(1/2*d*x+1/2*c)-1/d/a^2/(tan(1/
2*d*x+1/2*c)-1)+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c
)+1)-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.65, size = 145, normalized size = 1.79

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.41, size = 92, normalized size = 1.14

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}-\frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (2
*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) + (5*tan(c/2 + (d
*x)/2))/(2*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.61 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2} dx$

Optimal. Leaf size=119

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{8\tan(c+dx)\sec(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{\tan(c+dx)\sec(}{3d(a\cos(c+dx}$$

```
[Out] 7/2*arctanh(sin(d*x+c))/a^2/d-16/3*tan(d*x+c)/a^2/d+7/2*sec(d*x+c)*tan(d*x+
c)/a^2/d-8/3*sec(d*x+c)*tan(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*sec(d*x+c)*tan(
d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{8\tan(c+dx)\sec(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{\tan(c+dx)\sec(}{3d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (16*Tan[c + d*x])/(3*a^2*d) + (7*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*d) - (8*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(
1 + Cos[c + d*x])) - (Sec[c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(5a-3a\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{8\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(21a^2-16a^2\cos(c+dx\right.}{3a^4}$$
$$= -\frac{8\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{16\int \sec^2(c+dx)\,dx}{3a^2} + \frac{7\int}{}$$
$$= \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} +$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{16\tan(c+dx)}{3a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec(c+dx}{3a^2d(1+c}$$

Mathematica [B] time = 1.81, size = 292, normalized size = 2.45

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-2\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right) - 2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right) + 3\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-\frac{}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\right.\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*(-2*Sec[c/2]*Sin[(d*x)/2] - 40*Cos[(c + d*x)/2]^2*Sec[c/2
]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]^3*(-14*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) - (8*
Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - 2*Cos[(c + d*x)
/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 1.10, size = 162, normalized size = 1.36

$$\frac{21\left(\cos(dx+c)^4 + 2\cos(dx+c)^3 + \cos(dx+c)^2\right)\log(\sin(dx+c)+1) - 21\left(\cos(dx+c)^4 + 2\cos(dx+c)^3 +\right.}{12\left(a^2d\cos(dx+c)^4 + 2a^2d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x +
 c) + 1) - 21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin
(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 43*cos(d*x + c)^2 + 6*cos(d*x + c)
- 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*c
os(d*x + c)^2)
```

giac [A] time = 0.60, size = 122, normalized size = 1.03

$$\frac{\frac{21\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{21\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}+\frac{6\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{2}}-\frac{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+21\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4*ta
n(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.14, size = 162, normalized size = 1.36

$$-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d\,a^{2}}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}+\frac{1}{2d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{5}{2d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^{2}}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2*tan(1/2*d*x+1/2*c)+1/2/d/a^2/(tan
(1/2*d*x+1/2*c)-1)^2+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)-7/2/d/a^2*ln(tan(1/2*
d*x+1/2*c)-1)-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+5/2/d/a^2/(tan(1/2*d*x+1/2
*c)+1)+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.95, size = 190, normalized size = 1.60

$$-\frac{\frac{6\left(\frac{3\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{5\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}\right)}{a^{2}-\frac{2\,a^{2}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{a^{2}\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}}+\frac{\frac{21\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}}{a^{2}}-\frac{21\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^{2}}+\frac{21\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

mupad [B] time = 0.43, size = 122, normalized size = 1.03

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^2),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (3
*tan(c/2 + (d*x)/2) - 5*tan(c/2 + (d*x)/2)^3)/(d*(a^2*tan(c/2 + (d*x)/2)^4
- 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (7*tan(c/2 + (d*x)/2))/(2*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.62 $\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{4\tan^3(c+dx)}{a^2d}+\frac{12\tan(c+dx)}{a^2d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{10\tan(c+dx)\sec^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\mathrm{t}$$

```
[Out] -5*arctanh(sin(d*x+c))/a^2/d+12*tan(d*x+c)/a^2/d-5*sec(d*x+c)*tan(d*x+c)/a^
2/d-10/3*sec(d*x+c)^2*tan(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*sec(d*x+c)^2*tan(
d*x+c)/d/(a+a*cos(d*x+c))^2+4*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 3768, 3770}

$$\frac{4\tan^3(c+dx)}{a^2d}+\frac{12\tan(c+dx)}{a^2d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{10\tan(c+dx)\sec^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^2*d) + (12*Tan[c + d*x])/(a^2*d) - (5*Sec[c +
 d*x]*Tan[c + d*x])/(a^2*d) - (10*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1
+ Cos[c + d*x])) - (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2) + (4*Tan[c + d*x]^3)/(a^2*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(6a-4a\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(36a^2-30a^2\cos(c+\right.}{3a^4}$$

$$= -\frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{10\int \sec^3(c+dx)\,dx}{a^2} + 1$$

$$= -\frac{5\sec(c+dx)\tan(c+dx)}{a^2d} - \frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{12\tan(c+dx)}{a^2d} - \frac{5\sec(c+dx)\tan(c+dx)}{a^2d} - \frac{10\sec^2(c+}{3a^2d(1+}$$

Mathematica [B] time = 3.88, size = 343, normalized size = 2.58

$$\frac{960\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (960*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Sec[c
 + d*x]^3*(-3*Sin[(d*x)/2] + 155*Sin[(3*d*x)/2] - 153*Sin[c - (d*x)/2] + 21
*Sin[c + (d*x)/2] - 135*Sin[2*c + (d*x)/2] + 25*Sin[c + (3*d*x)/2] + 45*Sin
[2*c + (3*d*x)/2] - 85*Sin[3*c + (3*d*x)/2] + 99*Sin[c + (5*d*x)/2] + 21*Si
n[2*c + (5*d*x)/2] + 33*Sin[3*c + (5*d*x)/2] - 45*Sin[4*c + (5*d*x)/2] + 57
*Sin[2*c + (7*d*x)/2] + 18*Sin[3*c + (7*d*x)/2] + 24*Sin[4*c + (7*d*x)/2] -
 15*Sin[5*c + (7*d*x)/2] + 24*Sin[3*c + (9*d*x)/2] + 11*Sin[4*c + (9*d*x)/2
] + 13*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.96, size = 172, normalized size = 1.29

$$-\frac{15\left(\cos(dx+c)^5+2\cos(dx+c)^4+\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^5+2\cos(dx+c)^4+\right.}{6\left(a^2d\cos(dx+c)^5+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(15*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x +
 c) + 1) - 15*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 + cos(d*x + c)^3)*log(-sin
(d*x + c) + 1) - 2*(24*cos(d*x + c)^4 + 33*cos(d*x + c)^3 + 6*cos(d*x + c)^
2 - cos(d*x + c) + 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x
 + c)^4 + a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.93, size = 135, normalized size = 1.02

$$-\frac{\frac{30 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{30 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{4\left(15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3 a^2}-\frac{a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(30*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 30*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 4*(15*tan(1/2*d*x + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^3
 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (a^4*tan(
1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.10, size = 204, normalized size = 1.53

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d\,a^2}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{1}{3d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3}{2d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{5}{d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2*tan(1/2*d*x+1/2*c)-1/3/d/a^2/(tan(
1/2*d*x+1/2*c)-1)^3-3/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-5/d/a^2/(tan(1/2*d*x
+1/2*c)-1)+5/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a^2/(tan(1/2*d*x+1/2*c)+1
)^3+3/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2-5/d/a^2/(tan(1/2*d*x+1/2*c)+1)-5/d/a
^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.95, size = 234, normalized size = 1.76

$$\frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^2*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1) +
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

mupad [B] time = 0.45, size = 153, normalized size = 1.15

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}-\frac{10\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (10*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (
6*tan(c/2 + (d*x)/2) - (40*tan(c/2 + (d*x)/2)^3)/3 + 10*tan(c/2 + (d*x)/2)^
5)/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/
2 + (d*x)/2)^6 - a^2)) + (9*tan(c/2 + (d*x)/2))/(2*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.63 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{152\sin(c+dx)}{15a^3d}-\frac{76\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{13x}{2a^3}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{11 \mathrm{s}}{15}$$

```
[Out] 13/2*x/a^3-152/15*sin(d*x+c)/a^3/d+13/2*cos(d*x+c)*sin(d*x+c)/a^3/d-1/5*cos
(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-11/15*cos(d*x+c)^3*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^2-76/15*cos(d*x+c)^2*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2765, 2977, 2734}

$$-\frac{152\sin(c+dx)}{15a^3d}-\frac{76\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{13x}{2a^3}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{11 \mathrm{s}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (13*x)/(2*a^3) - (152*Sin[c + d*x])/(15*a^3*d) + (13*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3)
 - (11*Cos[c + d*x]^3*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (76*C
os[c + d*x]^2*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)(4a-7a\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(33a^2-43a^2\cos(c+\right.}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{76\cos^2(c+dx)\sin(c+d}{15d\left(a^3+a^3\cos(c+dx)\right.}$$

$$= \frac{13x}{2a^3} - \frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11}{1}$$

Mathematica [A] time = 0.57, size = 173, normalized size = 1.13

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(15(-12\sin(c+dx)+\sin(2(c+dx))+26dx)\cos^5\left(\frac{1}{2}(c+dx)\right)+46\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)-3\right.}{15a^3d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-3*Sec[c/2]*Sin[(d*x)/2] + 46*Cos[(c + d*x)/2]^2*Sec[c
/2]*Sin[(d*x)/2] - 508*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 15*Cos[(c
 + d*x)/2]^5*(26*d*x - 12*Sin[c + d*x] + Sin[2*(c + d*x)]) - 3*Cos[(c + d*x
)/2]*Tan[c/2] + 46*Cos[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Cos[c + d*x
])^3)
```

fricas [A] time = 1.02, size = 135, normalized size = 0.88

$$\frac{195\,dx\cos\left(dx+c\right)^{3}+585\,dx\cos\left(dx+c\right)^{2}+585\,dx\cos\left(dx+c\right)+195\,dx+\left(15\,\cos\left(dx+c\right)^{4}-45\,\cos\left(dx+c\right)}{30\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x + c
) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2 -
 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.38, size = 113, normalized size = 0.74

$$\frac{\frac{390\,(dx+c)}{a^{3}}-\frac{60\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{3}}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+465\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.06, size = 141, normalized size = 0.92

$$-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^{3}}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^{3}}-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{13\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+2/3/d/a^3*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*
tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-
5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+13/d/a^3*arctan(tan(1
/2*d*x+1/2*c))
```

maxima [A] time = 1.26, size = 184, normalized size = 1.20

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 0.47, size = 137, normalized size = 0.90

$$-\frac{3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-46\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+508\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\right.}{60\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^3,x)
```

```
[Out] -(3*sin(c/2 + (d*x)/2) - 46*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 508*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 +
 (d*x)/2) - 120*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) - 390*cos(c/2 + (d*
x)/2)^5*(c + d*x))/(60*a^3*d*cos(c/2 + (d*x)/2)^5)
```

sympy [A] time = 13.65, size = 473, normalized size = 3.09

$$\begin{cases} \frac{390dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}+\frac{780dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}+\frac{390dx}{60a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+120a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cdot} \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((390*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120
*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*d*x*tan(c/2 + d*x/2)**2/(60*
a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
390*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 6
0*a**3*d) - 3*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3
*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*tan(c/2 + d*x/2)**7/(60*a**3*d*tan
(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 388*tan(c/
2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)*
*2 + 60*a**3*d) - 1310*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*tan(c/2 + d*x/2)/(60*a**
3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d), Ne(d
, 0)), (x*cos(c)**5/(a*cos(c) + a)**3, True))
```

3.64 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=119

$$\frac{9\sin(c+dx)}{5a^3d}+\frac{3\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{3\sin(c+dx)\cos^2(c+dx)}{5ad(a\cos(c+dx)+a)^2}$$

```
[Out] -3*x/a^3+9/5*sin(d*x+c)/a^3/d-1/5*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^3-3/5*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+3*sin(d*x+c)/d/(a^3+
a^3*cos(d*x+c))
```

Rubi [A] time = 0.27, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2765, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{9\sin(c+dx)}{5a^3d}+\frac{3\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{3\sin(c+dx)\cos^2(c+dx)}{5ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 + (9*Sin[c + d*x])/(5*a^3*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(5*
d*(a + a*Cos[c + d*x])^3) - (3*Cos[c + d*x]^2*Sin[c + d*x])/(5*a*d*(a + a*C
os[c + d*x])^2) + (3*Sin[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(3a-6a\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(18a^2-27a^2\cos(c+dx\right.}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{18a^2\cos(c+dx)-27a^2\cos^2(c+d}{a+a\cos(c+dx)}}{15a^4}$$

$$= \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{45a^3\cos(c+}{a+a\cos(c+}}{15a^5}$$

$$= \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{3\int \frac{\cos(c+}{a+a\cos(c}}{a^2}$$

$$= -\frac{3x}{a^3} + \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{3\int}{}$$

$$= -\frac{3x}{a^3} + \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{}{d\left(a\right.}$$

Mathematica [A] time = 0.53, size = 161, normalized size = 1.35

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(20(\sin(c+dx)-3dx)\cos^5\left(\frac{1}{2}(c+dx)\right)-12\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\right.}{5a^3d(\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] - 12*Cos[(c + d*x)/2]^2*Sec[c/2]
*Sin[(d*x)/2] + 96*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 20*Cos[(c + d
*x)/2]^5*(-3*d*x + Sin[c + d*x]) + Cos[(c + d*x)/2]*Tan[c/2] - 12*Cos[(c +
d*x)/2]^3*Tan[c/2]))/(5*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 1.18, size = 126, normalized size = 1.06

$$-\frac{15\,dx\cos(dx+c)^3+45\,dx\cos(dx+c)^2+45\,dx\cos(dx+c)+15\,dx-\left(5\cos(dx+c)^3+39\cos(dx+c)^2+57\right.}{5\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*sin
(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)
```

giac [A] time = 0.55, size = 96, normalized size = 0.81

$$-\frac{\frac{60\,(dx+c)}{a^3}-\frac{40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}-\frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+85\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
+ 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3 +
 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.06, size = 107, normalized size = 0.90

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^3}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-1/2/d/a^3*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*t
an(1/2*d*x+1/2*c)+2/d/a^3*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a
^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.23, size = 137, normalized size = 1.15

$$\frac{\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 0.44, size = 113, normalized size = 0.95

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-}{20\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (sin(c/2 + (d*x)/2) - 12*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 96*cos(c
/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x
)/2) - 60*cos(c/2 + (d*x)/2)^5*(c + d*x))/(20*a^3*d*cos(c/2 + (d*x)/2)^5)
```

sympy [A] time = 8.56, size = 240, normalized size = 2.02

$$\begin{cases}-\frac{60dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d}-\frac{60dx}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d}+\frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d}-\frac{9\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d}+\frac{75\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2} \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-60*d*x*tan(c/2 + d*x/2)**2/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*
a**3*d) - 60*d*x/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + tan(c/2 + d*
x/2)**7/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) - 9*tan(c/2 + d*x/2)**5
/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + 75*tan(c/2 + d*x/2)**3/(20*a
**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + 125*tan(c/2 + d*x/2)/(20*a**3*d*ta
n(c/2 + d*x/2)**2 + 20*a**3*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**3,
True))
```

3.65 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=96

$$-\frac{29\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x}{a^3}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

```
[Out] x/a^3-1/5*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+7/15*sin(d*x+c)/a/d/
(a+a*cos(d*x+c))^2-29/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2765, 2968, 3019, 2735, 2648}

$$-\frac{29\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x}{a^3}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] x/a^3 - (Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (7*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (29*Sin[c + d*x])/(15*d*(a^3 +
 a^3*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
```

```
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(2a-5a\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2} \\
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{2a\cos(c+dx)-5a\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2} \\
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{-14a^2+15a^2\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{15a^4} \\
&= \frac{x}{a^3} - \frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{29\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2} \\
&= \frac{x}{a^3} - \frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{29\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx\right.}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 154, normalized size = 1.60

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(60dx\cos^5\left(\frac{1}{2}(c+dx)\right)+26\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)-3\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)-3\sec\left(\frac{c}{2}\right)\sin\right.}{15a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(60*d*x*Cos[(c + d*x)/2]^5 - 3*Sec[c/2]*Sin[(d*x)/2] +
26*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 128*Cos[(c + d*x)/2]^4*Sec[c/
2]*Sin[(d*x)/2] - 3*Cos[(c + d*x)/2]*Tan[c/2] + 26*Cos[(c + d*x)/2]^3*Tan[c
/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 1.75, size = 116, normalized size = 1.21

$$\frac{15\,dx\cos\left(dx+c\right)^{3}+45\,dx\cos\left(dx+c\right)^{2}+45\,dx\cos\left(dx+c\right)+15\,dx-\left(32\,\cos\left(dx+c\right)^{2}+51\,\cos\left(dx+c\right)+22\right.}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (32*cos(d*x + c)^2 + 51*cos(d*x + c) + 22)*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.47, size = 68, normalized size = 0.71

$$\frac{\frac{60\,(dx+c)}{a^{3}}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.06, size = 75, normalized size = 0.78

$$-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^{3}}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^{3}}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*t
an(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.69, size = 92, normalized size = 0.96

$$-\frac{\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1}-\frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 0.42, size = 81, normalized size = 0.84

$$\frac{x}{a^3}-\frac{\frac{32 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}-\frac{13 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{30}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (sin(c/2 + (d*x)/2)/20 - (13*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
))/30 + (32*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/15)/(a^3*d*cos(c/2 + (
d*x)/2)^5)
```

sympy [A] time = 5.15, size = 75, normalized size = 0.78

$$\begin{cases} \frac{x}{a^3}-\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d\neq 0 \\ \frac{x\cos^3(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/a**3 - tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)**3/(
3*a**3*d) - 7*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c
) + a)**3, True))
```

3.66 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{7\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{8\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] 1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-8/15*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+
7/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2758, 2750, 2648}

$$\frac{7\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{8\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d*(a + a
*Cos[c + d*x])^2) + (7*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{-3a+5a\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$
$$=\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{8\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{7\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2}$$
$$=\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{8\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{7\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

Mathematica [A] time = 0.19, size = 86, normalized size = 1.04

$$\frac{\sec\left(\frac{c}{2}\right)\left(-30\sin\left(c+\frac{dx}{2}\right)+20\sin\left(c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+40\sin\left(\frac{dx}{2}\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(40*Sin[(d*x)/2] - 30*Sin[c + (d*x)/2] + 20*Si
n[c + (3*d*x)/2] - 15*Sin[2*c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2]))/(240*
a^3*d)
```

fricas [A] time = 2.21, size = 75, normalized size = 0.90

$$\frac{\left(7\cos(dx+c)^2+6\cos(dx+c)+2\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.51, size = 46, normalized size = 0.55

$$\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

maple [A] time = 0.05, size = 45, normalized size = 0.54

$$\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

maxima [A] time = 1.01, size = 67, normalized size = 0.81

$$\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

mupad [B] time = 0.35, size = 45, normalized size = 0.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+15\right)}{60\,a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(3*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 15)
)/(60*a^3*d)
```

sympy [A] time = 3.36, size = 68, normalized size = 0.82

$$\begin{cases} \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**5/(20*a**3*d) - tan(c/2 + d*x/2)**3/(6*a**3*d)
 + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**3,
True))
```

3.67 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{\sin(c+dx)}{5d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\sin(c+dx)}{5ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] -1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/5*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+
1/5*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2750, 2650, 2648}

$$\frac{\sin(c+dx)}{5d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\sin(c+dx)}{5ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + Sin[c + d*x]/(5*a*d*(a + a*Cos
[c + d*x])^2) + Sin[c + d*x]/(5*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{3\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{5a}$$
$$= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{a+a\cos(c+dx)}\,dx}{5a^2}$$
$$= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{\sin(c+dx)}{5d\left(a^3+a^3\cos(c+dx)\right)}$$

Mathematica [A] time = 0.14, size = 71, normalized size = 0.86

$$\frac{\sec\left(\frac{c}{2}\right)\left(-5\sin\left(c+\frac{dx}{2}\right)+5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{80a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(5*Sin[(d*x)/2] - 5*Sin[c + (d*x)/2] + 5*Sin[c
 + (3*d*x)/2] + Sin[2*c + (5*d*x)/2]))/(80*a^3*d)
```

fricas [A] time = 0.76, size = 73, normalized size = 0.88

$$\frac{\left(\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{5\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.41, size = 31, normalized size = 0.37

$$-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{20\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
[Out] -1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

maple [A] time = 0.04, size = 32, normalized size = 0.39

$$\frac{-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*cos(d*x+c))^3,x)
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.48, size = 47, normalized size = 0.57

$$\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{20\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
[Out] 1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)
^5)/(a^3*d)
```

mupad [B] time = 0.34, size = 30, normalized size = 0.36

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}-5\right)}{20\,a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^3,x)
[Out] -(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^4 - 5))/(20*a^3*d)
```

sympy [A] time = 2.28, size = 48, normalized size = 0.58

$$\begin{cases} -\frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^{3}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^{3}d} & \text{for } d\neq 0\\ \frac{x\cos\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{3}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)/(4*a**3*d),
Ne(d, 0)), (x*cos(c)/(a*cos(c) + a)**3, True))
```

3.68 $\int \frac{1}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{2\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{2\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] 1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+2/15*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+
2/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$\frac{2\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{2\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-3),x]
```

```
[Out] Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + (2*Sin[c + d*x])/(15*a*d*(a + a
*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3}\,dx = \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{5a}$$
$$= \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{2\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2}$$
$$= \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{2\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{2\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 0.78

$$\frac{\left(10\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{15a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-3),x]
```

```
[Out] (Cos[(c + d*x)/2]*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*(c
 + d*x))/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.77, size = 75, normalized size = 0.90

$$\frac{\left(2\cos\left(dx+c\right)^2+6\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3a^3d\cos\left(dx+c\right)^2+3a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(d*x + c)^2 + 6*cos(d*x + c) + 7)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.46, size = 46, normalized size = 0.55

$$\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

maple [A] time = 0.04, size = 45, normalized size = 0.54

$$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

maxima [A] time = 0.33, size = 67, normalized size = 0.81

$$\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

mupad [B] time = 0.35, size = 45, normalized size = 0.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+15\right)}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(10*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + 15)
)/(60*a^3*d)
```

sympy [A] time = 1.63, size = 63, normalized size = 0.76

$$\begin{cases} \frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d\neq 0 \\ \frac{x}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)**3/(6*a**3*d)
 + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x/(a*cos(c) + a)**3, True))
```

3.69 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{22\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] arctanh(sin(d*x+c))/a^3/d-1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-7/15*sin(d*x+
c)/a/d/(a+a*cos(d*x+c))^2-22/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.20, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2766, 2978, 12, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{22\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^3*d) - Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) -
 (7*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (22*Sin[c + d*x])/(15*d
*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(5a-2a\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\left(15a^2-7a^2\cos(c+dx)\right)\sec(c+dx)}{a+a\cos(c+dx)}\,d}{15a^4}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{22\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)} +$$

$$= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{22\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)} +$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{22}{15d\left(a^3\right.}$$

Mathematica [B] time = 0.48, size = 201, normalized size = 2.07

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(14\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+3\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+60\cos^5\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(60*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Sec[c/2]*Sin[(d*
```

```
x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 88*Cos[(c + d*x)/2]^4
*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]*Tan[c/2] + 14*Cos[(c + d*x)/2]^
3*Tan[c/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 1.21, size = 158, normalized size = 1.63

$$\frac{15\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^3+3\cos(dx+\right.}{30\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(sin(d
*x + c) + 1) - 15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
log(-sin(d*x + c) + 1) - 2*(22*cos(d*x + c)^2 + 51*cos(d*x + c) + 32)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

giac [A] time = 0.59, size = 94, normalized size = 0.97

$$\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+20a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+105a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*a^12*tan(1/2*d*x + 1
/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.08, size = 96, normalized size = 0.99

$$-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^3}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^3}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*t
an(1/2*d*x+1/2*c)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tan(1/2*d*x+1
/2*c)+1)
```

maxima [A] time = 1.12, size = 119, normalized size = 1.23

$$-\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/a^3)/d
```

mupad [B] time = 0.40, size = 58, normalized size = 0.60

$$-\frac{105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-120\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] -(105*tan(c/2 + (d*x)/2) - 120*atanh(tan(c/2 + (d*x)/2)) + 20*tan(c/2 + (d*
x)/2)^3 + 3*tan(c/2 + (d*x)/2)^5)/(60*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x)
 + 1), x)/a**3
```

3.70 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=112

$$\frac{24\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3\tan(c+dx)}{5ad(a\cos(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] -3*arctanh(sin(d*x+c))/a^3/d+24/5*tan(d*x+c)/a^3/d-1/5*tan(d*x+c)/d/(a+a*co
s(d*x+c))^3-3/5*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^2-3*tan(d*x+c)/d/(a^3+a^3*c
os(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{24\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3\tan(c+dx)}{5ad(a\cos(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + (24*Tan[c + d*x])/(5*a^3*d) - Tan[c +
d*x]/(5*d*(a + a*Cos[c + d*x])^3) - (3*Tan[c + d*x])/(5*a*d*(a + a*Cos[c +
d*x])^2) - (3*Tan[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(6a-3a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\left(27a^2-18a^2\cos(c+dx)\right)\sec^2(c+dx)}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{3\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx)\right)} + \int($$

$$= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{3\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx)\right)} - 3\int$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{}{d\left(a\right.}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{24\tan(c+dx)}{5a^3d} - \frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c}{5ad(a+a\cos}$$

Mathematica [B] time = 1.15, size = 286, normalized size = 2.55

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(8\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+20\cos^5\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*
Sin[(d*x)/2] + 76*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 20*Cos[(c + d*
x)/2]^5*(3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]))) + Cos[(c + d*x)/2]*Tan[c/2] + 8*Cos[(c + d*x)/2]^3*Tan[c/2]))/(5*a^3*
d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.89, size = 190, normalized size = 1.70

$$-\frac{15\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^4\right.}{10\left(a^3d\cos(dx+c)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x +
 c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3 +
57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

giac [A] time = 0.52, size = 122, normalized size = 1.09

$$-\frac{\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^3}-\frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+85\,a^{12}\tan\left(\frac{1}{2}\right.}{a^{15}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a
^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85*a^
12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.08, size = 139, normalized size = 1.24

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^3}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}-\frac{1}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^3}-\frac{1}{d\,a^3\left(\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+1/2/d/a^3*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*t
an(1/2*d*x+1/2*c)-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d/a^3*ln(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 1.48, size = 165, normalized size = 1.47

$$\frac{\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)/a^3)/d
```

mupad [B] time = 0.40, size = 111, normalized size = 0.99

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,a^3\,d}-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(2*a^3*d) + tan(c/2 + (d*x)/2)^5/(20*a^3*d) - (6*atanh
(tan(c/2 + (d*x)/2)))/(a^3*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d
*x)/2)^2 - a^3)) + (17*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1), x)/a**3
```

3.71 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=156

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{76\tan(c+dx)\sec(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{11\tan(c+dx}{15ad(a\cos(}$$

```
[Out] 13/2*arctanh(sin(d*x+c))/a^3/d-152/15*tan(d*x+c)/a^3/d+13/2*sec(d*x+c)*tan(
d*x+c)/a^3/d-1/5*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^3-11/15*sec(d*x+c
)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^2-76/15*sec(d*x+c)*tan(d*x+c)/d/(a^3+a^3*
cos(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{76\tan(c+dx)\sec(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{11\tan(c+dx}{15ad(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (13*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (152*Tan[c + d*x])/(15*a^3*d) + (13*
Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - (Sec[c + d*x]*Tan[c + d*x])/(5*d*(a
+ a*Cos[c + d*x])^3) - (11*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c
+ d*x])^2) - (76*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
```

```
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(7a-4a\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\left(43a^2-33a^2\cos(c+dx)\right)\sec^3(c+a}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{76\sec(c+dx)\tan(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{76\sec(c+dx)\tan(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

$$= \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} -$$

$$= \frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{152\tan(c+dx)}{15a^3d} + \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec(c+dx}{5d(a+a\,c}$$

Mathematica [B] time = 3.85, size = 343, normalized size = 2.20

$$-\frac{24960\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/480*(24960*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec
[c]*Sec[c + d*x]^2*(-1235*Sin[(d*x)/2] + 3805*Sin[(3*d*x)/2] - 4329*Sin[c -
 (d*x)/2] + 1989*Sin[c + (d*x)/2] - 3575*Sin[2*c + (d*x)/2] - 475*Sin[c + (
3*d*x)/2] + 2005*Sin[2*c + (3*d*x)/2] - 2275*Sin[3*c + (3*d*x)/2] + 2673*Si
n[c + (5*d*x)/2] + 105*Sin[2*c + (5*d*x)/2] + 1593*Sin[3*c + (5*d*x)/2] - 9
75*Sin[4*c + (5*d*x)/2] + 1325*Sin[2*c + (7*d*x)/2] + 255*Sin[3*c + (7*d*x)
/2] + 875*Sin[4*c + (7*d*x)/2] - 195*Sin[5*c + (7*d*x)/2] + 304*Sin[3*c + (
9*d*x)/2] + 90*Sin[4*c + (9*d*x)/2] + 214*Sin[5*c + (9*d*x)/2]))/(a^3*d*(1
+ Cos[c + d*x])^3)
```

fricas [A] time = 1.15, size = 206, normalized size = 1.32

$$\frac{195\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-195\left(\cos(dx+c)^5\right.}{60\left(a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x +
 c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*c
os(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x + c
)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x +
 c)^3 + a^3*d*cos(d*x + c)^2)
```

giac [A] time = 0.72, size = 139, normalized size = 0.89

$$\frac{\frac{390 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{390 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3} + \frac{60\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2 a^3} - \frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40a^{12}\tan\left(\frac{1}{2}d\right.}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3 + 60*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 40
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.12, size = 181, normalized size = 1.16

$$-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^3} - \frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^3} - \frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3} + \frac{1}{2d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{7}{2d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{13\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-2/3/d/a^3*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*
tan(1/2*d*x+1/2*c)+1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2+7/2/d/a^3/(tan(1/2*d*
x+1/2*c)-1)-13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^3/(tan(1/2*d*x+1/2*
c)+1)^2+7/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)+13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1
)
```

maxima [A] time = 0.96, size = 211, normalized size = 1.35

$$-\frac{\frac{60\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

mupad [B] time = 0.39, size = 141, normalized size = 0.90

$$\frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,a^3\,d}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,a^3\,d}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}-\frac{31\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^3),x)
```

```
[Out] (13*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^5/(20*a^3*d) -
(2*tan(c/2 + (d*x)/2)^3)/(3*a^3*d) - (5*tan(c/2 + (d*x)/2) - 7*tan(c/2 + (d
*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^3))
 - (31*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1), x)/a**3
```

3.72 $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$-\frac{576\sin(c+dx)}{35a^4d}-\frac{43\sin(c+dx)\cos^3(c+dx)}{35a^4d(\cos(c+dx)+1)^2}-\frac{288\sin(c+dx)\cos^2(c+dx)}{35a^4d(\cos(c+dx)+1)}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}+\frac{21x}{2a^4}-$$

```
[Out] 21/2*x/a^4-576/35*sin(d*x+c)/a^4/d+21/2*cos(d*x+c)*sin(d*x+c)/a^4/d-43/35*c
os(d*x+c)^3*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-288/35*cos(d*x+c)^2*sin(d*x+c
)/a^4/d/(1+cos(d*x+c))-1/7*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-2/5
*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.38, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2765, 2977, 2734}

$$-\frac{576\sin(c+dx)}{35a^4d}-\frac{43\sin(c+dx)\cos^3(c+dx)}{35a^4d(\cos(c+dx)+1)^2}-\frac{288\sin(c+dx)\cos^2(c+dx)}{35a^4d(\cos(c+dx)+1)}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}+\frac{21x}{2a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (21*x)/(2*a^4) - (576*Sin[c + d*x])/(35*a^4*d) + (21*Cos[c + d*x]*Sin[c + d
*x])/(2*a^4*d) - (43*Cos[c + d*x]^3*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*
x])^2) - (288*Cos[c + d*x]^2*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])) -
(Cos[c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*Cos[c + d*x
]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^4(c+dx)(5a-9a\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)\left(56a^2-73a^2\cos(c+d\right.}{(a+a\cos(c+dx))^2}}{35a^4}$$

$$= -\frac{43\cos^3(c+dx)\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^4(c+dx)\sin(c+dx}{5ad(a+a\cos(c+dx))^3}$$

$$= -\frac{43\cos^3(c+dx)\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^4(c+dx)\sin(c+dx}{5ad(a+a\cos(c+dx))^3}$$

$$= \frac{21x}{2a^4} - \frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos^3(c+dx)\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} -$$

Mathematica [A] time = 0.58, size = 289, normalized size = 1.57

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(128730\sin\left(c+\frac{dx}{2}\right)-140826\sin\left(c+\frac{3dx}{2}\right)+44310\sin\left(2c+\frac{3dx}{2}\right)-60487\sin\left(2c+\frac{5d}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(102900*d*x*Cos[(d*x)/2] + 102900*d*x*Cos[c +
(d*x)/2] + 61740*d*x*Cos[c + (3*d*x)/2] + 61740*d*x*Cos[2*c + (3*d*x)/2] +
```

```
20580*d*x*Cos[2*c + (5*d*x)/2] + 20580*d*x*Cos[3*c + (5*d*x)/2] + 2940*d*x*
Cos[3*c + (7*d*x)/2] + 2940*d*x*Cos[4*c + (7*d*x)/2] - 179830*Sin[(d*x)/2]
+ 128730*Sin[c + (d*x)/2] - 140826*Sin[c + (3*d*x)/2] + 44310*Sin[2*c + (3*
d*x)/2] - 60487*Sin[2*c + (5*d*x)/2] + 1225*Sin[3*c + (5*d*x)/2] - 12001*Si
n[3*c + (7*d*x)/2] - 3185*Sin[4*c + (7*d*x)/2] - 315*Sin[4*c + (9*d*x)/2] -
 315*Sin[5*c + (9*d*x)/2] + 35*Sin[5*c + (11*d*x)/2] + 35*Sin[6*c + (11*d*x
)/2]))/(35840*a^4*d)
```

fricas [A] time = 1.73, size = 171, normalized size = 0.93

$$\frac{735\,dx\cos\left(dx+c\right)^4+2940\,dx\cos\left(dx+c\right)^3+4410\,dx\cos\left(dx+c\right)^2+2940\,dx\cos\left(dx+c\right)+735\,dx+\left(35\cos\right.}{70\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x +
 c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*x
+ c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) - 11
52)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*
cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.60, size = 128, normalized size = 0.70

$$\frac{\frac{2940\,(dx+c)}{a^4}-\frac{280\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4}+\frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+455\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3885\,a^{24}\tan\left(\right.}{a^{28}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2*c
)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 - 38
85*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

maple [A] time = 0.06, size = 160, normalized size = 0.87

$$\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^4}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}-\frac{111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}-\frac{9\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{7\tan}{d\,a^4\left(1+\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x)

[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5+13/8/d/a^4*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*tan(1/2*d*x+1/2*c)-9/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-7/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+21/d/a^4*arctan(tan(1/2*d*x+1/2*c))

maxima [A] time = 1.12, size = 204, normalized size = 1.11

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{4}+\frac{2a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{63\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{5\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{4}}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{4}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) - 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d

mupad [B] time = 0.52, size = 159, normalized size = 0.86

$$\frac{5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-78\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+596\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4408\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\right.}{280\,a^{4}\,d\cos\left(\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(a + a*cos(c + d*x))^4,x)

[Out] (5*sin(c/2 + (d*x)/2) - 78*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 596*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 4408*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) - 2520*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 560*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) + 2940*cos(c/2 + (d*x)/2)^7*(c + d*x))/(280*a^4*d*cos(c/2 + (d*x)/2)^7)

sympy [A] time = 29.67, size = 530, normalized size = 2.88

$$\begin{cases} \frac{2940dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{280a^4d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+560a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+280a^4d} + \frac{5880dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{280a^4d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+560a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+280a^4d} + \frac{2940dx}{280a^4d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+560a^4d\tan^2(} \\ \frac{x\cos^6(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((2940*d*x*tan(c/2 + d*x/2)**4/(280*a**4*d*tan(c/2 + d*x/2)**4 + 5
60*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) + 5880*d*x*tan(c/2 + d*x/2)**2/
(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4
*d) + 2940*d*x/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2
)**2 + 280*a**4*d) + 5*tan(c/2 + d*x/2)**11/(280*a**4*d*tan(c/2 + d*x/2)**4
 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) - 53*tan(c/2 + d*x/2)**9/(2
80*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d
) + 334*tan(c/2 + d*x/2)**7/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*ta
n(c/2 + d*x/2)**2 + 280*a**4*d) - 3038*tan(c/2 + d*x/2)**5/(280*a**4*d*tan(
c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) - 9835*tan(c
/2 + d*x/2)**3/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2
)**2 + 280*a**4*d) - 5845*tan(c/2 + d*x/2)/(280*a**4*d*tan(c/2 + d*x/2)**4
+ 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d), Ne(d, 0)), (x*cos(c)**6/(a*
cos(c) + a)**4, True))
```

3.73 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=150

$$\frac{244\sin(c+dx)}{105a^4d}-\frac{88\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4\sin(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{4x}{a^4}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{12\sin(c+dx}{35ad(a\cos(}$$

```
[Out] -4*x/a^4+244/105*sin(d*x+c)/a^4/d-88/105*cos(d*x+c)^2*sin(d*x+c)/a^4/d/(1+c
os(d*x+c))^2+4*sin(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*cos(d*x+c)^4*sin(d*x+c)/
d/(a+a*cos(d*x+c))^4-12/35*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.37, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2765, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{244\sin(c+dx)}{105a^4d}-\frac{88\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4\sin(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{4x}{a^4}-\frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{12\sin(c+dx}{35ad(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-4*x)/a^4 + (244*Sin[c + d*x])/(105*a^4*d) - (88*Cos[c + d*x]^2*Sin[c + d*
x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*Sin[c + d*x])/(a^4*d*(1 + Cos[c +
 d*x])) - (Cos[c + d*x]^4*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (12*
Cos[c + d*x]^3*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^3(c+dx)(4a-8a\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(36a^2-52a^2\cos(c+\right.}{(a+a\cos(c+dx))^2}}{35a^4}$$

$$= -\frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+dx)\sin(c+d}{35ad(a+a\cos(c+dx))}$$

$$= -\frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+dx)\sin(c+d}{35ad(a+a\cos(c+dx))}$$

$$= \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(}{35ad(a}$$

$$= \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(}{35ad(a}$$

$$= -\frac{4x}{a^4} + \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - $$

$$= -\frac{4x}{a^4} + \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - $$

Mathematica [**A**] time = 0.42, size = 263, normalized size = 1.75

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(46130\sin\left(c+\frac{dx}{2}\right)-46116\sin\left(c+\frac{3dx}{2}\right)+18060\sin\left(2c+\frac{3dx}{2}\right)-19292\sin\left(2c+\frac{5dx}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -1/26880*(Sec[c/2]*Sec[(c + d*x)/2]^7*(29400*d*x*Cos[(d*x)/2] + 29400*d*x*C
os[c + (d*x)/2] + 17640*d*x*Cos[c + (3*d*x)/2] + 17640*d*x*Cos[2*c + (3*d*x
)/2] + 5880*d*x*Cos[2*c + (5*d*x)/2] + 5880*d*x*Cos[3*c + (5*d*x)/2] + 840*
d*x*Cos[3*c + (7*d*x)/2] + 840*d*x*Cos[4*c + (7*d*x)/2] - 60830*Sin[(d*x)/2
] + 46130*Sin[c + (d*x)/2] - 46116*Sin[c + (3*d*x)/2] + 18060*Sin[2*c + (3*
d*x)/2] - 19292*Sin[2*c + (5*d*x)/2] + 2100*Sin[3*c + (5*d*x)/2] - 3791*Sin
[3*c + (7*d*x)/2] - 735*Sin[4*c + (7*d*x)/2] - 105*Sin[4*c + (9*d*x)/2] - 1
05*Sin[5*c + (9*d*x)/2]))/(a^4*d)
```

fricas [A] time = 0.65, size = 162, normalized size = 1.08

$$-\frac{420\,dx\cos\left(dx+c\right)^{4}+1680\,dx\cos\left(dx+c\right)^{3}+2520\,dx\cos\left(dx+c\right)^{2}+1680\,dx\cos\left(dx+c\right)+420\,dx-\left(105\cos\ldots\right.}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(420*d*x*cos(d*x + c)^4 + 1680*d*x*cos(d*x + c)^3 + 2520*d*x*cos(d*x
 + c)^2 + 1680*d*x*cos(d*x + c) + 420*d*x - (105*cos(d*x + c)^4 + 1184*cos(
d*x + c)^3 + 2636*cos(d*x + c)^2 + 2236*cos(d*x + c) + 664)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.49, size = 112, normalized size = 0.75

$$-\frac{\frac{3360\,(dx+c)}{a^{4}}-\frac{1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{4}}+\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-147\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+805\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5145\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x + 1
/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

maple [A] time = 0.06, size = 126, normalized size = 0.84

$$-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^{4}}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^{4}}-\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^{4}}+\frac{49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^{4}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{8\arctan\left(\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5-23/24/d/a^
4*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*tan(1/2*d*x+1/2*c)+2/d/a^4*tan(1/2*d*x+1/
2*c)/(1+tan(1/2*d*x+1/2*c)^2)-8/d/a^4*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.81, size = 158, normalized size = 1.05

$$\frac{\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1} - \frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4)/d
```

mupad [B] time = 0.47, size = 137, normalized size = 0.91

$$-\frac{15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-192\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1144\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6112\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin}{840\,a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^4,x)
```

```
[Out] -(15*sin(c/2 + (d*x)/2) - 192*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 114
4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 6112*cos(c/2 + (d*x)/2)^6*sin(c
/2 + (d*x)/2) - 1680*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 3360*cos(c/2
 + (d*x)/2)^7*(c + d*x))/(840*a^4*d*cos(c/2 + (d*x)/2)^7)
```

sympy [A] time = 19.07, size = 280, normalized size = 1.87

$$\begin{cases} -\frac{3360dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} - \frac{3360dx}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} - \frac{15\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} + \frac{132\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} - \frac{658\tan}{840a^4d\tan^2} \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-3360*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 +
840*a**4*d) - 3360*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15*t
an(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*tan(
c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*tan(c/2
 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*tan(c/2 +
 d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*tan(c/2 + d
*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*cos(c)**
5/(a*cos(c) + a)**4, True))
```

3.74 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=127

$$-\frac{43\sin(c+dx)}{21a^4d(\cos(c+dx)+1)}+\frac{11\sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2}+\frac{x}{a^4}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{2\sin(c+dx)\cos^2(c+dx)}{7ad(a\cos(c+dx)+a)^3}$$

```
[Out] x/a^4+11/21*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-43/21*sin(d*x+c)/a^4/d/(1+cos
(d*x+c))-1/7*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-2/7*cos(d*x+c)^2*
sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.28, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2968, 3019, 2735, 2648}

$$-\frac{43\sin(c+dx)}{21a^4d(\cos(c+dx)+1)}+\frac{11\sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2}+\frac{x}{a^4}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{2\sin(c+dx)\cos^2(c+dx)}{7ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] x/a^4 + (11*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*x])^2) - (43*Sin[c + d*x
])/(21*a^4*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^3*Sin[c + d*x])/(7*d*(a +
a*Cos[c + d*x])^4) - (2*Cos[c + d*x]^2*Sin[c + d*x])/(7*a*d*(a + a*Cos[c +
d*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
```

```
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(3a-7a\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(20a^2-35a^2\cos(c+dx}{(a+a\cos(c+dx))^2}}{35a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} - \frac{\int \frac{20a^2\cos(c+dx)-35a^2\cos^2(c+d}{(a+a\cos(c+dx))^2}}{35a^4}$$

$$= \frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} + $$

$$= \frac{x}{a^4} + \frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+d}{7ad(a+a\cos(c+dx))}$$

$$= \frac{x}{a^4} + \frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+d}{7ad(a+a\cos(c+dx))}$$

Mathematica [A] time = 0.34, size = 224, normalized size = 1.76

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(1652\sin\left(c+\frac{dx}{2}\right)-1428\sin\left(c+\frac{3dx}{2}\right)+756\sin\left(2c+\frac{3dx}{2}\right)-560\sin\left(2c+\frac{5dx}{2}\right)+168\,\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(735*d*x*Cos[(d*x)/2] + 735*d*x*Cos[c + (d*x)/
2] + 441*d*x*Cos[c + (3*d*x)/2] + 441*d*x*Cos[2*c + (3*d*x)/2] + 147*d*x*Co
s[2*c + (5*d*x)/2] + 147*d*x*Cos[3*c + (5*d*x)/2] + 21*d*x*Cos[3*c + (7*d*x
)/2] + 21*d*x*Cos[4*c + (7*d*x)/2] - 1988*Sin[(d*x)/2] + 1652*Sin[c + (d*x)
/2] - 1428*Sin[c + (3*d*x)/2] + 756*Sin[2*c + (3*d*x)/2] - 560*Sin[2*c + (5
*d*x)/2] + 168*Sin[3*c + (5*d*x)/2] - 104*Sin[3*c + (7*d*x)/2]))/(2688*a^4*
d)
```

fricas [A] time = 0.82, size = 152, normalized size = 1.20

$$\frac{21\,dx\cos(dx+c)^4 + 84\,dx\cos(dx+c)^3 + 126\,dx\cos(dx+c)^2 + 84\,dx\cos(dx+c) + 21\,dx - \left(52\,\cos(dx+c)^3 + \right.}{21\left(a^4d\cos(dx+c)^4 + 4\,a^4d\cos(dx+c)^3 + 6\,a^4d\cos(dx+c)^2 + 4\,a^4d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(21*d*x*cos(d*x + c)^4 + 84*d*x*cos(d*x + c)^3 + 126*d*x*cos(d*x + c)^
2 + 84*d*x*cos(d*x + c) + 21*d*x - (52*cos(d*x + c)^3 + 124*cos(d*x + c)^2
+ 107*cos(d*x + c) + 32)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(
d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.52, size = 83, normalized size = 0.65

$$\frac{\frac{168\,(dx+c)}{a^4}+\frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-315\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/2
*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x + 1
/2*c))/a^28)/d
```

maple [A] time = 0.05, size = 94, normalized size = 0.74

$$\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^4}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^4}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*
tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d
*x+1/2*c))
```

maxima [A] time = 1.26, size = 112, normalized size = 0.88

$$-\frac{\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d
```

mupad [B] time = 0.43, size = 102, normalized size = 0.80

$$\frac{x}{a^4}+\frac{-\frac{52\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{21}+\frac{16\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{21}-\frac{5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{28}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^4,x)
```

```
[Out] x/a^4 + (sin(c/2 + (d*x)/2)/56 - (5*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)
)/28 + (16*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/21 - (52*cos(c/2 + (d*x
)/2)^6*sin(c/2 + (d*x)/2))/21)/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

sympy [A] time = 12.00, size = 95, normalized size = 0.75

$$\begin{cases} \frac{x}{a^4}+\frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}-\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{11\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d}-\frac{15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d\neq 0 \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((x/a**4 + tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(
8*a**4*d) + 11*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*tan(c/2 + d*x/2)/(8*a**
4*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**4, True))
```

3.75 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=114

$$\frac{12\sin(c+dx)}{35a^4d(\cos(c+dx)+1)}-\frac{18\sin(c+dx)}{35a^4d(\cos(c+dx)+1)^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{8\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

```
[Out] -18/35*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2+12/35*sin(d*x+c)/a^4/d/(1+cos(d*x+
c))-1/7*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+8/35*sin(d*x+c)/a/d/(a
+a*cos(d*x+c))^3
```

Rubi [A] time = 0.20, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2765, 2968, 3019, 2750, 2648}

$$\frac{12\sin(c+dx)}{35a^4d(\cos(c+dx)+1)}-\frac{18\sin(c+dx)}{35a^4d(\cos(c+dx)+1)^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{8\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-18*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])^2) + (12*Sin[c + d*x])/(35*
a^4*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^2*Sin[c + d*x])/(7*d*(a + a*Cos[c
 + d*x])^4) + (8*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(2a-6a\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2} \\
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{2a\cos(c+dx)-6a\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2} \\
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{-24a^2+30a^2\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{35a^4} \\
&= -\frac{18\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \\
&= -\frac{18\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} +
\end{aligned}$$

Mathematica [A] time = 0.27, size = 112, normalized size = 0.98

$$\frac{\sec\left(\frac{c}{2}\right)\left(-210\sin\left(c+\frac{dx}{2}\right)+147\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+49\sin\left(2c+\frac{5dx}{2}\right)-35\sin\left(3c+\frac{5dx}{2}\right)+12\right.}{2240a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(210*Sin[(d*x)/2] - 210*Sin[c + (d*x)/2] + 147
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 49*Sin[2*c + (5*d*x)/2] -
35*Sin[3*c + (5*d*x)/2] + 12*Sin[3*c + (7*d*x)/2]))/(2240*a^4*d)
```

fricas [A] time = 0.59, size = 99, normalized size = 0.87

$$\frac{\left(12\cos\left(dx+c\right)^3+13\cos\left(dx+c\right)^2+8\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{35\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(12*cos(d*x + c)^3 + 13*cos(d*x + c)^2 + 8*cos(d*x + c) + 2)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.45, size = 59, normalized size = 0.52

$$-\frac{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d
*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.05, size = 58, normalized size = 0.51

$$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1
/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.86, size = 87, normalized size = 0.76

$$\frac{\frac{35 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\, a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

mupad [B] time = 0.39, size = 58, normalized size = 0.51

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-35\right)}{280\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^4,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 - 35))/(280*a^4*d)
```

sympy [A] time = 8.35, size = 88, normalized size = 0.77

$$\begin{cases} - \frac{\tan^{7}{\left(\frac{c}{2} + \frac{dx}{2} \right)}}{56 a^{4} d} + \frac{3 \tan^{5}{\left(\frac{c}{2} + \frac{dx}{2} \right)}}{40 a^{4} d} - \frac{\tan^{3}{\left(\frac{c}{2} + \frac{dx}{2} \right)}}{8 a^{4} d} + \frac{\tan{\left(\frac{c}{2} + \frac{dx}{2} \right)}}{8 a^{4} d} & \text{for}\: d \neq 0 \\ \frac{x \cos^{3}{\left(c \right)}}{\left(a \cos{\left(c \right)} + a\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*tan(c/2 + d*x/2)**5/(40*a**
4*d) - tan(c/2 + d*x/2)**3/(8*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d,
0)), (x*cos(c)**3/(a*cos(c) + a)**4, True))
```

3.76 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{13\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{13\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}-\frac{11\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] 1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-11/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
+13/105*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+13/105*sin(d*x+c)/d/(a^4+a^4*co
s(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2758, 2750, 2650, 2648}

$$\frac{13\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{13\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}-\frac{11\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) - (11*Sin[c + d*x])/(35*a*d*(a +
a*Cos[c + d*x])^3) + (13*Sin[c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) +
 (13*Sin[c + d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx &= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{\int \frac{-4a+7a\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}\\ &= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{11\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{13\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{35a^2}\\ &= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{11\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{13\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}+\\ &= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{11\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{13\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}+\end{aligned}$$

Mathematica [A] time = 0.26, size = 99, normalized size = 0.88

$$\frac{\sec\left(\frac{c}{2}\right)\left(-175\sin\left(c+\frac{dx}{2}\right)+168\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+91\sin\left(2c+\frac{5dx}{2}\right)+13\sin\left(3c+\frac{7dx}{2}\right)+280\right.}{6720a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(280*Sin[(d*x)/2] - 175*Sin[c + (d*x)/2] + 168
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 91*Sin[2*c + (5*d*x)/2] +
13*Sin[3*c + (7*d*x)/2]))/(6720*a^4*d)
```

fricas [A] time = 1.06, size = 99, normalized size = 0.88

$$\frac{\left(13\cos\left(dx+c\right)^3+52\cos\left(dx+c\right)^2+32\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.56, size = 59, normalized size = 0.53

$$\frac{15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 21 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 105 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{840 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*d
*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.04, size = 58, normalized size = 0.52

$$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7} - \frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5} - \frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d\, a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*
x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.84, size = 87, normalized size = 0.78

$$\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

mupad [B] time = 0.39, size = 58, normalized size = 0.52

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-105\right)}{840\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^4,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 - 1
5*tan(c/2 + (d*x)/2)^6 - 105))/(840*a^4*d)
```

sympy [A] time = 6.05, size = 87, normalized size = 0.78

$$\begin{cases} \frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d} - \frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d} - \frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d} & \text{for}\ d \neq 0 \\ \frac{x\cos^{2}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(40*a**4*d
) - tan(c/2 + d*x/2)**3/(24*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)
), (x*cos(c)**2/(a*cos(c) + a)**4, True))
```

3.77 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4} dx$

Optimal. Leaf size=112

$$\frac{8\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{8\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{4\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] -1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+4/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
+8/105*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+8/105*sin(d*x+c)/d/(a^4+a^4*cos(
d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2750, 2650, 2648}

$$\frac{8\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{8\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{4\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) + (4*Sin[c + d*x])/(35*a*d*(a +
a*Cos[c + d*x])^3) + (8*Sin[c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) +
(8*Sin[c + d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{7a}$$

$$= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{8\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{35a^2}$$

$$= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{8\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}$$

$$= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{8\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}$$

Mathematica [A] time = 0.24, size = 87, normalized size = 0.78

$$\frac{\sec\left(\frac{c}{2}\right)\left(-35\sin\left(c+\frac{dx}{2}\right)+2\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)+35\sin\left(\frac{dx}{2}\right)\right)\sec^7\left(\frac{1}{2}(c+dx\right.}{1680a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(35*Sin[(d*x)/2] - 35*Sin[c + (d*x)/2] + 2*(21
*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*c + (7*d*x)/2])))/(168
0*a^4*d)
```

fricas [A] time = 0.92, size = 99, normalized size = 0.88

$$\frac{\left(8\cos(dx+c)^3+32\cos(dx+c)^2+52\cos(dx+c)+13\right)\sin(dx+c)}{105\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.56, size = 59, normalized size = 0.53

$$-\frac{15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+21 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*
d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.04, size = 58, normalized size = 0.52

$$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5+1/3*tan(1/2*d
*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.20, size = 87, normalized size = 0.78

$$\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{21\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

mupad [B] time = 0.39, size = 58, normalized size = 0.52

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+105\right)}{840\,a^{4}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 - 15
*tan(c/2 + (d*x)/2)^6 + 105))/(840*a^4*d)
```

sympy [A] time = 4.46, size = 85, normalized size = 0.76

$$\begin{cases} -\frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d} - \frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d} + \frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(40*a**4*
d) + tan(c/2 + d*x/2)**3/(24*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0
)), (x*cos(c)/(a*cos(c) + a)**4, True))
```

3.78 $\int \frac{1}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{2\sin(c+dx)}{35d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2\sin(c+dx)}{35d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{3\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] 1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+3/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3+
2/35*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+2/35*sin(d*x+c)/d/(a^4+a^4*cos(d*x
+c))
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$\frac{2\sin(c+dx)}{35d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2\sin(c+dx)}{35d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{3\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-4),x]
```

```
[Out] Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) + (3*Sin[c + d*x])/(35*a*d*(a + a
*Cos[c + d*x])^3) + (2*Sin[c + d*x])/(35*d*(a^2 + a^2*Cos[c + d*x])^2) + (2
*Sin[c + d*x])/(35*d*(a^4 + a^4*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\cos(c+dx))^4}\,dx &= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{3\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{7a} \\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{3\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{6\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{35a^2} \\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{3\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2\sin(c+dx)}{35d\left(a^2+a^2\cos(c+dx)\right)^2} + \cdots \\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{3\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2\sin(c+dx)}{35d\left(a^2+a^2\cos(c+dx)\right)^2} + \cdots
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 77, normalized size = 0.69

$$
\frac{\left(35\sin\left(\frac{1}{2}(c+dx)\right)+21\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{70a^4d(\cos(c+dx)+1)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-4),x]
```

```
[Out] (Cos[(c + d*x)/2]*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin[(5
*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(70*a^4*d*(1 + Cos[c + d*x])^4)
```

fricas [A] time = 0.92, size = 99, normalized size = 0.88

$$
\frac{\left(2\cos(dx+c)^3+8\cos(dx+c)^2+13\cos(dx+c)+12\right)\sin(dx+c)}{35\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.44, size = 59, normalized size = 0.53

$$
\frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{280a^4d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d*
x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.04, size = 56, normalized size = 0.50

$$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/
2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.00, size = 87, normalized size = 0.78

$$\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{21\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{5\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{280\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

mupad [B] time = 0.38, size = 58, normalized size = 0.52

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+35\right)}{280\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 5*
tan(c/2 + (d*x)/2)^6 + 35))/(280*a^4*d)
```

sympy [A] time = 3.42, size = 83, normalized size = 0.74

$$\begin{cases} \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*tan(c/2 + d*x/2)**5/(40*a**4
*d) + tan(c/2 + d*x/2)**3/(8*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0
)), (x/(a*cos(c) + a)**4, True))
```

3.79 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{32\sin(c+dx)}{21a^4d(\cos(c+dx)+1)}-\frac{11\sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2}-\frac{2\sin(c+dx)}{7ad(a\cos(c+dx)+a)^3}-\frac{\sin(c+}{7d(a\cos(c+}$$

```
[Out] arctanh(sin(d*x+c))/a^4/d-11/21*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-32/21*sin
(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-2/7*sin(d*
x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.29, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2766, 2978, 12, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{32\sin(c+dx)}{21a^4d(\cos(c+dx)+1)}-\frac{11\sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2}-\frac{2\sin(c+dx)}{7ad(a\cos(c+dx)+a)^3}-\frac{\sin(c+}{7d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) - (11*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*
x])^2) - (32*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*x])) - Sin[c + d*x]/(7*
d*(a + a*Cos[c + d*x])^4) - (2*Sin[c + d*x])/(7*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(7a-3a\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\left(35a^2-20a^2\cos(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} + \cdots$$

$$= -\frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} - \cdots$$

$$= -\frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} - \cdots$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin\cdots}{7ad(a+\cdots}$$

Mathematica [A] time = 0.86, size = 185, normalized size = 1.54

$$\sec\left(\frac{c}{2}\right)\left(434\sin\left(c+\frac{dx}{2}\right)-525\sin\left(c+\frac{3dx}{2}\right)+147\sin\left(2c+\frac{3dx}{2}\right)-203\sin\left(2c+\frac{5dx}{2}\right)+21\sin\left(3c+\frac{5dx}{2}\right)-32\sin\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-1344*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-686*Sin[
(d*x)/2] + 434*Sin[c + (d*x)/2] - 525*Sin[c + (3*d*x)/2] + 147*Sin[2*c + (3
*d*x)/2] - 203*Sin[2*c + (5*d*x)/2] + 21*Sin[3*c + (5*d*x)/2] - 32*Sin[3*c
+ (7*d*x)/2]))/(84*a^4*d*(1 + Cos[c + d*x])^4)
```

fricas [A] time = 1.04, size = 202, normalized size = 1.68

$$\frac{21\left(\cos\left(dx+c\right)^{4}+4\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-21\left(\cos\left(dx+\right.\right.}{42\left(a^{4}d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x
+ c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6
*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos(d*
x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a^4*d
*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d
*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.69, size = 110, normalized size = 0.92

$$\frac{\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{4}}-\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^{4}}-\frac{3a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+21a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+77a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+315a^{24}\tan\left(\right.}{a^{28}}}{168d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*x
+ 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2*c)
)/a^28)/d
```

maple [A] time = 0.09, size = 115, normalized size = 0.96

$$-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^{4}}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}-\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^{4}}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^{4}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5-11/24/d/a^4
*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*tan(1/2*d*x+1/2*c)-1/d/a^4*ln(tan(1/2*d*x+
1/2*c)-1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 1.48, size = 139, normalized size = 1.16

$$-\frac{\frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1}+\frac{77 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 +
168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

mupad [B] time = 0.37, size = 83, normalized size = 0.69

$$-\frac{\frac{11 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4}+\frac{15 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^4),x)
```

```
[Out] -((11*tan(c/2 + (d*x)/2)^3)/(24*a^4) + tan(c/2 + (d*x)/2)^5/(8*a^4) + tan(c
/2 + (d*x)/2)^7/(56*a^4) - (2*atanh(tan(c/2 + (d*x)/2)))/a^4 + (15*tan(c/2
+ (d*x)/2))/(8*a^4))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)
**2 + 4*cos(c + d*x) + 1), x)/a**4
```

3.80 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=135

$$\frac{664\tan(c+dx)}{105a^4d}-\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{88\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{12\tan(c+dx)}{35ad(a\cos(c+dx)+}$$

```
[Out] -4*arctanh(sin(d*x+c))/a^4/d+664/105*tan(d*x+c)/a^4/d-88/105*tan(d*x+c)/a^4
/d/(1+cos(d*x+c))^2-4*tan(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*tan(d*x+c)/d/(a+a
*cos(d*x+c))^4-12/35*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.39, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{664\tan(c+dx)}{105a^4d}-\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{88\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{12\tan(c+dx)}{35ad(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-4*ArcTanh[Sin[c + d*x]])/(a^4*d) + (664*Tan[c + d*x])/(105*a^4*d) - (88*T
an[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - (4*Tan[c + d*x])/(a^4*d*(1
+ Cos[c + d*x])) - Tan[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) - (12*Tan[c +
d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(8a-4a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\left(52a^2-36a^2\cos(c+dx)\right)\sec^2(c+d}{(a+a\cos(c+dx))^2}}{35a^4}$$

$$= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} +$$

$$= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} -$$

$$= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} -$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - 35$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{664\tan(c+dx)}{105a^4d} - \frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan}{7d(a+a}$$

Mathematica [B] time = 4.15, size = 341, normalized size = 2.53

$$\frac{107520\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (107520*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Se
c[c + d*x]*(-10780*Sin[(d*x)/2] + 18788*Sin[(3*d*x)/2] - 20524*Sin[c - (d*x
)/2] + 14644*Sin[c + (d*x)/2] - 16660*Sin[2*c + (d*x)/2] - 4690*Sin[c + (3*
d*x)/2] + 14378*Sin[2*c + (3*d*x)/2] - 9100*Sin[3*c + (3*d*x)/2] + 11668*Si
n[c + (5*d*x)/2] - 630*Sin[2*c + (5*d*x)/2] + 9358*Sin[3*c + (5*d*x)/2] - 2
940*Sin[4*c + (5*d*x)/2] + 4228*Sin[2*c + (7*d*x)/2] + 315*Sin[3*c + (7*d*x
)/2] + 3493*Sin[4*c + (7*d*x)/2] - 420*Sin[5*c + (7*d*x)/2] + 664*Sin[3*c +
 (9*d*x)/2] + 105*Sin[4*c + (9*d*x)/2] + 559*Sin[5*c + (9*d*x)/2]))/(1680*a
^4*d*(1 + Cos[c + d*x])^4)
```

fricas [A] time = 0.90, size = 234, normalized size = 1.73

$$-\frac{210\left(\cos\left(dx+c\right)^{5}+4\,\cos\left(dx+c\right)^{4}+6\,\cos\left(dx+c\right)^{3}+4\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\cdots}{1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d
*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4*c
os(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(-si
n(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(d*x
+ c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a
^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4
*d*cos(d*x + c))
```

giac [A] time = 0.61, size = 139, normalized size = 1.03

$$-\frac{\frac{3360\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{4}}-\frac{3360\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{4}}+\frac{1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{4}}-\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+147\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+805\cdots}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/2*c
)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))/a^2
8)/d
```

maple [A] time = 0.09, size = 158, normalized size = 1.17

$$\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^{4}}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^{4}}+\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^{4}}+\frac{49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}-\frac{1}{d\,a^{4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4\ln\left(\tan\left(\frac{dx}{2}+\cdots\right.\right.}{d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5+23/24/d/a^4
*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*tan(1/2*d*x+1/2*c)-1/d/a^4/(tan(1/2*d*x+1/
```

```
2*c)-1)+4/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)-4/d
/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 1.47, size = 186, normalized size = 1.38

$$\frac{\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

mupad [B] time = 0.42, size = 130, normalized size = 0.96

$$\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^4\,d}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40\,a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4\,d}-\frac{8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^4\right)}+\frac{49\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^4),x)
```

```
[Out] (23*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) + (7*tan(c/2 + (d*x)/2)^5)/(40*a^4*d)
+ tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (8*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) -
 (2*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 - a^4)) + (49*tan(c/2
+ (d*x)/2))/(8*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d
*x)**2 + 4*cos(c + d*x) + 1), x)/a**4
```

3.81 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4} dx$

Optimal. Leaf size=185

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{21\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{288\tan(c+dx)\sec(c+dx)}{35a^4d(\cos(c+dx)+1)}-\frac{43\tan(c+}{35a^4d(\cos}$$

```
[Out] 21/2*arctanh(sin(d*x+c))/a^4/d-576/35*tan(d*x+c)/a^4/d+21/2*sec(d*x+c)*tan(
d*x+c)/a^4/d-43/35*sec(d*x+c)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-288/35*sec(
d*x+c)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos
(d*x+c))^4-2/5*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.43, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{21\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{288\tan(c+dx)\sec(c+dx)}{35a^4d(\cos(c+dx)+1)}-\frac{43\tan(c+}{35a^4d(\cos}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (576*Tan[c + d*x])/(35*a^4*d) + (21*
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - (43*Sec[c + d*x]*Tan[c + d*x])/(35*a
^4*d*(1 + Cos[c + d*x])^2) - (288*Sec[c + d*x]*Tan[c + d*x])/(35*a^4*d*(1 +
 Cos[c + d*x])) - (Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4)
- (2*Sec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
```

```
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(9a-5a\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\left(73a^2-56a^2\cos(c+dx)\right)\sec^3(c+dx}{(a+a\cos(c+dx))^2}}{35a^4}$$

$$= -\frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3}$$

$$= -\frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3}$$

$$= -\frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3}$$

$$= \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4}$$

$$= \frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{576\tan(c+dx)}{35a^4d} + \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec(c+}{35a^4d(1}$$

Mathematica [B] time = 6.27, size = 455, normalized size = 2.46

$$-\frac{168\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^4} + \frac{168\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^4} + \frac{\sec}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-168*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]])/(d
*(a + a*Cos[c + d*x])^4) + (168*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Se
c[c/2]*Sec[c]*Sec[c + d*x]^2*(24402*Sin[(d*x)/2] - 55556*Sin[(3*d*x)/2] + 6
1054*Sin[c - (d*x)/2] - 33614*Sin[c + (d*x)/2] + 51842*Sin[2*c + (d*x)/2] +
 12460*Sin[c + (3*d*x)/2] - 33716*Sin[2*c + (3*d*x)/2] + 34300*Sin[3*c + (3
*d*x)/2] - 39788*Sin[c + (5*d*x)/2] + 2940*Sin[2*c + (5*d*x)/2] - 26068*Sin
[3*c + (5*d*x)/2] + 16660*Sin[4*c + (5*d*x)/2] - 21351*Sin[2*c + (7*d*x)/2]
 - 1295*Sin[3*c + (7*d*x)/2] - 14911*Sin[4*c + (7*d*x)/2] + 5145*Sin[5*c +
(7*d*x)/2] - 7329*Sin[3*c + (9*d*x)/2] - 1225*Sin[4*c + (9*d*x)/2] - 5369*S
in[5*c + (9*d*x)/2] + 735*Sin[6*c + (9*d*x)/2] - 1152*Sin[4*c + (11*d*x)/2]
```

```
 - 280*Sin[5*c + (11*d*x)/2] - 872*Sin[6*c + (11*d*x)/2]))/(2240*d*(a + a*C
os[c + d*x])^4)
```

fricas [A] time = 1.09, size = 250, normalized size = 1.35

$$\frac{735\left(\cos\left(dx+c\right)^{6}+4\,\cos\left(dx+c\right)^{5}+6\,\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*
x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 + 4*
cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*log(
-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 4548*co
s(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

giac [A] time = 0.88, size = 155, normalized size = 0.84

$$\frac{\frac{2940\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{4}}-\frac{2940\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{4}}+\frac{280\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{4}}-\frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+63\,a^{24}\tan(}{}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)^7
+ 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 3885*a
^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

maple [A] time = 0.11, size = 200, normalized size = 1.08

$$-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^{4}}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^{4}}-\frac{13\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}-\frac{111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}+\frac{1}{2d\,a^{4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{}{2d\,a^{4}\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4
*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*tan(1/2*d*x+1/2*c)+1/2/d/a^4/(tan(1/2*d*x
+1/2*c)-1)^2+9/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)-21/2/d/a^4*ln(tan(1/2*d*x+1/2
*c)-1)-1/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2+9/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)+
21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 1.10, size = 231, normalized size = 1.25

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) + 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c)/(co
s(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^
4)/d
```

mupad [B] time = 0.47, size = 160, normalized size = 0.86

$$\frac{21\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4\,d}-\frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^4\,d}-\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^4\right)}-\frac{111\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^4),x)
```

```
[Out] (21*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (9*tan(c/2 + (d*x)/2)^5)/(40*a^4*d
) - tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (13*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) -
 (7*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^3)/(d*(a^4*tan(c/2 + (d*x)/2)
^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (111*tan(c/2 + (d*x)/2))/(8*a^4*d
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d
*x)**2 + 4*cos(c + d*x) + 1), x)/a**4
```

3.82 $\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=225

$$-\frac{7664\sin(c+dx)}{315a^5d}-\frac{3832\sin(c+dx)\cos^2(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{31\sin(c+dx)\cos(c+dx)}{2a^5d}+\frac{31x}{2a^5}-\frac{577\sin(c+dx)\cos^3(c+dx}{315a^3d(a\cos(c+dx)+a)^2}$$

```
[Out] 31/2*x/a^5-7664/315*sin(d*x+c)/a^5/d+31/2*cos(d*x+c)*sin(d*x+c)/a^5/d-1/9*c
os(d*x+c)^6*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-17/63*cos(d*x+c)^5*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^4-28/45*cos(d*x+c)^4*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^
3-577/315*cos(d*x+c)^3*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-3832/315*cos(d*x
+c)^2*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] time = 0.52, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2765, 2977, 2734}

$$-\frac{7664\sin(c+dx)}{315a^5d}-\frac{28\sin(c+dx)\cos^4(c+dx)}{45a^2d(a\cos(c+dx)+a)^3}-\frac{577\sin(c+dx)\cos^3(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}-\frac{3832\sin(c+dx)\cos^2(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (31*x)/(2*a^5) - (7664*Sin[c + d*x])/(315*a^5*d) + (31*Cos[c + d*x]*Sin[c +
 d*x])/(2*a^5*d) - (Cos[c + d*x]^6*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^
5) - (17*Cos[c + d*x]^5*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (28
*Cos[c + d*x]^4*Sin[c + d*x])/(45*a^2*d*(a + a*Cos[c + d*x])^3) - (577*Cos[
c + d*x]^3*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) - (3832*Cos[c +
 d*x]^2*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
```

```
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^5(c+dx)(6a-11a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^4(c+dx)\left(85a^2-111a^2\cos\right.}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\cos^4(c+dx)\sin(c+}{45a^2d(a+a\cos(c+dx}$$

$$= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\cos^4(c+dx)\sin(c+}{45a^2d(a+a\cos(c+dx}$$

$$= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\cos^4(c+dx)\sin(c+}{45a^2d(a+a\cos(c+dx}$$

$$= \frac{31x}{2a^5} - \frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} -$$

Mathematica [A] time = 0.78, size = 345, normalized size = 1.53

$$\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(7194600\sin\left(c+\frac{dx}{2}\right) - 7472241\sin\left(c+\frac{3dx}{2}\right) + 3432975\sin\left(2c+\frac{3dx}{2}\right) - 3871989\sin\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(4921560*d*x*Cos[(d*x)/2] + 4921560*d*x*Cos[c
+ (d*x)/2] + 3281040*d*x*Cos[c + (3*d*x)/2] + 3281040*d*x*Cos[2*c + (3*d*x)
/2] + 1406160*d*x*Cos[2*c + (5*d*x)/2] + 1406160*d*x*Cos[3*c + (5*d*x)/2] +
 351540*d*x*Cos[3*c + (7*d*x)/2] + 351540*d*x*Cos[4*c + (7*d*x)/2] + 39060*
d*x*Cos[4*c + (9*d*x)/2] + 39060*d*x*Cos[5*c + (9*d*x)/2] - 9163224*Sin[(d*
x)/2] + 7194600*Sin[c + (d*x)/2] - 7472241*Sin[c + (3*d*x)/2] + 3432975*Sin
[2*c + (3*d*x)/2] - 3871989*Sin[2*c + (5*d*x)/2] + 801675*Sin[3*c + (5*d*x)
/2] - 1186056*Sin[3*c + (7*d*x)/2] - 17640*Sin[4*c + (7*d*x)/2] - 175184*Si
n[4*c + (9*d*x)/2] - 45360*Sin[5*c + (9*d*x)/2] - 3465*Sin[5*c + (11*d*x)/2
] - 3465*Sin[6*c + (11*d*x)/2] + 315*Sin[6*c + (13*d*x)/2] + 315*Sin[7*c +
(13*d*x)/2]))/(1290240*a^5*d)
```

fricas [A] time = 0.88, size = 207, normalized size = 0.92

$$\frac{9765\,dx\cos\left(dx+c\right)^{5}+48825\,dx\cos\left(dx+c\right)^{4}+97650\,dx\cos\left(dx+c\right)^{3}+97650\,dx\cos\left(dx+c\right)^{2}+48825\,dx\cos}{630\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(d
*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x +
(315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440*co
s(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(d*x
+ c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c
)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

giac [A] time = 1.47, size = 145, normalized size = 0.64

$$\frac{\frac{78120\,(dx+c)}{a^{5}}-\frac{5040\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{5}}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-450\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+3024\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15750\,a^{4}}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(78120*(d*x + c)/a^5 - 5040*(11*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d
*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*tan(1/2*d*x +
1/2*c)^9 - 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2*c)
```

```
^5 - 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*c))/
a^45)/d
```

maple [A] time = 0.07, size = 179, normalized size = 0.80

$$-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^5}+\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^5}-\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d\,a^5}+\frac{25\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^5}-\frac{351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}-\frac{11\left(\tan^3\left(\frac{dx}{2}\right.\right.}{d\,a^5\left(1+\tan^2\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+5/56/d/a^5*tan(1/2*d*x+1/2*c)^7-3/5/d/a^5
*tan(1/2*d*x+1/2*c)^5+25/8/d/a^5*tan(1/2*d*x+1/2*c)^3-351/16/d/a^5*tan(1/2*
d*x+1/2*c)-11/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-9/d/a^5
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+31/d/a^5*arctan(tan(1/2*d*x+
1/2*c))
```

maxima [A] time = 2.43, size = 224, normalized size = 1.00

$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^5+\frac{2a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}-\frac{15750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3024\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{156240\arctan\left(\frac{\sin(dx+c}{\cos(dx+c)}\right.}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/(a^5 + 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) + 1)
- 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9)/a^5 - 156240*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^5)/d
```

mupad [B] time = 0.58, size = 181, normalized size = 0.80

$$-\frac{35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-590\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4584\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-23288\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*cos(c + d*x))^5,x)
```

```
[Out] -(35*sin(c/2 + (d*x)/2) - 590*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 458
4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 23288*cos(c/2 + (d*x)/2)^6*sin(
c/2 + (d*x)/2) + 129824*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 55440*cos
(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 10080*cos(c/2 + (d*x)/2)^12*sin(c/2
 + (d*x)/2) - 78120*cos(c/2 + (d*x)/2)^9*(c + d*x))/(5040*a^5*d*cos(c/2 + (
d*x)/2)^9)
```

sympy [A] time = 64.31, size = 588, normalized size = 2.61

$$\begin{cases} \frac{78120dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{5040a^5d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+10080a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+5040a^5d} + \frac{156240dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{5040a^5d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+10080a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+5040a^5d} + \frac{781}{5040a^5d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+1008} \\ \frac{x\cos^7(c)}{(a\cos(c)+a)^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((78120*d*x*tan(c/2 + d*x/2)**4/(5040*a**5*d*tan(c/2 + d*x/2)**4 +
 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) + 156240*d*x*tan(c/2 + d*x
/2)**2/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2
+ 5040*a**5*d) + 78120*d*x/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*
tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 35*tan(c/2 + d*x/2)**13/(5040*a**5*d*t
an(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) + 380*
tan(c/2 + d*x/2)**11/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/
2 + d*x/2)**2 + 5040*a**5*d) - 2159*tan(c/2 + d*x/2)**9/(5040*a**5*d*tan(c/
2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) + 10152*tan
(c/2 + d*x/2)**7/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 +
d*x/2)**2 + 5040*a**5*d) - 82089*tan(c/2 + d*x/2)**5/(5040*a**5*d*tan(c/2 +
 d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 260820*tan(c
/2 + d*x/2)**3/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*
x/2)**2 + 5040*a**5*d) - 155925*tan(c/2 + d*x/2)/(5040*a**5*d*tan(c/2 + d*x
/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d), Ne(d, 0)), (x*cos
(c)**7/(a*cos(c) + a)**5, True))
```

3.83 $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=191

$$\frac{181\sin(c+dx)}{63a^5d}+\frac{5\sin(c+dx)}{d\left(a^5\cos(c+dx)+a^5\right)}-\frac{5x}{a^5}-\frac{67\sin(c+dx)\cos^2(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}-\frac{29\sin(c+dx)\cos^3(c+dx)}{63a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+}{9d(a\,c}$$

```
[Out] -5*x/a^5+181/63*sin(d*x+c)/a^5/d-1/9*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x
+c))^5-5/21*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4-29/63*cos(d*x+c)
^3*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3-67/63*cos(d*x+c)^2*sin(d*x+c)/a^3/d/
(a+a*cos(d*x+c))^2+5*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] time = 0.49, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2765, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{181\sin(c+dx)}{63a^5d}-\frac{29\sin(c+dx)\cos^3(c+dx)}{63a^2d(a\cos(c+dx)+a)^3}-\frac{67\sin(c+dx)\cos^2(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}+\frac{5\sin(c+dx)}{d\left(a^5\cos(c+dx)+a^5\right)}-\frac{5x}{a^5}-\frac{\sin(c+}{9d(a\,c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (-5*x)/a^5 + (181*Sin[c + d*x])/(63*a^5*d) - (Cos[c + d*x]^5*Sin[c + d*x])/
(9*d*(a + a*Cos[c + d*x])^5) - (5*Cos[c + d*x]^4*Sin[c + d*x])/(21*a*d*(a +
 a*Cos[c + d*x])^4) - (29*Cos[c + d*x]^3*Sin[c + d*x])/(63*a^2*d*(a + a*Cos
[c + d*x])^3) - (67*Cos[c + d*x]^2*Sin[c + d*x])/(63*a^3*d*(a + a*Cos[c + d
*x])^2) + (5*Sin[c + d*x])/(d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^4(c+dx)(5a-10a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^3(c+dx)\left(60a^2-85a^2\cos(c-\right.}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c+dx)\sin(c+a}{63a^2d(a+a\cos(c+dx)}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c+dx)\sin(c+a}{63a^2d(a+a\cos(c+dx)}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c+dx)\sin(c+a}{63a^2d(a+a\cos(c+dx)}$$

$$= \frac{181\sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(}{63a^2d(}$$

$$= \frac{181\sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(}{63a^2d(}$$

$$= -\frac{5x}{a^5} + \frac{181\sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - $$

$$= -\frac{5x}{a^5} + \frac{181\sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - $$

Mathematica [A] time = 0.73, size = 319, normalized size = 1.67

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(143010\sin\left(c+\frac{dx}{2}\right) - 138726\sin\left(c+\frac{3dx}{2}\right) + 73290\sin\left(2c+\frac{3dx}{2}\right) - 70389\sin\left(2c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -1/64512*(Sec[c/2]*Sec[(c + d*x)/2]^9*(79380*d*x*Cos[(d*x)/2] + 79380*d*x*C
os[c + (d*x)/2] + 52920*d*x*Cos[c + (3*d*x)/2] + 52920*d*x*Cos[2*c + (3*d*x
)/2] + 22680*d*x*Cos[2*c + (5*d*x)/2] + 22680*d*x*Cos[3*c + (5*d*x)/2] + 56
70*d*x*Cos[3*c + (7*d*x)/2] + 5670*d*x*Cos[4*c + (7*d*x)/2] + 630*d*x*Cos[4
*c + (9*d*x)/2] + 630*d*x*Cos[5*c + (9*d*x)/2] - 175014*Sin[(d*x)/2] + 1430
```

```
10*Sin[c + (d*x)/2] - 138726*Sin[c + (3*d*x)/2] + 73290*Sin[2*c + (3*d*x)/2
] - 70389*Sin[2*c + (5*d*x)/2] + 20475*Sin[3*c + (5*d*x)/2] - 21141*Sin[3*c
 + (7*d*x)/2] + 1575*Sin[4*c + (7*d*x)/2] - 3091*Sin[4*c + (9*d*x)/2] - 567
*Sin[5*c + (9*d*x)/2] - 63*Sin[5*c + (11*d*x)/2] - 63*Sin[6*c + (11*d*x)/2]
))/(a^5*d)
```

fricas [A] time = 1.19, size = 198, normalized size = 1.04

$$-\frac{315\,dx\cos\left(dx+c\right)^{5}+1575\,dx\cos\left(dx+c\right)^{4}+3150\,dx\cos\left(dx+c\right)^{3}+3150\,dx\cos\left(dx+c\right)^{2}+1575\,dx\cos\left(dx+\right.}{63\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*co
s(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x + c)
^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d
*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5
*d*cos(d*x + c) + a^5*d)
```

giac [A] time = 0.74, size = 129, normalized size = 0.68

$$-\frac{\frac{5040\,(dx+c)}{a^{5}}-\frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{5}}-\frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+378\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1512\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8127\,a^{4}}{a^{45}}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x + 1/
2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c)^3
 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

maple [A] time = 0.06, size = 145, normalized size = 0.76

$$\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^{5}}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{14d\,a^{5}}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{5}}-\frac{3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{5}}+\frac{129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^{5}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x)
```

```
[Out] 1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-1/14/d/a^5*tan(1/2*d*x+1/2*c)^7+3/8/d/a^5*
tan(1/2*d*x+1/2*c)^5-3/2/d/a^5*tan(1/2*d*x+1/2*c)^3+129/16/d/a^5*tan(1/2*d*
x+1/2*c)+2/d/a^5*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-10/d/a^5*arcta
n(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.77, size = 178, normalized size = 0.93

$$\frac{\frac{2016\,\sin(dx+c)}{\left(a^5+\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{1512\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{72\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

mupad [B] time = 0.51, size = 159, normalized size = 0.83

$$\frac{7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-100\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+636\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2512\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin\left(\frac{c}{2}\right.}{1008\,a^5\,d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*cos(c + d*x))^5,x)
```

```
[Out] (7*sin(c/2 + (d*x)/2) - 100*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 636*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 2512*cos(c/2 + (d*x)/2)^6*sin(c/2
+ (d*x)/2) + 10096*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 2016*cos(c/2 +
 (d*x)/2)^10*sin(c/2 + (d*x)/2) - 5040*cos(c/2 + (d*x)/2)^9*(c + d*x))/(100
8*a^5*d*cos(c/2 + (d*x)/2)^9)
```

sympy [A] time = 42.52, size = 320, normalized size = 1.68

$$\begin{cases} -\frac{5040dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1008a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1008a^5d}-\frac{5040dx}{1008a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1008a^5d}+\frac{7\tan^{11}\left(\frac{c}{2}+\frac{dx}{2}\right)}{1008a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1008a^5d}-\frac{65\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{1008a^5d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1008a^5d}+\cdots \\ \frac{x\cos^6(c)}{(a\cos(c)+a)^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((-5040*d*x*tan(c/2 + d*x/2)**2/(1008*a**5*d*tan(c/2 + d*x/2)**2 +
 1008*a**5*d) - 5040*d*x/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) +
7*tan(c/2 + d*x/2)**11/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) - 65
*tan(c/2 + d*x/2)**9/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) + 306*
tan(c/2 + d*x/2)**7/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) - 1134*
tan(c/2 + d*x/2)**5/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) + 6615*
tan(c/2 + d*x/2)**3/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) + 10143
*tan(c/2 + d*x/2)/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d), Ne(d, 0)
), (x*cos(c)**6/(a*cos(c) + a)**5, True))
```

3.84 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=168

$$-\frac{661\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{x}{a^5}+\frac{173\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}-\frac{34\sin(c+dx)\cos^2(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^4(c+}{9d(a\cos(c+dx)+}$$

```
[Out] x/a^5-1/9*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-13/63*cos(d*x+c)^3*s
in(d*x+c)/a/d/(a+a*cos(d*x+c))^4-34/105*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+a*
cos(d*x+c))^3+173/315*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-661/315*sin(d*x+c
)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [**A**] time = 0.39, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2968, 3019, 2735, 2648}

$$-\frac{34\sin(c+dx)\cos^2(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}-\frac{661\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{173\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}+\frac{x}{a^5}-\frac{\sin(c+dx)\cos^4(c+}{9d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] x/a^5 - (Cos[c + d*x]^4*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (13*Co
s[c + d*x]^3*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (34*Cos[c + d*
x]^2*Sin[c + d*x])/(105*a^2*d*(a + a*Cos[c + d*x])^3) + (173*Sin[c + d*x])/
(315*a^3*d*(a + a*Cos[c + d*x])^2) - (661*Sin[c + d*x])/(315*d*(a^5 + a^5*C
os[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^3(c+dx)(4a-9a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)\left(39a^2-63a^2\cos(\right.}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(c+}{105a^2d(a+a\cos(c+d}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(c+}{105a^2d(a+a\cos(c+d}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(c+}{105a^2d(a+a\cos(c+d}$$

$$= \frac{x}{a^5} - \frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(}{105a^2d(a+a\cos(c}$$

$$= \frac{x}{a^5} - \frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(}{105a^2d(a+a\cos(c}$$

Mathematica [A] time = 0.49, size = 280, normalized size = 1.67

$$\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(100800\sin\left(c+\frac{dx}{2}\right) - 88284\sin\left(c+\frac{3dx}{2}\right) + 56700\sin\left(2c+\frac{3dx}{2}\right) - 43236\sin\left(2c+\frac{5}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(39690*d*x*Cos[(d*x)/2] + 39690*d*x*Cos[c + (d
*x)/2] + 26460*d*x*Cos[c + (3*d*x)/2] + 26460*d*x*Cos[2*c + (3*d*x)/2] + 11
340*d*x*Cos[2*c + (5*d*x)/2] + 11340*d*x*Cos[3*c + (5*d*x)/2] + 2835*d*x*Co
s[3*c + (7*d*x)/2] + 2835*d*x*Cos[4*c + (7*d*x)/2] + 315*d*x*Cos[4*c + (9*d
*x)/2] + 315*d*x*Cos[5*c + (9*d*x)/2] - 116676*Sin[(d*x)/2] + 100800*Sin[c
+ (d*x)/2] - 88284*Sin[c + (3*d*x)/2] + 56700*Sin[2*c + (3*d*x)/2] - 43236*
Sin[2*c + (5*d*x)/2] + 18900*Sin[3*c + (5*d*x)/2] - 12384*Sin[3*c + (7*d*x)
/2] + 3150*Sin[4*c + (7*d*x)/2] - 1726*Sin[4*c + (9*d*x)/2]))/(161280*a^5*d
)
```

fricas [A] time = 0.71, size = 188, normalized size = 1.12

$$\frac{315\,dx\cos(dx+c)^5 + 1575\,dx\cos(dx+c)^4 + 3150\,dx\cos(dx+c)^3 + 3150\,dx\cos(dx+c)^2 + 1575\,dx\cos(dx+}{315\left(a^5d\cos(dx+c)^5 + 5\,a^5d\cos(dx+c)^4 + 10\,a^5d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863*c
os(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x +
c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

giac [A] time = 0.55, size = 100, normalized size = 0.60

$$\frac{\frac{5040\,(dx+c)}{a^5} - \frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*tan
(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/2*
d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

maple [A] time = 0.05, size = 113, normalized size = 0.67

$$-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^5} + \frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^5} - \frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5d\,a^5} + \frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^5} - \frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5} + \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+3/56/d/a^5*tan(1/2*d*x+1/2*c)^7-1/5/d/a^5
*tan(1/2*d*x+1/2*c)^5+13/24/d/a^5*tan(1/2*d*x+1/2*c)^3-31/16/d/a^5*tan(1/2*
d*x+1/2*c)+2/d/a^5*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.70, size = 132, normalized size = 0.79

$$-\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1} - \frac{2730\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{10080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 10
080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

mupad [B] time = 0.48, size = 125, normalized size = 0.74

$$\frac{x}{a^5}-\frac{\frac{863\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{315}-\frac{356\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{315}+\frac{169\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{420}-\frac{41\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{504}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{144}}{a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^5,x)
```

```
[Out] x/a^5 - (sin(c/2 + (d*x)/2)/144 - (41*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/
2))/504 + (169*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/420 - (356*cos(c/2
+ (d*x)/2)^6*sin(c/2 + (d*x)/2))/315 + (863*cos(c/2 + (d*x)/2)^8*sin(c/2 +
(d*x)/2))/315)/(a^5*d*cos(c/2 + (d*x)/2)^9)
```

sympy [A] time = 28.10, size = 116, normalized size = 0.69

$$\begin{cases} \frac{x}{a^5}-\frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d}+\frac{3\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^5d}-\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{5a^5d}+\frac{13\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^5d}-\frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d\neq 0 \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((x/a**5 - tan(c/2 + d*x/2)**9/(144*a**5*d) + 3*tan(c/2 + d*x/2)**
7/(56*a**5*d) - tan(c/2 + d*x/2)**5/(5*a**5*d) + 13*tan(c/2 + d*x/2)**3/(24
*a**5*d) - 31*tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(
c) + a)**5, True))
```

3.85 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=155

$$\frac{83\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{142\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}+\frac{67\sin(c+dx)}{315a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}-$$

```
[Out] -1/9*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-11/63*cos(d*x+c)^2*sin(d*
x+c)/a/d/(a+a*cos(d*x+c))^4+67/315*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3-142/
315*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2+83/315*sin(d*x+c)/d/(a^5+a^5*cos(d*
x+c))
```

Rubi [A] time = 0.30, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2968, 3019, 2750, 2648}

$$\frac{83\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{142\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}+\frac{67\sin(c+dx)}{315a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -(Cos[c + d*x]^3*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (11*Cos[c + d
*x]^2*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) + (67*Sin[c + d*x])/(31
5*a^2*d*(a + a*Cos[c + d*x])^3) - (142*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[
c + d*x])^2) + (83*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)(3a-8a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos(c+dx)\left(22a^2-45a^2\cos(c+\right.}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{22a^2\cos(c+dx)-45a^2\cos^2(c+}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx)}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx)}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx)}$$

Mathematica [A] time = 0.28, size = 138, normalized size = 0.89

$$\frac{\sec\left(\frac{c}{2}\right)\left(-5040\sin\left(c+\frac{dx}{2}\right)+3612\sin\left(c+\frac{3dx}{2}\right)-3360\sin\left(2c+\frac{3dx}{2}\right)+1728\sin\left(2c+\frac{5dx}{2}\right)-1260\sin\left(3c+\frac{5dx}{2}\right.\right.}{80640a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(5418*Sin[(d*x)/2] - 5040*Sin[c + (d*x)/2] + 3
612*Sin[c + (3*d*x)/2] - 3360*Sin[2*c + (3*d*x)/2] + 1728*Sin[2*c + (5*d*x)
/2] - 1260*Sin[3*c + (5*d*x)/2] + 432*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c +
(7*d*x)/2] + 83*Sin[4*c + (9*d*x)/2]))/(80640*a^5*d)
```

fricas [A] time = 1.02, size = 123, normalized size = 0.79

$$\frac{\left(83\cos(dx+c)^4+100\cos(dx+c)^3+84\cos(dx+c)^2+40\cos(dx+c)+8\right)\sin(dx+c)}{315\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+c)+a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos(
d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

giac [A] time = 1.41, size = 72, normalized size = 0.46

$$\frac{35 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{9} - 180 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{7} + 378 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{5} - 420 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} + 315 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{5040\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

maple [A] time = 0.05, size = 71, normalized size = 0.46

$$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9} - \frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7} + \frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5} - \frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16d\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.99, size = 107, normalized size = 0.69

$$\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{420\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{378\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} - \frac{180\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}} + \frac{35\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{5040\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.43, size = 127, normalized size = 0.82

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-180\cos\right.}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^5,x)
```

```
[Out] (sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 - 1
80*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^4 - 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(5040*a
^5*d*cos(c/2 + (d*x)/2)^9)
```

sympy [A] time = 19.96, size = 107, normalized size = 0.69

$$\begin{cases} \frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d}-\frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{28a^5d}+\frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^5d}-\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{12a^5d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d\neq 0 \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**7/(28*a**5*
d) + 3*tan(c/2 + d*x/2)**5/(40*a**5*d) - tan(c/2 + d*x/2)**3/(12*a**5*d) +
tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**5, Tr
ue))
```

3.86 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=147

$$\frac{5\sin(c+dx)}{63d\left(a^5\cos(c+dx)+a^5\right)}+\frac{5\sin(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}-\frac{17\sin(c+dx)}{63a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}+\frac{}{7a}$$

```
[Out] -1/9*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^5+1/7*sin(d*x+c)/a/d/(a+a*c
os(d*x+c))^4-17/63*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+5/63*sin(d*x+c)/a^3/
d/(a+a*cos(d*x+c))^2+5/63*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] time = 0.23, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2968, 3019, 2750, 2650, 2648}

$$\frac{5\sin(c+dx)}{63d\left(a^5\cos(c+dx)+a^5\right)}+\frac{5\sin(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}-\frac{17\sin(c+dx)}{63a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}+\frac{}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -(Cos[c + d*x]^2*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) + Sin[c + d*x]/
(7*a*d*(a + a*Cos[c + d*x])^4) - (17*Sin[c + d*x])/(63*a^2*d*(a + a*Cos[c +
 d*x])^3) + (5*Sin[c + d*x])/(63*a^3*d*(a + a*Cos[c + d*x])^2) + (5*Sin[c +
 d*x])/(63*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos(c+dx)(2a-7a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{2a\cos(c+dx)-7a\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4} + \frac{\int \frac{-36a^2+49a^2\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4} - \frac{17\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4} - \frac{17\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4} - \frac{17\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3}$$

Mathematica [A] time = 0.24, size = 125, normalized size = 0.85

$$\frac{\sec\left(\frac{c}{2}\right)\left(-315\sin\left(c+\frac{dx}{2}\right)+273\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+117\sin\left(2c+\frac{5dx}{2}\right)-63\sin\left(3c+\frac{5dx}{2}\right)+45\right.}{16128a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(315*Sin[(d*x)/2] - 315*Sin[c + (d*x)/2] + 273
*Sin[c + (3*d*x)/2] - 147*Sin[2*c + (3*d*x)/2] + 117*Sin[2*c + (5*d*x)/2] -
 63*Sin[3*c + (5*d*x)/2] + 45*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)/2]
))/(16128*a^5*d)
```

fricas [A] time = 0.86, size = 123, normalized size = 0.84

$$\frac{\left(5\cos(dx+c)^4+25\cos(dx+c)^3+21\cos(dx+c)^2+10\cos(dx+c)+2\right)\sin(dx+c)}{63\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+c)+a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
```

```
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

giac [A] time = 0.46, size = 59, normalized size = 0.40

$$-\frac{7 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^9 - 18 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^7 + 42 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 63 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{1008\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

maple [A] time = 0.06, size = 58, normalized size = 0.39

$$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9} + \frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7} - \frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3} + \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16 d\, a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/2*d*x+1/2*c)^7-2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.90, size = 87, normalized size = 0.59

$$\frac{\frac{63 \sin(dx+c)}{\cos(dx+c)+1} - \frac{42 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.39, size = 58, normalized size = 0.39

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 42\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 63\right)}{1008\,a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^5,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 + 7
*tan(c/2 + (d*x)/2)^8 - 63))/(1008*a^5*d)
```

sympy [A] time = 15.35, size = 87, normalized size = 0.59

$$\begin{cases} -\frac{\tan^{9}\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^{5}d} + \frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{5}d} - \frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{5}d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^{5}d} & \text{for}\ d \neq 0 \\ \frac{x\cos^{3}(c)}{(a\cos(c)+a)^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**9/(144*a**5*d) + tan(c/2 + d*x/2)**7/(56*a**5
*d) - tan(c/2 + d*x/2)**3/(24*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d,
 0)), (x*cos(c)**3/(a*cos(c) + a)**5, True))
```

3.87 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=139

$$\frac{2\sin(c+dx)}{45d\left(a^5\cos(c+dx)+a^5\right)}+\frac{2\sin(c+dx)}{45a^3d(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{15a^2d(a\cos(c+dx)+a)^3}-\frac{2\sin(c+dx)}{9ad(a\cos(c+dx)+a)^4}+ 9d(a$$

```
[Out] 1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-2/9*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4+1
/15*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+2/45*sin(d*x+c)/a^3/d/(a+a*cos(d*x+
c))^2+2/45*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2758, 2750, 2650, 2648}

$$\frac{2\sin(c+dx)}{45d\left(a^5\cos(c+dx)+a^5\right)}+\frac{2\sin(c+dx)}{45a^3d(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{15a^2d(a\cos(c+dx)+a)^3}-\frac{2\sin(c+dx)}{9ad(a\cos(c+dx)+a)^4}+ 9d(a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) - (2*Sin[c + d*x])/(9*a*d*(a + a*
Cos[c + d*x])^4) + Sin[c + d*x]/(15*a^2*d*(a + a*Cos[c + d*x])^3) + (2*Sin[
c + d*x])/(45*a^3*d*(a + a*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(45*d*(a^5 +
 a^5*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\int \frac{-5a+9a\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{3a^2}$$

$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3} + \frac{2}{-}$$

$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3} + \frac{}{4}$$

$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3} + \frac{}{4}$$

Mathematica [A] time = 0.24, size = 110, normalized size = 0.79

$$\frac{\sec\left(\frac{c}{2}\right)\left(-45\sin\left(c+\frac{dx}{2}\right)+54\sin\left(c+\frac{3dx}{2}\right)-30\sin\left(2c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c\right.\right.}{5760a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(81*Sin[(d*x)/2] - 45*Sin[c + (d*x)/2] + 54*Si
n[c + (3*d*x)/2] - 30*Sin[2*c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Si
n[3*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2]))/(5760*a^5*d)
```

fricas [A] time = 0.73, size = 123, normalized size = 0.88

$$\frac{\left(2\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+21\,\cos\left(dx+c\right)^{2}+10\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{45\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+c\right)+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

giac [A] time = 0.50, size = 46, normalized size = 0.33

$$\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{720\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d*
x + 1/2*c))/(a^5*d)
```

maple [A] time = 0.06, size = 45, normalized size = 0.32

$$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1
/2*c))
```

maxima [A] time = 0.68, size = 67, normalized size = 0.48

$$\frac{\frac{45\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{18\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{5\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{720\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.36, size = 45, normalized size = 0.32

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-18\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+45\right)}{720\,a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(5*tan(c/2 + (d*x)/2)^8 - 18*tan(c/2 + (d*x)/2)^4 + 45)
)/(720*a^5*d)
```

sympy [A] time = 11.58, size = 68, normalized size = 0.49

$$\begin{cases} \frac{\tan^{9}{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{144a^{5}d} - \frac{\tan^{5}{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{40a^{5}d} + \frac{\tan{\left(\frac{c}{2}+\frac{dx}{2}\right)}}{16a^{5}d} & \text{for}\: d \neq 0 \\ \frac{x\cos^{2}{\left(c\right)}}{\left(a\cos{\left(c\right)}+a\right)^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**5/(40*a**5*
d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**
5, True))
```

3.88 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$\frac{2\sin(c+dx)}{63d\left(a^5\cos(c+dx)+a^5\right)}+\frac{2\sin(c+dx)}{63ad\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{\sin(c+dx)}{21a^2d(a\cos(c+dx)+a)^3}+\frac{5\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4}-\frac{}{9}$$

```
[Out] -1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5+5/63*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4
+1/21*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+2/63*sin(d*x+c)/a/d/(a^2+a^2*cos(
d*x+c))^2+2/63*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2750, 2650, 2648}

$$\frac{2\sin(c+dx)}{63d\left(a^5\cos(c+dx)+a^5\right)}+\frac{2\sin(c+dx)}{63ad\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{\sin(c+dx)}{21a^2d(a\cos(c+dx)+a)^3}+\frac{5\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4}-\frac{}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) + (5*Sin[c + d*x])/(63*a*d*(a +
a*Cos[c + d*x])^4) + Sin[c + d*x]/(21*a^2*d*(a + a*Cos[c + d*x])^3) + (2*Si
n[c + d*x])/(63*a*d*(a^2 + a^2*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(63*d*(a
^5 + a^5*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{5\int \frac{1}{(a+a\cos(c+dx))^4}\,dx}{9a}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}+\frac{5\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{21a^2}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}+\frac{\sin(c+dx)}{21a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}+\frac{\sin(c+dx)}{21a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4}+\frac{\sin(c+dx)}{21a^2d(a+a\cos(c+dx))^3}$$

Mathematica [A] time = 0.19, size = 97, normalized size = 0.68

$$\frac{\sec\left(\frac{c}{2}\right)\left(-63\sin\left(c+\frac{dx}{2}\right)+84\sin\left(c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)+63\sin\left(\frac{dx}{2}\right.\right.}{8064a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(63*Sin[(d*x)/2] - 63*Sin[c + (d*x)/2] + 84*Si
n[c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Sin[3*c + (7*d*x)/2] + Sin[4
*c + (9*d*x)/2]))/(8064*a^5*d)
```

fricas [A] time = 0.58, size = 123, normalized size = 0.86

$$\frac{\left(2\cos\left(dx+c\right)^4+10\cos\left(dx+c\right)^3+21\cos\left(dx+c\right)^2+25\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{63\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx+c\right)+a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*x
 + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

giac [A] time = 0.57, size = 59, normalized size = 0.41

$$-\frac{7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{1008\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

maple [A] time = 0.05, size = 58, normalized size = 0.41

$$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*cos(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-2/7*tan(1/2*d*x+1/2*c)^7+2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.97, size = 87, normalized size = 0.61

$$\frac{\frac{63\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{18\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{7\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{1008\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.40, size = 58, normalized size = 0.41

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+42\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+63\right)}{1008\,a^{5}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 - 7*
tan(c/2 + (d*x)/2)^8 + 63))/(1008*a^5*d)
```

sympy [A] time = 9.18, size = 85, normalized size = 0.59

$$\begin{cases} -\frac{\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^5d} - \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^5d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^5d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**7/(56*a**5
*d) + tan(c/2 + d*x/2)**3/(24*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d,
 0)), (x*cos(c)/(a*cos(c) + a)**5, True))
```

3.89 $\int \frac{1}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$\frac{8\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{8\sin(c+dx)}{315ad\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{4\sin(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}+\frac{4\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4}$$

```
[Out] 1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5+4/63*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4+
4/105*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+8/315*sin(d*x+c)/a/d/(a^2+a^2*cos
(d*x+c))^2+8/315*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$\frac{8\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{8\sin(c+dx)}{315ad\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{4\sin(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}+\frac{4\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-5),x]
```

```
[Out] Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) + (4*Sin[c + d*x])/(63*a*d*(a + a
*Cos[c + d*x])^4) + (4*Sin[c + d*x])/(105*a^2*d*(a + a*Cos[c + d*x])^3) + (
8*Sin[c + d*x])/(315*a*d*(a^2 + a^2*Cos[c + d*x])^2) + (8*Sin[c + d*x])/(31
5*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^5}\,dx = \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\int \frac{1}{(a+a\cos(c+dx))^4}\,dx}{9a}$$
$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{4\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{21a^2}$$
$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} +$$
$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} +$$
$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} +$$

Mathematica [A] time = 0.16, size = 89, normalized size = 0.62

$$\frac{\left(126\sin\left(\frac{1}{2}(c+dx)\right)+84\sin\left(\frac{3}{2}(c+dx)\right)+36\sin\left(\frac{5}{2}(c+dx)\right)+9\sin\left(\frac{7}{2}(c+dx)\right)+\sin\left(\frac{9}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+\right.}{315a^5d(\cos(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-5),x]
```

```
[Out] (Cos[(c + d*x)/2]*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*x))/2] + 36*Sin[
(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*x))/2]))/(315*a^5
*d*(1 + Cos[c + d*x])^5)
```

fricas [A] time = 1.08, size = 123, normalized size = 0.86

$$\frac{\left(8\cos(dx+c)^4+40\cos(dx+c)^3+84\cos(dx+c)^2+100\cos(dx+c)+83\right)\sin(dx+c)}{315\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+c)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(d
*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

giac [A] time = 0.38, size = 72, normalized size = 0.50

$$\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+180\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+378\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+315\tan\left(\frac{1}{2}dx+\right.}{5040a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

maple [A] time = 0.04, size = 71, normalized size = 0.50

$$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5+4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.01, size = 107, normalized size = 0.75

$$\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}+\frac{420\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{180\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.42, size = 127, normalized size = 0.89

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+180\cos\right.}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x))^5,x)
```

```
[Out] (sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 + 1
80*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^4 + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(5040*a
^5*d*cos(c/2 + (d*x)/2)^9)
```

sympy [A] time = 7.60, size = 102, normalized size = 0.71

$$\begin{cases} \frac{\tan^{9}\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^{5}d}+\frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{28a^{5}d}+\frac{3\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{5}d}+\frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{12a^{5}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^{5}d} & \text{for } d\neq 0 \\ \frac{x}{(a\cos(c)+a)^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) + tan(c/2 + d*x/2)**7/(28*a**5*
d) + 3*tan(c/2 + d*x/2)**5/(40*a**5*d) + tan(c/2 + d*x/2)**3/(12*a**5*d) +
tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x/(a*cos(c) + a)**5, True))
```

3.90 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=153

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{488\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{173\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}-\frac{34\sin(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}-\frac{}{63ad}$$

```
[Out] arctanh(sin(d*x+c))/a^5/d-1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-13/63*sin(d*x
+c)/a/d/(a+a*cos(d*x+c))^4-34/105*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3-173/3
15*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-488/315*sin(d*x+c)/d/(a^5+a^5*cos(d*
x+c))
```

Rubi [A] time = 0.38, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2766, 2978, 12, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{488\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{173\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}-\frac{34\sin(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}-\frac{}{63ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^5*d) - Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) -
 (13*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (34*Sin[c + d*x])/(105
*a^2*d*(a + a*Cos[c + d*x])^3) - (173*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c
 + d*x])^2) - (488*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\int \frac{(9a-4a\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{\int \frac{\left(63a^2-39a^2\cos(c+dx)\right)\sec(c+dx}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{}{105a^2}$$

Mathematica [A] time = 1.81, size = 211, normalized size = 1.38

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\left(-25515\sin\left(c+\frac{dx}{2}\right)+29757\sin\left(c+\frac{3dx}{2}\right)-11235\sin\left(2c+\frac{3dx}{2}\right)+14733\sin\left(2c+\frac{5dx}{2}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -1/2520*(Cos[(c + d*x)/2]*(80640*Cos[(c + d*x)/2]^9*(Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*(
35973*Sin[(d*x)/2] - 25515*Sin[c + (d*x)/2] + 29757*Sin[c + (3*d*x)/2] - 11
235*Sin[2*c + (3*d*x)/2] + 14733*Sin[2*c + (5*d*x)/2] - 2835*Sin[3*c + (5*d
*x)/2] + 4077*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c + (7*d*x)/2] + 488*Sin[4*c
 + (9*d*x)/2])))/(a^5*d*(1 + Cos[c + d*x])^5)
```

fricas [A] time = 1.24, size = 246, normalized size = 1.61

$$\frac{315\left(\cos\left(dx+c\right)^{5}+5\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+10\,\cos\left(dx+c\right)^{2}+5\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(
d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^
5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x +
c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x + c)^
3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*d*cos
(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*c
os(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

giac [A] time = 0.79, size = 126, normalized size = 0.82

$$\frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{5}}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^{5}}-\frac{35a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+270a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+1008a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+2730a}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1/2
```

```
*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d*x
+ 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

maple [A] time = 0.10, size = 134, normalized size = 0.88

$$-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^{5}}-\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^{5}}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{5d\,a^{5}}-\frac{13\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^{5}}-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*cos(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-3/56/d/a^5*tan(1/2*d*x+1/2*c)^7-1/5/d/a^5
*tan(1/2*d*x+1/2*c)^5-13/24/d/a^5*tan(1/2*d*x+1/2*c)^3-31/16/d/a^5*tan(1/2*
d*x+1/2*c)-1/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^5*ln(tan(1/2*d*x+1/2*c)+1
)
```

maxima [A] time = 0.96, size = 159, normalized size = 1.04

$$-\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1}+\frac{2730\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1008\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{270\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a^{5}}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{5}}+\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{5}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 50
40*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)/(co
s(d*x + c) + 1) - 1)/a^5)/d
```

mupad [B] time = 0.39, size = 99, normalized size = 0.65

$$-\frac{\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,a^{5}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{5\,a^{5}}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{56\,a^{5}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{144\,a^{5}}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^{5}}+\frac{31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a^{5}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^5),x)
```

```
[Out] -((13*tan(c/2 + (d*x)/2)^3)/(24*a^5) + tan(c/2 + (d*x)/2)^5/(5*a^5) + (3*ta
n(c/2 + (d*x)/2)^7)/(56*a^5) + tan(c/2 + (d*x)/2)^9/(144*a^5) - (2*atanh(ta
n(c/2 + (d*x)/2)))/a^5 + (31*tan(c/2 + (d*x)/2))/(16*a^5))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c + d*x
)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5
```

3.91 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=168

$$\frac{496\tan(c+dx)}{63a^5d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{5\tan(c+dx)}{d\left(a^5\cos(c+dx)+a^5\right)}-\frac{67\tan(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}-\frac{29\tan(c+a}{63a^2d(a\cos(c+a}$$

```
[Out] -5*arctanh(sin(d*x+c))/a^5/d+496/63*tan(d*x+c)/a^5/d-1/9*tan(d*x+c)/d/(a+a*
cos(d*x+c))^5-5/21*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^4-29/63*tan(d*x+c)/a^2/d
/(a+a*cos(d*x+c))^3-67/63*tan(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-5*tan(d*x+c)/
d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] time = 0.53, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{496\tan(c+dx)}{63a^5d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{5\tan(c+dx)}{d\left(a^5\cos(c+dx)+a^5\right)}-\frac{67\tan(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}-\frac{29\tan(c+a}{63a^2d(a\cos(c+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^5*d) + (496*Tan[c + d*x])/(63*a^5*d) - Tan[c
+ d*x]/(9*d*(a + a*Cos[c + d*x])^5) - (5*Tan[c + d*x])/(21*a*d*(a + a*Cos[c
 + d*x])^4) - (29*Tan[c + d*x])/(63*a^2*d*(a + a*Cos[c + d*x])^3) - (67*Tan
[c + d*x])/(63*a^3*d*(a + a*Cos[c + d*x])^2) - (5*Tan[c + d*x])/(d*(a^5 + a
^5*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
```

```
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\int \frac{(10a-5a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} + \frac{\int \frac{(85a^2-60a^2\cos(c+dx))\sec^2(c+dx}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3} +$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3} -$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3} -$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3} -$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} - 63a$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{496\tan(c+dx)}{63a^5d} - \frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(}{21ad(a+a}$$

Mathematica [B] time = 6.37, size = 453, normalized size = 2.70

$$\frac{160\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^5} - \frac{160\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^5} + \mathrm{se}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (160*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]])/(d
*(a + a*Cos[c + d*x])^5) - (160*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^5) + (Cos[c/2 + (d*x)/2]*S
ec[c/2]*Sec[c]*Sec[c + d*x]*(-33978*Sin[(d*x)/2] + 52002*Sin[(3*d*x)/2] - 5
6952*Sin[c - (d*x)/2] + 43722*Sin[c + (d*x)/2] - 47208*Sin[2*c + (d*x)/2] -
 18144*Sin[c + (3*d*x)/2] + 41796*Sin[2*c + (3*d*x)/2] - 28350*Sin[3*c + (3
*d*x)/2] + 34578*Sin[c + (5*d*x)/2] - 5691*Sin[2*c + (5*d*x)/2] + 28719*Sin
[3*c + (5*d*x)/2] - 11550*Sin[4*c + (5*d*x)/2] + 15517*Sin[2*c + (7*d*x)/2]
```

```
 - 504*Sin[3*c + (7*d*x)/2] + 13186*Sin[4*c + (7*d*x)/2] - 2835*Sin[5*c + (
7*d*x)/2] + 4149*Sin[3*c + (9*d*x)/2] + 252*Sin[4*c + (9*d*x)/2] + 3582*Sin
[5*c + (9*d*x)/2] - 315*Sin[6*c + (9*d*x)/2] + 496*Sin[4*c + (11*d*x)/2] +
63*Sin[5*c + (11*d*x)/2] + 433*Sin[6*c + (11*d*x)/2]))/(2016*d*(a + a*Cos[c
 + d*x])^5)
```

fricas [A] time = 1.05, size = 278, normalized size = 1.65

$$-\frac{315\left(\cos\left(dx+c\right)^{6}+5\,\cos\left(dx+c\right)^{5}+10\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+5\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/126*(315*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos
(d*x + c)^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 315*
(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 5*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(496*cos(d*x
+ c)^5 + 2165*cos(d*x + c)^4 + 3633*cos(d*x + c)^3 + 2840*cos(d*x + c)^2 +
946*cos(d*x + c) + 63)*sin(d*x + c))/(a^5*d*cos(d*x + c)^6 + 5*a^5*d*cos(d*
x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 5*a^5*d*cos(
d*x + c)^2 + a^5*d*cos(d*x + c))
```

giac [A] time = 0.55, size = 155, normalized size = 0.92

$$-\frac{\frac{5040\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{5}}-\frac{5040\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{5}}+\frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{5}}-\frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+378\,a^{4}}{}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^5 + 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 - 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 + 72*a^40*tan(1/2*d*x + 1/2*c)
^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 + 1512*a^40*tan(1/2*d*x + 1/2*c)^3 + 8
127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

maple [A] time = 0.09, size = 177, normalized size = 1.05

$$\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^{5}}+\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{14d\,a^{5}}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{5}}+\frac{3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{5}}+\frac{129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}-\frac{1}{d\,a^{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x)
```

```
[Out] 1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+1/14/d/a^5*tan(1/2*d*x+1/2*c)^7+3/8/d/a^5*
tan(1/2*d*x+1/2*c)^5+3/2/d/a^5*tan(1/2*d*x+1/2*c)^3+129/16/d/a^5*tan(1/2*d*
x+1/2*c)-1/d/a^5/(tan(1/2*d*x+1/2*c)-1)+5/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/
d/a^5/(tan(1/2*d*x+1/2*c)+1)-5/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 1.06, size = 206, normalized size = 1.23

$$\frac{\frac{2016\sin(dx+c)}{\left(a^5-\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1}+\frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\cdots}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

mupad [B] time = 0.45, size = 149, normalized size = 0.89

$$\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2a^5d}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8a^5d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{14a^5d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144a^5d}-\frac{10\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5d}-\frac{2\tan\left(\frac{c}{2}+\cdots\right.}{d\left(a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^5),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^3)/(2*a^5*d) + (3*tan(c/2 + (d*x)/2)^5)/(8*a^5*d) + t
an(c/2 + (d*x)/2)^7/(14*a^5*d) + tan(c/2 + (d*x)/2)^9/(144*a^5*d) - (10*ata
nh(tan(c/2 + (d*x)/2)))/(a^5*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^5*tan(c/2 +
(d*x)/2)^2 - a^5)) + (129*tan(c/2 + (d*x)/2))/(16*a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c +
d*x)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5
```

3.92 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5} dx$

Optimal. Leaf size=224

$$-\frac{7664\tan(c+dx)}{315a^5d}+\frac{31\tanh^{-1}(\sin(c+dx))}{2a^5d}+\frac{31\tan(c+dx)\sec(c+dx)}{2a^5d}-\frac{3832\tan(c+dx)\sec(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{577\tan}{315a^3d}$$

```
[Out] 31/2*arctanh(sin(d*x+c))/a^5/d-7664/315*tan(d*x+c)/a^5/d+31/2*sec(d*x+c)*ta
n(d*x+c)/a^5/d-1/9*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^5-17/63*sec(d*x
+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^4-28/45*sec(d*x+c)*tan(d*x+c)/a^2/d/(a+
a*cos(d*x+c))^3-577/315*sec(d*x+c)*tan(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-3832
/315*sec(d*x+c)*tan(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] time = 0.54, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{7664\tan(c+dx)}{315a^5d}+\frac{31\tanh^{-1}(\sin(c+dx))}{2a^5d}+\frac{31\tan(c+dx)\sec(c+dx)}{2a^5d}-\frac{3832\tan(c+dx)\sec(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{577\tan}{315a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (31*ArcTanh[Sin[c + d*x]])/(2*a^5*d) - (7664*Tan[c + d*x])/(315*a^5*d) + (3
1*Sec[c + d*x]*Tan[c + d*x])/(2*a^5*d) - (Sec[c + d*x]*Tan[c + d*x])/(9*d*(
a + a*Cos[c + d*x])^5) - (17*Sec[c + d*x]*Tan[c + d*x])/(63*a*d*(a + a*Cos[
c + d*x])^4) - (28*Sec[c + d*x]*Tan[c + d*x])/(45*a^2*d*(a + a*Cos[c + d*x]
)^3) - (577*Sec[c + d*x]*Tan[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) -
 (3832*Sec[c + d*x]*Tan[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\int \frac{(11a-6a\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{\int \frac{(111a^2-85a^2\cos(c+dx))\sec^3(}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\sec(c+dx)\tan(c+dx}{45a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\sec(c+dx)\tan(c+dx}{45a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\sec(c+dx)\tan(c+dx}{45a^2d(a+a\cos(c+dx))^3}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\sec(c+dx)\tan(c+dx}{45a^2d(a+a\cos(c+dx))^3}$$

$$= \frac{31\sec(c+dx)\tan(c+dx)}{2a^5d} - \frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4}$$

$$= \frac{31\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{7664\tan(c+dx)}{315a^5d} + \frac{31\sec(c+dx)\tan(c+dx)}{2a^5d} - \frac{\sec(c+}{9d(a+}$$

Mathematica [B] time = 6.35, size = 507, normalized size = 2.26

$$-\frac{496\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^5}+\frac{496\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^5}+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (-496*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]])/(
d*(a + a*Cos[c + d*x])^5) + (496*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/
2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^5) + (Cos[c/2 + (d*x)/2]*
Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(1472562*Sin[(d*x)/2] - 2822886*Sin[(3*d*x)/
2] + 3057654*Sin[c - (d*x)/2] - 1885854*Sin[c + (d*x)/2] + 2644362*Sin[2*c
+ (d*x)/2] + 867048*Sin[c + (3*d*x)/2] - 1868436*Sin[2*c + (3*d*x)/2] + 182
1498*Sin[3*c + (3*d*x)/2] - 2083537*Sin[c + (5*d*x)/2] + 339885*Sin[2*c + (
5*d*x)/2] - 1456687*Sin[3*c + (5*d*x)/2] + 966735*Sin[4*c + (5*d*x)/2] - 11
```

```
95641*Sin[2*c + (7*d*x)/2] + 46515*Sin[3*c + (7*d*x)/2] - 874341*Sin[4*c +
(7*d*x)/2] + 367815*Sin[5*c + (7*d*x)/2] - 494579*Sin[3*c + (9*d*x)/2] - 31
815*Sin[4*c + (9*d*x)/2] - 374879*Sin[5*c + (9*d*x)/2] + 87885*Sin[6*c + (9
*d*x)/2] - 128187*Sin[4*c + (11*d*x)/2] - 18585*Sin[5*c + (11*d*x)/2] - 998
37*Sin[6*c + (11*d*x)/2] + 9765*Sin[7*c + (11*d*x)/2] - 15328*Sin[5*c + (13
*d*x)/2] - 3150*Sin[6*c + (13*d*x)/2] - 12178*Sin[7*c + (13*d*x)/2]))/(4032
0*d*(a + a*Cos[c + d*x])^5)
```

fricas [A] time = 1.14, size = 294, normalized size = 1.31

$$\frac{9765 \left(\cos (dx+c)^7 + 5 \cos (dx+c)^6 + 10 \cos (dx+c)^5 + 10 \cos (dx+c)^4 + 5 \cos (dx+c)^3 + \cos (dx+c)^2\right) \log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/1260*(9765*(cos(d*x + c)^7 + 5*cos(d*x + c)^6 + 10*cos(d*x + c)^5 + 10*co
s(d*x + c)^4 + 5*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 9
765*(cos(d*x + c)^7 + 5*cos(d*x + c)^6 + 10*cos(d*x + c)^5 + 10*cos(d*x + c
)^4 + 5*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(15328*
cos(d*x + c)^6 + 66875*cos(d*x + c)^5 + 112119*cos(d*x + c)^4 + 87440*cos(d
*x + c)^3 + 28828*cos(d*x + c)^2 + 1575*cos(d*x + c) - 315)*sin(d*x + c))/(
a^5*d*cos(d*x + c)^7 + 5*a^5*d*cos(d*x + c)^6 + 10*a^5*d*cos(d*x + c)^5 + 1
0*a^5*d*cos(d*x + c)^4 + 5*a^5*d*cos(d*x + c)^3 + a^5*d*cos(d*x + c)^2)
```

giac [A] time = 0.79, size = 171, normalized size = 0.76

$$\frac{\frac{78120 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^5} - \frac{78120 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)}{a^5} + \frac{5040 \left(11 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 - 9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 - 1\right)^2 a^5} - \frac{35 a^{40} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^9 + 450 a^{40}}{}}{5040 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(78120*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 78120*log(abs(tan(1/
2*d*x + 1/2*c) - 1))/a^5 + 5040*(11*tan(1/2*d*x + 1/2*c)^3 - 9*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^5) - (35*a^40*tan(1/2*d*x + 1/2
*c)^9 + 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2*c)^5
+ 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*c))/a^4
5)/d
```

maple [A] time = 0.11, size = 219, normalized size = 0.98

$$-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^{5}}-\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^{5}}-\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d\,a^{5}}-\frac{25\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{5}}-\frac{351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}+\frac{1}{2d\,a^{5}\left(\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-5/56/d/a^5*tan(1/2*d*x+1/2*c)^7-3/5/d/a^5
*tan(1/2*d*x+1/2*c)^5-25/8/d/a^5*tan(1/2*d*x+1/2*c)^3-351/16/d/a^5*tan(1/2*
d*x+1/2*c)+1/2/d/a^5/(tan(1/2*d*x+1/2*c)-1)^2+11/2/d/a^5/(tan(1/2*d*x+1/2*c
)-1)-31/2/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^5/(tan(1/2*d*x+1/2*c)+1)^2
+11/2/d/a^5/(tan(1/2*d*x+1/2*c)+1)+31/2/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.69, size = 251, normalized size = 1.12

$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{11\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{5}-\frac{2a^{5}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{5}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}+\frac{15750\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3024\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{450\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a^{5}}-\frac{78120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\right.}{a^{5}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 11*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/(a^5 - 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) + 1)
+ 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9)/a^5 - 78120*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^5 + 78120*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

mupad [B] time = 0.48, size = 179, normalized size = 0.80

$$\frac{31\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^{5}\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{5\,a^{5}\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{56\,a^{5}\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{144\,a^{5}\,d}-\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8\,a^{5}\,d}-\frac{9\tan\left(\frac{c}{2}\right.}{d\left(a^{5}\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^5),x)
```

```
[Out] (31*atanh(tan(c/2 + (d*x)/2)))/(a^5*d) - (3*tan(c/2 + (d*x)/2)^5)/(5*a^5*d)
 - (5*tan(c/2 + (d*x)/2)^7)/(56*a^5*d) - tan(c/2 + (d*x)/2)^9/(144*a^5*d) -
 (25*tan(c/2 + (d*x)/2)^3)/(8*a^5*d) - (9*tan(c/2 + (d*x)/2) - 11*tan(c/2 +
 (d*x)/2)^3)/(d*(a^5*tan(c/2 + (d*x)/2)^4 - 2*a^5*tan(c/2 + (d*x)/2)^2 + a^
5)) - (351*tan(c/2 + (d*x)/2))/(16*a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c +
d*x)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5
```

3.93 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

Optimal. Leaf size=184

$$\frac{146\sin(c+dx)}{693a^6d(\cos(c+dx)+1)}-\frac{268\sin(c+dx)}{693a^6d(\cos(c+dx)+1)^2}+\frac{130\sin(c+dx)}{693a^6d(\cos(c+dx)+1)^3}-\frac{118\sin(c+dx)\cos^2(c+dx)}{693a^2d(a\cos(c+dx)+a)^4}-$$

```
[Out] 130/693*sin(d*x+c)/a^6/d/(1+cos(d*x+c))^3-268/693*sin(d*x+c)/a^6/d/(1+cos(d
*x+c))^2+146/693*sin(d*x+c)/a^6/d/(1+cos(d*x+c))-1/11*cos(d*x+c)^4*sin(d*x+
c)/d/(a+a*cos(d*x+c))^6-14/99*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^
5-118/693*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^4
```

Rubi [A] time = 0.41, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2968, 3019, 2750, 2648}

$$-\frac{118\sin(c+dx)\cos^2(c+dx)}{693a^2d(a\cos(c+dx)+a)^4}+\frac{146\sin(c+dx)}{693a^6d(\cos(c+dx)+1)}-\frac{268\sin(c+dx)}{693a^6d(\cos(c+dx)+1)^2}+\frac{130\sin(c+dx)}{693a^6d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^6,x]
```

```
[Out] (130*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c + d*x])^3) - (268*Sin[c + d*x])/(6
93*a^6*d*(1 + Cos[c + d*x])^2) + (146*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c +
 d*x])) - (Cos[c + d*x]^4*Sin[c + d*x])/(11*d*(a + a*Cos[c + d*x])^6) - (14
*Cos[c + d*x]^3*Sin[c + d*x])/(99*a*d*(a + a*Cos[c + d*x])^5) - (118*Cos[c
+ d*x]^2*Sin[c + d*x])/(693*a^2*d*(a + a*Cos[c + d*x])^4)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{\int \frac{\cos^3(c+dx)(4a-10a\cos(c+dx))}{(a+a\cos(c+dx))^5}\,dx}{11a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)(42a^2-76a^2\cos(}{(a+a\cos(c+dx))^4}}{99a^4}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{118\cos^2(c+dx)\sin(c+}{693a^2d(a+a\cos(c+d}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{118\cos^2(c+dx)\sin(c+}{693a^2d(a+a\cos(c+d}$$

$$= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5}$$

$$= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5}$$

$$= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5}$$

Mathematica [A] time = 0.36, size = 164, normalized size = 0.89

$$\sec\left(\frac{c}{2}\right)\left(-33726\sin\left(c+\frac{dx}{2}\right)+25080\sin\left(c+\frac{3dx}{2}\right)-23100\sin\left(2c+\frac{3dx}{2}\right)+12540\sin\left(2c+\frac{5dx}{2}\right)-11550\sin\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^6,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^11*(33726*Sin[(d*x)/2] - 33726*Sin[c + (d*x)/2]
+ 25080*Sin[c + (3*d*x)/2] - 23100*Sin[2*c + (3*d*x)/2] + 12540*Sin[2*c + (
5*d*x)/2] - 11550*Sin[3*c + (5*d*x)/2] + 4565*Sin[3*c + (7*d*x)/2] - 3465*S
in[4*c + (7*d*x)/2] + 913*Sin[4*c + (9*d*x)/2] - 693*Sin[5*c + (9*d*x)/2] +
 146*Sin[5*c + (11*d*x)/2]))/(709632*a^6*d)
```

fricas [A] time = 0.80, size = 147, normalized size = 0.80

$$\frac{\left(146\cos(dx+c)^5+183\cos(dx+c)^4+184\cos(dx+c)^3+124\cos(dx+c)^2+48\cos(dx+c)+\right.}{693\left(a^6d\cos(dx+c)^6+6a^6d\cos(dx+c)^5+15a^6d\cos(dx+c)^4+20a^6d\cos(dx+c)^3+15a^6d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] 1/693*(146*cos(d*x + c)^5 + 183*cos(d*x + c)^4 + 184*cos(d*x + c)^3 + 124*c
os(d*x + c)^2 + 48*cos(d*x + c) + 8)*sin(d*x + c)/(a^6*d*cos(d*x + c)^6 + 6
*a^6*d*cos(d*x + c)^5 + 15*a^6*d*cos(d*x + c)^4 + 20*a^6*d*cos(d*x + c)^3 +
 15*a^6*d*cos(d*x + c)^2 + 6*a^6*d*cos(d*x + c) + a^6*d)
```

giac [A] time = 0.47, size = 85, normalized size = 0.46

$$-\frac{63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-385\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+990\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1386\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1155\,\tan\left(\frac{1}{2}\,a\right.}{22176\,a^{6}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="giac")
```

```
[Out] -1/22176*(63*tan(1/2*d*x + 1/2*c)^11 - 385*tan(1/2*d*x + 1/2*c)^9 + 990*tan
(1/2*d*x + 1/2*c)^7 - 1386*tan(1/2*d*x + 1/2*c)^5 + 1155*tan(1/2*d*x + 1/2*
c)^3 - 693*tan(1/2*d*x + 1/2*c))/(a^6*d)
```

maple [A] time = 0.06, size = 84, normalized size = 0.46

$$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}+\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{10\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x)
```

```
[Out] 1/32/d/a^6*(-1/11*tan(1/2*d*x+1/2*c)^11+5/9*tan(1/2*d*x+1/2*c)^9-10/7*tan(1
/2*d*x+1/2*c)^7+2*tan(1/2*d*x+1/2*c)^5-5/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x
+1/2*c))
```

maxima [A] time = 0.67, size = 127, normalized size = 0.69

$$\frac{\frac{693\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{1155\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1386\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{990\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{385\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{63\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{22176\,a^{6}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] 1/22176*(693*sin(d*x + c)/(cos(d*x + c) + 1) - 1155*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 1386*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 990*sin(d*x + c)^
```

```
7/(cos(d*x + c) + 1)^7 + 385*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 63*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11)/(a^6*d)
```

mupad [B] time = 0.88, size = 75, normalized size = 0.41

$$\frac{\frac{495\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{8}+\frac{495\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{16}+\frac{275\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{8}+\frac{55\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{8}+\frac{73\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)}{16}}{22176\,a^6\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^6,x)
```

```
[Out] ((495*sin((3*c)/2 + (3*d*x)/2))/8 + (495*sin((5*c)/2 + (5*d*x)/2))/16 + (27
5*sin((7*c)/2 + (7*d*x)/2))/8 + (55*sin((9*c)/2 + (9*d*x)/2))/8 + (73*sin((
11*c)/2 + (11*d*x)/2))/16)/(22176*a^6*d*cos(c/2 + (d*x)/2)^11)
```

sympy [A] time = 39.01, size = 129, normalized size = 0.70

$$\begin{cases} -\frac{\tan^{11}\left(\frac{c}{2}+\frac{dx}{2}\right)}{352a^6d}+\frac{5\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{288a^6d}-\frac{5\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{112a^6d}+\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^6d}-\frac{5\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{96a^6d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32a^6d} & \text{for } d \neq 0 \\ \frac{x\cos^5(c)}{(a\cos(c)+a)^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**6,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**11/(352*a**6*d) + 5*tan(c/2 + d*x/2)**9/(288*
a**6*d) - 5*tan(c/2 + d*x/2)**7/(112*a**6*d) + tan(c/2 + d*x/2)**5/(16*a**6
*d) - 5*tan(c/2 + d*x/2)**3/(96*a**6*d) + tan(c/2 + d*x/2)/(32*a**6*d), Ne(
d, 0)), (x*cos(c)**5/(a*cos(c) + a)**6, True))
```

3.94 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx$

Optimal. Leaf size=176

$$\frac{61\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)}+\frac{61\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^2}-\frac{241\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^3}+\frac{9\sin(c+dx)}{77a^2d(a\cos(c+dx)+a)^4}-$$

```
[Out] -241/1155*sin(d*x+c)/a^6/d/(1+cos(d*x+c))^3+61/1155*sin(d*x+c)/a^6/d/(1+cos
(d*x+c))^2+61/1155*sin(d*x+c)/a^6/d/(1+cos(d*x+c))-1/11*cos(d*x+c)^3*sin(d*
x+c)/d/(a+a*cos(d*x+c))^6-4/33*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))
^5+9/77*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^4
```

Rubi [A] time = 0.32, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2765, 2977, 2968, 3019, 2750, 2650, 2648}

$$\frac{61\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)}+\frac{61\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^2}-\frac{241\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^3}+\frac{9\sin(c+dx)}{77a^2d(a\cos(c+dx)+a)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^6,x]
```

```
[Out] (-241*Sin[c + d*x])/(1155*a^6*d*(1 + Cos[c + d*x])^3) + (61*Sin[c + d*x])/(
1155*a^6*d*(1 + Cos[c + d*x])^2) + (61*Sin[c + d*x])/(1155*a^6*d*(1 + Cos[c
 + d*x])) - (Cos[c + d*x]^3*Sin[c + d*x])/(11*d*(a + a*Cos[c + d*x])^6) - (
4*Cos[c + d*x]^2*Sin[c + d*x])/(33*a*d*(a + a*Cos[c + d*x])^5) + (9*Sin[c +
 d*x])/(77*a^2*d*(a + a*Cos[c + d*x])^4)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{\int \frac{\cos^2(c+dx)(3a-9a\cos(c+dx))}{(a+a\cos(c+dx))^5}\,dx}{11a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos(c+dx)\left(24a^2-57a^2\cos(c+dx\right.}{(a+a\cos(c+dx))^4}}{99a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} - \frac{\int \frac{24a^2\cos(c+dx)-57a^2\cos^2(c+d}{(a+a\cos(c+dx))^4}}{99a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} + \frac{9\sin(c+dx)}{77a^2d(a+a\cos(c+dx))^4}$$

$$= -\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5}$$

$$= -\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5}$$

$$= -\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5}$$

Mathematica [A] time = 0.32, size = 151, normalized size = 0.86

$$\frac{\sec\left(\frac{c}{2}\right)\left(-12936\sin\left(c+\frac{dx}{2}\right)+10890\sin\left(c+\frac{3dx}{2}\right)-9240\sin\left(2c+\frac{3dx}{2}\right)+6600\sin\left(2c+\frac{5dx}{2}\right)-3465\sin\left(3c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^6,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^11*(15246*Sin[(d*x)/2] - 12936*Sin[c + (d*x)/2]
+ 10890*Sin[c + (3*d*x)/2] - 9240*Sin[2*c + (3*d*x)/2] + 6600*Sin[2*c + (5*
d*x)/2] - 3465*Sin[3*c + (5*d*x)/2] + 2200*Sin[3*c + (7*d*x)/2] - 1155*Sin[
4*c + (7*d*x)/2] + 671*Sin[4*c + (9*d*x)/2] + 61*Sin[5*c + (11*d*x)/2]))/(1
182720*a^6*d)
```

fricas [A] time = 0.84, size = 147, normalized size = 0.84

$$\frac{\left(61\cos(dx+c)^5+366\cos(dx+c)^4+368\cos(dx+c)^3+248\cos(dx+c)^2+96\cos(dx+c)+16\right.}{1155\left(a^6d\cos(dx+c)^6+6a^6d\cos(dx+c)^5+15a^6d\cos(dx+c)^4+20a^6d\cos(dx+c)^3+15a^6d\cos(dx+c)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="fricas")

[Out] 1/1155*(61*cos(d*x + c)^5 + 366*cos(d*x + c)^4 + 368*cos(d*x + c)^3 + 248*cos(d*x + c)^2 + 96*cos(d*x + c) + 16)*sin(d*x + c)/(a^6*d*cos(d*x + c)^6 + 6*a^6*d*cos(d*x + c)^5 + 15*a^6*d*cos(d*x + c)^4 + 20*a^6*d*cos(d*x + c)^3 + 15*a^6*d*cos(d*x + c)^2 + 6*a^6*d*cos(d*x + c) + a^6*d)

giac [A] time = 0.71, size = 85, normalized size = 0.48

$$\frac{105 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{11} - 385 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{9} + 330 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{7} + 462 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{5} - 1155 \tan\left(\frac{1}{2}\,a\right.}{36960\,a^6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="giac")

[Out] 1/36960*(105*tan(1/2*d*x + 1/2*c)^11 - 385*tan(1/2*d*x + 1/2*c)^9 + 330*tan(1/2*d*x + 1/2*c)^7 + 462*tan(1/2*d*x + 1/2*c)^5 - 1155*tan(1/2*d*x + 1/2*c)^3 + 1155*tan(1/2*d*x + 1/2*c))/(a^6*d)

maple [A] time = 0.05, size = 84, normalized size = 0.48

$$\frac{\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11} - \frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3} + \frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7} + \frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5} - \left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32 d\, a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x)

[Out] 1/32/d/a^6*(1/11*tan(1/2*d*x+1/2*c)^11-1/3*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/2*d*x+1/2*c)^7+2/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))

maxima [A] time = 1.49, size = 127, normalized size = 0.72

$$\frac{\frac{1155 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1155 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{462 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{330 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{385 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} + \frac{105 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{36960\,a^6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="maxima")

[Out] 1/36960*(1155*sin(d*x + c)/(cos(d*x + c) + 1) - 1155*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 462*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 330*sin(d*x + c)^

```
7/(cos(d*x + c) + 1)^7 - 385*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 105*sin(
d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^6*d)
```

mupad [B] time = 0.46, size = 151, normalized size = 0.86

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(1155\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-1155\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+462\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+330\,c\right.}{36960\,a^6\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^6,x)
```

```
[Out] (sin(c/2 + (d*x)/2)*(1155*cos(c/2 + (d*x)/2)^10 + 105*sin(c/2 + (d*x)/2)^10
 - 385*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 330*cos(c/2 + (d*x)/2)^4
*sin(c/2 + (d*x)/2)^6 + 462*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 - 115
5*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2))/(36960*a^6*d*cos(c/2 + (d*x)/
2)^11)
```

sympy [A] time = 30.36, size = 124, normalized size = 0.70

$$\begin{cases} \frac{\tan^{11}\left(\frac{c}{2}+\frac{dx}{2}\right)}{352a^6d}-\frac{\tan^{9}\left(\frac{c}{2}+\frac{dx}{2}\right)}{96a^6d}+\frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{112a^6d}+\frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{80a^6d}-\frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{32a^6d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32a^6d} & \text{for}\ d\neq 0 \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**6,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**11/(352*a**6*d) - tan(c/2 + d*x/2)**9/(96*a**6
*d) + tan(c/2 + d*x/2)**7/(112*a**6*d) + tan(c/2 + d*x/2)**5/(80*a**6*d) -
tan(c/2 + d*x/2)**3/(32*a**6*d) + tan(c/2 + d*x/2)/(32*a**6*d), Ne(d, 0)),
(x*cos(c)**4/(a*cos(c) + a)**6, True))
```

3.95 $\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{32\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{64\sin(c+dx)}{3}$$

```
[Out] 32/105*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+32/45*a*sin(d*x+c)/d/(a+a*cos(
d*x+c))^(1/2)+16/63*a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*
a*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-64/315*sin(d*x+c)*(a+a*c
os(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2770, 2759, 2751, 2646}

$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{32\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{64\sin(c+dx)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (32*a*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a*Cos[c + d*x]^3*
Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Cos[c + d*x]^4*Sin[c +
 d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (64*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(315*d) + (32*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
```

```
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{8}{9}\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{16}{21}\int \cos^2$$

$$= \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{32(a+a\,\mathrm{c}}{}$$

$$= \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} - \frac{64\sqrt{a+a\,\mathrm{c}}}{}$$

$$= \frac{32a\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^4(c+dx}{9d\sqrt{a+a\,\mathrm{c}}}$$

Mathematica [A] time = 0.28, size = 92, normalized size = 0.58

$$\frac{\left(1890\sin\left(\frac{1}{2}(c+dx)\right)+420\sin\left(\frac{3}{2}(c+dx)\right)+252\sin\left(\frac{5}{2}(c+dx)\right)+45\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)\sec}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(1890*Sin[(c + d*x)/2] + 420*S
in[(3*(c + d*x))/2] + 252*Sin[(5*(c + d*x))/2] + 45*Sin[(7*(c + d*x))/2] +
35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

fricas [A] time = 0.72, size = 72, normalized size = 0.46

$$\frac{2\left(35\,\cos\left(dx+c\right)^4+40\,\cos\left(dx+c\right)^3+48\,\cos\left(dx+c\right)^2+64\,\cos\left(dx+c\right)+128\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+\right.}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 48*cos(d*x + c)^2 + 64*cos(d
*x + c) + 128)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.65, size = 129, normalized size = 0.82

$$\frac{1}{2520}\sqrt{2}\sqrt{a}\left(\frac{35\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}{d}+\frac{45\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{252\,\mathrm{sgn}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*sqrt(a)*(35*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c)/d
 + 45*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 252*sgn(cos(1/2*d*
x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 420*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*
d*x + 3/2*c)/d + 1890*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d)
```

maple [A] time = 0.18, size = 97, normalized size = 0.61

$$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(560\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-800\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+552\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-104\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}{315\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*cos(1/2*d*x+1/2*c)^8-800
*cos(1/2*d*x+1/2*c)^6+552*cos(1/2*d*x+1/2*c)^4-104*cos(1/2*d*x+1/2*c)^2+107
)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.13, size = 79, normalized size = 0.50

$$\frac{\left(35\,\sqrt{2}\,\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+45\,\sqrt{2}\,\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+252\,\sqrt{2}\,\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+420\,\sqrt{2}\,\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+1890\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2520*(35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c) +
 252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 1890
*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.96 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{12\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{8\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{4a\sin(}{5d\sqrt{a\cos(}}$$

```
[Out] 12/35*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+4/5*a*sin(d*x+c)/d/(a+a*cos(d*x
+c))^(1/2)+2/7*a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-8/35*sin(
d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2770, 2759, 2751, 2646}

$$\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{12\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{8\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{4a\sin(}{5d\sqrt{a\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (4*a*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Cos[c + d*x]^3*Sin
[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) - (8*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(35*d) + (12*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
```

```
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{6}{7}\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{12(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35ad} + \frac{12 \ldots}{}$$

$$= \frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{8\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{35d} + \frac{12(a+ \ldots}{}$$

$$= \frac{4a\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{8\sqrt{a+a\cos(c+ \ldots}}{35 \ldots}$$

Mathematica [A] time = 0.16, size = 80, normalized size = 0.66

$$\frac{\left(105\sin\left(\frac{1}{2}(c+dx)\right)+35\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+5\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx) \ldots}}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(105*Sin[(c + d*x)/2] + 35*Sin
[(3*(c + d*x))/2] + 7*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2]))/(140*
d)
```

fricas [A] time = 0.68, size = 62, normalized size = 0.51

$$\frac{2\left(5\cos(dx+c)^3+6\cos(dx+c)^2+8\cos(dx+c)+16\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{35\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 1.26, size = 105, normalized size = 0.86

$$\frac{1}{140}\sqrt{2}\sqrt{a}\left(\frac{5\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{7\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{35\,\mathrm{sgn}\left(\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/140*sqrt(2)*sqrt(a)*(5*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d +
 7*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 35*sgn(cos(1/2*d*x +
1/2*c))*sin(3/2*d*x + 3/2*c)/d + 105*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
+ 1/2*c)/d)
```

maple [A] time = 0.17, size = 84, normalized size = 0.69

$$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(40\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+22\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+9\right)\sqrt{2}}{35\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/35*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(40*cos(1/2*d*x+1/2*c)^6-36*co
s(1/2*d*x+1/2*c)^4+22*cos(1/2*d*x+1/2*c)^2+9)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)
^2)^(1/2)/d
```

maxima [A] time = 1.45, size = 65, normalized size = 0.53

$$\frac{\left(5\sqrt{2}\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+7\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+35\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+105\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/140*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35
*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^3\sqrt{a+a\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.97 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}-\frac{4\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{14a\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/5*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+14/15*a*sin(d*x+c)/d/(a+a*cos(d*x
+c))^(1/2)-4/15*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2759, 2751, 2646}

$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}-\frac{4\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{14a\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (14*a*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (4*Sqrt[a + a*Cos[c +
 d*x]]*Sin[c + d*x])/(15*d) + (2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
5*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
```

```
[m, -2^(-1)]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}+\frac{2\int\left(\frac{3a}{2}-a\cos(c+dx)\right)\sqrt{a+a\cos}}{5a}$$
$$=-\frac{4\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}$$
$$=\frac{14a\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}-\frac{4\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2(a+a\cos}{}$$

Mathematica [A] time = 0.10, size = 68, normalized size = 0.79

$$\frac{\left(30\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sin[(c + d*x)/2] + 5*Sin[(
3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)
```

fricas [A] time = 0.82, size = 52, normalized size = 0.60

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\left(3\cos\left(dx+c\right)^2+4\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sin(d
*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.53, size = 81, normalized size = 0.94

$$\frac{1}{30}\sqrt{2}\sqrt{a}\left(\frac{3\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{5\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}{d}+\frac{30\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*sqrt(a)*(3*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d +
5*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 30*sgn(cos(1/2*d*x + 1
/2*c))*sin(1/2*d*x + 1/2*c)/d)
```

maple [A] time = 0.17, size = 71, normalized size = 0.83

$$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\right)\sqrt{2}}{15\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*cos(1/2*d*x+1/2*c)^4-4*cos
(1/2*d*x+1/2*c)^2+7)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.40, size = 51, normalized size = 0.59

$$\frac{\left(3\sqrt{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c) + 30*
sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**2, x)
```

3.98 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2a\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/3*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+a*cos(d*x+c))^(
1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2751, 2646}

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2a\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(3*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx &= \frac{2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}\int \sqrt{a+a\cos(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.96

$$\frac{\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sin[(c + d*x)/2] + Sin[(3*(
c + d*x))/2]))/(3*d)
```

fricas [A] time = 1.13, size = 40, normalized size = 0.71

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) + 2)*sin(d*x + c)/(d*cos(d*x + c
) + d)
```

giac [A] time = 0.47, size = 56, normalized size = 1.00

$$\frac{1}{3}\sqrt{2}\sqrt{a}\left(\frac{\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)}{d}+\frac{3\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*sqrt(a)*(sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 3*s
gn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d)
```

maple [A] time = 0.16, size = 58, normalized size = 1.04

$$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\sqrt{2}}{3\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(2*cos(1/2*d*x+1/2*c)^2+1)*2^(1
/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.88, size = 36, normalized size = 0.64

$$\frac{\left(\sqrt{2}\,\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)
/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x), x)
```

3.99 $\int \sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.12

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d
```

fricas [A] time = 1.04, size = 32, normalized size = 1.23

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.39, size = 30, normalized size = 1.15

$$\frac{2\sqrt{2}\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d
```

maple [A] time = 0.00, size = 43, normalized size = 1.65

$$\frac{2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2),x)
[Out] 2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

maxima [A] time = 1.64, size = 20, normalized size = 0.77

$$\frac{2\sqrt{2}\sqrt{a}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

mupad [B] time = 0.46, size = 33, normalized size = 1.27

$$\frac{2\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2),x)
[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*(cos(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cos\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*cos(c + d*x) + a), x)
```

3.100 $\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] 2*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2773, 206}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 1.35

$$\frac{\sqrt{2}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(
c + d*x)/2])/d
```

fricas [A] time = 1.02, size = 146, normalized size = 3.95

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2d}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{a\cos(dx+c)+a}\sqrt{-a}\sin(dx+c}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x
+ c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 +
cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sin
(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

giac [A] time = 0.53, size = 58, normalized size = 1.57

$$-\frac{\sqrt{a}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] -sqrt(a)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin
(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c))/d
```

maple [B] time = 0.50, size = 180, normalized size = 4.86

$$\frac{\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\ln\left(\frac{4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}+4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\right.}{-2\cos\left(\frac{dx}{2}+\right.}\right.\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))/
sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cos(dx+c)+a}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+a\,\cos(c+d\,x)}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\cos(c+dx)+1)}\,\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x), x)
```

3.101 $\int \sqrt{a+a\cos(c+dx)}\,\sec^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*tan(d*x+c)/d
/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2772, 2773, 206}

$$\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*T
an[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}\,\sec^2(c+dx)\,dx &= \frac{a\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{1}{2}\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx\\ &= \frac{a\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{a\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 1.27

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\,\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\,\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*ArcTanh[
Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*Sin[(c + d*x)/2]))/(2*d)
```

fricas [B] time = 1.28, size = 140, normalized size = 2.26

$$\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\cos}}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*co
s(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*
x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [A] time = 0.75, size = 104, normalized size = 1.68

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1}\right)\sqrt{a}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sq
rt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*sgn(cos(1/2*
d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/
d
```

maple [B] time = 0.48, size = 379, normalized size = 6.11

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-2a\left(\ln\left(\frac{4\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}+4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{-2\cos\left(\frac{dx}{2}\right.}\right.\right.\right.}{\sqrt{a}\left(2\cos\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*si
n(1/2*d*x+1/2*c)^2+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+ln(4/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 2.59, size = 1170, normalized size = 18.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((4*sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (4*sqrt(2)*sin(1/2*d*x +
 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*
sin(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sq
rt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 2*(2*sqrt(2)*sin(3/2*d*x + 3/
2*c) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) - 4*(sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*sin(5/2*d*x + 5/2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) + 4*sqrt
(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + a\cos(c + d\,x)}}{\cos(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**2, x)
```

3.102 $\int \sqrt{a+a\cos(c+dx)}\,\sec^3(c+dx)\,dx$

Optimal. Leaf size=102

$$\frac{3a\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 3/4*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+3/4*a*tan(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+
c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2772, 2773, 206}

$$\frac{3a\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (3*a*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Sec[c + d*x]*Tan[c
 + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,\sec^3(c+dx)\,dx = \frac{a\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{4}\int \sqrt{a+a\cos(c+dx)}\,\sec^2(c+dx)\,dx$$
$$= \frac{3a\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{8}\int \sqrt{a+a\cos(c$$
$$= \frac{3a\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\right.}{}$$
$$= \frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{3a\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec(c+dx)\,\mathrm{ta}}{2d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.18, size = 94, normalized size = 0.92

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{3}{2}(c+dx)\right)+3\sqrt{2}\,\cos^2(c+dx)\tanh^{-1}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(3*Sqrt[2]*ArcT
anh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + Sin[(c + d*x)/2] + 3*Sin[(3*
(c + d*x))/2]))/(8*d)
```

fricas [A] time = 1.15, size = 155, normalized size = 1.52

$$\frac{3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\,\mathrm{c}}}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
c) + a)*(3*cos(d*x + c) + 2)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x +
c)^2)
```

giac [A] time = 1.01, size = 131, normalized size = 1.28

$$-\frac{\sqrt{2}\left(3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(6\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-5\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\cdots}{\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}}\cdots}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2
*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(6*sgn(co
s(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 5*sgn(cos(1/2*d*x + 1/2*c))*si
n(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

maple [B] time = 0.53, size = 545, normalized size = 5.34

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12a\left(\ln\left(\frac{4\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}+4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\cdots}{-2\cos\left(\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*a*(ln(4/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(
1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
)*sin(1/2*d*x+1/2*c)^4+(-12*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-1
2*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-12*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+3*ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+3*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+10*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*
d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 11.91, size = 2642, normalized size = 25.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*c
os(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)*
cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(
2*d*x + 2*c) + 2*(6*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
```

```
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(2*d*x + 2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*si
n(5/2*d*x + 5/2*c) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(4*d*x + 4*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*
sin(1/2*d*x + 1/2*c) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d*
x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) +
3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*c
os(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 12
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**3, x)
```

3.103 $\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{5a\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{5a\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 5/8*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+5/8*a*tan(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+5/12*a*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x
+c))^(1/2)+1/3*a*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2772, 2773, 206}

$$\frac{5a\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{5a\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

```
[Out] (5*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (5*a*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (5*a*Sec[c + d*x]*Tan
[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,\sec^4(c+dx)\,dx = \frac{a\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{6}\int \sqrt{a+a\cos(c+dx)}\,\sec^3(c+dx)\,dx$$

$$= \frac{5a\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{8}\int \sqrt{a+a\,c}$$

$$= \frac{5a\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec^2(c+dx)\tan}{3d\sqrt{a+a\cos(c}}$$

$$= \frac{5a\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec^2(c+dx)\tan}{3d\sqrt{a+a\cos(c}}$$

$$= \frac{5\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{5a\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\sec(c+dx)\,t}{12d\sqrt{a+a\,\mathrm{co}}}$$

Mathematica [A] time = 0.33, size = 109, normalized size = 0.79

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\,\left(42\sin\left(\frac{1}{2}(c+dx)\right)+5\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)+3}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(30*Sqrt[2]*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 42*Sin[(c + d*x)/2] + 5*(Si
n[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

fricas [A] time = 0.74, size = 165, normalized size = 1.20

$$\frac{15\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a}}{96\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(15*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*
sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
 c) + a)*(15*cos(d*x + c)^2 + 10*cos(d*x + c) + 8)*sin(d*x + c))/(d*cos(d*x
 + c)^4 + d*cos(d*x + c)^3)
```

giac [A] time = 0.69, size = 154, normalized size = 1.12

$$-\frac{\sqrt{2}\left(15\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(60\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-80\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\right.\right.\right.}{\left(2\sin\left(\frac{1}{2}dx\right.\right.}\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(15*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(
2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(60*sgn(
cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 80*sgn(cos(1/2*d*x + 1/2*c))
*sin(1/2*d*x + 1/2*c)^3 + 33*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)
)/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

maple [B] time = 0.58, size = 709, normalized size = 5.14

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-120a\left(\ln\left(\frac{4\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}+4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{-2\cos\ldots}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-120*a*(ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
)))*sin(1/2*d*x+1/2*c)^6+60*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*c
```

```
os(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-90*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a-90*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-
160*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+15
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+15*ln(4/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))*a+66*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a
^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/si
n(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 171.93, size = 5115, normalized size = 37.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(15*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqr
t(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(6*d*x + 6*c)^2
 + 135*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(
2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*c)^2 +
 135*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c)^2 + 1
5*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*lo
```

```
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*sin(6*d*x + 6*c)^2 + 135*
(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c)^2 + 135*(s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c)^2 - 120*(sin
(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/
2*c) + 2*(45*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
 sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*
c) + 45*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt
(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) +
15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*sqrt(2)*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*log(2*cos(1/2*d*x +
```

1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 60*sin(11/2*d*x + 11/2*c) + 200*sin(9/2*
d*x + 9/2*c) + 168*sin(7/2*d*x + 7/2*c) + 12*sin(5/2*d*x + 5/2*c) - 20*sin(
3/2*d*x + 3/2*c) - 120*sin(1/2*d*x + 1/2*c))*cos(6*d*x + 6*c) - 360*(sin(4*
d*x + 4*c) + sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) - 1200*(sin(4*d*x + 4
*c) + sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 6*(45*(sqrt(2)*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 8*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 15*sqrt(2)*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + 15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - 15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 168*sin(7/2*d*x + 7/2*c) + 12*sin(5/2*d*x + 5/2*c) - 20*sin(3/2*d*x +
3/2*c) - 120*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*c) + 30*(3*sqrt(2)*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*sqrt(2)*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 3*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - 4*sin(3/2*d*x + 3/2*c) - 24*sin(1/2*d*x + 1/2*c))*cos(2*d*x +
 2*c) + 120*(cos(6*d*x + 6*c) + 3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1
)*sin(13/2*d*x + 13/2*c) + 2*(45*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(
2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/
2*c))*sin(4*d*x + 4*c) + 45*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt

```
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))
*sin(2*d*x + 2*c) - 60*cos(11/2*d*x + 11/2*c) - 200*cos(9/2*d*x + 9/2*c) -
168*cos(7/2*d*x + 7/2*c) - 12*cos(5/2*d*x + 5/2*c) + 20*cos(3/2*d*x + 3/2*c
))*sin(6*d*x + 6*c) + 120*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin
(11/2*d*x + 11/2*c) + 400*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin
(9/2*d*x + 9/2*c) + 6*(45*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
 sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*s
in(2*d*x + 2*c) - 168*cos(7/2*d*x + 7/2*c) - 12*cos(5/2*d*x + 5/2*c) + 20*c
os(3/2*d*x + 3/2*c))*sin(4*d*x + 4*c) + 336*(3*cos(2*d*x + 2*c) + 1)*sin(7/
2*d*x + 7/2*c) + 24*(3*cos(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c) - 1008*co
s(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 72*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2
*c) + 120*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 15*sqrt(2)*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - 40*sin(3/2*d*x + 3/2*c) - 120*sin(1/2*d*x + 1/2*c))*sqrt(a)/((sqr
t(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*
x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18
*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 +
 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) +
6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**4,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**4, x)
```

3.104 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{34a^2\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{68a^2\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{68\sin(c+dx)(a\cos(c+}{105d}$$

```
[Out] 68/105*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+68/45*a^2*sin(d*x+c)/d/(a+a*cos(
d*x+c))^(1/2)+34/63*a^2*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/
9*a^2*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-136/315*a*sin(d*x+c)
*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2763, 21, 2770, 2759, 2751, 2646}

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{34a^2\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{68a^2\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{68\sin(c+dx)(a\cos(c+}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (68*a^2*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (34*a^2*Cos[c + d*x
]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Cos[c + d*x]^4*S
in[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (136*a*Sqrt[a + a*Cos[c + d*x
]]*Sin[c + d*x])/(315*d) + (68*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10
5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{2}{9}\int \frac{\cos^3(c+dx)\left(\frac{17a^2}{2}+\frac{17}{2}a^2\cos(c\right.}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{9}(17a)\int \cos^3(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{21}(3$$

$$= \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{68(a}{}$$

$$= \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} - \frac{136a}{}$$

$$= \frac{68a^2\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c}{9d\sqrt{a}}$$

Mathematica [A] time = 0.26, size = 93, normalized size = 0.57

$$\frac{a\left(3780\sin\left(\frac{1}{2}(c+dx)\right)+1050\sin\left(\frac{3}{2}(c+dx)\right)+378\sin\left(\frac{5}{2}(c+dx)\right)+135\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right.}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3780*Sin[(c + d*x)/2] + 105
0*Sin[(3*(c + d*x))/2] + 378*Sin[(5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2
] + 35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

fricas [A] time = 1.54, size = 78, normalized size = 0.48

$$\frac{2\left(35\,a\cos\left(dx+c\right)^4+85\,a\cos\left(dx+c\right)^3+102\,a\cos\left(dx+c\right)^2+136\,a\cos\left(dx+c\right)+272\,a\right)\sqrt{a\cos\left(dx+c\right)+a}}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a*cos(d*x + c)^4 + 85*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2 + 1
36*a*cos(d*x + c) + 272*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
 + c) + d)
```

giac [A] time = 0.67, size = 134, normalized size = 0.83

$$\frac{1}{2520}\sqrt{2}\left(\frac{35\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}{d}+\frac{135\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{378\,a\mathrm{sgn}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*a*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c)/d + 135
*a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 378*a*sgn(cos(1/2*d*x
 + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 1050*a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/
2*d*x + 3/2*c)/d + 3780*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d)
*sqrt(a)
```

maple [A] time = 0.15, size = 99, normalized size = 0.61

$$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-220\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+114\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+47\left(\cos^{2}\left(\frac{dx}{2}+\right.\right.\right.}{315\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*cos(1/2*d*x+1/2*c)^8-2
20*cos(1/2*d*x+1/2*c)^6+114*cos(1/2*d*x+1/2*c)^4+47*cos(1/2*d*x+1/2*c)^2+94
)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.05, size = 84, normalized size = 0.52

$$\frac{\left(35\,\sqrt{2}\,a\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+135\,\sqrt{2}\,a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+378\,\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+1050\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2520*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x + 7/2
*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.105 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{152a^2\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}-\frac{4\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{38a\sin(c+dx)}{}$$

```
[Out] -4/35*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(a+a*cos(d*x+c))^(5/2)*sin(d*
x+c)/a/d+152/105*a^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+38/105*a*sin(d*x+c
)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2647, 2646}

$$\frac{152a^2\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}-\frac{4\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{38a\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (152*a^2*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (38*a*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(105*d) - (4*(a + a*Cos[c + d*x])^(3/2)*Sin[c +
 d*x])/(35*d) + (2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} + \frac{2\int\left(\frac{5a}{2}-a\cos(c+dx)\right)(a+a\,c}{7a}$$

$$= -\frac{4(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+}{7ad}$$

$$= \frac{38a\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{105d} - \frac{4(a+a\cos(c+dx))^{3/2}\sin(c+}{35d}$$

$$= \frac{152a^2\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{38a\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{105d} - \frac{4(a}{}$$

Mathematica [A] time = 0.17, size = 81, normalized size = 0.70

$$\frac{a\left(735\sin\left(\frac{1}{2}(c+dx)\right)+175\sin\left(\frac{3}{2}(c+dx)\right)+63\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(}}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(735*Sin[(c + d*x)/2] + 175*
Sin[(3*(c + d*x))/2] + 63*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c + d*x))/2]))/
(420*d)
```

fricas [A] time = 0.99, size = 67, normalized size = 0.58

$$\frac{2\left(15\,a\cos\left(dx+c\right)^3+39\,a\cos\left(dx+c\right)^2+52\,a\cos\left(dx+c\right)+104\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104*
a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.64, size = 109, normalized size = 0.94

$$\frac{1}{420}\sqrt{2}\left(\frac{15\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{63\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{175\,a\mathrm{sgn}\left(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 63*a
*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 175*a*sgn(cos(1/2*d*x +
 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 735*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d
*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.16, size = 86, normalized size = 0.74

$$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(60\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+19\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+38\right)\sqrt{2}}{105\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(60*cos(1/2*d*x+1/2*c)^6-12
*cos(1/2*d*x+1/2*c)^4+19*cos(1/2*d*x+1/2*c)^2+38)*2^(1/2)/(a*cos(1/2*d*x+1/
2*c)^2)^(1/2)/d
```

maxima [A] time = 1.01, size = 69, normalized size = 0.59

$$\frac{\left(15\,\sqrt{2}\,a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\,\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+175\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+735\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/420*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c
) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^2 (a+a\cos(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.106 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=86

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

```
[Out] 2/5*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+8/5*a^2*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)+2/5*a*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2647, 2646}

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(5*d) + (2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(5*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{3}{5}\int (a+a\cos(c+dx))^{3/2}\,dx$$
$$= \frac{2a\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{5d}+\frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx}{5d}$$
$$= \frac{8a^2\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}+\frac{2a\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{5d}+\frac{2(a+a}{}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.78

$$\frac{a\left(20\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(20*Sin[(c + d*x)/2] + 5*Sin
[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(10*d)
```

fricas [A] time = 0.59, size = 55, normalized size = 0.64

$$\frac{2\left(a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+6\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 6*a)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.52, size = 83, normalized size = 0.97

$$\frac{1}{10}\sqrt{2}\left(\frac{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{5\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}{d}+\frac{20\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2)*(a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 5*a*sgn(
cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 20*a*sgn(cos(1/2*d*x + 1/2*c
))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.16, size = 71, normalized size = 0.83

$$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\sqrt{2}}{5\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 4/5*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(2*cos(1/2*d*x+1/2*c)^4+cos(1
/2*d*x+1/2*c)^2+2)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.13, size = 53, normalized size = 0.62

$$\frac{\left(\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 2
0*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*cos(c + d*x), x)
```

3.107 $\int (a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[Out] 8/3*a^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*(a+a*cos(d*x+c
))^(1/2)/d
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}\,dx &= \frac{2a\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}(4a)\int\sqrt{a+a\cos(c+dx)}\,dx\\ &= \frac{8a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.93

$$\frac{a\left(9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(9*Sin[(c + d*x)/2] + Sin[(3
*(c + d*x))/2]))/(3*d)
```

fricas [A] time = 0.97, size = 44, normalized size = 0.75

$$\frac{2\left(a\cos\left(dx+c\right)+5\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c) + 5*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
 + c) + d)
```

giac [A] time = 0.41, size = 58, normalized size = 0.98

$$\frac{1}{3}\sqrt{2}\left(\frac{a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}{d}+\frac{9\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 9*a*sgn(c
os(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.14, size = 58, normalized size = 0.98

$$\frac{4a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2+\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{2}}{3\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 4/3*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(2+cos(1/2*d*x+1/2*c)^2)*2^(1
/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.13, size = 38, normalized size = 0.64

$$\frac{\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a+a\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + a)**(3/2), x)
```

3.108 $\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

Optimal. Leaf size=66

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2*a^(3/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2*a^2*sin(d*
x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2763, 21, 2773, 206}

$$\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2
*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx &= \frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+2\int \frac{\left(\frac{a^2}{2}+\frac{1}{2}a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx\\
&= \frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+a\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx\\
&= \frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\
&= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 65, normalized size = 0.98

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcTanh[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*Sin[(c + d*x)/2]))/d
```

fricas [B] time = 1.68, size = 127, normalized size = 1.92

$$\frac{(a\cos(dx+c)+a)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\sqrt{a\cos(dx+c)}}{2(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*a*sin(d*
x + c))/(d*cos(d*x + c) + d)
```

giac [B] time = 5.34, size = 1884, normalized size = 28.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*(sqrt(2)*(a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(
1/4*c)^6 - 6*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^5 + 3*a*sg
n(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^6 - 15*a*sgn(cos(1/2*d*x +
1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4 + 18*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*
c)^2*tan(1/4*c)^5 - 3*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 +
 20*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^3 - 45*a*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^4 + 18*a*sgn(cos(1/2*d*x + 1/2*c))*
tan(1/2*c)*tan(1/4*c)^5 - a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^6 + 15*a*s
gn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2 - 60*a*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 45*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2
*c)*tan(1/4*c)^4 - 6*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^5 - 6*a*sgn(cos
(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c) + 45*a*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/2*c)^2*tan(1/4*c)^2 - 60*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(
1/4*c)^3 + 15*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^4 - a*sgn(cos(1/2*d*x
+ 1/2*c))*tan(1/2*c)^3 + 18*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/
4*c) - 45*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*a*sgn(co
s(1/2*d*x + 1/2*c))*tan(1/4*c)^3 - 3*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)
^2 + 18*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c) - 15*a*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*c)^2 + 3*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c) - 6
*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c) + a*sgn(cos(1/2*d*x + 1/2*c)))*log(
abs(-2*tan(1/4*d*x + c)*tan(1/2*c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*
tan(1/2*c)^3 - 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(
1/2*c) - 6*tan(1/2*c)^2 - 2*tan(1/4*d*x + c) + 6*tan(1/2*c) + 2)/abs(-2*tan
(1/4*d*x + c)*tan(1/2*c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*tan(1/2*c)
^3 + 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6
*tan(1/2*c)^2 - 2*tan(1/4*d*x + c) + 6*tan(1/2*c) + 2))/((tan(1/4*c)^6 + 3*
tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1)*(tan(1/2*c)^2 + 1)^(3/2)) + sqrt(2)*(a*s
gn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^6 + 6*a*sgn(cos(1/2*d*x +
1/2*c))*tan(1/2*c)^3*tan(1/4*c)^5 - 3*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c
```

```
)^2*tan(1/4*c)^6 - 15*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4
 + 18*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^5 - 3*a*sgn(cos(1
/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 - 20*a*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/2*c)^3*tan(1/4*c)^3 + 45*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(
1/4*c)^4 - 18*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^5 + a*sgn(c
os(1/2*d*x + 1/2*c))*tan(1/4*c)^6 + 15*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*
c)^3*tan(1/4*c)^2 - 60*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^
3 + 45*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*a*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*c)^5 + 6*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*t
an(1/4*c) - 45*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^2 + 60*a
*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 - 15*a*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/4*c)^4 - a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3 + 18*a*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c) - 45*a*sgn(cos(1/2*d*x + 1/2
*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^3
+ 3*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2 - 18*a*sgn(cos(1/2*d*x + 1/2*c
))*tan(1/2*c)*tan(1/4*c) + 15*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 3*
a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c) - 6*a*sgn(cos(1/2*d*x + 1/2*c))*tan(
1/4*c) - a*sgn(cos(1/2*d*x + 1/2*c)))*log(abs(-2*tan(1/4*d*x + c)*tan(1/2*c
)^3 - 6*tan(1/4*d*x + c)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 - 2*sqrt(2)*(tan(1/2
*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6*tan(1/2*c)^2 + 2*tan(1
/4*d*x + c) - 6*tan(1/2*c) + 2)/abs(-2*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*ta
n(1/4*d*x + c)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 + 2*sqrt(2)*(tan(1/2*c)^2 + 1)
^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6*tan(1/2*c)^2 + 2*tan(1/4*d*x + c
) - 6*tan(1/2*c) + 2))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1
)*(tan(1/2*c)^2 + 1)^(3/2)) - 8*(a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x +
c)*tan(1/4*c)^6 - 15*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c
)^4 + 6*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^5 + 15*a*sgn(cos(1/2*d*x + 1
/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^2 - 20*a*sgn(cos(1/2*d*x + 1/2*c))*tan(1
/4*c)^3 - a*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c) + 6*a*sgn(cos(1/2*d*
x + 1/2*c))*tan(1/4*c))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 +
1)*(tan(1/4*d*x + c)^2 + 1)))/d
```

maple [B] time = 0.45, size = 207, normalized size = 3.14

$$\frac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*2^(1/2)*(a*sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/sin(1/2*d*x+1/2*c)/(a*
cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\cos(c+dx)+1))^{\frac{3}{2}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*sec(c + d*x), x)
```

3.109 $\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 3*a^(3/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+a^2*tan(d*x+
c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 21, 2773, 206}

$$\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (3*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a
^2*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
```

```
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx &= \frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - a\int \frac{\left(-\frac{3a}{2}-\frac{3}{2}a\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx \\ &= \frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}(3a)\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx \\ &= \frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\ &= \frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 81, normalized size = 1.25

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\,\left(2\sin\left(\frac{1}{2}(c+dx)\right)+3\sqrt{2}\,\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*ArcT
anh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*Sin[(c + d*x)/2]))/(2*d)
```

fricas [B] time = 1.04, size = 146, normalized size = 2.25

$$\frac{3\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a}}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*
sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
 c) + a)*a*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.44, size = 381, normalized size = 5.86

$$\frac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-6a\left(\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{-2}\right.\right.\right.}{\left(2\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-6*a*(ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a)))*sin(1/2*d*x+1/2*c)^2+3*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 1.67, size = 1314, normalized size = 20.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(5
/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sq
rt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt(2)*a*sin(
3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c
)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) -
 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin
(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/2*d*x
+ 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/2*d*x + 1/
2*c))*sin(2*d*x + 2*c))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + a\,\cos(c + d\,x))^{3/2}}{\cos(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.110 $\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{7a^2\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 7/4*a^(3/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+7/4*a^2*ta
n(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(
d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 21, 2772, 2773, 206}

$$\frac{7a^2\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (7*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (7*a^2*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sec[c + d*x]*T
an[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
```

```
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx &= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{1}{2}a\int \frac{\left(-\frac{7a}{2}-\frac{7}{2}a\cos(c+dx)\right)\sec^2(c+d\\
&= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(7a)\int \sqrt{a+a\cos(c+dx)}\,\sec^2(c+dx\\
&= \frac{7a^2\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(7a)\int \sqrt{a+\\
&= \frac{7a^2\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\left(7a^2\right)\text{Subst}\left(\int\\
&= \frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{7a^2\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec(c+d\\
\end{aligned}$$

Mathematica [A] time = 0.23, size = 97, normalized size = 0.92

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)+7\sqrt{2}\cos^2(c+dx)\tan\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(7*Sqrt[2]*Ar
cTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7*Sin
[(3*(c + d*x))/2]))/(8*d)
```

fricas [A] time = 1.89, size = 162, normalized size = 1.53

$$\frac{7\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4(7}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d*x
+ c) + 2*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*co
s(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.57, size = 545, normalized size = 5.14

$$\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(28a\left(\ln\left(\frac{4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}+4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}{-2\cos}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(28*a*(ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-28*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a-28*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-28*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+7*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a+7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*a+18*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)
-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*
d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 3.70, size = 3216, normalized size = 30.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*
sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 4*a*sin(5/2*d*x + 5/2*c) - 12
*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*c)^2 + 4
*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*s
in(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c)^2 + (7*sqrt(2)*a*log(2*cos(1/2*d*x +
```

1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) + 4*a*sin(5/2*d*x + 5/2*c) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*
d*x + 1/2*c))*sin(4*d*x + 4*c)^2 - 160*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2
*c) - 168*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 72*a*cos(3/2*d*x + 3/2*
c)*sin(2*d*x + 2*c) + 4*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*
sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 12*a*sin(3/
2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c)^2 + 7*sqrt(2)*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) + 4*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*
cos(13/2*d*x + 13/2*c) - 12*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos
(11/2*d*x + 11/2*c) - 48*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(9/
2*d*x + 9/2*c) + 2*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
+ 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 2*(7*sqrt(2)*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/

```
2*c))*cos(2*d*x + 2*c) + 40*a*sin(7/2*d*x + 7/2*c) + 2*(4*a*cos(2*d*x + 2*c
) + 23*a)*sin(5/2*d*x + 5/2*c) + 6*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*
x + 1/2*c))*cos(4*d*x + 4*c) + 4*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 6*
a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) - 4*(a
*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(13/2*d*x + 13/2*c) + 12*(
a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(11/2*d*x + 11/2*c) + 48*
(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(9/2*d*x + 9/2*c) + 4*(4
*a*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 20*a*cos(7/2*d*x + 7/2*c) - 21*a
*cos(5/2*d*x + 5/2*c) - 9*a*cos(3/2*d*x + 3/2*c) + (7*sqrt(2)*a*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*sin(4*d*x + 4*c) + 80*(2*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*x
+ 7/2*c) + 8*(2*a*cos(2*d*x + 2*c)^2 + 2*a*sin(2*d*x + 2*c)^2 + 23*a*cos(2*
d*x + 2*c) + 11*a)*sin(5/2*d*x + 5/2*c) + 24*a*sin(3/2*d*x + 3/2*c) - 56*a*
sin(1/2*d*x + 1/2*c))*sqrt(a)/((sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(
2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\cos(c + dx))^{3/2}}{\cos(c + dx)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.111 $\int (a + a\cos(c + dx))^{3/2} \sec^4(c + dx)\, dx$

Optimal. Leaf size=144

$$\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{11a^2\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\tan(c+dx)\sec(c+dx}{12d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 11/8*a^(3/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+11/8*a^2*
tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+11/12*a^2*sec(d*x+c)*tan(d*x+c)/d/(a+a*
cos(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 21, 2772, 2773, 206}

$$\frac{11a^2\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\tan(c+dx)\sec(c+dx}{12d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

```
[Out] (11*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
 + (11*a^2*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (11*a^2*Sec[c + d
*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sec[c + d*x]^2*Tan
[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
```

```
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx = \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{1}{3}a\int \frac{\left(-\frac{11a}{2}-\frac{11}{2}a\cos(c+dx)\right)\sec^3(c$$

$$= \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(11a)\int \sqrt{a+a\cos(c+dx)}\,\sec^3(c+$$

$$= \frac{11a^2\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(11a)$$

$$= \frac{11a^2\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec^2(c+}{3d\sqrt{a+a}}$$

$$= \frac{11a^2\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec^2(c+}{3d\sqrt{a+a}}$$

$$= \frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{11a^2\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\sec(c}{12d\sqrt{a+}}$$

Mathematica [A] time = 0.35, size = 110, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(54\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right.\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(66*Sqrt[2]*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] + 11*
(Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

fricas [A] time = 0.88, size = 173, normalized size = 1.20

$$\frac{33\left(a\cos(dx+c)^4+a\cos(dx+c)^3\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4}{96\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
```

```
2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos(d*
x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/
(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.58, size = 710, normalized size = 4.93

$$\frac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-264a\left(\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{d}{2}\right.\right.}\right.}{-}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-264*a*(ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+132*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-22*(16*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a))*a+9*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2
*c)^2+33*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+33*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+126*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\cos(c + d\,x))^{3/2}}{\cos(c + d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**4,x)
[Out] Timed out
```

3.112 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$\frac{46a^3\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{710a^3\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{284a^3\sin(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\cos^4(c+$$

```
[Out] 284/231*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+284/99*a^3*sin(d*x+c)/d/(a+a*
cos(d*x+c))^(1/2)+710/693*a^3*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1
/2)+46/99*a^3*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-568/693*a^2*
sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2/11*a^2*cos(d*x+c)^4*sin(d*x+c)*(a+a*c
os(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.36, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2763, 2981, 2770, 2759, 2751, 2646}

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}+\frac{46a^3\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{710a^3\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (284*a^3*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*x]]) + (710*a^3*Cos[c + d
*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (46*a^3*Cos[c + d*x]
^4*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*x]]) - (568*a^2*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(693*d) + (2*a^2*Cos[c + d*x]^4*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(11*d) + (284*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x]
)/(231*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{2a^2\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{11d} + \frac{2}{11}\int \cos^3(c+dx)\sqrt{\ldots}$$

$$= \frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,\text{si}\ldots}{11d}$$

$$= \frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\ldots}{}$$

$$= \frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\ldots}{}$$

$$= \frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} - \frac{568a\ldots}{}$$

$$= \frac{284a^3\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\cos^4(\ldots}{99d\sqrt{a\ldots}}$$

Mathematica [A] time = 0.50, size = 107, normalized size = 0.53

$$\frac{a^2\left(31878\sin\left(\frac{1}{2}(c+dx)\right)+8778\sin\left(\frac{3}{2}(c+dx)\right)+3465\sin\left(\frac{5}{2}(c+dx)\right)+1287\sin\left(\frac{7}{2}(c+dx)\right)+385\sin\left(\frac{9}{2}(c+\ldots\right.\right.}{11088d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(31878*Sin[(c + d*x)/2] +
8778*Sin[(3*(c + d*x))/2] + 3465*Sin[(5*(c + d*x))/2] + 1287*Sin[(7*(c + d*
x))/2] + 385*Sin[(9*(c + d*x))/2] + 63*Sin[(11*(c + d*x))/2]))/(11088*d)
```

fricas [A] time = 0.98, size = 101, normalized size = 0.50

$$\frac{2\left(63\,a^2\cos(dx+c)^5+224\,a^2\cos(dx+c)^4+355\,a^2\cos(dx+c)^3+426\,a^2\cos(dx+c)^2+568\,a^2\cos(dx+c)+\ldots\right.}{693\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^2*cos(d*x + c)^5 + 224*a^2*cos(d*x + c)^4 + 355*a^2*cos(d*x + c
)^3 + 426*a^2*cos(d*x + c)^2 + 568*a^2*cos(d*x + c) + 1136*a^2)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 2.03, size = 171, normalized size = 0.84

$$\frac{1}{11088}\sqrt{2}\left(\frac{63\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)}{d}+\frac{385\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}{d}+\frac{128\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/11088*sqrt(2)*(63*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c)/d
+ 385*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c)/d + 1287*a^2*sgn(c
os(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 3465*a^2*sgn(cos(1/2*d*x + 1/
2*c))*sin(5/2*d*x + 5/2*c)/d + 8778*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d
*x + 3/2*c)/d + 31878*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d)
*sqrt(a)
```

maple [A] time = 0.18, size = 112, normalized size = 0.55

$$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(504\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-364\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+178\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+75\left(\cos^{4}\left(\frac{dx}{2}\cdots\right.\right.\right.}{693\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 8/693*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(504*cos(1/2*d*x+1/2*c)^10-
364*cos(1/2*d*x+1/2*c)^8+178*cos(1/2*d*x+1/2*c)^6+75*cos(1/2*d*x+1/2*c)^4+1
00*cos(1/2*d*x+1/2*c)^2+200)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.22, size = 111, normalized size = 0.55

$$\frac{\left(63\sqrt{2}\,a^2\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+385\sqrt{2}\,a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+1287\sqrt{2}\,a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+3465\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx\cdots\right.\right.}{11088\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11088*(63*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d*
x + 9/2*c) + 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin(5
/2*d*x + 5/2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*a^2
*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.113 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=146

$$\frac{832a^3\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{208a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}-\frac{4\sin(c+\cdots}{}$$

```
[Out] 26/105*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d-4/63*(a+a*cos(d*x+c))^(5/2)*si
n(d*x+c)/d+2/9*(a+a*cos(d*x+c))^(7/2)*sin(d*x+c)/a/d+832/315*a^3*sin(d*x+c)
/d/(a+a*cos(d*x+c))^(1/2)+208/315*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2647, 2646}

$$\frac{208a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{832a^3\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}-\frac{4\sin(c+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (832*a^3*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (208*a^2*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (26*a*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(105*d) - (4*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) +
 (2*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{2(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad} + \frac{2\int\left(\frac{7a}{2}-a\cos(c+dx)\right)(a+a\,\mathrm{co}}{9a}$$
$$= -\frac{4(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2(a+a\cos(c+dx))^{7/2}\sin(c+a}{9ad}$$
$$= \frac{26a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d} - \frac{4(a+a\cos(c+dx))^{5/2}\sin(c+}{63d}$$
$$= \frac{208a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{315d} + \frac{26a(a+a\cos(c+dx))^{3/2}\sin(c}{105d}$$
$$= \frac{832a^3\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{208a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{315d} + \frac{26}{}$$

Mathematica [A] time = 0.29, size = 95, normalized size = 0.65

$$\frac{a^2\left(8190\sin\left(\frac{1}{2}(c+dx)\right)+2100\sin\left(\frac{3}{2}(c+dx)\right)+756\sin\left(\frac{5}{2}(c+dx)\right)+225\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right.}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(8190*Sin[(c + d*x)/2] + 2
100*Sin[(3*(c + d*x))/2] + 756*Sin[(5*(c + d*x))/2] + 225*Sin[(7*(c + d*x))
/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

fricas [A] time = 0.73, size = 88, normalized size = 0.60

$$\frac{2\left(35\,a^2\cos(dx+c)^4+130\,a^2\cos(dx+c)^3+219\,a^2\cos(dx+c)^2+292\,a^2\cos(dx+c)+584\,a^2\right)\sqrt{a\cos(dx+c)}}{315\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x + c
)^2 + 292*a^2*cos(d*x + c) + 584*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/(d*cos(d*x + c) + d)
```

giac [A] time = 1.33, size = 144, normalized size = 0.99

$$\frac{1}{2520}\sqrt{2}\left(\frac{35\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}{d}+\frac{225\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{756\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c)/d + 2
25*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 756*a^2*sgn(cos(1
/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 2100*a^2*sgn(cos(1/2*d*x + 1/2*c)
)*sin(3/2*d*x + 3/2*c)/d + 8190*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x +
 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.18, size = 99, normalized size = 0.68

$$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+39\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+52\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}{315\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*cos(1/2*d*x+1/2*c)^8-2
0*cos(1/2*d*x+1/2*c)^6+39*cos(1/2*d*x+1/2*c)^4+52*cos(1/2*d*x+1/2*c)^2+104)
*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.18, size = 94, normalized size = 0.64

$$\frac{\left(35\sqrt{2}\,a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+225\sqrt{2}\,a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+756\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+2100\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\right.\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2520*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2*d*x +
 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*sin(3/2*d
*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,(a+a\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.114 $\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{64a^3\sin(c+dx)}{21d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d}+\frac{2\sin(c+d$$

```
[Out] 2/7*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(a+a*cos(d*x+c))^(5/2)*sin(d*
x+c)/d+64/21*a^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/21*a^2*sin(d*x+c)*(
a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2647, 2646}

$$\frac{64a^3\sin(c+dx)}{21d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{7d}+\frac{2\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(7*d) + (2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{5}{7}\int (a+a\cos(c+dx))^{5/2}\,dx$$
$$= \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx}{7d}$$
$$= \frac{16a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+}{7d}$$
$$= \frac{64a^3\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a(a+}{}$$

Mathematica [A] time = 0.23, size = 84, normalized size = 0.72

$$\frac{a^2\left(315\sin\left(\frac{1}{2}(c+dx)\right)+77\sin\left(\frac{3}{2}(c+dx)\right)+3\left(7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c}}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(315*Sin[(c + d*x)/2] + 77
*Sin[(3*(c + d*x))/2] + 3*(7*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2])))
/(84*d)
```

fricas [A] time = 0.60, size = 75, normalized size = 0.65

$$\frac{2\left(3\,a^2\cos\left(dx+c\right)^3+12\,a^2\cos\left(dx+c\right)^2+23\,a^2\cos\left(dx+c\right)+46\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) +
46*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.64, size = 117, normalized size = 1.01

$$\frac{1}{84}\sqrt{2}\left(\frac{3\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{21\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{77\,a^2\mathrm{sgn}\left(\mathrm{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/84*sqrt(2)*(3*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 21*a
^2*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 77*a^2*sgn(cos(1/2*d*
x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 315*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(
1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.14, size = 86, normalized size = 0.74

$$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)\sqrt{2}}{21\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 8/21*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(6*cos(1/2*d*x+1/2*c)^6+3*co
s(1/2*d*x+1/2*c)^4+4*cos(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

maxima [A] time = 0.97, size = 77, normalized size = 0.66

$$\frac{\left(3\sqrt{2}a^{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\sqrt{2}a^{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+77\sqrt{2}a^{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+315\sqrt{2}a^{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/84*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2
*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1
/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(c+d\,x\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.115 $\int (a + a\cos(c + dx))^{5/2} \, dx$

Optimal. Leaf size=89

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

```
[Out] 2/5*a*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+64/15*a^3*sin(d*x+c)/d/(a+a*cos(d
*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(5*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\,dx = \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}(8a)\int (a+a\cos(c+dx))^{3/2}\,dx$$
$$= \frac{16a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{15}(32$$
$$= \frac{64a^3\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2a(a+a\cos(c+a}{5a}$$

Mathematica [A] time = 0.10, size = 71, normalized size = 0.80

$$\frac{a^2\left(150\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(150*Sin[(c + d*x)/2] + 25
*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)
```

fricas [A] time = 0.92, size = 62, normalized size = 0.70

$$\frac{2\left(3\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+43\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.72, size = 90, normalized size = 1.01

$$\frac{1}{30}\sqrt{2}\left(\frac{3\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{25\,a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}{d}+\frac{150\,a^2\mathrm{sgn}\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 25*a
^2*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 150*a^2*sgn(cos(1/2*d
*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.00, size = 73, normalized size = 0.82

$$\frac{8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)\sqrt{2}}{15\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 8/15*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(3*cos(1/2*d*x+1/2*c)^4+4*co
s(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 0.99, size = 60, normalized size = 0.67

$$\frac{\left(3\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.116 $\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[Out] 2*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+14/3*a^3*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/
d
```

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2763, 2981, 2773, 206}

$$\frac{14a^3\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (1
4*a^3*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int\sqrt{a+a\cos(c+dx)}\left(\frac{3a^2}{2}+\right.$$

$$=\frac{14a^3\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}+a^2\int\sqrt{}$$

$$=\frac{14a^3\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{(2a^3)\,S}{}$$

$$=\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\,\mathrm{co}}}{}$$

Mathematica [A] time = 0.51, size = 89, normalized size = 0.91

$$\frac{2a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{1-\cos(c+dx)}\,(\cos(c+dx)+8)+3\tanh^{-1}\left(\sqrt{1-\cos(c+dx)}\right)\right)}{3d\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (2*a^2*Sqrt[a*(1 + Cos[c + d*x])]*(3*ArcTanh[Sqrt[1 - Cos[c + d*x]]] + Sqrt
[1 - Cos[c + d*x]]*(8 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(3*d*Sqrt[1 - Cos[
c + d*x]])
```

fricas [A] time = 1.22, size = 147, normalized size = 1.50

$$\frac{3\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(a^2\cos\left(dx+\right.\right.}{6\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
 + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c
) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c) + 8*a^2)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

giac [B] time = 56.70, size = 5671, normalized size = 57.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*sqrt(a)*(3*sqrt(2)*(a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*
tan(1/4*c)^6 - 6*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^5 +
3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^6 - 15*a^2*sgn(cos(
1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4 + 18*a^2*sgn(cos(1/2*d*x + 1/2*
c))*tan(1/2*c)^2*tan(1/4*c)^5 - 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*
tan(1/4*c)^6 + 20*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^3 -
 45*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^4 + 18*a^2*sgn(co
s(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^5 - a^2*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/4*c)^6 + 15*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2
- 60*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 45*a^2*sgn(c
os(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*a^2*sgn(cos(1/2*d*x + 1/2*
c))*tan(1/4*c)^5 - 6*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)
+ 45*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^2 - 60*a^2*sgn(c
os(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 + 15*a^2*sgn(cos(1/2*d*x + 1/2
*c))*tan(1/4*c)^4 - a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3 + 18*a^2*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c) - 45*a^2*sgn(cos(1/2*d*x + 1
/2*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c
)^3 - 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2 + 18*a^2*sgn(cos(1/2*d*x
 + 1/2*c))*tan(1/2*c)*tan(1/4*c) - 15*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4
*c)^2 + 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c) - 6*a^2*sgn(cos(1/2*d*x
+ 1/2*c))*tan(1/4*c) + a^2*sgn(cos(1/2*d*x + 1/2*c)))*log(abs(-2*tan(1/4*d*
x + c)*tan(1/2*c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 - 2*
sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6*tan(1/
```

2*c)^2 - 2*tan(1/4*d*x + c) + 6*tan(1/2*c) + 2)/abs(-2*tan(1/4*d*x + c)*tan
(1/2*c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*tan(1/2*c)^3 + 2*sqrt(2)*(t
an(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6*tan(1/2*c)^2 - 2
*tan(1/4*d*x + c) + 6*tan(1/2*c) + 2))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*
tan(1/4*c)^2 + 1)*(tan(1/2*c)^2 + 1)^(3/2)) + 3*sqrt(2)*(a^2*sgn(cos(1/2*d*
x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^6 + 6*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan
(1/2*c)^3*tan(1/4*c)^5 - 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1
/4*c)^6 - 15*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4 + 18*a
^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^5 - 3*a^2*sgn(cos(1/2*
d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 - 20*a^2*sgn(cos(1/2*d*x + 1/2*c))*ta
n(1/2*c)^3*tan(1/4*c)^3 + 45*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan
(1/4*c)^4 - 18*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^5 + a^2*
sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^6 + 15*a^2*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/2*c)^3*tan(1/4*c)^2 - 60*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*ta
n(1/4*c)^3 + 45*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*a
^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^5 + 6*a^2*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/2*c)^3*tan(1/4*c) - 45*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*ta
n(1/4*c)^2 + 60*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 - 15*
a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^4 - a^2*sgn(cos(1/2*d*x + 1/2*c))*
tan(1/2*c)^3 + 18*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c) - 4
5*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*a^2*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*c)^3 + 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2
 - 18*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c) + 15*a^2*sgn(cos(
1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)
 - 6*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c) - a^2*sgn(cos(1/2*d*x + 1/2*c
)))*log(abs(-2*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)*tan(1/2*c
)^2 + 2*tan(1/2*c)^3 - 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x +
 c)*tan(1/2*c) - 6*tan(1/2*c)^2 + 2*tan(1/4*d*x + c) - 6*tan(1/2*c) + 2)/ab
s(-2*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)*tan(1/2*c)^2 + 2*ta
n(1/2*c)^3 + 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1/
2*c) - 6*tan(1/2*c)^2 + 2*tan(1/4*d*x + c) - 6*tan(1/2*c) + 2))/((tan(1/4*c
)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1)*(tan(1/2*c)^2 + 1)^(3/2)) - 8*(3
*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)^6*tan(1/4*c)^6
 - 45*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)^6*tan(1/4
*c)^4 + 18*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^6*ta
n(1/4*c)^5 + 63*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)
^4*tan(1/4*c)^6 - 36*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1
/2*c)^5*tan(1/4*c)^6 + 14*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*
tan(1/2*c)^6*tan(1/4*c)^6 + 45*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x +
c)^5*tan(1/2*c)^6*tan(1/4*c)^2 - 60*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d
*x + c)^4*tan(1/2*c)^6*tan(1/4*c)^3 - 945*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan
(1/4*d*x + c)^5*tan(1/2*c)^4*tan(1/4*c)^4 + 540*a^2*sgn(cos(1/2*d*x + 1/2*c
))*tan(1/4*d*x + c)^4*tan(1/2*c)^5*tan(1/4*c)^4 - 210*a^2*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^6*tan(1/4*c)^4 + 378*a^2*sgn(cos(1/2
*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^4*tan(1/4*c)^5 - 288*a^2*sgn(c

os(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^5*tan(1/4*c)^5 + 108*a^2
*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^6*tan(1/4*c)^5 - 2
7*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)^2*tan(1/4*c)^
6 + 120*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^3*tan(1
/4*c)^6 + 6*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^4*t
an(1/4*c)^6 + 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6
*tan(1/4*c)^6 - 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/2*
c)^6 + 18*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^6*tan
(1/4*c) + 945*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)^4
*tan(1/4*c)^2 - 540*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/
2*c)^5*tan(1/4*c)^2 + 210*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*
tan(1/2*c)^6*tan(1/4*c)^2 - 1260*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x
+ c)^4*tan(1/2*c)^4*tan(1/4*c)^3 + 960*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/
4*d*x + c)^3*tan(1/2*c)^5*tan(1/4*c)^3 - 360*a^2*sgn(cos(1/2*d*x + 1/2*c))*
tan(1/4*d*x + c)^2*tan(1/2*c)^6*tan(1/4*c)^3 + 405*a^2*sgn(cos(1/2*d*x + 1/
2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)^2*tan(1/4*c)^4 - 1800*a^2*sgn(cos(1/2*d
*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^3*tan(1/4*c)^4 - 90*a^2*sgn(cos(
1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^4*tan(1/4*c)^4 - 45*a^2*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6*tan(1/4*c)^4 - 162*a^2
*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^2*tan(1/4*c)^5 + 9
60*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^3*tan(1/4*c)
^5 - 324*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^4*tan(
1/4*c)^5 + 42*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c)^5 + 9*a
^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/4*c)^6 - 36*a^2*sgn(c
os(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)*tan(1/4*c)^6 + 66*a^2*sg
n(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^2*tan(1/4*c)^6 + 63*a
^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c)^6 - 1
2*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^6 - 63*a^2*sgn(cos(
1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)^4 + 36*a^2*sgn(cos(1/2*d*x
+ 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^5 - 14*a^2*sgn(cos(1/2*d*x + 1/2*c)
)*tan(1/4*d*x + c)^3*tan(1/2*c)^6 + 378*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1
/4*d*x + c)^4*tan(1/2*c)^4*tan(1/4*c) - 288*a^2*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/4*d*x + c)^3*tan(1/2*c)^5*tan(1/4*c) + 108*a^2*sgn(cos(1/2*d*x + 1/2*c
))*tan(1/4*d*x + c)^2*tan(1/2*c)^6*tan(1/4*c) - 405*a^2*sgn(cos(1/2*d*x + 1
/2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)^2*tan(1/4*c)^2 + 1800*a^2*sgn(cos(1/2*
d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^3*tan(1/4*c)^2 + 90*a^2*sgn(cos
(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^4*tan(1/4*c)^2 + 45*a^2*sg
n(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6*tan(1/4*c)^2 + 540*a^
2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^2*tan(1/4*c)^3 -
3200*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^3*tan(1/4*
c)^3 + 1080*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^4*t
an(1/4*c)^3 - 140*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c)^3 -
 135*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/4*c)^4 + 540*a^
2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)*tan(1/4*c)^4 - 99
0*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^2*tan(1/4*c)^

4 - 945*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c)^4 + 180*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^4 + 54*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/4*c)^5 - 288*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)*tan(1/4*c)^5 + 756*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^2*tan(1/4*c)^5 + 18*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^5 + 10*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/4*c)^6 - 27*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^6 + 40*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^6 + 27*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/2*c)^2 - 120*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^3 - 6*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^4 - 3*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^6 - 162*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)^2*tan(1/4*c) + 960*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^3*tan(1/4*c) - 324*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^4*tan(1/4*c) + 42*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^6*tan(1/4*c) + 135*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5*tan(1/4*c)^2 - 540*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c)*tan(1/4*c)^2 + 990*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^2*tan(1/4*c)^2 + 945*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4*tan(1/4*c)^2 - 180*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^5*tan(1/4*c)^2 - 180*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/4*c)^3 + 960*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)*tan(1/4*c)^3 - 2520*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^2*tan(1/4*c)^3 - 60*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c)^3 - 150*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/4*c)^4 + 405*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^4 - 600*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4 + 36*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/4*c)^5 + 198*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^5 + 9*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^6 - 12*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 - 9*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^5 + 36*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/2*c) - 66*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)^2 - 63*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^4 + 12*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^5 + 54*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^4*tan(1/4*c) - 288*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/2*c)*tan(1/4*c) + 756*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/2*c)^2*tan(1/4*c) + 18*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^4*tan(1/4*c) + 150*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^3*tan(1/4*c)^2 - 405*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)^2*tan(1/4*c)^2 + 600*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2 - 120*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/4*c)^3 - 660*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 - 135*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^4 + 180*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 + 30*a^2*sgn(c

```
os(1/2*d*x + 1/2*c))*tan(1/4*c)^5 - 10*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/
4*d*x + c)^3 + 27*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/2*c)
^2 - 40*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3 + 36*a^2*sgn(cos(1/2*d*x
 + 1/2*c))*tan(1/4*d*x + c)^2*tan(1/4*c) + 198*a^2*sgn(cos(1/2*d*x + 1/2*c)
)*tan(1/2*c)^2*tan(1/4*c) + 135*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x +
 c)*tan(1/4*c)^2 - 180*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^
2 - 100*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^3 - 9*a^2*sgn(cos(1/2*d*x
+ 1/2*c))*tan(1/4*d*x + c) + 12*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c) +
30*a^2*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c))/((tan(1/2*c)^6*tan(1/4*c)^6 +
3*tan(1/2*c)^6*tan(1/4*c)^4 + 3*tan(1/2*c)^4*tan(1/4*c)^6 + 3*tan(1/2*c)^6*
tan(1/4*c)^2 + 9*tan(1/2*c)^4*tan(1/4*c)^4 + 3*tan(1/2*c)^2*tan(1/4*c)^6 +
tan(1/2*c)^6 + 9*tan(1/2*c)^4*tan(1/4*c)^2 + 9*tan(1/2*c)^2*tan(1/4*c)^4 +
tan(1/4*c)^6 + 3*tan(1/2*c)^4 + 9*tan(1/2*c)^2*tan(1/4*c)^2 + 3*tan(1/4*c)^
4 + 3*tan(1/2*c)^2 + 3*tan(1/4*c)^2 + 1)*(tan(1/4*d*x + c)^2 + 1)^3))/d
```

maple [B] time = 0.56, size = 244, normalized size = 2.49

$$\frac{a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} \left(-4\sqrt{a}\ \sqrt{2}\ \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+18\sqrt{2}\ \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x)
```

```
[Out] 1/3*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+18*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a+3*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/sin(1/2*d*x+1/2*c
)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x),x)
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c),x)
[Out] Timed out
```

3.117 $\int (a + a\cos(c + dx))^{5/2} \sec^2(c + dx)\,dx$

Optimal. Leaf size=92

$$\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} + \frac{a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}} + \frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

```
[Out] 5*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+a^3*sin(d*x+
c)/d/(a+a*cos(d*x+c))^(1/2)+a^2*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 2981, 2773, 206}

$$\frac{a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}} + \frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d} + \frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] (5*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a
^3*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c + d*x
]]*Tan[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \frac{a^2\sqrt{a+a\cos(c+dx)}\,\tan(c+dx)}{d} - a\int\left(-\frac{5a}{2}-\frac{1}{2}a\cos(c+dx)\right)\sqrt{a$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\tan(c+dx)}{d} + \frac{1}{2}\left(5a^2\right)$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\tan(c+dx)}{d} - \frac{\left(5a^3\right)\mathrm{Su}}{}$$

$$= \frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos}}{}$$

Mathematica [C] time = 36.23, size = 1547, normalized size = 16.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] ((-5/32 + (5*I)/32)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16
*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*
x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)
*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*S
qrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) +
```

```
(8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2
)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*
x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt
[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*(a*(1
+ Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(((-1 - I) + Sqrt[2]*E^((I/2)*
c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x
)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)) + I*E^((2*I)*(c + d*x)))^2) - (((5*I
)/8)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d
*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)
/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*d) - (((5
*I)/8)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 +
(d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x
)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*d) - (5*
(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*S
in[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(16*Sqrt[2]*d) - (5*(a*(1 + Cos[c
+ d*x]))^(5/2)*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)
/2]]*Sec[c/2 + (d*x)/2]^5)/(16*Sqrt[2]*d) + (Cos[(d*x)/2]*(a*(1 + Cos[c + d
*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[c/2])/(2*d) - (((5*I)/4)*ArcTan[((2*I)
*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2
+ 4*Sin[c/2]^2]]*(a*(1 + Cos[c + d*x]))^(5/2)*Cot[c/2]*Sec[c/2 + (d*x)/2]^5
)/(d*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (5*(a*(1 + Cos[c + d*x]))^(5
/2)*Csc[c/2]*Sec[c/2 + (d*x)/2]^5*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[
(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcT
an[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Co
s[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2])
)/(4*Sqrt[2]*d*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + (Cos[c/2]*(a*(1 + Cos[c + d
*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(2*d) + ((a*(1 + Cos[c + d*x
]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(8*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])) - ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(8*d*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 1.20, size = 164, normalized size = 1.78

$$\frac{5\left(a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,(}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*a^2*cos(d
*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d
*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.48, size = 408, normalized size = 4.43

$$\frac{a^{\frac{3}{2}} \cos\left(\frac{dx}{2}+\frac{c}{2}\right) \sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\left(\left(-8\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,-10\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}-a\sqrt{2}\,\cos\left(\right.\right.}{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x)
```

```
[Out] a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-8*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-10*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a))*a-10*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/
2*c)^2+6*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+5*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+5*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*
d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.118 $\int (a + a\cos(c + dx))^{5/2} \sec^3(c + dx) \, dx$

Optimal. Leaf size=106

$$\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

```
[Out] 19/4*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+9/4*a^3*t
an(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)*(a+a*cos(d*x+c))^(1/2
)*tan(d*x+c)/d
```

Rubi [A] time = 0.22, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 2980, 2773, 206}

$$\frac{9a^3\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] (19*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
 + (9*a^3*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Co
s[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}a\int\left(-\frac{9a}{2}-\frac{5}{2}a\cos(\right.$$

$$= \frac{9a^3\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d}$$

$$= \frac{9a^3\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d}$$

$$= \frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{9a^3\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos}}{}$$

Mathematica [C] time = 36.01, size = 1693, normalized size = 15.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] ((-19/128 + (19*I)/128)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16
- 16*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2
)*d*x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((
```

```
3*I)*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*
I)*Sqrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x
) + (8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*
I)/2)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c
+ d*x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) +
Sqrt[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*(a
*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(((-1 - I) + Sqrt[2]*E^((I
/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c +
 d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)) + I*E^((2*I)*(c + d*x)))^2) - ((
(19*I)/32)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/
4 + (d*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 +
 (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*d)
- (((19*I)/32)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Si
n[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/
4 + (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*
d) - (19*(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] -
Sqrt[2]*Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(64*Sqrt[2]*d) - (19*(a*(
1 + Cos[c + d*x]))^(5/2)*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c
/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(64*Sqrt[2]*d) - (((19*I)/16)*ArcTan[(
(2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/
2]^2 + 4*Sin[c/2]^2]]*(a*(1 + Cos[c + d*x]))^(5/2)*Cot[c/2]*Sec[c/2 + (d*x)
/2]^5)/(d*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (19*(a*(1 + Cos[c + d*x
]))^(5/2)*Csc[c/2]*Sec[c/2 + (d*x)/2]^5*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] +
2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2
]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2
+ 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/
2]^2]))/(16*Sqrt[2]*d*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + ((a*(1 + Cos[c + d*x
]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(16*d*(Cos[c/2] - Sin[c/2])*(C
os[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a*(1 + Cos[c + d*x]))^(5/2)*
Sec[c/2 + (d*x)/2]^5*(11*Cos[c/2] - 9*Sin[c/2]))/(32*d*(Cos[c/2] - Sin[c/2]
)*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + ((a*(1 + Cos[c + d*x]))^(5/2
)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(16*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
 (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2
+ (d*x)/2]^5*(-11*Cos[c/2] - 9*Sin[c/2]))/(32*d*(Cos[c/2] + Sin[c/2])*(Cos[
c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.67, size = 170, normalized size = 1.60

$$\frac{19\left(a^{2}\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right) -}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x +
c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a^2*
cos(d*x + c) + 2*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2)
```

giac [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B]   time = 0.53, size = 545, normalized size = 5.14

$$\frac{a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(76a\left(\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{-2\cos\left(\right.}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(76*a*(ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-76*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a-76*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-44*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+19*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+19*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))*a+26*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*
c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/
2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B]   time = 13.58, size = 3667, normalized size = 34.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(150*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 154*sqrt(2)*
a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - (3*sqrt(2)*a^2*sin(7/2*d*x + 7
/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x +
 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - (3*
sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*
sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
 + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
```

```
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)
^2 - 3*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(
15/2*d*x + 15/2*c) - 5*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*
d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 11*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*s
qrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 45*(sqrt(2)*a^2*sin(4
*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) - (11*sq
rt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 38*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(
1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(4*sqrt(2)*a^2*co
s(2*d*x + 2*c) + 27*sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + (20*sqrt(2)*a^2*cos
(2*d*x + 2*c) + 87*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) - 2*
(11*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)
+ 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/2*d*x + 15/2*c) + 5*(sqr
t(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*s
in(13/2*d*x + 13/2*c) - 11*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*co
s(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/2*d*x + 11/2*c) - 45*(sqrt(2)*a^2*cos(
4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/2*d*x +
9/2*c) - (12*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 20*sqrt(2)
*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 75*sqrt(2)*a^2*cos(7/2*d*x + 7
```

```
/2*c) - 77*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) - 45*sqrt(2)*a^2*cos(3/2*d*x +
3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) - 4*(17*sqrt(2)*a^2*sin(3/2*d*
x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*
d*x + 2*c))*sin(4*d*x + 4*c) - 6*(2*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 2*sqrt
(2)*a^2*sin(2*d*x + 2*c)^2 + 27*sqrt(2)*a^2*cos(2*d*x + 2*c) + 13*sqrt(2)*a
^2)*sin(7/2*d*x + 7/2*c) - 2*(10*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 10*sqrt(2
)*a^2*sin(2*d*x + 2*c)^2 + 87*sqrt(2)*a^2*cos(2*d*x + 2*c) + 41*sqrt(2)*a^2
)*sin(5/2*d*x + 5/2*c) + 2*(45*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 11*sqrt(2
)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sqrt(a)/((2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\cos(c + d\,x))^{5/2}}{\cos(c + d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
[Out] Timed out
```

3.119 $\int (a + a\cos(c + dx))^{5/2} \sec^4(c + dx)\, dx$

Optimal. Leaf size=144

$$\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{25a^3\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] 25/8*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+25/8*a^3*
tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+13/12*a^3*sec(d*x+c)*tan(d*x+c)/d/(a+a*
cos(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.28, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 2980, 2772, 2773, 206}

$$\frac{25a^3\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{13a^3\tan}{12d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] (25*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
 + (25*a^3*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (13*a^3*Sec[c + d
*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c +
 d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{1}{3}a\int\left(-\frac{13a}{2} - \frac{9}{2}a\,c\right.$$

$$= \frac{13a^3\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sec^2(c+dx)\tan}{3d}$$

$$= \frac{25a^3\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\,\mathrm{co}}}{}$$

$$= \frac{25a^3\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\,\mathrm{co}}}{}$$

$$= \frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{25a^3\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\sec(c}{12d\sqrt{a+}}$$

Mathematica [C] time = 36.11, size = 1825, normalized size = 12.67

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] ((-25/256 + (25*I)/256)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16
- 16*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2
)*d*x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((
3*I)*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*
I)*Sqrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x
) + (8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*
I)/2)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c
+ d*x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) +
Sqrt[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*(a
*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(((-1 - I) + Sqrt[2]*E^((I
/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c +
 d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)) + I*E^((2*I)*(c + d*x)))^2) - ((
(25*I)/64)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/
4 + (d*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 +
 (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*d)
- (((25*I)/64)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Si
n[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/
4 + (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*
d) - (25*(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] -
Sqrt[2]*Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(128*Sqrt[2]*d) - (25*(a*
(1 + Cos[c + d*x]))^(5/2)*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[
c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(128*Sqrt[2]*d) - (((25*I)/32)*ArcTan
[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[
c/2]^2 + 4*Sin[c/2]^2]]*(a*(1 + Cos[c + d*x]))^(5/2)*Cot[c/2]*Sec[c/2 + (d*
x)/2]^5)/(d*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (25*(a*(1 + Cos[c + d
*x]))^(5/2)*Csc[c/2]*Sec[c/2 + (d*x)/2]^5*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2]
+ 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt
[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-
2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[
c/2]^2]))/(32*Sqrt[2]*d*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + ((a*(1 + Cos[c + d
*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(48*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*
x)/2])^3) + (5*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/
2])/(32*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2
) + (5*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(5*Cos[c/2] - 3*Si
n[c/2]))/(64*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])) - ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(48*d*(Cos[c/2 +
 (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (5*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/
```

```
2 + (d*x)/2]^5*Sin[(d*x)/2])/(32*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2])^2) - (5*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x
)/2]^5*(5*Cos[c/2] + 3*Sin[c/2]))/(64*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.96, size = 183, normalized size = 1.27

$$\frac{75\left(a^2\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right) \cdots}{96\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+ \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x +
c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a^2*
cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.56, size = 709, normalized size = 4.92

$$a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-600a\left(\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin^2\left(\cdots\right.\right.}\right.}{-\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-600*a*(ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
```

```
))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+300*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-450*ln(-4/(-2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-450*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a))*a-736*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+
1/2*c)^2+75*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+75*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+234*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^
3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 131.61, size = 6703, normalized size = 46.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(4176*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 2430*a^2*cos(5/2*d*
x + 5/2*c)*sin(2*d*x + 2*c) + 678*a^2*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c)
 - 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 75*
sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2
)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 75*sqrt(2)*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - (75*sqrt(2)*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 75*sqrt(2)*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 10*a^2*sin(9/2*d*x + 9/2*c) + 30*a^2*sin(7/2
*d*x + 7/2*c) + 78*a^2*sin(5/2*d*x + 5/2*c) - 170*a^2*sin(3/2*d*x + 3/2*c)
- 600*a^2*sin(1/2*d*x + 1/2*c))*cos(6*d*x + 6*c)^2 - 9*(75*sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
```

+ 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 75*sqrt(2)*a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) + 30*a^2*sin(7/2*d*x + 7/2*c) + 78*a^2*sin(5/
2*d*x + 5/2*c) - 170*a^2*sin(3/2*d*x + 3/2*c) - 600*a^2*sin(1/2*d*x + 1/2*c
))*cos(4*d*x + 4*c)^2 - 45*(15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 34*a^2*sin(3/2*d*x + 3/2*c) - 120*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x +
2*c)^2 - (75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7
5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt
(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 10*a^2*sin(9/
2*d*x + 9/2*c) + 30*a^2*sin(7/2*d*x + 7/2*c) + 78*a^2*sin(5/2*d*x + 5/2*c)
- 170*a^2*sin(3/2*d*x + 3/2*c) - 600*a^2*sin(1/2*d*x + 1/2*c))*sin(6*d*x +
6*c)^2 - 9*(75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqr
t(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 30*a^2*sin(7
/2*d*x + 7/2*c) + 78*a^2*sin(5/2*d*x + 5/2*c) - 170*a^2*sin(3/2*d*x + 3/2*c
) - 600*a^2*sin(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c)^2 - 45*(15*sqrt(2)*a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*sqrt(2)*a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 34*a^2*sin(3/2*d*x + 3/2*c) - 120*a^2*si

n(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c)^2 - 56*a^2*sin(3/2*d*x + 3/2*c) + 600*
a^2*sin(1/2*d*x + 1/2*c) + 10*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c
) + 3*a^2*sin(2*d*x + 2*c))*cos(21/2*d*x + 21/2*c) - 30*(a^2*sin(6*d*x + 6*
c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(19/2*d*x + 19/2*c
) - 48*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2
*c))*cos(17/2*d*x + 17/2*c) + 80*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x +
4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(15/2*d*x + 15/2*c) + 396*(a^2*sin(6*d*x
+ 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(13/2*d*x + 13
/2*c) - 6*(25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - 25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 2
5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 25*sqrt
(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 170*a^2*sin(1
1/2*d*x + 11/2*c) - 19*a^2*sin(3/2*d*x + 3/2*c) - 200*a^2*sin(1/2*d*x + 1/2
*c) + (75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 75*sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 30*a^2*sin(7/2*d*
x + 7/2*c) + 78*a^2*sin(5/2*d*x + 5/2*c) - 170*a^2*sin(3/2*d*x + 3/2*c) - 6
00*a^2*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*c) + 5*(15*sqrt(2)*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 34*a^2*sin(3/2*d*x + 3/2*c) - 120*a^2*sin(1/2*d*
x + 1/2*c))*cos(2*d*x + 2*c) - 10*(a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2
*c) - 25*a^2)*sin(9/2*d*x + 9/2*c) + 2*(15*a^2*cos(2*d*x + 2*c) + 121*a^2)*
sin(7/2*d*x + 7/2*c) + (78*a^2*cos(2*d*x + 2*c) + 161*a^2)*sin(5/2*d*x + 5/
2*c))*cos(6*d*x + 6*c) + 3060*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))
*cos(11/2*d*x + 11/2*c) + 4560*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c)
)*cos(9/2*d*x + 9/2*c) - 18*(25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - 25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + 25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*

x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2 *c) + 2) - 25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/ 2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*sin(3/2*d*x + 3/2*c) - 200*a^2*sin(1/2*d*x + 1/2*c) + 5*(15*sqr t(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a ^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*sqrt(2)*a^2*log (2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d* x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos (1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/ 2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 34*a^2*sin(3/2*d*x + 3/2*c) - 120*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 2*(15*a^2*cos(2*d*x + 2*c) + 121*a^2)*sin(7/2*d*x + 7/2*c) + (78*a^2*cos(2*d*x + 2*c) + 161*a^2)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) - 18*(25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2) *sin(1/2*d*x + 1/2*c) + 2) + 25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1 /2*d*x + 1/2*c) + 2) - 25*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*sin(3/2*d*x + 3/2*c) - 200*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) - 10*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3* a^2*cos(2*d*x + 2*c) + a^2)*sin(21/2*d*x + 21/2*c) + 30*(a^2*cos(6*d*x + 6* c) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(19/2*d*x + 19/2*c) + 48*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d *x + 2*c) + a^2)*sin(17/2*d*x + 17/2*c) - 80*(a^2*cos(6*d*x + 6*c) + 3*a^2* cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(15/2*d*x + 15/2*c) - 3 96*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(13/2*d*x + 13/2*c) - 2*(90*a^2*sin(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 234*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 510*a^2*cos(11/2*d*x + 11/2*c) - 760*a^2*cos(9/2*d*x + 9/2*c) - 696*a^2*cos(7/2*d*x + 7/2*c) - 405*a^2*cos(5/2*d*x + 5/2*c) - 113*a^2*cos(3/2*d*x + 3/2*c) - 30*(a^2*sin(4 *d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(9/2*d*x + 9/2*c) + 3*(75*sqrt(2)*a^ 2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1 /2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 75*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2 *c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 75*sqrt(2)*a^2*log(2*cos(1/2*d* x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 30*a^2*sin(7/2*d*x + 7/2*c) + 78*a^2* sin(5/2*d*x + 5/2*c) - 170*a^2*sin(3/2*d*x + 3/2*c) - 600*a^2*sin(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 15*(15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2

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 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) + 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - 34*a^2*sin(3/2*d*x + 3/2*c) - 120*a^2*sin(1/2*d*x + 1/2*c))*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) - 1020*(3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*
x + 2*c) + a^2)*sin(11/2*d*x + 11/2*c) + 10*(9*a^2*cos(4*d*x + 4*c)^2 + 9*a
^2*cos(2*d*x + 2*c)^2 + 9*a^2*sin(4*d*x + 4*c)^2 + 18*a^2*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 9*a^2*sin(2*d*x + 2*c)^2 - 450*a^2*cos(2*d*x + 2*c) - 15
1*a^2 + 18*(a^2*cos(2*d*x + 2*c) - 25*a^2)*cos(4*d*x + 4*c))*sin(9/2*d*x +
9/2*c) - 6*(90*a^2*sin(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 234*a^2*sin(5/2*
d*x + 5/2*c)*sin(2*d*x + 2*c) - 696*a^2*cos(7/2*d*x + 7/2*c) - 405*a^2*cos(
5/2*d*x + 5/2*c) - 113*a^2*cos(3/2*d*x + 3/2*c) + 15*(15*sqrt(2)*a^2*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*sqrt(2)*a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 34*a^2*sin(3/2*d*x + 3/2*c) - 120*a^2*sin(1/2
*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 18*(15*a^2*cos(2*d*x +
2*c)^2 + 15*a^2*sin(2*d*x + 2*c)^2 + 242*a^2*cos(2*d*x + 2*c) + 79*a^2)*sin
(7/2*d*x + 7/2*c) - 6*(117*a^2*cos(2*d*x + 2*c)^2 + 117*a^2*sin(2*d*x + 2*c
)^2 + 483*a^2*cos(2*d*x + 2*c) + 148*a^2)*sin(5/2*d*x + 5/2*c))*sqrt(a)/((s
qrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*
d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 +
18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2
 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*co
s(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c)
+ 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\cos(c + d\,x))^{5/2}}{\cos(c + d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.120 $\int (a + a\cos(c+dx))^{5/2} \sec^5(c+dx)\,dx$

Optimal. Leaf size=182

$$\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{163a^3\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{17a^3\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{163a^3\tan(c+dx)\sec(c}{96d\sqrt{a\cos(c+dx)+}}$$

```
[Out] 163/64*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+163/64*
a^3*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+163/96*a^3*sec(d*x+c)*tan(d*x+c)/d/
(a+a*cos(d*x+c))^(1/2)+17/24*a^3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))
^(1/2)+1/4*a^2*sec(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.34, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 2980, 2772, 2773, 206}

$$\frac{163a^3\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{17a^3\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sec^3(c+}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

```
[Out] (163*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(64*
d) + (163*a^3*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + (163*a^3*Sec[
c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (17*a^3*Sec[c + d*
x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c
+ d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a + a\cos(c + dx))^{5/2}\sec^5(c + dx)\,dx = \frac{a^2\sqrt{a + a\cos(c + dx)}\ \sec^3(c + dx)\tan(c + dx)}{4d} - \frac{1}{4}a\int\left(-\frac{17a}{2} - \frac{13}{2}a\,\mathrm{c}\right.$$

$$= \frac{17a^3\sec^2(c + dx)\tan(c + dx)}{24d\sqrt{a + a\cos(c + dx)}} + \frac{a^2\sqrt{a + a\cos(c + dx)}\ \sec^3(c + dx)\tan}{4d}$$

$$= \frac{163a^3\sec(c + dx)\tan(c + dx)}{96d\sqrt{a + a\cos(c + dx)}} + \frac{17a^3\sec^2(c + dx)\tan(c + dx)}{24d\sqrt{a + a\cos(c + dx)}} + \frac{a^2\sqrt{a}}{}$$

$$= \frac{163a^3\tan(c + dx)}{64d\sqrt{a + a\cos(c + dx)}} + \frac{163a^3\sec(c + dx)\tan(c + dx)}{96d\sqrt{a + a\cos(c + dx)}} + \frac{17a^3\sec^2(c}{24d\sqrt{a -}}$$

$$= \frac{163a^3\tan(c + dx)}{64d\sqrt{a + a\cos(c + dx)}} + \frac{163a^3\sec(c + dx)\tan(c + dx)}{96d\sqrt{a + a\cos(c + dx)}} + \frac{17a^3\sec^2(c}{24d\sqrt{a -}}$$

$$= \frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{163a^3\tan(c + dx)}{64d\sqrt{a + a\cos(c + dx)}} + \frac{163a^3\sec(}{96d\sqrt{a}}$$

Mathematica [C] time = 35.84, size = 2069, normalized size = 11.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

```
[Out] ((-163/2048 + (163*I)/2048)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) +
(16 - 16*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*
I)/2)*d*x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*
E^((3*I)*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4
+ 4*I)*Sqrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)
*d*x) + (8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(
((3*I)/2)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)
*(c + d*x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)
) + Sqrt[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*
x*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(((-1 - I) + Sqrt[2]*E
^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*
(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)) + I*E^((2*I)*(c + d*x)))^2)
- (((163*I)/512)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*
Sin[c/4 + (d*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin
[c/4 + (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[
2]*d) - (((163*I)/512)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sq
rt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4]
```

```
- Sin[c/4 + (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(
Sqrt[2]*d) - (163*(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 - Sqrt[2]*Cos[c/2 + (d
*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(1024*Sqrt[2]*d)
 - (163*(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - S
qrt[2]*Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(1024*Sqrt[2]*d) - (((163*
I)/256)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sq
rt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*(a*(1 + Cos[c + d*x]))^(5/2)*Cot[c/2]
*Sec[c/2 + (d*x)/2]^5)/(d*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (163*(a
*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2]*Sec[c/2 + (d*x)/2]^5*(-(d*x*Cos[c/2]) +
 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2
] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(
d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[
c/2]^2 + 4*Sin[c/2]^2]))/(256*Sqrt[2]*d*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + ((
a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(64*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + ((a*(1 + Cos
[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(23*Cos[c/2] - 17*Sin[c/2]))/(384*d*
(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (43*(a
*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(256*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a*(1 + Cos
[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(163*Cos[c/2] - 77*Sin[c/2]))/(512*d
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + ((a*(1
+ Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(64*d*(Cos[c/2] +
 Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + ((a*(1 + Cos[c +
d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(-23*Cos[c/2] - 17*Sin[c/2]))/(384*d*(Cos
[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (43*(a*(1
+ Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(256*d*(Cos[c/2]
+ Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + ((a*(1 + Cos[c +
 d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(-163*Cos[c/2] - 77*Sin[c/2]))/(512*d*(C
os[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 1.47, size = 196, normalized size = 1.08

$$\frac{489\left(a^2\cos(dx+c)^5+a^2\cos(dx+c)^4\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{768\left(d\cos(dx+c)^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/768*(489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(489*a
^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)
*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^
4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.60, size = 872, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x)
[Out] 1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7824*a*(ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-7824*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)+2*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+1304*(11*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+9*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a+9*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1
/2*c)^4+(-3912*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-3912*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-9212*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+489*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+489*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2094*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \cos (c + d x))^{5/2}}{\cos (c + d x)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.121 $\int (a + a\cos(c + dx))^{7/2}\, dx$

Optimal. Leaf size=119

$$\frac{256a^4\sin(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{64a^3\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{24a^2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a\sin(c+$$

```
[Out] 24/35*a^2*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*a*(a+a*cos(d*x+c))^(5/2)*
sin(d*x+c)/d+256/35*a^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+64/35*a^3*sin(d
*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{256a^4\sin(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{64a^3\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{24a^2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (256*a^4*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (64*a^3*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(35*d) + (24*a^2*(a + a*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(35*d) + (2*a*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{7/2}\,dx = \frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}(12a)\int (a+a\cos(c+dx))^{5/2}\,dx$$
$$= \frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{35}$$
$$= \frac{64a^3\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{35d} + \frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + 2a$$
$$= \frac{256a^4\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{64a^3\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{35d} + 24a^2(a+a\cos(c$$

Mathematica [A] time = 0.24, size = 83, normalized size = 0.70

$$\frac{a^3\left(1225\sin\left(\frac{1}{2}(c+dx)\right)+245\sin\left(\frac{3}{2}(c+dx)\right)+49\sin\left(\frac{5}{2}(c+dx)\right)+5\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos}}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (a^3*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(1225*Sin[(c + d*x)/2] + 2
45*Sin[(3*(c + d*x))/2] + 49*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2])
)/(140*d)
```

fricas [A] time = 0.70, size = 75, normalized size = 0.63

$$\frac{2\left(5\,a^3\cos\left(dx+c\right)^3+27\,a^3\cos\left(dx+c\right)^2+71\,a^3\cos\left(dx+c\right)+177\,a^3\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^3*cos(d*x + c)^3 + 27*a^3*cos(d*x + c)^2 + 71*a^3*cos(d*x + c) +
177*a^3)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.80, size = 117, normalized size = 0.98

$$\frac{1}{140}\sqrt{2}\left(\frac{5\,a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{49\,a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{245\,a^3\mathrm{sgn}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/140*sqrt(2)*(5*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 49*
a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 245*a^3*sgn(cos(1/2*
d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 1225*a^3*sgn(cos(1/2*d*x + 1/2*c))*s
in(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.16, size = 86, normalized size = 0.72

$$\frac{16a^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(5\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+16\right)\sqrt{2}}{35\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 16/35*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(5*cos(1/2*d*x+1/2*c)^6+6*c
os(1/2*d*x+1/2*c)^4+8*cos(1/2*d*x+1/2*c)^2+16)*2^(1/2)/(a*cos(1/2*d*x+1/2*c
)^2)^(1/2)/d
```

maxima [A] time = 1.07, size = 77, normalized size = 0.65

$$\frac{\left(5\sqrt{2}\,a^3\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+49\sqrt{2}\,a^3\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+245\sqrt{2}\,a^3\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+1225\sqrt{2}\,a^3\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\cdot}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/140*(5*sqrt(2)*a^3*sin(7/2*d*x + 7/2*c) + 49*sqrt(2)*a^3*sin(5/2*d*x + 5/
2*c) + 245*sqrt(2)*a^3*sin(3/2*d*x + 3/2*c) + 1225*sqrt(2)*a^3*sin(1/2*d*x
+ 1/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\cos(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.122 $\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{62\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}-\frac{148\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+}}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)-148/105*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/35*cos(d*x+c)^2*sin(d*x
+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))
^(1/2)+62/105*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.37, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2778, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{62\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}-\frac{148\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d) - (148*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*Co
s[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Cos[c + d*x
]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (62*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(105*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)(-6a+a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}dx}{7a}$$

$$= -\frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)\left(2a^2-\frac{31}{2}a^2\cos(c\cdots\right.}{\sqrt{a+a\cos(c+dx)}}}{35a^2}$$

$$= -\frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{2\int \frac{2a^2\cos(c+dx)-\frac{31}{2}a^2\cos^2(c\cdots}{\sqrt{a+a\cos(c+dx)}}}{35a^2}$$

$$= -\frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{62\sqrt{a+a\cos(c+dx)}\ \mathrm{si}\cdots}{105ad}$$

$$= -\frac{148\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{148\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{148\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2\cos^2(c+dx)\sin(c+d\cdots}{35d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.20, size = 130, normalized size = 0.75

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-525\sin\left(\frac{1}{2}(c+dx)\right)+175\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)-420\log\left(\cdots\right.\right.}{210d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-420*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 420*Log[
Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] - 525*Sin[(c + d*x)/2] + 175*Sin[(3*(c
 + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c + d*x))/2]))/(210*d*Sq
rt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.29, size = 153, normalized size = 0.88

$$\frac{4\left(15\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)^{2}+31\cos\left(dx+c\right)-43\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)+\frac{105\sqrt{2}\left(a\cos(dx+c)+\cdots\right.}{\cdots}}{210\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/210*(4*(15*cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 31*cos(d*x + c) - 43)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c) + 105*sqrt(2)*(a*cos(d*x + c) + a)*log(-(
cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*co
s(d*x + c) + a*d)
```

giac [A] time = 1.25, size = 118, normalized size = 0.68

$$-\frac{\sqrt{2}\left(\frac{105\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{8\left(35a^{3}+\left(23a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+28a^{3}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{7}{2}}}\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/105*sqrt(2)*(105*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a)))/sqrt(a) + 8*(35*a^3 + (23*a^3*tan(1/2*d*x + 1/2*c)^2
+ 28*a^3)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^3/(a*tan(1/2*d*x + 1
/2*c)^2 + a)^(7/2))/d
```

maple [A] time = 0.46, size = 194, normalized size = 1.11

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-240\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+336\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\cdots\right.}{105a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/105*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+336*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-280*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+105*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+
1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.123 $\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}+\frac{28\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{2}\sqrt{a\cos(c+a}}\right.}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)+28/15*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*cos(d*x+c)^2*sin(d*x+c
)/d/(a+a*cos(d*x+c))^(1/2)-2/15*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.24, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2778, 2968, 3023, 2751, 2649, 206}

$$\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}+\frac{28\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{2}\sqrt{a\cos(c+a}}\right.}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d)) + (28*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C
os[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) - (2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\cos(c+dx)(-4a+a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-4a\cos(c+dx)+a\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15ad} - \frac{2\int \frac{\frac{a^2}{2}-7a^2\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}}{15a^2}$$

$$= \frac{28\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2\sqrt{a+a\cos(c+dx)}\,\sin(c}{15ad}$$

$$= \frac{28\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2\sqrt{a+a\cos(c+dx)}\,\sin(c}{15ad}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{28\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^2(c+dx)\sin(c+}{5d\sqrt{a+a\cos(c+d}}$$

Mathematica [A] time = 0.17, size = 118, normalized size = 0.84

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(60\sin\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+30\log\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c\right.\right.\right.}{15d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(30*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 30*Log[Cos
[(c + d*x)/4] + Sin[(c + d*x)/4]] + 60*Sin[(c + d*x)/2] - 5*Sin[(3*(c + d*x
))/2] + 3*Sin[(5*(c + d*x))/2]))/(15*d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 0.68, size = 143, normalized size = 1.02

$$\frac{4\sqrt{a\cos(dx+c)+a}\left(3\cos(dx+c)^2-\cos(dx+c)+13\right)\sin(dx+c)+\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos}}{}}{\cos(dx+c}\right.}{\sqrt{a}}}{30(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(4*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*sin
(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.29, size = 116, normalized size = 0.83

$$\frac{\sqrt{2}\left(\frac{15\,\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{2\left(\left(17\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+20\,a^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,a^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{5}{2}}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/sqrt(a) + 2*((17*a^2*tan(1/2*d*x + 1/2*c)^2 + 20*a^2)*ta
n(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)
^2 + a)^(5/2))/d
```

maple [A] time = 0.39, size = 183, normalized size = 1.31

$$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}}{15a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/15*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+20*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+15*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-30*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.124 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}-\frac{4\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)-4/3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+a*cos(d*x+c
))^(1/2)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2649, 206}

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}-\frac{4\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d) - (4*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{a}{2}-a\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= -\frac{4\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} + \int \frac{1}{\sqrt{a+a\cos(c+d}}$$

$$= -\frac{4\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,\right.}{d}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{4\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sqrt{a+a\cos(c+dx)}\,\sin(}{3ad}$$

Mathematica [A] time = 0.11, size = 104, normalized size = 1.00

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)-3\log\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)+3\log\left(\sin\right.\right.}{3d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-3*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 3*Log[Co
s[(c + d*x)/4] + Sin[(c + d*x)/4]] - 3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))
/2]))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.14, size = 131, normalized size = 1.26

$$\frac{4\sqrt{a\cos(dx+c)+a}(\cos(dx+c)-1)\sin(dx+c)+\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{\sqrt{a}}}{6(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2)
*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.67, size = 79, normalized size = 0.76

$$-\frac{\sqrt{2}\left(\frac{4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}+\frac{3\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/
2) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a)))/sqrt(a))/d
```

maple [A] time = 0.29, size = 135, normalized size = 1.30

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4\sqrt{a}\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right.\right.}{3a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+3*2^(1/2)*ln(4/cos(1/2*d
```

```
*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*
x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 0.38, size = 97, normalized size = 0.93

$$\frac{2\sin(c+d\,x)\,\sqrt{a+a\,\cos(c+d\,x)}}{3\,a\,d}-\frac{2\left(4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)-3\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\right)\sqrt{\frac{a+a\,\cos(c+d\,x)}{2\,a}}}{3\,a^2\,d\,\sqrt{a+a\,\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*sin(c + d*x)*(a + a*cos(c + d*x))^(1/2))/(3*a*d) - (2*(4*a^2*ellipticE(c
/2 + (d*x)/2, 1) - 3*a^2*ellipticF(c/2 + (d*x)/2, 1))*((a + a*cos(c + d*x))
/(2*a))^(1/2))/(3*a^2*d*(a + a*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.125 $\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)+2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2649, 206}

$$\frac{2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.73

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(ArcTanh[Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2]))/(d*S
qrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.31, size = 122, normalized size = 1.67

$$\frac{\frac{\sqrt{2}\,(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}} + 4\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/
(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.06, size = 74, normalized size = 1.01

$$\frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a)))/sqrt(a) + 2*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))/d
```

maple [A] time = 0.31, size = 120, normalized size = 1.64

$$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a-2\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(
1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 0.40, size = 60, normalized size = 0.82

$$\frac{2\left(2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)-\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\right)\sqrt{\frac{a+a\,\cos(c+d\,x)}{2\,a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(2*ellipticE(c/2 + (d*x)/2, 1) - ellipticF(c/2 + (d*x)/2, 1))*((a + a*co
s(c + d*x))/(2*a))^(1/2))/(d*(a + a*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.126 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.87

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])
])
```

fricas [A] time = 0.88, size = 126, normalized size = 2.74

$$\left[\frac{\sqrt{2}\,\log\left(-\frac{\cos(dx+c)^2-\frac{2\,\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{a}}-2\,\cos(dx+c)-3}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)}{2\,\sqrt{a}\,d}, -\frac{\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{-\frac{1}{a}}}{\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sqrt(-1/a)/sin(d*x + c))/d]
```

giac [B] time = 0.62, size = 93, normalized size = 2.02

$$\frac{\frac{\sqrt{2}\,\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{2}\,\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/(s
qrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c)
+ sin(1/2*d*x + 1/2*c) - 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

maple [C] time = 0.06, size = 54, normalized size = 1.17

$$\frac{\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}|1\right)}{d\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\mathrm{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/csgn(cos(1/2*d*x+1/2*c))*cos(1/2
*d*x+1/2*c)*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

maxima [B] time = 1.31, size = 90, normalized size = 1.96

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx\right.\right.}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

mupad [B] time = 0.36, size = 45, normalized size = 0.98

$$\frac{\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\cos(c+d\,x))}{a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*
cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cos(c + d*x) + a), x)
```

3.127 $\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/2*
sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2780, 2649, 206, 2773}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx &= \frac{\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx}{a} - \int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx\\ &= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 0.76

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*(ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]])
*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [B] time = 0.73, size = 164, normalized size = 1.93

$$\frac{\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\mathrm{cc}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*
cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)))/(a*d)
```

giac [B] time = 1.87, size = 162, normalized size = 1.91

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{a}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{a}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))/abs(a) + log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2)/sqrt(a))/d
```

maple [B] time = 0.61, size = 224, normalized size = 2.64

$$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)-\ln\left(\frac{4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}+4a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)}{\sqrt{a}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))-ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))/a^(1/2
)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/sqrt(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.128 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1/2*s
in(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+tan(d*x
+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2779, 2985, 2649, 206, 2773}

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]/(Sqrt[a]*d)) + (
Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])
/(Sqrt[a]*d) + Tan[c + d*x]/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx &= \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(a-a\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a} \\
&= \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx}{2a} + \int \frac{1}{\sqrt{a+a\cos(c+ \cdots}} \\
&= \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx \cdots\right)}{d} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 27.56, size = 1540, normalized size = 14.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((1/4 - I/4)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*I)*E^(
((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x) - (3
4 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*c + ((
5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sqrt[2]*
E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (8*I)*E
^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)*(c +
d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x)) - 1
6*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[2]*E^(
(4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2 + (d*
x)/2])/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((
I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))
 + I*E^((2*I)*(c + d*x)))^2*Sqrt[a*(1 + Cos[c + d*x])]) + (I*ArcTan[(Cos[c/
4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(-Cos[c/4 +
 (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*
x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + (I*ArcTan[(Cos[c/4 + (d*x)/
4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] +
 Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/2])/(Sqr
t[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) - (2*Cos[c/2 + (d*x)/2]*Log[Cos[c/4 + (d
*x)/4] - Sin[c/4 + (d*x)/4]])/(d*Sqrt[a*(1 + Cos[c + d*x])]) + (2*Cos[c/2 +
 (d*x)/2]*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]])/(d*Sqrt[a*(1 + Cos[
c + d*x])]) + (Cos[c/2 + (d*x)/2]*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt
[2]*Sin[c/2 + (d*x)/2]])/(2*Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + (Cos[c/
2 + (d*x)/2]*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2
]])/(2*Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + ((2*I)*ArcTan[((2*I)*Cos[c/2
] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[
c/2]^2]]*Cos[c/2 + (d*x)/2]*Cot[c/2])/(d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-2
 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) - (Sqrt[2]*Cos[c/2 + (d*x)/2]*Csc[c/2]*(-(
d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d
*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*
Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/S
qrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]))/(d*Sqrt[a*(1 + Cos[c + d*x])]*(4*Co
s[c/2]^2 + 4*Sin[c/2]^2)) + Cos[c/2 + (d*x)/2]/(d*Sqrt[a*(1 + Cos[c + d*x])
]*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - Cos[c/2 + (d*x)/2]/(d*Sqrt[a
*(1 + Cos[c + d*x])]*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [B] time = 0.90, size = 236, normalized size = 2.19

$$\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\,\sqrt{2}\left(a\cos\cdots\right.}{\cdots}}{4\left(ad\cos\left(dx+c\right)^2+ad\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*co
s(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*
x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c
)^2 + a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1))/sqrt(a) + 4*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a*d*cos(d*x
 + c)^2 + a*d*cos(d*x + c))
```

giac [B] time = 2.71, size = 290, normalized size = 2.69

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{a}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{2\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)}{\sqrt{a}}-\frac{\cdots}{\left(\sqrt{a}\tan\left(\cdots\right.\right.}\cdots\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)
*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))/abs(a) + 2*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - a^(3/2))/((sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

maple [B] time = 0.67, size = 466, normalized size = 4.31

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-2a\left(2\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)-\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}-a\sqrt{2}\,\cos\left(\cdots\right)\cdots\right)}{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\cdots\right)\cdots\right)\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(2*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))-ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))
*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a))*a+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
-ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.129 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=147

$$-\frac{\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 7/4*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/
2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-1/4*
tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*
x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2779, 2984, 2985, 2649, 206, 2773}

$$-\frac{\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d) - Tan[c + d*x]/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (Sec[c + d*
x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(a-3a\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\left(-\frac{7a^2}{2}+\frac{1}{2}a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,d}{4a^2}$$

$$= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{7\int \sqrt{a+a\cos(c+dx)}\,\sec(c}{8a}$$

$$= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(}{\sqrt{a+a\,c}}\right.}{4d}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 31.17, size = 1791, normalized size = 12.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-7/16 + (7*I)/16)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16
*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*
x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)
*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*S
qrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) +
(8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2
)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*
x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt
[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/
2 + (d*x)/2])/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[
2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c
+ d*x)) + I*E^((2*I)*(c + d*x)))^2*Sqrt[a*(1 + Cos[c + d*x])]) - (((7*I)/4)
*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/
4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])
]*Cos[c/2 + (d*x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) - (((7*I)/4)*A
rcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4]
)/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*C
```

```
os[c/2 + (d*x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + (2*Cos[c/2 + (d
*x)/2]*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(d*Sqrt[a*(1 + Cos[c +
 d*x])]) - (2*Cos[c/2 + (d*x)/2]*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4
]])/(d*Sqrt[a*(1 + Cos[c + d*x])]) - (7*Cos[c/2 + (d*x)/2]*Log[2 - Sqrt[2]*
Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(8*Sqrt[2]*d*Sqrt[a*(1 +
Cos[c + d*x])]) - (7*Cos[c/2 + (d*x)/2]*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2]
- Sqrt[2]*Sin[c/2 + (d*x)/2]])/(8*Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) - (
((7*I)/2)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/
Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]*Cot[c/2])/(d*Sqr
t[a*(1 + Cos[c + d*x])]*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (7*Cos[c/
2 + (d*x)/2]*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Sin
[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*Co
s[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4
*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]))/(2*Sqrt[2]
*d*Sqrt[a*(1 + Cos[c + d*x])]*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + (Cos[c/2 + (
d*x)/2]*Sin[(d*x)/2])/(2*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2] - Sin[c/2])
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 + (d*x)/2]*(-Cos[c
/2] + 3*Sin[c/2]))/(4*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2] - Sin[c/2])*(C
os[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c/2 + (d*x)/2]*Sin[(d*x)/2]
)/(2*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 + (d*x)/2]*(Cos[c/2] + 3*Sin[c/2]))/(4
*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2]))
```

fricas [B] time = 0.80, size = 251, normalized size = 1.71

$$\frac{7\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)-4\,\sqrt{a\cos\cdots}}{16\left(ad\cos\left(dx+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x +
c) + a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*sqrt(2)*(a*cos(d*x + c)^3 + a*c
os(d*x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

giac [B] time = 2.34, size = 371, normalized size = 2.52

$$\sqrt{2}\left(\frac{7\sqrt{2}\sqrt{a}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{8\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)}{\sqrt{a}}-\frac{8\left(17\left(\sqrt{a}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(7*sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))/abs(a) + 8*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(17*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 57*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 19*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 3*a^(7/2))/
((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a
^2)^2)/d
```

maple [B] time = 0.68, size = 671, normalized size = 4.56

$$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4a\left(-8\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)+7\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\mathrm{c}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*a*(-8*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+7*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+7*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
```

```
)*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-32*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+28*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a+28*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-4*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+8*2^(1/2)*ln(4
/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-7*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+
1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\,\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.130 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{7\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\tan}{12}$$

```
[Out] -9/8*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1
/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+7/8
*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1/12*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(
d*x+c))^(1/2)+1/3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2779, 2984, 2985, 2649, 206, 2773}

$$\frac{7\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}\,d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\tan}{12}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-9*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*Sqrt[a]*d)
 + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]
])])/(Sqrt[a]*d) + (7*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) - (Sec[c
 + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (Sec[c + d*x]^2*Tan
[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(a-5a\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{6a}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\left(-\frac{21a^2}{2}+\frac{3}{2}a^2\cos(c+dx)\right)\sec^2(c+}{\sqrt{a+a\cos(c+dx)}}}{12a^2}$$

$$= \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \int$$

$$= \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - 9\int$$

$$= \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{9\,\mathrm{Su}}{}$$

$$= -\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx}}$$

Mathematica [C] time = 30.13, size = 1921, normalized size = 10.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((9/32 - (9*I)/32)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*
I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x
) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*
c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sq
rt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (
8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)
*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x
)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[
2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2
 + (d*x)/2])/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2
]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c +
 d*x)) + I*E^((2*I)*(c + d*x)))^2*Sqrt[a*(1 + Cos[c + d*x])]) + (((9*I)/8)*
ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4
```

```
])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]
*Cos[c/2 + (d*x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + (((9*I)/8)*Ar
cTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])
/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Co
s[c/2 + (d*x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) - (2*Cos[c/2 + (d*
x)/2]*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(d*Sqrt[a*(1 + Cos[c +
d*x])]) + (2*Cos[c/2 + (d*x)/2]*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]
])/(d*Sqrt[a*(1 + Cos[c + d*x])]) + (9*Cos[c/2 + (d*x)/2]*Log[2 - Sqrt[2]*C
os[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(16*Sqrt[2]*d*Sqrt[a*(1 +
Cos[c + d*x])]) + (9*Cos[c/2 + (d*x)/2]*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2]
- Sqrt[2]*Sin[c/2 + (d*x)/2]])/(16*Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) +
(((9*I)/4)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])
/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]*Cot[c/2])/(d*Sq
rt[a*(1 + Cos[c + d*x])]*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) - (9*Cos[c
/2 + (d*x)/2]*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Si
n[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*C
os[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 +
4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]))/(4*Sqrt[2
]*d*Sqrt[a*(1 + Cos[c + d*x])]*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + Cos[c/2 + (
d*x)/2]/(6*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*
x)/2])^3) - (Cos[c/2 + (d*x)/2]*Sin[(d*x)/2])/(4*d*Sqrt[a*(1 + Cos[c + d*x]
)]*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Co
s[c/2 + (d*x)/2]*(7*Cos[c/2] - 9*Sin[c/2]))/(8*d*Sqrt[a*(1 + Cos[c + d*x])]
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - Cos[c/2
 + (d*x)/2]/(6*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
 (d*x)/2])^3) - (Cos[c/2 + (d*x)/2]*Sin[(d*x)/2])/(4*d*Sqrt[a*(1 + Cos[c +
d*x])]*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) +
 (Cos[c/2 + (d*x)/2]*(-7*Cos[c/2] - 9*Sin[c/2]))/(8*d*Sqrt[a*(1 + Cos[c + d
*x])]*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.85, size = 263, normalized size = 1.45

$$\frac{27\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\,\sqrt{a}}{96\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(27*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*
sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
 c) + a)*(21*cos(d*x + c)^2 - 2*cos(d*x + c) + 8)*sin(d*x + c) + 48*sqrt(2)
```

```
*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x +
 c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c
)^3)
```

giac [B] time = 3.73, size = 451, normalized size = 2.49

$$-\frac{\sqrt{2}\left(\frac{27\sqrt{2}\sqrt{a}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{48\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{a}}-\frac{8\left(165\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(27*sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/abs(a) + 48*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(165*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(a) - 1323*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*a^(3/2) + 3906*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(5/2) - 21
18*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^
(7/2) + 393*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2*a^(9/2) - 31*a^(11/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

maple [B] time = 0.62, size = 875, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*a*(-16*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+9*ln(4/(2
```

```
*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+9*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+(576*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-324*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2*a))*a-324*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+168*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^4+(-288*2^(1
/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+1
62*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+162*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a-160*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
))*sin(1/2*d*x+1/2*c)^2+48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+a))*a-27*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*
a))*a-27*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+54*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(
1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.131 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{13\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{10a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{9\sin(c+dx)\cos^2(c}{10ad\sqrt{a\cos(c+dx}}$$

```
[Out] -1/2*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-15/4*arctanh(1/2*sin(
d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+31/5*sin(d
*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+9/10*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(
d*x+c))^(1/2)-13/10*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.40, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2765, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{13\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{10a^2d}-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{9\sin(c+dx)\cos^2(c}{10ad\sqrt{a\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x
])^(3/2)) + (31*Sin[c + d*x])/(5*a*d*Sqrt[a + a*Cos[c + d*x]]) + (9*Cos[c +
 d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]]) - (13*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(10*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos^2(c+dx)\left(3a-\frac{9}{2}a\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(-9a^2+\frac{39}{4}a^2\cos(c+\right.}{\sqrt{a+a\cos(c+dx)}}}{5a^3}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-9a^2\cos(c+dx)+\frac{39}{4}a^2\cos^2(c}{\sqrt{a+a\cos(c+dx)}}}{5a^3}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{13\sqrt{a+a\cos(c+dx)}\ \mathrm{si}}{10a^2d}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx}}$$

Mathematica [A] time = 1.36, size = 226, normalized size = 1.23

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(200\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)-20\sin\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)+4\sin\left(\frac{5c}{2}\right)\cos\left(\frac{5dx}{2}\right)+200\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-20\,\mathrm{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(150*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 150*Log
[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] + 200*Cos[(d*x)/2]*Sin[c/2] - 20*Cos[
(3*d*x)/2]*Sin[(3*c)/2] + 4*Cos[(5*d*x)/2]*Sin[(5*c)/2] + 200*Cos[c/2]*Sin[
(d*x)/2] - 20*Cos[(3*c)/2]*Sin[(3*d*x)/2] + 4*Cos[(5*c)/2]*Sin[(5*d*x)/2] +
```

```
5/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - 5/(Cos[(c + d*x)/4] + Sin[(c +
d*x)/4])^2))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.98, size = 184, normalized size = 1.01

$$\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+4\left(\cdots\right.}{40\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(d\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x +
 c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x + c) + 49)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 1.27, size = 137, normalized size = 0.75

$$\frac{\frac{75\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(5\sqrt{2}a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+127\sqrt{2}a\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+175\sqrt{2}a\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+85\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{5}{2}}}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/20*(75*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/a^(3/2) + (((5*sqrt(2)*a*tan(1/2*d*x + 1/2*c)^2 + 127*sq
rt(2)*a)*tan(1/2*d*x + 1/2*c)^2 + 175*sqrt(2)*a)*tan(1/2*d*x + 1/2*c)^2 + 8
5*sqrt(2)*a)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

maple [A] time = 0.32, size = 265, normalized size = 1.45

$$-\frac{\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(32\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-32\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^{4}\left(\frac{dx}{2}+\cdots\right.\right.\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/20/cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(32*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6-32*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+80*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)*sin(1/2*d*x+1/2*c)^2-75*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a))*a*sin(1/2*d*x+1/2*c)^2-85*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+75*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+a))*a)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.132 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{13\sin(c+dx)}{3ad\sqrt{a\cos(c+dx)+}}$$

```
[Out] -1/2*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+11/4*arctanh(1/2*sin(
d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-13/3*sin(d
*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+7/6*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/
d
```

Rubi [A] time = 0.26, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2968, 3023, 2751, 2649, 206}

$$\frac{7\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{13\sin(c+dx)}{3ad\sqrt{a\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (11*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2)) - (13*Sin[c + d*x])/(3*a*d*Sqrt[a + a*Cos[c + d*x]]) + (7*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(6*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(2a-\frac{7}{2}a\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{2a\cos(c+dx)-\frac{7}{2}a\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{6a^2d} - \frac{\int \frac{-\frac{7a^2}{4}+\frac{13}{2}a^2\cos(c+}{\sqrt{a+a\cos(c+dx)}}}{3a^3}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + \frac{7\sqrt{a+a\cos(c+dx)}\,\sin(}{6a^2d}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + \frac{7\sqrt{a+a\cos(c+dx)}\,\sin(}{6a^2d}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx}}$$

Mathematica [A] time = 0.96, size = 196, normalized size = 1.35

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-72\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)+8\sin\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)-72\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+8\cos\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)-\frac{}{(\cos(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(-66*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 66*Log[
Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] - 72*Cos[(d*x)/2]*Sin[c/2] + 8*Cos[(3*
d*x)/2]*Sin[(3*c)/2] - 72*Cos[c/2]*Sin[(d*x)/2] + 8*Cos[(3*c)/2]*Sin[(3*d*x
)/2] - 3/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 + 3/(Cos[(c + d*x)/4] + Si
n[(c + d*x)/4])^2))/(6*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.68, size = 174, normalized size = 1.20

$$\frac{33\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4}{24\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d
*x + c) + a)*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sin(d*x + c))/(a^2*d
*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 1.35, size = 115, normalized size = 0.79

$$-\frac{\frac{\left(\left(3\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+46\,\sqrt{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+27\,\sqrt{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}}+\frac{33\,\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2 + 46*sqrt(2))*tan(1/2*d*x + 1/2*c
)^2 + 27*sqrt(2))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)
 + 33*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a)))/a^(3/2))/d
```

maple [A] time = 0.32, size = 234, normalized size = 1.61

$$\frac{\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(16\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/12/cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+8*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2-33*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*sin(1/2*d*x+1/2*c)^2-27*a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+33*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+a))*a)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.133 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-7/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*
2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+2*sin(d*x+c)/a/d/(a+a*cos
(d*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2751, 2649, 206}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (2*Sin
[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{-\frac{3a}{2}+2a\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{7\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{7\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\,\mathrm{s}}{\sqrt{a+}}\right.}{2ad}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.46, size = 164, normalized size = 1.56

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(16\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)+16\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{1}{\left(\sin\left(\frac{1}{4}(c+dx)\right)+\cos\left(\frac{1}{4}(c+dx)\right)\right.}\right.}{2d(a(\cos(c+dx)+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(14*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 14*Log[C
os[(c + d*x)/4] + Sin[(c + d*x)/4]] + 16*Cos[(d*x)/2]*Sin[c/2] + 16*Cos[c/2
]*Sin[(d*x)/2] + (Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^(-2) - (Cos[(c + d*x
)/4] + Sin[(c + d*x)/4])^(-2)))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.70, size = 164, normalized size = 1.56

$$\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{a}\ldots}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*co
s(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*x
 + c) + a)*(4*cos(d*x + c) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 2.05, size = 102, normalized size = 0.97

$$\frac{\frac{\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}+\frac{9\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{7\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a + 9*sqrt(2)/a)*tan(1/2*d*x + 1/2*c)/
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + 7*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x
 + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2))/d
```

maple [A] time = 0.31, size = 173, normalized size = 1.65

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-7\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\sqrt{a}\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos^2\left(\ldots\right.\right.}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-7*2^(1/2)*ln(2*(2*a
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*
```

```
x+1/2*c)^2+8*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2
*c)^2+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/sin(1/2*d*x+1
/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.134 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+3/4*arctanh(1/2*sin(d*x+c)*a^(1/2)
*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2750, 2649, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$
$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}$$
$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.10, size = 54, normalized size = 0.70

$$\frac{3\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{1}{2}\sin(c+dx)}{d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 - Sin[c + d*x]/2)/(d*(a*(1
+ Cos[c + d*x]))^(3/2))
```

fricas [B] time = 0.88, size = 154, normalized size = 2.00

$$\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\sqrt{a}}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*co
s(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2
*d)
```

giac [A] time = 1.43, size = 81, normalized size = 1.05

$$-\frac{\frac{3\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a)))/a^(3/2) + sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(
1/2*d*x + 1/2*c)/a^2)/d
```

maple [B] time = 0.30, size = 140, normalized size = 1.82

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\right)}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2-2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)/a^(5/2)/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)/(a + a*cos(c + d*x))^(3/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (c + dx)}{(a (\cos (c + dx) + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.135 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*
2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[
2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.82

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + Tan[(c +
d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [B] time = 1.78, size = 153, normalized size = 1.99

$$\frac{\sqrt{2}\left(\cos(dx+c)^2+2\,\cos(dx+c)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)+4\,\sqrt{a\,c}}{8\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(
d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
)
```

giac [A] time = 1.24, size = 81, normalized size = 1.05

$$-\frac{\frac{\sqrt{2}\,\log\left(\left|-\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\,\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a)))/a^(3/2) - sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/
2*d*x + 1/2*c)/a^2)/d
```

maple [B] time = 0.00, size = 138, normalized size = 1.79

$$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\right)}{4a^{\frac{5}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(2
*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1
/2*d*x+1/2*c)^2+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x
+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + a)**(-3/2), x)
```

3.136 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 2*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-1/2*sin(d*x+
c)/d/(a+a*cos(d*x+c))^(3/2)-5/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a
*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2766, 2985, 2649, 206, 2773}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) -
(5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
```

```
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2a-\frac{1}{2}a\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx}{a^2} - \frac{5\int \frac{1}{\sqrt{a+a\cos(c}}}{4a}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} + \frac{5\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/}}$$

Mathematica [C] time = 23.75, size = 1787, normalized size = 15.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]

[Out] ((-1 + I)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*I)*E^(((3
*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x) - (34 -
 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*c + ((5*I
)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sqrt[2]*E^(
(4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (8*I)*E^((
I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)*(c + d*x
)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x)) - 16*S
qrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[2]*E^((4*
I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2 + (d*x)/
2]^3)/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I
/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))
+ I*E^((2*I)*(c + d*x)))^2*(a*(1 + Cos[c + d*x]))^(3/2)) - ((2*I)*Sqrt[2]*A
rcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4]
)/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*
Cos[c/2 + (d*x)/2]^3)/(d*(a*(1 + Cos[c + d*x]))^(3/2)) + (5*Cos[c/2 + (d*x)
/2]^3*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(d*(a*(1 + Cos[c + d*x]
))^(3/2)) - (5*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x
)/4]])/(d*(a*(1 + Cos[c + d*x]))^(3/2)) - (Sqrt[2]*Cos[c/2 + (d*x)/2]^3*Log
[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(d*(a*(1 + C
os[c + d*x]))^(3/2)) + ((1 - I)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x
)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 +
(d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/2]^3*((1 + I)*Cos[c/4] + Sq
rt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4])*((-1 - I)*Cos[c/4]
+ Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4]))/(Sqrt[2]*d*(a*
(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] + Sin[c/2])) - ((1/2 + I/2)*Cos[c/2 + (
d*x)/2]^3*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]]*
((1 + I)*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4
])*((-1 - I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin
[c/4]))/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] + Sin[c/2])) - ((
8*I)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[
-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]^3*Cot[c/2])/(d*(a*(1
+ Cos[c + d*x]))^(3/2)*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (4*Sqrt[2]
*Cos[c/2 + (d*x)/2]^3*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[(d*
x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcTan[
((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c
/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]))/(
d*(a*(1 + Cos[c + d*x]))^(3/2)*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) - Cos[c/2 + (
d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 +
 (d*x)/4])^2) + Cos[c/2 + (d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos
[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2)
```

fricas [B] time = 1.54, size = 254, normalized size = 2.23

$$\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+4\left(c\right.}{8\left(a^{2}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*co
s(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
- 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [B] time = 2.76, size = 189, normalized size = 1.66

$$\frac{\frac{5\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)}{a^{\frac{3}{2}}}-\frac{2\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{2}}+\frac{8\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}\right.}\right.\right.\right.}{a^{\frac{3}{2}}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(5*sqrt(2)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^2)/a^(3/2) - 2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1
/2*d*x + 1/2*c)/a^2 + 8*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(3/2) - 8*log(abs((sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt
(2) - 3)))/a^(3/2))/d
```

maple [B] time = 0.67, size = 290, normalized size = 2.54

$$-\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(5\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\ln\left(-\frac{4\left(a\sqrt{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}\right.\right.}\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\right.\right.}{4a^{\frac{5}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2-4*ln(-4*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-4*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a+2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/
(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+a\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -3*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+9/4*arctanh
(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1
/2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+3/2*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^
(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2984, 2985, 2649, 206, 2773}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 (9*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (3*Tan
[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(3a-\frac{3}{2}a\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-3a^2+\frac{3}{2}a^2\cos(c+dx)\right)\sec(c+d}{\sqrt{a+a\cos(c+dx)}}}{2a^3}$$

$$= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{3\int \sqrt{a+a\cos(c+dx)}\,\text{se}}{2a^2}$$

$$= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{3\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{}{\sqrt{a}}\right.}{ad}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\tan(c+dx)}{2d(a+a\cos(c+dx))}$$

Mathematica [A] time = 0.48, size = 103, normalized size = 0.72

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(2\sec(c+dx)+3)+9\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\right.}{2ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (9*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] - 6*Sqrt[2]*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]]*Cos[(c + d*x)/2] + (3 + 2*Sec[c + d*x])*Tan[(c + d*x)/2])/
(2*a*d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [B] time = 1.21, size = 286, normalized size = 1.99

$$\frac{9\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*l
og(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x
```

```
+ c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 6*(
cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c
) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*co
s(d*x + c) + a)*(3*cos(d*x + c) + 2)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 +
2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.63, size = 567, normalized size = 3.94

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(18\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\cdots\right.\right.}\right.}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(18*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4-12*ln(-4
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-12*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a-9*2^(1/2)*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*
d*x+1/2*c)^2+6*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1
/2*c)^2+6*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2
*a+6*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a-2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*
c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}{\left(c+dx\right)}}{\left(a\left(\cos{\left(c+dx\right)}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.138 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{7\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{\tan(c+dx)\sec(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}-\frac{\tan(}{2d(a}$$

```
[Out] 19/4*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-13/4*arct
anh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2
)-1/2*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-7/4*tan(d*x+c)/a/d/(a+
a*cos(d*x+c))^(1/2)+sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2984, 2985, 2649, 206, 2773}

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{7\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{\tan(c+dx)\sec(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}-\frac{\tan(}{2d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*a^(3/2)*d)
 - (13*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/
(2*Sqrt[2]*a^(3/2)*d) - (7*Tan[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]) -
 (Sec[c + d*x]*Tan[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (Sec[c + d*
x]*Tan[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(4a-\frac{5}{2}a\cos(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-7a^2+6a^2\cos(c+dx)\right)\sec^2(c+dx}{\sqrt{a+a\cos(c+dx)}}}{4a^3}$$

$$= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \int$$

$$= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + 1$$

$$= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - 1$$

$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{13\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx}}$$

Mathematica [C] time = 28.43, size = 1941, normalized size = 10.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((-19/8 + (19*I)/8)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16
*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*
x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)
*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*S
qrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) +
(8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2
)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*
x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt
[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/
2 + (d*x)/2]^3)/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqr
t[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(
c + d*x)) + I*E^((2*I)*(c + d*x)))^2*(a*(1 + Cos[c + d*x]))^(3/2)) - (((19*
I)/2)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (
```

```
d*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x
)/4])]*Cos[c/2 + (d*x)/2]^3)/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)) - (((
19*I)/2)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4
+ (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d
*x)/4])]*Cos[c/2 + (d*x)/2]^3)/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)) + (
13*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(d*(a
*(1 + Cos[c + d*x]))^(3/2)) - (13*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/
4] + Sin[c/4 + (d*x)/4]])/(d*(a*(1 + Cos[c + d*x]))^(3/2)) - (19*Cos[c/2 +
(d*x)/2]^3*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]]
)/(4*Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)) - (19*Cos[c/2 + (d*x)/2]^3*Log
[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(4*Sqrt[2]*d
*(a*(1 + Cos[c + d*x]))^(3/2)) - ((19*I)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[
2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[
c/2 + (d*x)/2]^3*Cot[c/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*Sqrt[-2 + 4*Cos[
c/2]^2 + 4*Sin[c/2]^2]) + (19*Cos[c/2 + (d*x)/2]^3*Csc[c/2]*(-(d*x*Cos[c/2]
) + 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[
c/2] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Ta
n[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*C
os[c/2]^2 + 4*Sin[c/2]^2]))/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)*(4*Cos[
c/2]^2 + 4*Sin[c/2]^2)) - Cos[c/2 + (d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^
(3/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) + Cos[c/2 + (d*x)/2]^3/(
2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^
2) + (Cos[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(C
os[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c/2
 + (d*x)/2]^3*(-5*Cos[c/2] + 7*Sin[c/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c/
2 + (d*x)/2]^3*Sin[(d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] + Si
n[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 + (d*x)/2]^
3*(5*Cos[c/2] + 7*Sin[c/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] +
Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 1.63, size = 302, normalized size = 1.63

$$\frac{26\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(d}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(26*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(
a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(
d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
 19*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos
(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(
```

```
d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sq
rt(a*cos(d*x + c) + a)*(7*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)*sin(d*x + c)
)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos((d*t_nostep+c)/2))]Discontinuities at zeroes of
cos((d*t_nostep+c)/2) were not checkedUnable to divide, perhaps due to roun
ding error%%%{%%{[%%%{%%{[350488137400481480704,0]:[1,0,-2]%%},[16]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[0]%%%} / %%%{%%%{%%{[18446744073709551616,0]:[1,0,
-2]%%},[16]%%%},[0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.05, size = 807, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(104*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*a-76*ln(
-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a-76*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^6*a-104*2^(1/2)*ln(
2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(
1/2*d*x+1/2*c)^4+28*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*
d*x+1/2*c)^4+76*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/
2*c)^4*a+76*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^
4*a+26*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*
d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2-22*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-19*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/
```

```
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2
)))*cos(1/2*d*x+1/2*c)^2*a-19*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*c
os(1/2*d*x+1/2*c)^2*a+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(
5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}{\left(c + dx \right)}}{\left(a \left(\cos{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**3/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.139 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{95\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}-\frac{197\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\cos^3(c}{4d(a\cos(c+dx)+}$$

```
[Out] -1/4*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-17/16*cos(d*x+c)^2*si
n(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+163/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2
^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-197/24*sin(d*x+c)/a^2/d/(a
+a*cos(d*x+c))^(1/2)+95/48*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.41, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2765, 2977, 2968, 3023, 2751, 2649, 206}

$$\frac{95\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}-\frac{197\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}+\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^3(c}{4d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (163*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*
x])^(5/2)) - (17*Cos[c + d*x]^2*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^
(3/2)) - (197*Sin[c + d*x])/(24*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + (95*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(48*a^3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos^2(c+dx)\left(3a-\frac{11}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(17a^2-\frac{95}{4}a^2\cos(c\right.}{\sqrt{a+a\cos(c+dx)}}}{8a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{17a^2\cos(c+dx)-\frac{95}{4}a^2\cos^2(c}{\sqrt{a+a\cos(c+dx)}}}{8a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{95\sqrt{a+a\cos(c+dx)}\ \text{s}}{48a^3d}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{197\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{197\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+}{16ad(a+a\cos(c+dx}$$

Mathematica [B] time = 6.35, size = 587, normalized size = 3.21

$$-\frac{40\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(a(\cos(c+dx)+1))^{5/2}}+\frac{8\sin\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d(a(\cos(c+dx)+1))^{5/2}}-\frac{40\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(a(\cos(c+dx)+1))^{5/2}}+\frac{8\cos}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-163*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(4
*d*(a*(1 + Cos[c + d*x]))^(5/2)) + (163*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 +
(d*x)/4] + Sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) - (40*Co
s[(d*x)/2]*Cos[c/2 + (d*x)/2]^5*Sin[c/2])/(d*(a*(1 + Cos[c + d*x]))^(5/2))
+ (8*Cos[(3*d*x)/2]*Cos[c/2 + (d*x)/2]^5*Sin[(3*c)/2])/(3*d*(a*(1 + Cos[c +
 d*x]))^(5/2)) - (40*Cos[c/2]*Cos[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(d*(a*(1 +
```

```
Cos[c + d*x]))^(5/2)) + (8*Cos[(3*c)/2]*Cos[c/2 + (d*x)/2]^5*Sin[(3*d*x)/2
])/(3*d*(a*(1 + Cos[c + d*x]))^(5/2)) + Cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + C
os[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^4) - (29*Cos[
c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - S
in[c/4 + (d*x)/4])^2) - Cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5
/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^4) + (29*Cos[c/2 + (d*x)/2]^5
)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4
])^2)
```

fricas [A] time = 0.83, size = 208, normalized size = 1.14

$$\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{192\left(a^3d\cos(dx+c)^3+3a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)) + 4*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 - 503*cos(d*x + c) - 299)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 3.96, size = 146, normalized size = 0.80

$$\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}-\frac{23\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{668\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{465\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}-\frac{489\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}\right.}\right.\right.}{a^{\frac{5}{2}}}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a - 23*sqrt(2)/a)*tan(1/2*d*x +
 1/2*c)^2 - 668*sqrt(2)/a)*tan(1/2*d*x + 1/2*c)^2 - 465*sqrt(2)/a)*tan(1/2*
d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 489*sqrt(2)*log(abs(-sq
rt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/
d
```

maple [A] time = 0.32, size = 242, normalized size = 1.32

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(128\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+489\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos\right.\right.}{96\cos\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(128*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+489*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4-512*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-87*a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+6*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)^3/a^(7/2)/sin(1/2*d*x+
1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.140 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{9\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{13\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -1/4*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+13/16*sin(d*x+c)/a/d/
(a+a*cos(d*x+c))^(3/2)-75/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*co
s(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+9/4*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1
/2)
```

Rubi [A] time = 0.27, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2968, 3019, 2751, 2649, 206}

$$\frac{9\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{13\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^2*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*
x])^(5/2)) + (13*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (9*Sin
[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos(c+dx)\left(2a-\frac{9}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{2a\cos(c+dx)-\frac{9}{2}a\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{-\frac{39a^2}{4}+9a^2\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{8a^4}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+a}$$

Mathematica [A] time = 4.05, size = 216, normalized size = 1.49

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(128\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)+128\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{21}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{21}{\left(\sin\left(\frac{1}{4}(c+dx)\right)+\cos\left(\frac{1}{4}(c+d\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(150*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 150*Log
[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] + 128*Cos[(d*x)/2]*Sin[c/2] + 128*Cos
[c/2]*Sin[(d*x)/2] - (Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^(-4) + 21/(Cos[(
c + d*x)/4] - Sin[(c + d*x)/4])^2 + (Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^(
-4) - 21/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2))/(8*d*(a*(1 + Cos[c + d*x
]))^(5/2))
```

fricas [A] time = 1.12, size = 198, normalized size = 1.37

$$\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) + 4*sqrt(a*cos(d*x + c) + a)*(32*cos(d*x + c)^2 + 85*cos(d*x + c) + 49)*
sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

giac [A] time = 3.13, size = 124, normalized size = 0.86

$$-\frac{\frac{\left(\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{2}}-\frac{17\sqrt{2}}{a^{2}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\frac{83\sqrt{2}}{a^{2}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}-\frac{75\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(((2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a^2 - 17*sqrt(2)/a^2)*tan(1/2*d*x
 + 1/2*c)^2 - 83*sqrt(2)/a^2)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a) - 75*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

maple [A] time = 0.42, size = 208, normalized size = 1.43

$$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-75\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\cos\right.\right.}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-75*2^(1/2)*ln(2*
(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/
2*d*x+1/2*c)^4+64*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*
x+1/2*c)^4+21*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)^2-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1/2*d*
x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^3}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.141 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-13/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c
))^(3/2)+19/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2
))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2750, 2649, 206}

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (13*S
in[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{-\frac{5a}{2}+4a\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{19\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{19\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\right.}{16a^2d}$$

$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx)}$$

Mathematica [A] time = 1.13, size = 103, normalized size = 0.96

$$\frac{-18\sin(c+dx) - 13\sin(2(c+dx)) - 152\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{4}(c+dx)\right) - \sin\left(\frac{1}{4}(c+dx)\right)\right) - \log\left(\sin\right.\right.}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-152*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - Log[Co
s[(c + d*x)/4] + Sin[(c + d*x)/4]]) - 18*Sin[c + d*x] - 13*Sin[2*(c + d*x)]
)/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [B] time = 1.04, size = 188, normalized size = 1.76

$$\frac{19\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) - 4*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c) + 9)*sin(d*x + c))/(a^3*d*
cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.86, size = 103, normalized size = 0.96

$$\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{3}}-\frac{11\sqrt{2}}{a^{3}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{19\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+\right.\right.}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/
a^3 - 11*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 19*sqrt(2)*log(abs(-sqrt(a)*ta
n(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

maple [A] time = 0.36, size = 174, normalized size = 1.63

$$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(19\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-13\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos^{2}\left(\right.\right.\right.}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(19*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4-13*a^(1
```

```
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+2*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)^3/a^(7/2)/sin(1/2
*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.142 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] -1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+5/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c
))^(3/2)+5/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2)
)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2750, 2650, 2649, 206}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (5*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \cdot\right.}{16a^2d}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))}$$

Mathematica [A] time = 0.25, size = 65, normalized size = 0.61

$$\frac{2\sin(c+dx) + 5\sin(2(c+dx)) + 40\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (40*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 2*Sin[c + d*x] + 5*Sin[2
*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [B] time = 1.02, size = 188, normalized size = 1.76

$$\frac{5\sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sin(dx+c) - 2a\,c}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right.}{64\left(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + 3a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*s
in(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
) + 4*sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c) + 1)*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.46, size = 103, normalized size = 0.96

$$-\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}-\frac{3\sqrt{2}}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{5\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\cdots}\right.\right.}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/a^3 - 3*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) + 5*sqrt(2)*log(abs(-sqrt(a)*tan
(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

maple [A] time = 0.31, size = 174, normalized size = 1.63

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(5\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos^2\left(\frac{dx}{2}\cdots\right.\right.\right.}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*2^(1/2)*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*
d*x+1/2*c)^4+5*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1
/2*c)^2-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a(\cos(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.143 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+3/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c)
)^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))
/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx}{8a}$$
$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}$$
$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\right.}{16a^2d}$$
$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))}$$

Mathematica [A] time = 0.14, size = 65, normalized size = 0.61

$$\frac{14\sin(c+dx)+3\sin(2(c+dx))+24\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (24*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 14*Sin[c + d*x] + 3*Sin[
2*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [B] time = 1.20, size = 188, normalized size = 1.76

$$\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2ac}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*s
in(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
```

```
) + 4*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c) + 7)*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 2.36, size = 103, normalized size = 0.96

$$\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}+\frac{5\,\sqrt{2}}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,\sqrt{2}\,\log\left(\left|-\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right.\right.}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/
a^3 + 5*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 3*sqrt(2)*log(abs(-sqrt(a)*tan(
1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

maple [A] time = 0.30, size = 174, normalized size = 1.63

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos^2\left(\frac{dx}{2}\right.\right.\right.}{32a^{\frac{7}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2)
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*
cos(1/2*d*x+1/2*c)^4+3*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)^2+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d
*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(c + d*x))^(5/2),x)
[Out] int(1/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2),x)
[Out] Integral((a*cos(c + d*x) + a)**(-5/2), x)
```

3.144 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{11\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 2*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*sin(d*x+
c)/d/(a+a*cos(d*x+c))^(5/2)-11/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-43/
32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d
*2^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2985, 2649, 206, 2773}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{11\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) -
(43*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (11*S
in[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(4a-\frac{3}{2}a\cos(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(8a^2-\frac{11}{4}a^2\cos(c+dx)\right)\sec(c+}{\sqrt{a+a\cos(c+dx)}}}{8a^4}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\cos(c+dx)}\,\mathrm{sec}}{a^3}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\right.}{a^2d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [C] time = 24.14, size = 1919, normalized size = 13.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((-2 + 2*I)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*I)*E^((
(3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x) - (34
 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*c + ((5
*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sqrt[2]*E
^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (8*I)*E^
((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)*(c + d
*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x)) - 16
*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[2]*E^((
4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2 + (d*x
)/2]^5)/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^(
(I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)
) + I*E^((2*I)*(c + d*x)))^2*(a*(1 + Cos[c + d*x]))^(5/2)) - ((4*I)*Sqrt[2]
*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/
4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])
]*Cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2)) + (43*Cos[c/2 + (d
*x)/2]^5*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c +
 d*x]))^(5/2)) - (43*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] + Sin[c/4
```

```
+ (d*x)/4]])/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) - (2*Sqrt[2]*Cos[c/2 + (d*x
)/2]^5*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(d
*(a*(1 + Cos[c + d*x]))^(5/2)) + ((1 - I)*Sqrt[2]*ArcTan[(Cos[c/4 + (d*x)/4
] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] +
Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/2]^5*((1
+ I)*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4])*(
(-1 - I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4
]))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2] + Sin[c/2])) - ((1 + I)*Cos[c
/2 + (d*x)/2]^5*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x
)/2]]*((1 + I)*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*S
in[c/4])*((-1 - I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[
2]*Sin[c/4]))/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2] + Sin[c/2])
) - ((16*I)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4]
)/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]^5*Cot[c/2])/(d
*(a*(1 + Cos[c + d*x]))^(5/2)*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (8*
Sqrt[2]*Cos[c/2 + (d*x)/2]^5*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*
Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*
ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 +
4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]
^2]))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) - Cos[
c/2 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - Si
n[c/4 + (d*x)/4])^4) - (11*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x])
)^(5/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) + Cos[c/2 + (d*x)/2]^5
/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]
)^4) + (11*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4
 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2)
```

fricas [B] time = 1.06, size = 298, normalized size = 2.07

$$\frac{43 \sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 + 2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c) - 2a}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) - 4*sqrt(a*cos(d*x + c) + a)*(11*cos(d*x + c) + 15)*sin(d*x + c))/(a^3*d*
cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 3.59, size = 211, normalized size = 1.47

$$-\frac{2\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{3}}+\frac{13\sqrt{2}}{a^{3}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{43\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}\right.\right.}{a^{\frac{5}{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/a^3 + 13*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 43*sqrt(2)*log((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^(5/2) - 64*log
(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2
- a*(2*sqrt(2) + 3)))/a^(5/2) + 64*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(5/2))/d
```

maple [B] time = 0.61, size = 325, normalized size = 2.26

$$-\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(43\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-32\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\cos}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*2^(1/
2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*
a*cos(1/2*d*x+1/2*c)^4-32*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1
/2*d*x+1/2*c)^4*a-32*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d
*x+1/2*c)^4*a+11*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)^2+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2
*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(a*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a(\cos(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.145 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{35\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{15\tan(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -5*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+115/32*arct
anh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2
)-1/4*tan(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-15/16*tan(d*x+c)/a/d/(a+a*cos(d*x
+c))^(3/2)+35/16*tan(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773}

$$\frac{35\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{15\tan(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 (115*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(
16*Sqrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (15
*Tan[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (35*Tan[c + d*x])/(16*
a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
```

```
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(5a-\frac{5}{2}a\cos(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{35a^2}{2}-\frac{45}{4}a^2\cos(c+dx)\right)\sec^2(}{\sqrt{a+a\cos(c+dx)}}}{8a^4}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\cos(c+dx)}$$

Mathematica [C] time = 24.10, size = 2051, normalized size = 11.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((5 - 5*I)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*I)*E^(((
3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x) - (34
- 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*c + ((5*
I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sqrt[2]*E^
((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (8*I)*E^(
(I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)*(c + d*
x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x)) - 16*
```

```
Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[2]*E^((4
*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2 + (d*x)
/2]^5)/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((
I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))
 + I*E^((2*I)*(c + d*x)))^2*(a*(1 + Cos[c + d*x]))^(5/2)) + ((10*I)*Sqrt[2]
*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/
4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])
]*Cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2)) - (115*Cos[c/2 + (
d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c
+ d*x]))^(5/2)) + (115*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] + Sin[c/
4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) + (5*Sqrt[2]*Cos[c/2 + (d
*x)/2]^5*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/
(d*(a*(1 + Cos[c + d*x]))^(5/2)) - ((5 - 5*I)*ArcTan[(Cos[c/4 + (d*x)/4] +
Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt
[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/2]^5*((1 + I)
*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4])*((-1
- I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4]))/
(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2] + Sin[c/2])) + ((5/2 + (5
*I)/2)*Cos[c/2 + (d*x)/2]^5*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Si
n[c/2 + (d*x)/2]]*((1 + I)*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] -
 I*Sqrt[2]*Sin[c/4])*((-1 - I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/
4] - I*Sqrt[2]*Sin[c/4]))/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2]
 + Sin[c/2])) + ((40*I)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*
Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]^5*
Cot[c/2])/(d*(a*(1 + Cos[c + d*x]))^(5/2)*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/
2]^2]) - (20*Sqrt[2]*Cos[c/2 + (d*x)/2]^5*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log
[Sqrt[2] + 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((
4*I)*Sqrt[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4
])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2
 + 4*Sin[c/2]^2]))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(4*Cos[c/2]^2 + 4*Sin[c/
2]^2)) + Cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 +
(d*x)/4] - Sin[c/4 + (d*x)/4])^4) + (19*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 +
Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) - Cos[c/2
 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] + Sin[c
/4 + (d*x)/4])^4) - (19*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(
5/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2) + (4*Cos[c/2 + (d*x)/2]^5
)/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])
) - (4*Cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [B] time = 1.99, size = 330, normalized size = 1.90

$$115\sqrt{2}\left(\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(dx+c\right)^{2}-2\sqrt{2}\sqrt{a\cos\left(dx+c\right)+a}\ \cdots}{\cos\left(dx+c\right)^{2}+2\ \mathrm{co}\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + c
os(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqr
t(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d
*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*(35*cos(d*x + c)^
2 + 55*cos(d*x + c) + 16)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.60, size = 601, normalized size = 3.45

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(230\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-160\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\mathrm{co}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/16*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(230*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*a-160*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^6*a-160*ln(-4*
(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a-115*2^(1/2)*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*co
s(1/2*d*x+1/2*c)^4+70*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/
2*d*x+1/2*c)^4+80*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1
```

```
/2*c)^4*a+80*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c
)^4*a-15*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^
2-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/cos(1/2*d*x+1/2
*c)^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2
*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\,{\left(a+a\,\cos(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a(\cos(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.146 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin(c+$$

```
[Out] 6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d
+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2635, 2639, 2641}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^{\frac{5}{2}}(c+dx)\,dx + a\int \cos^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(3a)\int$$

$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin}{5d}$$

$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+}{21d}$$

Mathematica [C] time = 6.17, size = 490, normalized size = 4.41

$$a\left(-\frac{3\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{10d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*Cot[c])/
(5*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[3
*d*x]*Sin[3*c])/(28*d) + (23*Cos[c]*Sin[d*x])/(84*d) + (Cos[2*c]*Sin[2*d*x]
)/(10*d) + (Cos[3*c]*Sin[3*d*x])/(28*d)) - (5*(1 + Cos[c + d*x])*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
```

```
 ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*(1 + Cos[c + d*x])*Csc[c]
*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.42, size = 270, normalized size = 2.43

$$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(
```

```
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.76, size = 87, normalized size = 0.78

$$-\frac{2\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2F_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,a\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\;{}_2F_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x)),x)
[Out] - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*cos(c + d*x)^(9/2)*sin(c + d*x
)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c)),x)
[Out] Timed out
```

3.147 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2
/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2635, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^{\frac{3}{2}}(c+dx)\,dx + a\int \cos^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \frac{}{\sqrt{c}}$$

$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

Mathematica [C] time = 5.62, size = 232, normalized size = 2.67

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-18\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[
c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*
Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 5.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\cos\left(dx+c\right)^2 + a\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.55, size = 219, normalized size = 2.52

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-28\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2}\right.}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x)
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.13, size = 80, normalized size = 0.92

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}-\frac{2\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;\cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d
*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.148 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2639, 2635, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))\,dx = a\int \sqrt{\cos(c+dx)}\,dx + a\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+a}}$$
$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx}{3d}$$

Mathematica [C] time = 5.01, size = 222, normalized size = 3.64

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right){}_2F}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos[c
 + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*S
in[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x + ArcTa
n[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d
*x]])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+a)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.56, size = 225, normalized size = 3.69

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.12, size = 53, normalized size = 0.87

$$\frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*
d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{\cos\left(c+dx\right)}\,dx+\int\cos^{\frac{3}{2}}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(cos(c + d*x)), x) + Integral(cos(c + d*x)**(3/2), x))
```

3.149 $\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=35

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2748, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + a\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [C] time = 24.64, size = 155, normalized size = 4.43

$$\frac{a\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}} - 2\sin(c)\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d*
x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x + A
rcTan[Tan[c]]] - (HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(
2*d)
```

fricas [F] time = 1.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\cos(dx+c)+a}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.40, size = 150, normalized size = 4.29

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.46, size = 27, normalized size = 0.77

$$\frac{2\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a*(ellipticE(c/2 + (d*x)/2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx+\int\sqrt{\cos(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(cos(c + d*x)), x) + Integral(sqrt(cos(c + d*x)), x))
```

3.150 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \sqrt{\cos(c+dx)}\,dx\\ &= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 9.67, size = 209, normalized size = 3.67

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c - d
*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[S
ec[c]^2] - 4*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x -
 ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[S
in[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.45, size = 146, normalized size = 2.56

$$-\frac{2a\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*a*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.64, size = 60, normalized size = 1.05

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*sin(c + d*x)*hypergeom([-1/4, 1/
2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(cos(c + d*x)**(-3/2), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

3.151 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*
a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2636, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - a\int \sqrt{\cos(c+dx)}\,dx\\ &= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 6.15, size = 444, normalized size = 5.35

$$a\left(\frac{\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\,\mathrm{co}}}\right.}{2d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((Csc[c]*Sec[
c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c + d*x]*(Sin[
c] + 3*Sin[d*x]))/(3*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
```

```
/(3*d*Sqrt[1 + Cot[c]^2]) + ((1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(2*d))
```

fricas [F] time = 1.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

maple [B] time = 0.94, size = 369, normalized size = 4.45

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.78, size = 87, normalized size = 1.05

$$\frac{2\,a\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2]
, 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.152 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)+
2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3a)\int \frac{1}{\cos^{\frac{3}{2}}(c+d}\\
&= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(3a)\\
&= -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6}{5d}\end{aligned}$$

Mathematica [C] time = 6.17, size = 477, normalized size = 4.30

$$a\left(\frac{3\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{10d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*Se
c[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*x]
```

```
^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*Sin
[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
t[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
10*d))
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

maple [B] time = 0.83, size = 384, normalized size = 3.46

$$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{40\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{d}{}\right.}{5\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(s\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 0.87, size = 87, normalized size = 0.78

$$\frac{2\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\;-\frac{1}{4};\;\cos(c+d\,x)^2\right)}{5\,d\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c +
 d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-5/4, 1/
2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.153 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=147

$$\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\,\mathrm{s}}{}$$

```
[Out] 32/15*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+20/21*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/45*a^2*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+4/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*s
in(d*x+c)/d+20/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2635, 2639, 2641}

$$\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2,
2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^{\frac{5}{2}}(c+dx)+2a^2\cos^{\frac{7}{2}}(c+dx)+a^2\cos^{\frac{9}{2}}(c+dx)\right)dx$$

$$= a^2\int \cos^{\frac{5}{2}}(c+dx)\,dx + a^2\int \cos^{\frac{9}{2}}(c+dx)\,dx + \left(2a^2\right)\int \cos^{\frac{7}{2}}(c+dx)$$

$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{4a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos}{}$$

$$= \frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{20a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+}{4}$$

$$= \frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20a^2\sqrt{\cos(c+dx)}\,\sin}{21d}$$

Mathematica [C] time = 6.14, size = 532, normalized size = 3.62

$$-\frac{4\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c)\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+}}}\right.}{15d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((-8*Cot[c])
/(15*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (37*Cos[2*d*x]*Sin[2*c])/(360*d) +
(Cos[3*d*x]*Sin[3*c])/(28*d) + (Cos[4*d*x]*Sin[4*c])/(144*d) + (23*Cos[c]*S
in[d*x])/(84*d) + (37*Cos[2*c]*Sin[2*d*x])/(360*d) + (Cos[3*c]*Sin[3*d*x])/
```

```
(28*d) + (Cos[4*c]*Sin[4*d*x])/(144*d)) - (5*(a + a*Cos[c + d*x])^2*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (4*(a + a*Cos[c + d*x])^2
*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

fricas [F] time = 2.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*s
qrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

maple [A] time = 0.51, size = 260, normalized size = 1.77

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-960\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+608\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*cos
(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*cos
(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.77, size = 136, normalized size = 0.93

$$-\frac{2a^2\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}-\frac{4a^2\cos(c+dx)^{9/2}\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\right.}{9d\sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] - (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^2*cos(c + d*x)^(9/2)*sin(c +
 d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1
/2)) - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4,
 cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.154 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=121

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin(}{}$$

```
[Out] 12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+8/7*a^2*sin(d*x+c)*cos(d*x+c)^(
1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2635, 2641, 2639}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2]
)/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx &= \int \left(a^2\cos^{\frac{3}{2}}(c+dx)+2a^2\cos^{\frac{5}{2}}(c+dx)+a^2\cos^{\frac{7}{2}}(c+dx)\right)dx\\
&= a^2\int \cos^{\frac{3}{2}}(c+dx)\,dx+a^2\int \cos^{\frac{7}{2}}(c+dx)\,dx+\left(2a^2\right)\int \cos^{\frac{5}{2}}(c+dx)\\
&= \frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^2\,\mathrm{cc}}{}\\
&= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sin(c}{7d}\\
&= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sin(c}{7d}
\end{aligned}$$

Mathematica [C] time = 6.13, size = 500, normalized size = 4.13

$$-\frac{3\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c)\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+}}}\right.}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((-3*Cot[c])
/(5*d) + (17*Cos[d*x]*Sin[c])/(56*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[
3*d*x]*Sin[3*c])/(56*d) + (17*Cos[c]*Sin[d*x])/(56*d) + (Cos[2*c]*Sin[2*d*x
])/(10*d) + (Cos[3*c]*Sin[3*d*x])/(56*d)) - (2*(a + a*Cos[c + d*x])^2*Csc[c
```

```
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]
*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (3*(a + a*Cos[c + d*x])^
2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

fricas [F] time = 1.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos\left(dx+c\right)^3+2\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.46, size = 272, normalized size = 2.25

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
```

```
)^6+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.65, size = 129, normalized size = 1.07

$$\frac{2\left(a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)\right)}{3\,d}-\frac{4\,a^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\right.}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (4*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*cos(c + d*x)^(9/2)*
sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x
)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.155 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=95

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)/d+4/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2639, 2635, 2641}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\,dx &= \int \left(a^2\sqrt{\cos(c+dx)}+2a^2\cos^{\frac{3}{2}}(c+dx)+a^2\cos^{\frac{5}{2}}(c+dx)\right)dx\\ &= a^2\int \sqrt{\cos(c+dx)}\,dx+a^2\int \cos^{\frac{5}{2}}(c+dx)\,dx+\left(2a^2\right)\int \cos^{\frac{3}{2}}(c+d\\ &= \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a^2\cos^{\frac{3}{2}}(c+\\ &= \frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sin(\end{aligned}$$

Mathematica [C] time = 5.62, size = 235, normalized size = 2.47

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-24\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20
*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[
c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c + d*x
)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan
[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.45, size = 250, normalized size = 2.63

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-13*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.74, size = 104, normalized size = 1.09

$$\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a^2\,\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2}\right.}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (4*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d*x
)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin
(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.156 $\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2641, 2639, 2635}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx &= \int \left(\frac{a^2}{\sqrt{\cos(c+dx)}} + 2a^2\sqrt{\cos(c+dx)} + a^2\cos^{\frac{3}{2}}(c+dx)\right)dx \\ &= a^2\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + a^2\int \cos^{\frac{3}{2}}(c+dx)\,dx + \left(2a^2\right)\int \sqrt{\cos(c+dx)}\,dx \\ &= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}a^2 \\ &= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [C] time = 5.11, size = 224, normalized size = 3.34

$$a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8*C
os[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]]*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d*x +
ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[
c + d*x]])
```

fricas [F] time = 2.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\cos\left(dx+c\right)^2 + 2\,a^2\cos\left(dx+c\right) + a^2}{\sqrt{\cos\left(dx+c\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.48, size = 228, normalized size = 3.40

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.68, size = 59, normalized size = 0.88

$$\frac{4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{8\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

```
[Out] (4*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (8*a^2*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.157 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2639, 2641}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*EllipticF[(c + d*x)/2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \left(\frac{a^2}{\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2}{\sqrt{\cos(c+dx)}}+a^2\sqrt{\cos(c+dx)}\right)dx\\ &= a^2\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+a^2\int \sqrt{\cos(c+dx)}\,dx+\left(2a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-a^2\int \sqrt{\cos(c+dx)}\\ &= \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 39, normalized size = 0.89

$$\frac{2a^2\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
[Out] (2*a^2*(2*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Sqrt[Cos[c + d*x]]))/d
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.50, size = 104, normalized size = 2.36

$$-\frac{4a^2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

```
[Out] -4*a^2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.80, size = 82, normalized size = 1.86

$$\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.158 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3
/2)+4*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2641, 2639}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \int \left(\frac{a^2}{\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^2}{\cos^{\frac{3}{2}}(c+dx)}+\frac{a^2}{\sqrt{\cos(c+dx)}}\right)dx\\ &= a^2\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx+a^2\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+\left(2a^2\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{1}{3}a^2\int \frac{1}{\sqrt{\cos(c+dx)}}\,d\\ &= -\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 6.16, size = 454, normalized size = 4.99

$$\frac{\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\ \mathrm{co}}}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((Csc[c]*Sec
[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(Sin
[c] + 6*Sin[d*x]))/(6*d)) - (2*(a + a*Cos[c + d*x])^2*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Se
```

```
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + ((a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 +
 (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(2*d)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2 \cos(dx+c)^2 + 2a^2 \cos(dx+c) + a^2}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

maple [B] time = 0.82, size = 371, normalized size = 4.08

$$\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2 \left(4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\, \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.87, size = 109, normalized size = 1.20

$$\frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\right.}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*a^2*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*co
s(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.159 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*sin(d*x+c)/d/cos(d*x+c)
^(5/2)+4/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+16/5*a^2*sin(d*x+c)/d/cos(d*x+
c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2639, 2641}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \int \left(\frac{a^2}{\cos^{\frac{7}{2}}(c+dx)}+\frac{2a^2}{\cos^{\frac{5}{2}}(c+dx)}+\frac{a^2}{\cos^{\frac{3}{2}}(c+dx)}\right)dx \\
&= a^2\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx+a^2\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\left(2a^2\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{1}{5}\left(3a^2\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,d \\
&= -\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} \\
&= -\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [C] time = 6.20, size = 487, normalized size = 4.02

$$\frac{2\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((4*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d*
```

```
x]^2*(3*Sin[c] + 10*Sin[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 12
*Sin[d*x]))/(15*d)) - ((a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(3*d*Sqrt[1 + Cot[c]^2]) + (2*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)
/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(5*d)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^(7/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

maple [B] time = 0.82, size = 386, normalized size = 3.19

$$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-\frac{4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}+\frac{17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{d}{2}\right.\right.}}{30\sqrt{-2\left(\sin^4\left(\frac{d}{}\right.\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-4/5*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d
*x+1/2*c)^2)^(1/2)+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 0.99, size = 114, normalized size = 0.94

$$\frac{6\,a^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)+20\,a^2\,\cos(c+d\,x)\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos(c+d\,x)^2\right.}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^(7/2),x)
```

```
[Out] (6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*a^2*c
os(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*a
^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))
/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.160 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=147

$$\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a^3}{}$$

```
[Out] 68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+44/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+68/45*a^3*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+6/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^3*cos(d*x+c)^(7/2)*s
in(d*x+c)/d+44/21*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2635, 2641, 2639}

$$\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx = \int \left(a^3\cos^{\frac{3}{2}}(c+dx)+3a^3\cos^{\frac{5}{2}}(c+dx)+3a^3\cos^{\frac{7}{2}}(c+dx)+a^3\cos^{\frac{9}{2}}(c\right.$$

$$= a^3\int \cos^{\frac{3}{2}}(c+dx)\,dx + a^3\int \cos^{\frac{9}{2}}(c+dx)\,dx + \left(3a^3\right)\int \cos^{\frac{5}{2}}(c+dx)\,d$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{6a^3\cos}{}$$

$$= \frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{44a^3\sqrt{\cos(c+dx)}\,\sin(c}{21d}$$

$$= \frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{44a^3\sqrt{\cos(c+dx)}\,\sin(}{21d}$$

Mathematica [C] time = 6.14, size = 532, normalized size = 3.62

$$-\frac{17\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c)\right)\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+}}}\right.}{60d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*((-17*Cot[c]
)/(30*d) + (97*Cos[d*x]*Sin[c])/(336*d) + (73*Cos[2*d*x]*Sin[2*c])/(720*d)
+ (3*Cos[3*d*x]*Sin[3*c])/(112*d) + (Cos[4*d*x]*Sin[4*c])/(288*d) + (97*Cos
[c]*Sin[d*x])/(336*d) + (73*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*Cos[3*c]*Sin[
```

```
3*d*x])/(112*d) + (Cos[4*c]*Sin[4*d*x])/(288*d)) - (11*(a + a*Cos[c + d*x])
^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (17*(a + a*Cos[
c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d)
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\cos(dx+c)^4 + 3a^3\cos(dx+c)^3 + 3a^3\cos(dx+c)^2 + a^3\cos(dx+c)\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2
+ a^3*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.73, size = 260, normalized size = 1.77

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^7\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.78, size = 206, normalized size = 1.40

$$\frac{2\left(a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}-\frac{2\left(\frac{33\,a^3\,\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}-\frac{5\,a^3\,\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}\right){}_2\mathrm{F}_1\left(\right.}{77\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (2*((33*a^3*cos(c + d*x)^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2
) - (5*a^3*cos(c + d*x)^(11/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hyperg
eom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(77*d) - (2*a^3*cos(c + d*x)^(9/2)*
sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x
)^2)^(1/2)) - (104*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/
4], 19/4, cos(c + d*x)^2))/(385*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.161 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=121

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52a^3}{}$$

```
[Out] 28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+6/5*a^3*cos(d*x+c)^(3/2)*sin(d
*x+c)/d+2/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+52/21*a^3*sin(d*x+c)*cos(d*x+
c)^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2757, 2639, 2635, 2641}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2
])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3\,dx = \int \left(a^3\sqrt{\cos(c+dx)} + 3a^3\cos^{\frac{3}{2}}(c+dx) + 3a^3\cos^{\frac{5}{2}}(c+dx) + a^3\cos^{\frac{7}{2}}(c$$

$$= a^3\int \sqrt{\cos(c+dx)}\,dx + a^3\int \cos^{\frac{7}{2}}(c+dx)\,dx + \left(3a^3\right)\int \cos^{\frac{3}{2}}(c+dx)$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx}{5d}$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{52a^3\sqrt{\cos(c+dx)}\,\sin(c}{21d}$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{52a^3\sqrt{\cos(c+dx)}\,\sin}{21d}$$

Mathematica [C] time = 6.12, size = 500, normalized size = 4.13

$$-\frac{7\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right.}{\sqrt{\tan^2(c)+1}\,\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\,\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\,\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*((-7*Cot[c])
/(10*d) + (107*Cos[d*x]*Sin[c])/(336*d) + (3*Cos[2*d*x]*Sin[2*c])/(40*d) +
(Cos[3*d*x]*Sin[3*c])/(112*d) + (107*Cos[c]*Sin[d*x])/(336*d) + (3*Cos[2*c]
*Sin[2*d*x])/(40*d) + (Cos[3*c]*Sin[3*d*x])/(112*d)) - (13*(a + a*Cos[c + d
*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
```

```
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (7*(a + a*C
os[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sq
rt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) -
((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\cos\left(dx+c\right)^3 + 3\,a^3\cos\left(dx+c\right)^2 + 3\,a^3\cos\left(dx+c\right) + a^3\right)\sqrt{\cos\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(dx+c\right) + a\right)^3 \sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.52, size = 272, normalized size = 2.25

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^6+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+65*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
```

```
icE(cos(1/2*d*x+1/2*c),2^(1/2))-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.65, size = 143, normalized size = 1.18

$$\frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)\right)}{d}-\frac{6\,a^3\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\right.}{7\,d\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3
*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (6*a^3*cos(c + d*x)^(7/2)*sin(c + d*
x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)
) - (2*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(
c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.162 $\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

```
[Out] 36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2641, 2639, 2635}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2]
)/d + (2*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \left(\frac{a^3}{\sqrt{\cos(c+dx)}} + 3a^3\sqrt{\cos(c+dx)} + 3a^3\cos^{\frac{3}{2}}(c+dx) + a^3\cos^{\frac{5}{2}}(c+dx)\right) dx$$

$$= a^3\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + a^3\int \cos^{\frac{5}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sqrt{\cos(c+dx)}\,dx + \left(3a^3\right)$$

$$= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^3\text{co}}{}$$

$$= \frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^3\text{c}}{}$$

Mathematica [C] time = 5.71, size = 233, normalized size = 2.56

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-18\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*
Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c
]]]*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(40*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^3\cos\left(dx+c\right)^3 + 3\,a^3\cos\left(dx+c\right)^2 + 3\,a^3\cos\left(dx+c\right) + a^3}{\sqrt{\cos\left(dx+c\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.92, size = 250, normalized size = 2.75

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{5\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.62, size = 104, normalized size = 1.14

$$\frac{6a^3 \mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{4a^3 \mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d}-\frac{2a^3\cos(c+dx)^{7/2}\,\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\right.}{7d\sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

```
[Out] (6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^3*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2)*
sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x
)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.163 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2
)+2/3*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2757, 2636, 2639, 2641, 2635}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int\left(\frac{a^3}{\cos^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\cos(c+dx)}}+3a^3\sqrt{\cos(c+dx)}+a^3\cos^{\frac{3}{2}}(c+dx)\right)dx$$

$$= a^3\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + a^3\int\cos^{\frac{3}{2}}(c+dx)\,dx + \left(3a^3\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(3a^3\right)$$

$$= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^3\sqrt{\cos(c+dx)}\,\sin}{3d}$$

$$= \frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^3\sqrt{\cos(c+dx)}\,\mathrm{s}}{3d}$$

Mathematica [C] time = 4.67, size = 240, normalized size = 2.64

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right){}_2F\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-3*Cos[d*x]*Csc[c] - 9*Cos[2*
c + d*x]*Csc[c] + 9*Cos[c - d*x - ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] + 3
```

```
*Cos[c + d*x + ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt
[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Sin[
2*(c + d*x)] - 6*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Ar
cTan[Tan[c]]]^2]))/(24*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\cos\left(dx+c\right)+a\right)^{3}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.56, size = 172, normalized size = 1.89

$$-\frac{4a^{3}\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\mathrm{co}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.64, size = 104, normalized size = 1.14

$$\frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{20\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}+\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos(c\right.}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

```
[Out] (6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (20*a^3*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c + d*
x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(
c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.164 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^3*sin(d*x+c)/d/cos(d*x+c)^(
3/2)+6*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2641, 2639}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (6*a^3*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\cos^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\cos^{\frac{3}{2}}(c+dx)} + \frac{3a^3}{\sqrt{\cos(c+dx)}} + a^3\sqrt{\cos(c+dx)}\right) dx$$

$$= a^3\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a^3\int \sqrt{\cos(c+dx)}\,dx + \left(3a^3\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right)$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}$$

$$= -\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 6.20, size = 463, normalized size = 5.09

$$\frac{\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\,\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\,\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\,\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\,\cos(}\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/8*((-5 +
 Cos[2*c])*Csc[c]*Sec[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*d) + (Se
c[c]*Sec[c + d*x]*(Sin[c] + 9*Sin[d*x]))/(12*d)) - (5*(a + a*Cos[c + d*x])^
3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
```

```
Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + ((a + a*Cos[c + d
*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

fricas [F] time = 1.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^3\cos(dx+c)^3 + 3\,a^3\cos(dx+c)^2 + 3\,a^3\cos(dx+c) + a^3}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

maple [B] time = 0.93, size = 371, normalized size = 4.08

$$\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(10*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2-18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.02, size = 126, normalized size = 1.38

$$\frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{6\,a^3\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a^3\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\right.}{3\,d\cos(c+d\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^3*ellipticF(c/2 + (d*x)/2, 2)))/d
 + (6*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.165 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(
5/2)+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)+36/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(
1/2)
```

Rubi [A] time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2639, 2641}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2
])/d + (2*a^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a^3*Sin[c + d*x])
/(d*Cos[c + d*x]^(3/2)) + (36*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int\left(\frac{a^3}{\cos^{\frac{7}{2}}(c+dx)}+\frac{3a^3}{\cos^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\cos^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\sqrt{\cos(c+dx)}}\right)dx$$
$$= a^3\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a^3\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(3a^3\right)\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \left(3a^3\right)$$
$$= \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{5}\left(3a\right.$$
$$= -\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} +$$
$$= -\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] time = 6.21, size = 485, normalized size = 4.15

$$\frac{9\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\,\mathrm{co}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*((9*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(20*d) + (Sec[c]*Sec[c + d
*x]^2*(Sin[c] + 5*Sin[d*x]))/(20*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 18*S
in[d*x]))/(20*d)) - ((a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2
*d*Sqrt[1 + Cot[c]^2]) + (9*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2
]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(20*d)
```

fricas [F] time = 2.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^3\cos(dx+c)^3 + 3\,a^3\cos(dx+c)^2 + 3\,a^3\cos(dx+c) + a^3}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

maple [B] time = 0.91, size = 386, normalized size = 3.30

$$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\; a^3 \left(\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{10\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} - \frac{\cos(\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.10, size = 154, normalized size = 1.32

$$\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a^3\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a^3\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x\right.}{d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^3*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-5/
4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1
/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.166 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+}{5d\sqrt{\cos(c+}}$$

```
[Out] -28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^3*sin(d*x+c)/d/cos(d*x+
c)^(7/2)+6/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)+52/21*a^3*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+28/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2757, 2636, 2641, 2639}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+}{5d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^
(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\cos^{\frac{9}{2}}(c+dx)} + \frac{3a^3}{\cos^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\cos^{\frac{5}{2}}(c+dx)} + \frac{a^3}{\cos^{\frac{3}{2}}(c+dx)}\right) dx$$

$$= a^3 \int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}\,dx + a^3 \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right) \int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + \left(3a^3\right)$$

$$= \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{7}\left(5a^3\right)$$

$$= -\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+d}{5d\cos^{\frac{5}{2}}(c+d}$$

Mathematica [C] time = 6.25, size = 515, normalized size = 3.50

$$\frac{7\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*((7*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[c]*Sec[c + d
```

```
*x]^3*(5*Sin[c] + 21*Sin[d*x]))/(140*d) + (Sec[c]*Sec[c + d*x]^2*(63*Sin[c]
 + 130*Sin[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(65*Sin[c] + 147*Sin[d*x])
)/(210*d)) - (13*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*
Sqrt[1 + Cot[c]^2]) + (7*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(20*d)
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

maple [B] time = 0.95, size = 439, normalized size = 2.99

$$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(-\frac{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{160\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{10\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-3/160*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^3-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-
2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+53/105*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/20*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/448*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-13/
168*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

mupad [B] time = 1.21, size = 145, normalized size = 0.99

$$\frac{\frac{2\,a^3\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+d\,x)^2\right)}{7}+\frac{6\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)}{5}+2\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(9/2),x)
```

```
[Out] ((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*a
^3*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/
5 + 2*a^3*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d
*x)^2) + 2*a^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(
c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.167 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=173

$$\frac{904a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{11d}+\frac{8a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{15}{}$$

```
[Out] 128/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+904/231*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+128/45*a^4*cos(d*x+c)^(3/2
)*sin(d*x+c)/d+150/77*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+8/9*a^4*cos(d*x+c)^
(7/2)*sin(d*x+c)/d+2/11*a^4*cos(d*x+c)^(9/2)*sin(d*x+c)/d+904/231*a^4*sin(d
*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2635, 2641, 2639}

$$\frac{904a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{11d}+\frac{8a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{15}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (128*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (904*a^4*EllipticF[(c + d*x)/2
, 2])/(231*d) + (904*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (128*a^
4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (150*a^4*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(77*d) + (8*a^4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*a^4*
Cos[c + d*x]^(9/2)*Sin[c + d*x])/(11*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx = \int \left(a^4\cos^{\frac{3}{2}}(c+dx)+4a^4\cos^{\frac{5}{2}}(c+dx)+6a^4\cos^{\frac{7}{2}}(c+dx)+4a^4\cos^{\frac{9}{2}}(c\right.$$

$$= a^4\int \cos^{\frac{3}{2}}(c+dx)\,dx + a^4\int \cos^{\frac{11}{2}}(c+dx)\,dx + \left(4a^4\right)\int \cos^{\frac{5}{2}}(c+dx)\,d$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{8a^4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{12a^4\,\mathrm{co}}{}$$

$$= \frac{24a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{74a^4\sqrt{\cos(c+dx)}\,\sin(c}{21d}$$

$$= \frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{74a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{904a^4\sqrt{\cos(c+dx)}\,\mathrm{si}}{231d}$$

$$= \frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{904a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d} + \frac{904a^4\sqrt{\cos(c+dx)}\,\mathrm{s}}{231d}$$

Mathematica [C] time = 3.62, size = 271, normalized size = 1.57

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{59136\sec(c)\left(\csc(c)\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(3\cos\left(c-\tan^{-1}(\tan(c))-dx\right)+\cos\left(c+\tan^{-1}(\tan(c))+dx\right.\right.\right.}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+a\right.}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(-108480*Cos[c + d*x]*Sqrt[Cos
[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
```

```
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c +
d*x]*(-236544*Cot[c] + 122610*Sin[c + d*x] + 45584*Sin[2*(c + d*x)] + 14445
*Sin[3*(c + d*x)] + 3080*Sin[4*(c + d*x)] + 315*Sin[5*(c + d*x)]) + (59136*
Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c +
d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Sec
[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(443520*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4\cos(dx+c)^5 + 4\,a^4\cos(dx+c)^4 + 6\,a^4\cos(dx+c)^3 + 4\,a^4\cos(dx+c)^2 + a^4\cos(dx+c)\right)\sqrt{\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^5 + 4*a^4*cos(d*x + c)^4 + 6*a^4*cos(d*x + c)^3
+ 4*a^4*cos(d*x + c)^2 + a^4*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.49, size = 273, normalized size = 1.58

$$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(5040\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5320\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1740\left(\cos^9\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x)
```

```
[Out] -8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*c
os(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9+3
26*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)^3+
1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*cos(1/2*
```

```
d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \cos(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.88, size = 221, normalized size = 1.28

$$\frac{2\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^4\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{8\,a^4\,\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (2*a^4*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a^4*cos(c + d*x)^(1/2)*sin(c
 + d*x))/(3*d) - (8*a^4*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4
], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^4*cos(c + d*x
)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin
(c + d*x)^2)^(1/2)) - (8*a^4*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/
2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*a^4*cos
(c + d*x)^(13/2)*sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d*x)^2))
/(13*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.168 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=147

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{152a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{122}{}$$

```
[Out] 152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+32/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+122/45*a^4*cos(d*x+c)^(3/2)*s
in(d*x+c)/d+8/7*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^4*cos(d*x+c)^(7/2)*
sin(d*x+c)/d+32/7*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2639, 2635, 2641}

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{152a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{122}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (152*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (32*a^4*EllipticF[(c + d*x)/2,
 2])/(7*d) + (32*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (122*a^4*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (8*a^4*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^4\,dx = \int \left(a^4\sqrt{\cos(c+dx)} + 4a^4\cos^{\frac{3}{2}}(c+dx) + 6a^4\cos^{\frac{5}{2}}(c+dx) + 4a^4\cos^{\frac{7}{2}}\right.$$
$$= a^4\int \sqrt{\cos(c+dx)}\,dx + a^4\int \cos^{\frac{9}{2}}(c+dx)\,dx + \left(4a^4\right)\int \cos^{\frac{3}{2}}(c+dx)$$
$$= \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^4\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{12a^4\cos^{\frac{3}{2}}(c+d}{5d}$$
$$= \frac{46a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{32a^4\sqrt{\cos(c+dx)}\,\sin(c}{7d}$$
$$= \frac{152a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{32a^4\sqrt{\cos(c+dx)}\,\sin}{7d}$$

Mathematica [C] time = 6.15, size = 532, normalized size = 3.62

$$-\frac{19\csc(c)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^4\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c)\right)\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+}}}\right.}{60d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*((-19*Cot[c]
)/(30*d) + (17*Cos[d*x]*Sin[c])/(56*d) + (127*Cos[2*d*x]*Sin[2*c])/(1440*d)
 + (Cos[3*d*x]*Sin[3*c])/(56*d) + (Cos[4*d*x]*Sin[4*c])/(576*d) + (17*Cos[c
]*Sin[d*x])/(56*d) + (127*Cos[2*c]*Sin[2*d*x])/(1440*d) + (Cos[3*c]*Sin[3*d
*x])/(56*d) + (Cos[4*c]*Sin[4*d*x])/(576*d)) - (2*(a + a*Cos[c + d*x])^4*Cs
c[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2 + (d*x)/2]^8*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]
```

```
]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + S
in[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (19*(a + a*Cos[c + d*
x])^4*Csc[c]*Sec[c/2 + (d*x)/2]^8*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4\cos\left(dx+c\right)^4+4\,a^4\cos\left(dx+c\right)^3+6\,a^4\cos\left(dx+c\right)^2+4\,a^4\cos\left(dx+c\right)+a^4\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(dx+c\right)+a\right)^4\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.54, size = 260, normalized size = 1.77

$$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(280\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+34\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x)
```

```
[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos
(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(
1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399
```

```
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos (dx+c)+a)^4\sqrt{\cos (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.81, size = 223, normalized size = 1.52

$$\frac{2\left(3\,a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,a^4\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}-\frac{2\left(\frac{66\,a^4\,{\cos(c+d\,x)}^{7/2}\,\sin(c+d\,x)}{\sqrt{{\sin(c+d\,x)}^2}}-\frac{17\,a^4\,\cos}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (2*(3*a^4*ellipticE(c/2 + (d*x)/2, 2) + 4*a^4*ellipticF(c/2 + (d*x)/2, 2) +
 4*a^4*cos(c + d*x)^(1/2)*sin(c + d*x)))/(3*d) - (2*((66*a^4*cos(c + d*x)^(
7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2) - (17*a^4*cos(c + d*x)^(11/2)*sin
(c + d*x))/(sin(c + d*x)^2)^(1/2))*hypergeom([1/2, 11/4], 15/4, cos(c + d*x
)^2))/(77*d) - (8*a^4*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4],
 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (208*a^4*cos(c + d*x
)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 19/4, cos(c + d*x)^2))/(385*d*
(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.169 $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=121

$$\frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{94a}{}$$

```
[Out] 64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a^4*cos(d*x+c)^(3/2)*sin(
d*x+c)/d+2/7*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+94/21*a^4*sin(d*x+c)*cos(d*x
+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2641, 2639, 2635}

$$\frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{94a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2,
2])/(21*d) + (94*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (8*a^4*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^4*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx = \int \left(\frac{a^4}{\sqrt{\cos(c+dx)}} + 4a^4\sqrt{\cos(c+dx)} + 6a^4\cos^{\frac{3}{2}}(c+dx) + 4a^4\cos^{\frac{5}{2}}(c+dx) + a^4\,\mathrm{c}$$

$$= a^4\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + a^4\int \cos^{\frac{7}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sqrt{\cos(c+dx)}\,dx + \left(4a^4\right)$$

$$= \frac{8a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^4\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d} + \frac{8a^4\,\mathrm{co}}{}$$

$$= \frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{6a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{94a^4\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{8a^4}{}$$

$$= \frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{94a^4\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{8}{}$$

Mathematica [C] time = 6.17, size = 500, normalized size = 4.13

$$-\frac{2\csc(c)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^4\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right.}{\sqrt{\tan^2(c)+1}\,\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\,\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\,\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*((-4*Cot[c])
/(5*d) + (191*Cos[d*x]*Sin[c])/(672*d) + (Cos[2*d*x]*Sin[2*c])/(20*d) + (Co
s[3*d*x]*Sin[3*c])/(224*d) + (191*Cos[c]*Sin[d*x])/(672*d) + (Cos[2*c]*Sin[
```

```
2*d*x])/(20*d) + (Cos[3*c]*Sin[3*d*x])/(224*d)) - (17*(a + a*Cos[c + d*x])^
4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^8*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (2*(a + a*Cos[c
+ d*x])^4*Csc[c]*Sec[c/2 + (d*x)/2]^8*((HypergeometricPFQ[{-1/2, -1/4}, {3/
4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
- Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\cos(dx+c)^4+4a^4\cos(dx+c)^3+6a^4\cos(dx+c)^2+4a^4\cos(dx+c)+a^4}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a\cos(dx+c)+a)^4}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.49, size = 272, normalized size = 2.25

$$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-258\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x)
```

```
[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^4}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.71, size = 146, normalized size = 1.21

$$\frac{2\left(4a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+2a^4\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)\right)}{d}-\frac{8a^4\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}}{7d\sqrt{\sin(c+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*(4*a^4*ellipticE(c/2 + (d*x)/2, 2) + 3*a^4*ellipticF(c/2 + (d*x)/2, 2) +
 2*a^4*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (8*a^4*cos(c + d*x)^(7/2)*sin(
c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)
^(1/2)) - (2*a^4*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4
, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.170 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^4}{d\sqrt{c}}$$

```
[Out] 56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^4*cos(d*x+c)^(3/2)*sin(d*
x+c)/d+2*a^4*sin(d*x+c)/d/cos(d*x+c)^(1/2)+8/3*a^4*sin(d*x+c)*cos(d*x+c)^(1
/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2757, 2636, 2639, 2641, 2635}

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^4}{d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(3/2),x]
```

```
[Out] (56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (8*a^4*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*d) + (2*a^4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int\left(\frac{a^4}{\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sqrt{\cos(c+dx)}}+6a^4\sqrt{\cos(c+dx)}+4a^4\cos^{\frac{3}{2}}(c+dx)+a^4\cos^{\frac{5}{2}}$$

$$= a^4\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+a^4\int\cos^{\frac{5}{2}}(c+dx)\,dx+\left(4a^4\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx+\left(4a^4\right)$$

$$= \frac{12a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{8a^4\sqrt{\cos(c+dx)}\,\mathrm{s}}{3d}$$

$$= \frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{8a^4\sqrt{\cos(c+dx)}}{3d}$$

Mathematica [C] time = 6.20, size = 497, normalized size = 4.18

$$-\frac{7\csc(c)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^4\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(3/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(-1/80*((23
+ 33*Cos[2*c])*Csc[c]*Sec[c])/d + (Cos[d*x]*Sin[c])/(6*d) + (Cos[2*d*x]*Sin
[2*c])/(80*d) + (Cos[c]*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*Sin[d*x])/(8
*d) + (Cos[2*c]*Sin[2*d*x])/(80*d)) - (2*(a + a*Cos[c + d*x])^4*Csc[c]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*
x)/2]^8*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (7*(a + a*Cos[c + d*x])^4*Csc[
c]*Sec[c/2 + (d*x)/2]^8*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\cos(dx+c)^4 + 4\,a^4\cos(dx+c)^3 + 6\,a^4\cos(dx+c)^2 + 4\,a^4\cos(dx+c) + a^4}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c) + a)^4}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.52, size = 194, normalized size = 1.63

$$-\frac{8a^4\left(-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 26\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right) + 20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x)

[Out] -8/15*a^4*(-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+26*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)

mupad [B] time = 0.79, size = 149, normalized size = 1.25

$$\frac{12\,a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{32\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{8\,a^4\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}+\frac{2\,a^4\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos(c\right.}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(3/2),x)

[Out] (12*a^4*ellipticE(c/2 + (d*x)/2, 2))/d + (32*a^4*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (8*a^4*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^4*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) - (2*a^4*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(3/2),x)

[Out] Timed out

3.171 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{40a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 40/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))/d+2/3*a^4*sin(d*x+c)/d/cos(d*x+c)^(3/2)+8*a^4*sin(d*x+c
)/d/cos(d*x+c)^(1/2)+2/3*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2757, 2636, 2641, 2639, 2635}

$$\frac{40a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(5/2),x]
```

```
[Out] (40*a^4*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^4*Sin[c + d*x])/(3*d*Cos[c
 + d*x]^(3/2)) + (8*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^4*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^4}{\sqrt{\cos(c+dx)}} + 4a^4\sqrt{\cos(c+dx)} + a^4\cos^{\frac{3}{2}}(c\right.$$

$$= a^4\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a^4\int \cos^{\frac{3}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \left(4a^4\right)\int$$

$$= \frac{8a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{12a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} +$$

$$= \frac{40a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^4\sqrt{\cos(c+dx)}\,\sin(c}{3d}$$

Mathematica [A] time = 0.32, size = 70, normalized size = 0.71

$$\frac{a^4\left(5\sin(c+dx) + 24\sin(2(c+dx)) + \sin(3(c+dx)) + 80\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^4*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[c + d*x] + 24
*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 2.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^{4}\cos\left(dx+c\right)^{4}+4\,a^{4}\cos\left(dx+c\right)^{3}+6\,a^{4}\cos\left(dx+c\right)^{2}+4\,a^{4}\cos\left(dx+c\right)+a^{4}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)
```

maple [B] time = 0.86, size = 292, normalized size = 2.98

$$\frac{8\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^{4}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x)
```

```
[Out] 8/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^2-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+7
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\cos\left(dx+c\right)+a\right)^{4}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.84, size = 145, normalized size = 1.48

$$\frac{2\left(12\,a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+19\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^4\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}+\frac{8\,a^4\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \mathrm{co}\right.}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*(12*a^4*ellipticE(c/2 + (d*x)/2, 2) + 19*a^4*ellipticF(c/2 + (d*x)/2, 2)
 + a^4*cos(c + d*x)^(1/2)*sin(c + d*x)))/(3*d) + (8*a^4*sin(c + d*x)*hyperg
eom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^
2)^(1/2)) + (2*a^4*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2)
)/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.172 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{66a^4\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^4*sin(d*x+c)/d/cos(d*x+c
)^(5/2)+8/3*a^4*sin(d*x+c)/d/cos(d*x+c)^(3/2)+66/5*a^4*sin(d*x+c)/d/cos(d*x
+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2639, 2641}

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{66a^4\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2,
2])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (8*a^4*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2)) + (66*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*
x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^4}{\cos^{\frac{5}{2}}(c+dx)} + \frac{6a^4}{\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\cos(c+dx)}} + a^4\sqrt{\cos(c+dx}\right.$$

$$= a^4\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a^4\int \sqrt{\cos(c+dx)}\,dx + \left(4a^4\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \left(4a^4\right)$$

$$= \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{10a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] time = 4.34, size = 283, normalized size = 2.34

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(-\frac{168\sec(c)\cos^2(c+dx)\left(\csc(c)\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(3\cos\left(c-\tan^{-1}(\tan(c))-dx\right)+\cos\left(c+\tan^{-1}\right.\right.\right.}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan\right.}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*((363*Cos[d*x] + 141*Cos[2*c +
 d*x] + 40*Cos[c + 2*d*x] - 40*Cos[3*c + 2*d*x] + 183*Cos[2*c + 3*d*x] - 15
*Cos[4*c + 3*d*x])*Csc[c] - 640*Cos[c + d*x]^3*Sqrt[Cos[d*x - ArcTan[Cot[c]
]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] - (168*Cos[c + d*x]^2*Sec[c]*(
-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + Ar
cTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Sec[c]^2]*S
qrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(960*d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 1.76, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)
```

maple [B] time = 0.90, size = 386, normalized size = 3.19

$$-\frac{32\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(-\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\operatorname{Ellip}\right.}{20\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(-7/20*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+41/60*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/320*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^3-33/40*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1
/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/24*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.30, size = 202, normalized size = 1.67

$$\frac{2\left(a^4\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{2\left(\frac{34\,a^4\,\sin(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{a^4\,\sin(c+d\,x)}{\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c\right.}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*(a^4*ellipticE(c/2 + (d*x)/2, 2) + 4*a^4*ellipticF(c/2 + (d*x)/2, 2)))/d
 + (2*((34*a^4*sin(c + d*x))/(cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) +
(a^4*sin(c + d*x))/(cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)))*hypergeom([
-1/4, 1/2], 3/4, cos(c + d*x)^2))/(5*d) + (8*a^4*sin(c + d*x)*hypergeom([-3
/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1
/2)) - (8*a^4*sin(c + d*x)*hypergeom([-1/4, 1/2], 7/4, cos(c + d*x)^2))/(15
*d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.173 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{94a^4\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{64a^4\sin(c+}{5d\sqrt{\cos(c+}}$$

```
[Out] -64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^4*sin(d*x+c)/d/cos(d*x
+c)^(7/2)+8/5*a^4*sin(d*x+c)/d/cos(d*x+c)^(5/2)+94/21*a^4*sin(d*x+c)/d/cos(
d*x+c)^(3/2)+64/5*a^4*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2641, 2639}

$$\frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{94a^4\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{64a^4\sin(c+}{5d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2,
 2])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Cos[c + d*x]^(5/2)) + (94*a^4*Sin[c + d*x])/(21*d*Cos[c + d*x]
^(3/2)) + (64*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\cos^{\frac{9}{2}}(c+dx)} + \frac{4a^4}{\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^4}{\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\cos^{\frac{3}{2}}(c+dx)} + \frac{a^4}{\sqrt{\cos(c+dx)}}\right.$$

$$= a^4\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}\,dx + a^4\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(4a^4\right)\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + \left(4a^4\right.$$

$$= \frac{2a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^4\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{8a^4}{d\sqrt{}}$$

$$= -\frac{8a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{6a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+}{5d\cos^{\frac{5}{2}}(c+}$$

Mathematica [C] time = 5.14, size = 298, normalized size = 2.03

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1344\sec(c)\cos^3(c+dx)\left(\csc(c)\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(3\cos\left(c-\tan^{-1}(\tan(c))-dx\right)+\cos\left(c+\tan\right.\right.\right.}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}\right.}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(9/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*((2016*Cos[c] + 295*Cos[d*x] -
 295*Cos[2*c + d*x] + 2184*Cos[c + 2*d*x] + 504*Cos[3*c + 2*d*x] + 235*Cos[
2*c + 3*d*x] - 235*Cos[4*c + 3*d*x] + 672*Cos[3*c + 4*d*x])*Csc[c] - 2720*C
os[c + d*x]^4*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[
c]]]*Sin[c] - (1344*Cos[c + d*x]^3*Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4
}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c
 - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d
*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]
)))/(6720*d*Cos[c + d*x]^(7/2))
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\cos(dx+c)^4 + 4a^4\cos(dx+c)^3 + 6a^4\cos(dx+c)^2 + 4a^4\cos(dx+c) + a^4}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

maple [B] time = 1.20, size = 439, normalized size = 2.99

$$-\frac{32\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(\frac{253\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{420\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} - \frac{\cos(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(253/420*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/896*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-47/672
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-
1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

mupad [B] time = 1.34, size = 199, normalized size = 1.35

$$\frac{2\,a^4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{8\,a^4\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{4\,a^4\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\right.}{d\,\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(9/2),x)
```

```
[Out] (2*a^4*ellipticF(c/2 + (d*x)/2, 2))/d + (8*a^4*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (4*a^4*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (8*a^4*sin(c + d*x)*hypergeom([-5/
4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1
/2)) + (2*a^4*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/(7
*d*cos(c + d*x)^(7/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.174 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{c}}{3ad}$$

```
[Out] 21/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+7/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d-
cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))-5/3*sin(d*x+c)*cos(d*x+c)^(1
/2)/a/d
```

Rubi [A] time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2767, 2748, 2635, 2641, 2639}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{c}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (21*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + (7*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*a*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-\frac{7}{2}a\cos(c+dx)\right)dx}{a^2} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{5\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{7\int \cos^{\frac{5}{2}}(c+dx)\,dx}{2a} \\
&= -\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} \\
&= \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)}{5a}
\end{aligned}$$

Mathematica [C] time = 1.81, size = 315, normalized size = 2.46

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\csc(c)\sqrt{\cos(c+dx)}\left(5\sin(2c)\sin(dx)+10\sin^2(c)\cos(dx)-6\cos(c)\left(\sin^2(c)\cos(2dx)-8\right)-3\sin(c)\cos(2c)\sin(2dx)+30\sin\left(\frac{c}{2}\right)s\right.}{d}\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - (2*Sqrt[Cos[c + d*x]]*Csc[c]*(15 + 10*Cos[d*x]*Sin[c]^2 -
6*Cos[c]*(-8 + Cos[2*d*x]*Sin[c]^2) + 30*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x
)/2] + 5*Sin[2*c]*Sin[d*x] - 3*Cos[2*c]*Sin[c]*Sin[2*d*x]))/d))/(15*a*(1 +
Cos[c + d*x]))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{a\cos\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.53, size = 229, normalized size = 1.79

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\left(25\,\text{Ellipti}\right.\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4
+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{7/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.175 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

```
[Out] -3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos
(d*x+c))+5/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2767, 2748, 2639, 2635, 2641}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d
) + (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) - (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \sqrt{\cos(c+dx)}\left(\frac{3a}{2}-\frac{5}{2}a\cos(c+dx)\right)dx}{a^2}$$
$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int \sqrt{\cos(c+dx)}\,dx}{2a} + \frac{5\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a}$$
$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{5\int \frac{}{\sqrt{\mathrm{c}}}$$
$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{d(a+a\cos(}$$

Mathematica [C] time = 1.28, size = 289, normalized size = 2.89

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\csc(c)\sqrt{\cos(c+dx)}\left(\sin(2c)\sin(dx)+2\sin^2(c)\cos(dx)+6\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+6\cos(c)+3\right)}{d} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(9\left(-1+e^{2}\right.\right.}{}\right.}{3a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
```

```
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))
/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*
(c + d*x))]) + (2*Sqrt[Cos[c + d*x]]*Csc[c]*(3 + 6*Cos[c] + 2*Cos[d*x]*Sin[
c]^2 + 6*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x)/2] + Sin[2*c]*Sin[d*x]))/d))/(
3*a*(1 + Cos[c + d*x]))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\cos\left(dx+c\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.63, size = 215, normalized size = 2.15

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(9\,\text{EllipticE}\left(\right.\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(9*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-
8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{5/2}}{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x)),x)
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
[Out] Timed out
```

3.176 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=72

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+
c))
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2767, 2748, 2641, 2639}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) - (Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{\int \frac{\frac{a}{2}-\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{3\int \sqrt{\cos(c+dx)}\,dx}{2a}\\ &= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [C] time = 2.64, size = 264, normalized size = 3.67

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\sqrt{\cos(c+dx)}\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+2\cot(c)+\csc(c)\right)}{d}+\frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c\cdot}\right.\right.}{\left(-1+e^{2ic}\right.}\right.}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d
*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c
+ d*x))]) - (2*Sqrt[Cos[c + d*x]]*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(c + d*
x)/2]*Sin[(d*x)/2]))/d))/(a*(1 + Cos[c + d*x]))
```

fricas [F] time = 1.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.61, size = 199, normalized size = 2.76

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\mathrm{EllipticF}\left(\cos\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}}{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x)),x)
[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
[Out] Timed out
```

3.177 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))/a/d+sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c
))
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2769, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a*d)) + EllipticF[(c + d*x)/2, 2]/(a*d) + (Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2769

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a
*f*(a + b*Sin[e + f*x])), x] + Dist[(d*n)/(a*b), Int[(c + d*Sin[e + f*x])^(
n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n
] || EqQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{2a^2}\\
&= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}-\frac{\int \sqrt{\cos(c+dx)}\,dx}{2a}\\
&= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [C] time = 1.05, size = 256, normalized size = 3.66

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\sqrt{\cos(c+dx)}\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c)\right)}{d}-\frac{2i\sqrt{2}\,e^{-i(c+dx)}\big(\big(-1+e^{2ic}\big)\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\big(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\big)+\big(-1+e \cdots}{\big(-1+e^{2ic}\big)d\sqrt{e^{-i(c+dx)}\big(\cdots}}}{a(\cos(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^
((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I
*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x
))]) + (2*Sqrt[Cos[c + d*x]]*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/
2]))/d))/(a*(1 + Cos[c + d*x]))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.48, size = 198, normalized size = 2.83

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\mathrm{EllipticF}\left(c\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\cos(c+dx)}}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(cos(c + d*x) + 1), x)/a
```

3.178 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=70

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c)
)
```

Rubi [A] time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2768, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a*d) + EllipticF[(c + d*x)/2, 2]/(a*d) - (Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx &= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{\int \frac{-\frac{a}{2}-\frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}\\
&= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{\int \sqrt{\cos(c+dx)}\,dx}{2a}\\
&= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}
\end{aligned}$$

Mathematica [C] time = 1.04, size = 257, normalized size = 3.67

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\sqrt{\cos(c+dx)}\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c)\right)}{d}+\frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-(-1 \cdots\right.}{\left(-1+e^{2ic}\right)d\sqrt{e^{-i(c+d \cdots}}}\right.}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(
(2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*
(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)
)]) - (2*Sqrt[Cos[c + d*x]]*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2
]))/d))/(a*(1 + Cos[c + d*x]))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 0.48, size = 200, normalized size = 2.86

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\mathrm{EllipticF}\left(c\right.\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\ \left(a+a\ \cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(1/(cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x)/a
```

3.179 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=96

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)}$$

```
[Out] -3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+3*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)-sin(d*x+
c)/d/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2768, 2748, 2636, 2639, 2641}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) + (3
*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]
]*(a + a*Cos[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))} - \frac{\int \frac{-\frac{3a}{2}+\frac{1}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{3\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,d}{2a}$$

$$= -\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{}{d\sqrt{\cos(c+}}$$

Mathematica [C] time = 2.09, size = 297, normalized size = 3.09

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(2\cos\left(\frac{1}{2}(c-dx)\right)+\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{2d\sqrt{\cos(c+dx)}} - \frac{2i\sqrt{2}\,e^{-i(c+dx)}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\ {}_2F\right.}{}\right.}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(
d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c
 + d*x))]) + ((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c + 3*d*x)/
2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Sqrt[Cos[c + d*x]])))/(a*(1 +
Cos[c + d*x]))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

maple [A] time = 0.53, size = 253, normalized size = 2.64

$$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\text{EllipticF}\left(\cos\left(\right.\right.\right.}{a\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
```

```
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+a\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
[Out] Integral(1/(cos(c + d*x)**(5/2) + cos(c + d*x)**(3/2)), x)/a
```

3.180 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=124

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-sin
(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))-3*sin(d*x+c)/a/d/cos(d*x+c)^(1/
2)
```

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2768, 2748, 2636, 2641, 2639}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d)
 + (5*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (3*Sin[c + d*x])/(a*d*Sqrt
[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{\int \frac{-\frac{5a}{2}+\frac{3}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{3\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{5\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,d}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+}{ad\sqrt{\cos(c}}$$

Mathematica [C] time = 3.94, size = 332, normalized size = 2.68

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(10\cos\left(\frac{1}{2}(c-dx)\right)+8\cos\left(\frac{1}{2}(3c+dx)\right)+4\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+9\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{4d\cos^{\frac{3}{2}}(c+dx)}\right.}{3a(\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/
(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(
c + d*x))]) - ((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4*Cos[(c + 3*d
*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]
*Sec[(c + d*x)/2])/(4*d*Cos[c + d*x]^(3/2))))/(3*a*(1 + Cos[c + d*x]))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 0.89, size = 413, normalized size = 3.33

$$\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*x
+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d
*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{5/2}\,\left(a+a\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)+\cos^{\frac{5}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(1/(cos(c + d*x)**(7/2) + cos(c + d*x)**(5/2)), x)/a
```

3.181 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=160

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)}{a}$$

```
[Out] 56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+56/15*cos(d*x+c)^(3/2)*sin(d*x+c)/
a^2/d-3*cos(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^(7/
2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2-5*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.22, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (56*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(a
^2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d) + (56*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(15*a^2*d) - (3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1
+ Cos[c + d*x])) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*
x])^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2}-\frac{11}{2}a\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{45a^2}{2} - 2\right.}{3a^4}$$

$$= -\frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{15\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a^2} +$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c}{a^2d(1+\cos(c+d}$$

$$= \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c}{}$$

Mathematica [C] time = 2.63, size = 367, normalized size = 2.29

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\csc(c)\sqrt{\cos(c+dx)}\left(40\sin^2(c)\cos(dx)-6\sin(c)\sin(2c)\cos(2dx)+8\cos(c)(5\sin(c)\sin(dx)+27)-6\sin(c)\cos(2c)\sin(2dx)-1\right.}{3d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(56*(1 + E^((2*I)*(c + d*x))) + 56*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - (2*Sqrt[Cos[c + d*x]]*Csc[c]*(120 + 40*Cos[d*x]*Sin[c]^2 -
 6*Cos[2*d*x]*Sin[c]*Sin[2*c] + 240*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x)/2]
- 10*Sec[(c + d*x)/2]^3*Sin[c/2]*Sin[(d*x)/2] + 8*Cos[c]*(27 + 5*Sin[c]*Sin
[d*x]) - 6*Cos[2*c]*Sin[c]*Sin[2*d*x] - 5*Sec[(c + d*x)/2]^2*Sin[c]*Tan[c/2
]))/(3*d)))/(5*a^2*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos(dx+c)^{\frac{9}{2}}}{a^2\cos(dx+c)^2 + 2a^2\cos(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(9/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^2, x)
```

maple [A] time = 0.54, size = 283, normalized size = 1.77

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d
*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{9/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**2,x)
[Out] Timed out
```

3.182 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{\sin(c+dx)\cos}{3d(a\cos(c+d}$$

```
[Out] -7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-7/3*cos(d*x+c)^(3/2)*sin(d*x+c)/a
^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+10
/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2748, 2639, 2635, 2641}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{\sin(c+dx)\cos}{3d(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*
a^2*d) + (10*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) - (7*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^(5/2)*Sin[c
 + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-\frac{9}{2}a\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \sqrt{\cos(c+dx)}\left(\frac{21a^2}{2}-1\right.}{3a^4}$$

$$= -\frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{7\int \sqrt{\cos(c+dx)}\,dx}{2a^2} + 5$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \mathrm{c}$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\cos^{\frac{3}{2}}(c}{3a^2d(1}$$

Mathematica [C] time = 2.03, size = 337, normalized size = 2.44

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc(c)\sqrt{\cos(c+dx)}\left(72\cos\left(\frac{1}{2}(c-dx)\right)+54\cos\left(\frac{1}{2}(3c+dx)\right)+33\cos\left(\frac{1}{2}(c+3dx)\right)+9\cos\left(\frac{1}{2}(5c+3dx)\right)+\cos\left(\frac{1}{2}(3c+5dx)\right)-\cos\left(\frac{1}{2}(7c+5\right.\right.}{2d}\right.}{3a^2(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^
(I*(c + d*x))]) + (Sqrt[Cos[c + d*x]]*(72*Cos[(c - d*x)/2] + 54*Cos[(3*c +
d*x)/2] + 33*Cos[(c + 3*d*x)/2] + 9*Cos[(5*c + 3*d*x)/2] + Cos[(3*c + 5*d*x
)/2] - Cos[(7*c + 5*d*x)/2])*Csc[c]*Sec[(c + d*x)/2]^3)/(2*d)))/(3*a^2*(1 +
 Cos[c + d*x])^2)
```

fricas [F] time = 2.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(7/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x)
```

maple [A] time = 0.54, size = 270, normalized size = 1.96

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(16\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d*
x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x
+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*d*x+
1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{7/2}}{(a+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**2,x)
[Out] Timed out
```

3.183 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] 4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(
a+a*cos(d*x+c))^2-5/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1+cos(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2765, 2977, 2748, 2641, 2639}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a^
2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-\frac{7}{2}a\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\frac{5a^2}{2}-6a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{3a^4}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{6a^2} + \frac{2\int \sqrt{\cdots}}{\cdots}$$

$$= \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)}{3d(a+a\cos\cdots}$$

Mathematica [C] time = 2.88, size = 319, normalized size = 2.85

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(20\cos\left(\frac{1}{2}(c-dx)\right)+16\cos\left(\frac{1}{2}(3c+dx)\right)+9\cos\left(\frac{1}{2}(c+3dx)\right)+3\cos\left(\frac{1}{2}(5c+3dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{2d}+\right.}{3a^2(\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I
*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*(20*Cos[(c - d*x)/2] + 16*Cos[(3*c + d*
x)/2] + 9*Cos[(c + 3*d*x)/2] + 3*Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Se
c[(c + d*x)/2]^3)/(2*d)))/(3*a^2*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

maple [A] time = 0.56, size = 257, normalized size = 2.29

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(24\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\;\mathrm{Ell}\cdots\right.}{6a^2\sqrt{-2\left(\mathrm{si}\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x
+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d*x+1
/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.184 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1
+cos(d*x+c))-1/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.19, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2765, 2978, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a^2*d)) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2
*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - (Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\frac{a}{2}-\frac{5}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{-a^2+\frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{3a^4}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2} - \frac{\int \sqrt{\cos}}{}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx}}{3d(a+a\cos}$$

Mathematica [C] time = 6.32, size = 640, normalized size = 5.87

$$-\frac{4\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\cot(c))\right)}}{3d\sqrt{\cot^2(c)+1}\,(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((-1/2*I)*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4
*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*((4*Csc[c])/d + (4
*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/
2]^3*Sin[(d*x)/2])/(3*d) - (2*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a
*Cos[c + d*x])^2
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

maple [A]   time = 0.58, size = 257, normalized size = 2.36

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{E}}{6a^{2}\sqrt{-2\left(\mathrm{s}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x+1/2
*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.185 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] 1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/a^2/d+1/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2764, 21, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{\frac{a}{2}+\frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx}{3a^2}\\ &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{6a^2}\\ &= \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.21, size = 63, normalized size = 1.11

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

maple [B] time = 0.48, size = 188, normalized size = 3.30

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\cos\left(c+dx\right)}}{\cos^2\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.186 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1+
cos(d*x+c))-1/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.18, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2766, 2978, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - (Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{5a}{2}-\frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{a^2+\frac{3}{2}a^2\cos}{\sqrt{\cos(c+}}}{3a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}}{3a^2}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \sqrt{}$$

Mathematica [C] time = 2.04, size = 304, normalized size = 2.79

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(7\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{2d}+\frac{4i\sqrt{2}e^{-i(c+dx)}\left(3\left(-1+\right.\right.}{}\right.}{3a^2(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/
(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(
c + d*x))]) - (Sqrt[Cos[c + d*x]]*(7*Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2
] + 3*Cos[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(2*d)))/(3*
a^2*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 2.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^3+2a^2\cos(dx+c)^2+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 + a^
2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

maple [A] time = 0.52, size = 257, normalized size = 2.36

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{Ell}\right.}{6a^2\cos\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{5}{2}}{\left(c+dx\right)}+2\cos^{\frac{3}{2}}{\left(c+dx\right)}+\sqrt{\cos{\left(c+dx\right)}}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(5/2) + 2*cos(c + d*x)**(3/2) + sqrt(cos(c + d*x)
)), x)/a**2
```

3.187 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+4*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2
)-5/3*sin(d*x+c)/a^2/d/(1+cos(d*x+c))/cos(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+
a*cos(d*x+c))^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2978, 2748, 2636, 2639, 2641}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (-4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (4*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (5*Sin[c + d*x])/(3*a
^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[Cos[c
+ d*x]]*(a + a*Cos[c + d*x])^2)
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}dx = -\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{7a}{2}-\frac{3}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}dx}{3a^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\cos(c+$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\cos(c+$$

$$= -\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos$$

$$= -\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{}{3a^2d\sqrt{}}$$

Mathematica [C] time = 1.96, size = 334, normalized size = 2.46

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(29\cos\left(\frac{1}{2}(c-dx)\right)+19\cos\left(\frac{1}{2}(3c+dx)\right)+31\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+12\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec^3\left(\frac{1}{2}(c+d\right.}{4d\sqrt{\cos(c+dx)}}\right.}{3a^2(\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) + ((29*Cos[(c - d*x)/2] + 19*Cos[(3*c + d*x)/2] + 31*Cos[(c
+ 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 12*Cos[(3*c + 5*d*x)/2])*Csc[c/2]*Se
c[c/2]*Sec[(c + d*x)/2]^3)/(4*d*Sqrt[Cos[c + d*x]])))/(3*a^2*(1 + Cos[c + d
*x])^2)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 + a^
2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.60, size = 405, normalized size = 2.98

$$-\frac{2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{7}{2}}{\left(c+dx \right)}+2\cos^{\frac{5}{2}}{\left(c+dx \right)}+\cos^{\frac{3}{2}}{\left(c+dx \right)}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(7/2) + 2*cos(c + d*x)**(5/2) + cos(c + d*x)**(3/
2)), x)/a**2
```

3.188 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

```
[Out] 7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(
3/2)-7/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(1+cos(d*x+c))-1/3*sin(d*x+c)/d/
cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-7*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2978, 2748, 2636, 2641, 2639}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) - (7*Sin[c + d*x])/(
a^2*d*Sqrt[Cos[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1
 + Cos[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x
])^2)
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}dx = -\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{9a}{2}-\frac{5}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}dx}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+a}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+a}$$

$$= \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c-}$$

$$= \frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(}{a^2d\sqrt{co}}$$

Mathematica [C] time = 5.85, size = 364, normalized size = 2.25

$$\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(82\cos\left(\frac{1}{2}(c-dx)\right)+65\cos\left(\frac{1}{2}(3c+dx)\right)+68\cos\left(\frac{1}{2}(c+3dx)\right)+37\cos\left(\frac{1}{2}(5c+3dx)\right)+53\cos\left(\frac{1}{2}(3c+5dx)\right)+10\cos\left(\frac{1}{2}(\right.\right.}{8d\cos^{\frac{3}{2}}(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - ((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] + 68*Cos[(c
+ 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2] + 10*Cos[(7
*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2
]^3)/(8*d*Cos[c + d*x]^(3/2))))/(3*a^2*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^5 + 2a^2\cos(dx+c)^4 + a^2\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^5 + 2*a^2*cos(d*x + c)^4 + a^
2*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 1.03, size = 413, normalized size = 2.55

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+16*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{9}{2}}{\left(c+dx \right)}+2\cos^{\frac{7}{2}}{\left(c+dx \right)}+\cos^{\frac{5}{2}}{\left(c+dx \right)}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(9/2) + 2*cos(c + d*x)**(7/2) + cos(c + d*x)**(5/
2)), x)/a**2
```

3.189 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{63\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c}{}$$

```
[Out] 231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+77/10*cos(d*x+c)^(3/2)*sin(d*
x+c)/a^3/d-1/5*cos(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-4/5*cos(d*x
+c)^(7/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-63/10*cos(d*x+c)^(5/2)*sin(d*x+
c)/d/(a^3+a^3*cos(d*x+c))-21/2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.34, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2748, 2635, 2641, 2639}

$$-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{63\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (231*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (21*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) - (21*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) + (77*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(10*a^3*d) - (Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d
*(a + a*Cos[c + d*x])^3) - (4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*a*d*(a +
a*Cos[c + d*x])^2) - (63*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Cos[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(\frac{9a}{2}-\frac{15}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}dx}{5a^2}$$

$$= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(42a^2-\frac{105}{2}a^2\cos\right.}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+}{10d\left(a^3+a^3\cos(c+dx\right.}$$

$$= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+}{10d\left(a^3+a^3\cos(c+dx\right.}$$

$$= -\frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{9}{2}}(c+dx)\sin(c}{5d(a+a\cos(c+d}$$

$$= \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos}{}$$

Mathematica [C] time = 2.75, size = 388, normalized size = 1.87

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\sqrt{\cos(c+dx)}\left(\frac{1}{16}\sec\left(\frac{c}{2}\right)\left(-770\sin\left(c+\frac{dx}{2}\right)+840\sin\left(c+\frac{3dx}{2}\right)-150\sin\left(2c+\frac{3dx}{2}\right)+23\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((42*I)*Sqrt[2]*(11*(1 + E^((2*I)*(c + d*x))) + 11*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - Sqrt[Cos[c + d*x]]*(264*Cot[c] + 198*Csc[c] + (Sec[c/2]*Se
c[(c + d*x)/2]^5*(1210*Sin[(d*x)/2] - 770*Sin[c + (d*x)/2] + 840*Sin[c + (3
*d*x)/2] - 150*Sin[2*c + (3*d*x)/2] + 238*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c
 + (5*d*x)/2] + 5*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (7*d*x)/2] - Sin[4*c +
 (9*d*x)/2] - Sin[5*c + (9*d*x)/2]))/16)))/(5*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{11}{2}}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(11/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3
*a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{11}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/(a*cos(d*x + c) + a)^3, x)
```

maple [A] time = 0.64, size = 296, normalized size = 1.43

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(64\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-288\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-76\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*d
*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos(1/2
*d*x+1/2*c)^2-1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{11}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/(a*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{11/2}}{\left(a+a\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(11/2)/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.190 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{119\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{\sin(c+dx)}{5d(a\cos(c}$$

```
[Out] -119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*cos(d*x+c)^(7/2)*sin(d*x
+c)/d/(a+a*cos(d*x+c))^3-2/3*cos(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c
))^2-119/30*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))+11/2*sin(d*x
+c)*cos(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.32, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2748, 2639, 2635, 2641}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{119\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{\sin(c+dx)}{5d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2]
)/(2*a^3*d) + (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) - (Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*Cos[c + d*x]^(5/2)*
Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - (119*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2}-\frac{13}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(25a^2-\frac{69}{2}a^2\cos(c+\right.}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+}{30d\left(a^3+a^3\cos(c+dx\right.}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+}{30d\left(a^3+a^3\cos(c+dx\right.}$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - 2$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} - \frac{\cos^{\frac{7}{2}}(c}{5d(a+}$$

Mathematica [C] time = 1.90, size = 369, normalized size = 2.04

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc(c)\sqrt{\cos(c+dx)}\left(1961\cos\left(\frac{1}{2}(c-dx)\right)+1609\cos\left(\frac{1}{2}(3c+dx)\right)+1165\cos\left(\frac{1}{2}(c+3dx)\right)+620\cos\left(\frac{1}{2}(5c+3dx)\right)+292\cos\left(\frac{1}{2}(3c+5dx)\right)\right.}{12d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x))) + 119*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/
E^(I*(c + d*x))]) + (Sqrt[Cos[c + d*x]]*(1961*Cos[(c - d*x)/2] + 1609*Cos[(
3*c + d*x)/2] + 1165*Cos[(c + 3*d*x)/2] + 620*Cos[(5*c + 3*d*x)/2] + 292*Co
s[(3*c + 5*d*x)/2] + 65*Cos[(7*c + 5*d*x)/2] + 5*Cos[(5*c + 7*d*x)/2] - 5*C
os[(9*c + 7*d*x)/2])*Csc[c]*Sec[(c + d*x)/2]^5)/(12*d)))/(5*a^3*(1 + Cos[c
+ d*x])^3)
```

fricas [F] time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(9/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x)
```

maple [A] time = 0.47, size = 283, normalized size = 1.56

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(160\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*
d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(
1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^{9/2}}{(a+a\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.191 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{8\sin(c+dx)}{15ad(a\cos(}$$

```
[Out] 49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*cos(d*x+c)^(5/2)*sin(d*x+c
)/d/(a+a*cos(d*x+c))^3-8/15*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c)
)^2-13/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2765, 2977, 2748, 2641, 2639}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{8\sin(c+dx)}{15ad(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/
(6*a^3*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3)
- (8*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (13
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-\frac{11}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(12a^2-\frac{41}{2}a^2\cos\right.}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{13\sqrt{\cos(c+dx)}\,\sin(c+}{6d\left(a^3+a^3\cos(c+dx\right.}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{13\sqrt{\cos(c+dx)}\,\sin(c+}{6d\left(a^3+a^3\cos(c+dx\right.}$$

$$= \frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+}{15ad(a+a}$$

Mathematica [C] time = 4.20, size = 349, normalized size = 2.25

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(806\cos\left(\frac{1}{2}(c-dx)\right)+664\cos\left(\frac{1}{2}(3c+dx)\right)+470\cos\left(\frac{1}{2}(c+3dx)\right)+265\cos\left(\frac{1}{2}(5c+3dx)\right)+117\cos\left(\frac{1}{2}(3\right.\right.}{8d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-
1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*(806*Cos[(c - d*x)/2] + 664*Cos[(3*c
 + d*x)/2] + 470*Cos[(c + 3*d*x)/2] + 265*Cos[(5*c + 3*d*x)/2] + 117*Cos[(3
*c + 5*d*x)/2] + 30*Cos[(7*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2
]^5)/(8*d)))/(15*a^3*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{7}{2}}}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(7/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x)
```

maple [A] time = 0.75, size = 270, normalized size = 1.74

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(348\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{F}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d
*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2
*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{7/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

3.192 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{2\sin(c+dx)\sqrt{\cos(c}}{5ad(a\cos(c+dx)}$$

```
[Out] -9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*cos(d*x+c)^(3/2)*sin(d*x+c)
/d/(a+a*cos(d*x+c))^3-2/5*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^
2+9/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2978, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{2\sin(c+dx)\sqrt{\cos(c}}{5ad(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^
3*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^2) + (9*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-\frac{9}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{3a^2-\frac{21}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,}{15a^4}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{9\sqrt{\cos(c+dx)}\,\sin(c+d}{10d\left(a^3+a^3\cos(c+dx\right)}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{9\sqrt{\cos(c+dx)}\,\sin(c+d}{10d\left(a^3+a^3\cos(c+dx\right)}$$

$$= -\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}}{5ad(a+a\cos}$$

Mathematica [C] time = 6.41, size = 705, normalized size = 4.55

$$-\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\cot(c))\right)}}{d\sqrt{\cot^2(c)+1}\,(a\cos(c+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (((-9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
 (2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[
```

```
1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*((36*Csc[c])/(5*d
) + (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (12*Sec[c/2]*Sec[
c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sin
[(d*x)/2])/(5*d) - (12*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) + (2*Sec[c/2 +
(d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x)
```

maple [A] time = 0.48, size = 270, normalized size = 1.74

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2*d*x
```

```
+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{5/2}}{(a+a\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.193 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos(c}{5d(a\cos(c+dx)+}$$

```
[Out] -1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1/2)
/d/(a+a*cos(d*x+c))^3+4/15*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))
^2+1/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2765, 2978, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos(c}{5d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (4*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\frac{a}{2}-\frac{7}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{-\frac{a^2}{2}-2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx}}{15a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+}{10d\left(a^3+a^3\cos(c+d\right.}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+}{10d\left(a^3+a^3\cos(c+d\right.}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx}}{15ad(a+a\,c}$$

Mathematica [C] time = 3.95, size = 334, normalized size = 2.15

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(14\cos\left(\frac{1}{2}(c-dx)\right)+16\cos\left(\frac{1}{2}(3c+dx)\right)+20\cos\left(\frac{1}{2}(c+3dx)\right)-5\cos\left(\frac{1}{2}(5c+3dx)\right)+3\cos\left(\frac{1}{2}(3c+5dx)\right)\right)s}{8d}\right.}{15a^3(\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))
/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*
(c + d*x))]) + (Sqrt[Cos[c + d*x]]*(14*Cos[(c - d*x)/2] + 16*Cos[(3*c + d*x
)/2] + 20*Cos[(c + 3*d*x)/2] - 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)
/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d)))/(15*a^3*(1 + Cos[c + d*x
])^3)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{3}{2}}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

maple [A] time = 0.52, size = 270, normalized size = 1.74

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{El}}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^{3/2}}{(a+a\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

3.194 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)\sqrt{\cos(c+d}}{5d(a\cos(c+dx)+a)}$$

```
[Out] 1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*sin(d*x+c)*cos(d*x+c)^(1/2)/
d/(a+a*cos(d*x+c))^3+1/15*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^
2-1/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2764, 2978, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)\sqrt{\cos(c+d}}{5d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
+ (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{a}{2}+\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{2a^2+\frac{1}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx}{15a^4}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\,\sin}{15ad(a+a\cos(}$$

Mathematica [C] time = 3.39, size = 334, normalized size = 2.15

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(4\cos\left(\frac{1}{2}(c-dx)\right)+26\cos\left(\frac{1}{2}(3c+dx)\right)+10\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+3\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\mathrm{se}}{8d}\right.}{15a^3(\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/
(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(
c + d*x))]) - (Sqrt[Cos[c + d*x]]*(4*Cos[(c - d*x)/2] + 26*Cos[(3*c + d*x)/
2] + 10*Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)/2
])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d)))/(15*a^3*(1 + Cos[c + d*x])
^3)
```

fricas [F] time = 1.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

maple [A] time = 0.50, size = 270, normalized size = 1.74

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{El}}{60a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

3.195 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{5ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos}}{5d(a\cos(c+dx}$$

```
[Out] 9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1/2)/
d/(a+a*cos(d*x+c))^3-2/5*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2
-9/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2766, 2978, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{5ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos}}{5d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3
*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^2) - (9*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{9a}{2}-\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{21a^2}{2}\ldots}{\sqrt{\cos(c\ldots}}}{\ldots}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{9\sqrt{\cos(c\ldots}}{10d\left(a^3\ldots\right.}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{9\sqrt{\cos(c\ldots}}{10d\left(a^3\ldots\right.}$$

$$= \frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \ldots$$

Mathematica [C] time = 6.37, size = 705, normalized size = 4.55

$$-\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\cot(c)\ldots\right.}}{d\sqrt{\cot^2(c)+1}\,(a\cos(c+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (((9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
(2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[1
```

```
+ Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*((-36*Csc[c])/(5*d
) - (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (8*Sec[c/2]*Sec[c
/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sin[
(d*x)/2])/(5*d) - (8*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) - (2*Sec[c/2 + (d
*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

maple [A] time = 0.60, size = 268, normalized size = 1.73

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(36\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\;\text{Elli}\cdots\right.}{20a^{3}\,c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
```

```
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{7}{2}}{\left(c+dx \right)}+3\cos^{\frac{5}{2}}{\left(c+dx \right)}+3\cos^{\frac{3}{2}}{\left(c+dx \right)}+\sqrt{\cos{\left(c+dx \right)}}}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(7/2) + 3*cos(c + d*x)**(5/2) + 3*cos(c + d*x)**(
3/2) + sqrt(cos(c + d*x))), x)/a**3
```

3.196 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}-\frac{}{15ad\sqrt{\text{co}}}$$

```
[Out] -49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+49/10*sin(d*x+c)/a^3/d/cos(d*
x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2)-8/15*sin(d*
x+c)/a/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)-13/6*sin(d*x+c)/d/(a^3+a^3*cos
(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2978, 2748, 2636, 2639, 2641}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}-\frac{}{15ad\sqrt{\text{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (-49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])
/(6*a^3*d) + (49*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]
/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d
*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) - (13*Sin[c + d*x])/(6*d*Sqrt[C
os[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}dx = -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{11a}{2}-\frac{5}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\cos(c$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\cos(c$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\cos(c$$

$$= -\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{co}$$

$$= -\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{}{5d}$$

Mathematica [C] time = 2.01, size = 364, normalized size = 2.01

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}(c+3dx)\right)+374\cos\left(\frac{1}{2}(5c+3dx)\right)+670\cos\left(\frac{1}{2}(3c+5dx)\right)+65\,\mathrm{c}}{16d\sqrt{\cos(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/
E^(I*(c + d*x))]) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 1243
*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2] +
```

```
 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[
(c + d*x)/2]^5)/(16*d*Sqrt[Cos[c + d*x]])))/(15*a^3*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{5}+3\,a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{3}+a^{3}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 3*
a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.73, size = 555, normalized size = 3.07

$$-\frac{-2\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(65\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
```

```
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{9}{2}}{\left(c+dx \right)}+3\cos^{\frac{7}{2}}{\left(c+dx \right)}+3\cos^{\frac{5}{2}}{\left(c+dx \right)}+\cos^{\frac{3}{2}}{\left(c+dx \right)}}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(9/2) + 3*cos(c + d*x)**(7/2) + 3*cos(c + d*x)**(
5/2) + cos(c + d*x)**(3/2)), x)/a**3
```

3.197 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{119\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin}{10a^3d\sqrt{c}}$$

```
[Out] 119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*sin(d*x+c)/a^3/d/cos(d*x
+c)^(3/2)-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3-2/3*sin(d*x+
c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-119/30*sin(d*x+c)/d/cos(d*x+c)^(
3/2)/(a^3+a^3*cos(d*x+c))-119/10*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2978, 2748, 2636, 2641, 2639}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{119\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin}{10a^3d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Cos[c + d*x]^(3/2)) - (119*Sin[c +
d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*
(a + a*Cos[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a
*Cos[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*
Cos[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{13a}{2}-\frac{7}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c$$

$$= \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos$$

$$= \frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{11}{10a^3}$$

Mathematica [C] time = 2.64, size = 394, normalized size = 1.90

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(5134\cos\left(\frac{1}{2}(c-dx)\right)+4148\cos\left(\frac{1}{2}(3c+dx)\right)+4664\cos\left(\frac{1}{2}(c+3dx)\right)+2476\cos\left(\frac{1}{2}(5c+3dx)\right)+3340\cos\left(\frac{1}{2}(3c+5dx\right.\right.}{96d\cos^{\frac{3}{2}}(c+dx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x))) + 119*(-
1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] - 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - ((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/2] + 4664
```

```
*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c + 5*d*x)/2]
 + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Cos[(9*c + 7*
d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(
96*d*Cos[c + d*x]^(3/2))))/(5*a^3*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 2.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^6+3a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+a^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^6 + 3*a^3*cos(d*x + c)^5 + 3*
a^3*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

maple [A] time = 1.12, size = 453, normalized size = 2.19

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{32\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{118\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(32/15*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+11
8/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
```

```
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+48*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*
c)^2)^(1/2)+1/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)^5-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{11}{2}}{\left(c+dx \right)}+3\cos^{\frac{9}{2}}{\left(c+dx \right)}+3\cos^{\frac{7}{2}}{\left(c+dx \right)}+\cos^{\frac{5}{2}}{\left(c+dx \right)}}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(11/2) + 3*cos(c + d*x)**(9/2) + 3*cos(c + d*x)**
(7/2) + cos(c + d*x)**(5/2)), x)/a**3
```

3.198 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{5a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx}}{8d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 5/8*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+5/12*a*cos(
d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*cos(d*x+c)^(5/2)*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+5/8*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*c
os(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2770, 2774, 216}

$$\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{5a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx}}{8d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (5*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d) +
 (5*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (5*
a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Cos
[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{6}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{5a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{8}\int \sqrt{\cos$$

$$= \frac{5a\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{5}{2}}(c}{3d\sqrt{a}}$$

$$= \frac{5a\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{5}{2}}(c}{3d\sqrt{a}}$$

$$= \frac{5\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{5a\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\cos^{\frac{3}{2}}(c}{12d\sqrt{a}}$$

Mathematica [A] time = 0.35, size = 105, normalized size = 0.68

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(14\sin\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{3}{2}(c+d\right.\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*ArcSin[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(14*Sin[(c + d*x)/2] + 3*Sin[(3*(c +
d*x))/2] + 2*Sin[(5*(c + d*x))/2])))/(48*d)
```

fricas [A] time = 0.98, size = 108, normalized size = 0.70

$$\frac{\sqrt{a\cos(dx+c)+a}\left(8\cos(dx+c)^2+10\cos(dx+c)+15\right)\sqrt{\cos(dx+c)}\sin(dx+c)-15\sqrt{a}(\cos(dx+c)+1}{24(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 10*cos(d*x + c) + 15)*sq
rt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x +
 c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

maple [A] time = 0.28, size = 196, normalized size = 1.27

$$-\frac{\left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{3}\left(8\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)+10\sqrt{\frac{c}{1+}}\right.}{24d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/24/d*cos(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(8*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+10*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+15*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
)))/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6
```

maxima [B] time = 2.01, size = 1921, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
```

```
 + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arct
an2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
```

```
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{5/2}\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.199 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 3/4*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+1/2*a*cos(d
*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+3/4*a*sin(d*x+c)*cos(d*x+c)
^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2770, 2774, 216}

$$\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d) +
 (3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{8}\int \frac{\sqrt{a+\cdots}}{\sqrt{c\cdots}}$$

$$= \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{3\,\mathrm{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{3\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{3}{2}}(c+\cdots}{2d\sqrt{a+\cdots}}$$

Mathematica [A] time = 0.21, size = 91, normalized size = 0.78

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\cdots}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x)
)/2])))/(8*d)
```

fricas [A] time = 1.75, size = 98, normalized size = 0.84

$$\frac{\sqrt{a\cos(dx+c)+a}\,(2\cos(dx+c)+3)\sqrt{\cos(dx+c)}\sin(dx+c) - 3\sqrt{a}\,(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\cdots}{\sqrt{a}\sin(d\cdots}\right.}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d
*x + c) - 3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.17, size = 161, normalized size = 1.39

$$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a\,(1+\cos(dx+c))}\,(-1+\cos(dx+c))^2\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+3\sqrt{\frac{\cos(dx\cdots}{1+\cos(\cdots}}\cdots\right.}{4d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)+3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c)))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

maxima [B] time = 1.71, size = 1059, normalized size = 9.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
```

```
x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{3/2}\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\cos(c+dx)+1)}\,\cos^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/2), x)
```

3.200 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)*co
s(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2770, 2774, 216}

$$\frac{\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

$$= \frac{\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 1.07

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c
 + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

fricas [A] time = 0.89, size = 88, normalized size = 1.22

$$-\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.18, size = 123, normalized size = 1.71

$$-\frac{\left(\sqrt{\cos}\ (dx+c)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \sin\left(dx+c\right)+\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\right)\left(-1+\cos\left(d\right.\right.}{d\sin\left(dx+c\right)^{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*((cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c)))*(-1+cos(d*x+c))/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

maxima [B] time = 1.84, size = 791, normalized size = 10.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
```

```
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x)), x)
```

3.201 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2774, 216}

$$\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 1.35

$$\frac{\sqrt{2}\,\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c
 + d*x)/2])/d
```

fricas [A] time = 1.97, size = 119, normalized size = 3.22

$$\left[\frac{\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{a\cos(dx+c)+a}\,\sqrt{-a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d}, -\frac{2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sqr
t(cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))/d, -
2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.14, size = 80, normalized size = 2.16

$$\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
[Out] 2/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c)))^(
1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))
```

maxima [B] time = 1.73, size = 146, normalized size = 3.95

$$\sqrt{a}\,\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sqrt(cos(c + d*x)), x)
```

3.202 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2771}

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 1.08

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

fricas [A] time = 0.79, size = 49, normalized size = 1.36

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^
2 + d*cos(d*x + c))
```

giac [A] time = 0.78, size = 58, normalized size = 1.61

$$\frac{4\sqrt{2}\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)}{\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/(sqrt(tan(
1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)
```

maple [A] time = 0.15, size = 42, normalized size = 1.17

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{d\sin\left(dx+c\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*(-1+cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

maxima [B] time = 1.33, size = 98, normalized size = 2.72

$$\frac{2\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

mupad [B] time = 0.45, size = 41, normalized size = 1.14

$$\frac{2\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*cos(c + d*x)^(1/2)*(cos(c
+ d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/cos(c + d*x)**(3/2), x)
```

3.203 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/3*a*sin(d*x+c)
/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2772, 2771}

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2}{3}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.66

$$\frac{2(2\cos(c+dx)+1)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 2*Cos[c + d*x])*Tan[(c + d*x)/2])/(3*d*C
os[c + d*x]^(3/2))
```

fricas [A] time = 0.90, size = 61, normalized size = 0.79

$$\frac{2\sqrt{a\cos(dx+c)+a}\,(2\cos(dx+c)+1)\sqrt{\cos(dx+c)}\,\sin(dx+c)}{3\left(d\cos(dx+c)^{3}+d\cos(dx+c)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [A] time = 0.96, size = 87, normalized size = 1.13

$$\frac{4\sqrt{2}\left(\left(3\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-10\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+3\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)}{3\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{4}-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+1\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*((3*tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2 + 3)*sq
rt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)
^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

maple [A] time = 0.18, size = 54, normalized size = 0.70

$$-\frac{2\left(2\left(\cos^{2}\left(dx+c\right)\right)-\cos\left(dx+c\right)-1\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(2*cos(d*x+c)^2-cos(d*x+c)-1)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/co
s(d*x+c)^(3/2)
```

maxima [B] time = 1.44, size = 190, normalized size = 2.47

$$\frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2
)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 1))
```

mupad [B] time = 1.27, size = 82, normalized size = 1.06

$$\frac{4\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (4*(a*(cos(c + d*x) + 1))^(1/2)*(sin(c + d*x) + sin(2*c + 2*d*x) + sin(3*c
+ 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + c
os(3*c + 3*d*x) + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/cos(c + d*x)**(5/2), x)
```

3.204 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/15*a*sin(d*x+c
)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)/d/cos(d*x+c)
^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2772, 2771}

$$\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a
*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*S
in[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{4}{5}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{8}{15}\int$$
$$= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{15d}$$

Mathematica [A] time = 0.09, size = 66, normalized size = 0.57

$$\frac{2\left(5\sin\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(5*Sin[(c + d*x)/2] + 2*Sin[
(5*(c + d*x))/2]))/(15*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 1.07, size = 71, normalized size = 0.62

$$\frac{2\sqrt{a\cos(dx+c)+a}\left(8\cos(dx+c)^2+4\cos(dx+c)+3\right)\sqrt{\cos(dx+c)}\sin(dx+c)}{15\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt(
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

giac [A] time = 1.40, size = 116, normalized size = 1.01

$$\frac{4\sqrt{2}\left(\left(\left(5\left(3\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-20\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+282\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-100\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+15\right)}{15\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 4/15*sqrt(2)*(((5*(3*tan(1/4*d*x + 1/4*c)^2 - 20)*tan(1/4*d*x + 1/4*c)^2 +
282)*tan(1/4*d*x + 1/4*c)^2 - 100)*tan(1/4*d*x + 1/4*c)^2 + 15)*sqrt(a)*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6*ta
n(1/4*d*x + 1/4*c)^2 + 1)^(5/2)*d)
```

maple [A] time = 0.16, size = 64, normalized size = 0.56

$$-\frac{2\left(8\left(\cos^{3}\left(dx+c\right)\right)-4\left(\cos^{2}\left(dx+c\right)\right)-\cos\left(dx+c\right)-3\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{15d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(8*cos(d*x+c)^3-4*cos(d*x+c)^2-cos(d*x+c)-3)*(a*(1+cos(d*x+c)))^(1/
2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

maxima [B] time = 1.11, size = 237, normalized size = 2.06

$$\frac{2\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

mupad [B] time = 2.37, size = 132, normalized size = 1.15

$$\frac{8\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(7\,\sin\left(c+d\,x\right)+4\,\sin\left(2\,c+2\,d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+2\,\sin\left(4\,c+4\,d\,x\right)+2\,\sin\left(5\,c\right.\right.}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c+5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (8*(a*(cos(c + d*x) + 1))^(1/2)*(7*sin(c + d*x) + 4*sin(2*c + 2*d*x) + 9*si
n(3*c + 3*d*x) + 2*sin(4*c + 4*d*x) + 2*sin(5*c + 5*d*x)))/(15*d*cos(c + d*
x)^(1/2)*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos
(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.205 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{16a\sin(c+dx)}{35d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/7*a*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+12/35*a*sin(d*x+
c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+16/35*a*sin(d*x+c)/d/cos(d*x+c
)^(3/2)/(a+a*cos(d*x+c))^(1/2)+32/35*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*c
os(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2772, 2771}

$$\frac{16a\sin(c+dx)}{35d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (12*
a*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*
Sin[c + d*x])/(35*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (32*a*Si
n[c + d*x])/(35*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{6}{7}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{24}{35}\int$$
$$= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{}{35d}$$
$$= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{}{35d}$$

Mathematica [A] time = 0.13, size = 66, normalized size = 0.43

$$\frac{2\left(7\sin\left(\frac{3}{2}(c+dx)\right)+2\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{35d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(7*Sin[(3*(c + d*x))/2] + 2*
Sin[(7*(c + d*x))/2]))/(35*d*Cos[c + d*x]^(7/2))
```

fricas [A] time = 0.99, size = 81, normalized size = 0.53

$$\frac{2\left(16\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+6\cos\left(dx+c\right)+5\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*
x + c)^4)
```

giac [A] time = 1.64, size = 143, normalized size = 0.93

$$\frac{4\sqrt{2}\left(\left(\left(\left(7\left(5\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-10\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+267\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-3684\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+186\right.\right.\right.}{35\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] 4/35*sqrt(2)*(((((7*(5*(tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2
+ 267)*tan(1/4*d*x + 1/4*c)^2 - 3684)*tan(1/4*d*x + 1/4*c)^2 + 1869)*tan(1/
4*d*x + 1/4*c)^2 - 350)*tan(1/4*d*x + 1/4*c)^2 + 35)*sqrt(a)*sgn(cos(1/2*d*
x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
 1/4*c)^2 + 1)^(7/2)*d)
```

maple [A] time = 0.16, size = 74, normalized size = 0.48

$$-\frac{2\left(16\left(\cos^4(dx+c)\right)-8\left(\cos^3(dx+c)\right)-2\left(\cos^2(dx+c)\right)-\cos(dx+c)-5\right)\sqrt{a(1+\cos(dx+c))}}{35d\sin(dx+c)\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/35/d*(16*cos(d*x+c)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)*(a*(1+
cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

maxima [B] time = 1.48, size = 283, normalized size = 1.85

$$\frac{2\left(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{58\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right.}{35\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2
```

```
/(cos(d*x + c) + 1)^2 + 1)^4/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

mupad [**B**] time = 5.64, size = 415, normalized size = 2.71

$$\frac{\sqrt{a + a\left(\frac{e^{-c\,1i-d\,x\,1}}{2}\right.}}{\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,1i+d\,x\,1i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,2i+d\,x\,2i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,3i+d\,x\,3i}\sqrt{\frac{e^{-c\,1i-}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2),x)
```

```
[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(32i/(35*d)
+ (exp(c*2i + d*x*2i)*16i)/(5*d) - (exp(c*5i + d*x*5i)*16i)/(5*d) - (exp(c*
7i + d*x*7i)*32i)/(35*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)
^(1/2) + exp(c*1i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)
^(1/2) + 3*exp(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2)^(1/2) + 3*exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2)^(1/2) + 3*exp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2)^(1/2) + 3*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2)^(1/2) + exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2)^(1/2) + exp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.206 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=160

$$\frac{11a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sqrt{\cos(}}{8d\sqrt{a\cos(c+dx)}}$$

```
[Out] 11/8*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+11/12*a^2*
cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^(5/
2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+11/8*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)
/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2763, 21, 2770, 2774, 216}

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{11a^2\sin(c+dx)\sqrt{\cos(}}{8d\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (11*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) +
 (11*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
 (a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{11a^2}{2}+\frac{11}{2}a^2\cos(c+\right.}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(11a)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}$$

$$= \frac{11a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(11a)$$

$$= \frac{11a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\,\mathrm{c}}{3}$$

$$= \frac{11a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\,\mathrm{c}}{3}$$

$$= \frac{11a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{11a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\,\mathrm{c}}{12}$$

Mathematica [A] time = 0.37, size = 106, normalized size = 0.66

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(33\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(26\sin\left(\frac{1}{2}(c+dx)\right)+9\sin\left(\frac{3}{2}(c+d\right.\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(33*Sqrt[2]*ArcSin[Sqrt[2]*S
in[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(26*Sin[(c + d*x)/2] + 9*Sin[(3*(c
+ d*x))/2] + 2*Sin[(5*(c + d*x))/2])))/(48*d)
```

fricas [A] time = 2.23, size = 114, normalized size = 0.71

$$\frac{\left(8\,a\cos\left(dx+c\right)^2+22\,a\cos\left(dx+c\right)+33\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-33\left(a\cos\left(dx+c\right)+\right.}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((8*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 33*a)*sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))*sin(d*x + c) - 33*(a*cos(d*x + c) + a)*sqrt(a)*arcta
n(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*c
os(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.21, size = 197, normalized size = 1.23

$$\frac{(-1+\cos(dx+c))^2\left(8\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\sin(dx+c)+22\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+33}{24d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*
sin(d*x+c)+22*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+33*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+33*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/
2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4*a
```

maxima [B] time = 2.07, size = 1942, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x
+ 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
```

(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
1*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a)
 + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - a
*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
 cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arctan2
((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*

```
c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^
(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
 cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))/
d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{3/2}\,(a+a\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.207 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=120

$$\frac{7a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 7/4*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+1/2*a^2*cos
(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+7/4*a^2*sin(d*x+c)*cos(d*
x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2763, 21, 2770, 2774, 216}

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d) +
 (7*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (
a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
```

```
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}\,dx = \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}\int \frac{\sqrt{\cos(c+dx)}\left(\frac{7a^2}{2}+\frac{7}{2}a^2\cos(c\right.}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(7a)\int \sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c}$$

$$= \frac{7a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(7a$$

$$= \frac{7a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(7a)\,S}{}$$

$$= \frac{7a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{7a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos}{2d}$$

Mathematica [A] time = 0.23, size = 92, normalized size = 0.77

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(7\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(6\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(7*Sqrt[2]*ArcSin[Sqrt[2]*Si
n[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(6*Sin[(c + d*x)/2] + Sin[(3*(c + d*
x))/2])))/(8*d)
```

fricas [A] time = 1.05, size = 103, normalized size = 0.86

$$\frac{(2\,a\cos(dx+c)+7\,a)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-7\,(a\cos(dx+c)+a)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx}}{\sqrt{a}}\right.}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*cos(d*x + c) + 7*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*s
in(d*x + c) - 7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.17, size = 160, normalized size = 1.33

$$-\frac{(-1+\cos(dx+c))\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+7\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+7\arctan\left(\frac{\sin(dx+c)\sqrt{}}{\cos(}\right.\right.}{4d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(
d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(
d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*a
```

maxima [B] time = 1.91, size = 1080, normalized size = 9.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+d\,x)}\,\left(a+a\,\cos(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x)), x)
```

3.208 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 3*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+a^2*sin(d*x+c
)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2763, 21, 2774, 216}

$$\frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a^
2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\int \frac{\frac{3a^2}{2}+\frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx\\
&= \frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{1}{2}(3a)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\
&= \frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 79, normalized size = 1.05

$$
\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\,\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Si
n[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

fricas [A] time = 1.12, size = 90, normalized size = 1.20

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}\,a\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - 3*(a*cos(d*x
+ c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.16, size = 168, normalized size = 2.24

$$\frac{\left(3\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+\cos\left(dx+c\right)\sin\left(dx+c\right)+3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\cdots\right.\right.}{d\sqrt{\cos\left(dx+c\right)}\,\left(1+\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+cos(d*x+c)*sin(d*x+c)+3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(1+cos(d*x+c))*a
```

maxima [B] time = 1.96, size = 803, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \left(\cos{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}{\sqrt{\cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/sqrt(cos(c + d*x)), x)
```

3.209 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2*a^2*sin(d*x
+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2762, 21, 2774, 216}

$$\frac{2a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*
a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}-(2a)\int \frac{-\frac{a}{2}-\frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx\\
&= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}+a\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}-\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\
&= \frac{2a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 85, normalized size = 1.12

$$
\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right)}{d\sqrt{\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d*x
]])
```

fricas [A] time = 1.41, size = 109, normalized size = 1.43

$$\frac{2\left(\sqrt{a\cos\left(dx+c\right)+a}\,a\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\cdots}{\sqrt{a}\,\sin(dx\cdots}\right.\right.}{d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - (a*cos(d*x
+ c)^2 + a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.17, size = 249, normalized size = 3.28

$$-\frac{2\left(\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+2\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\cdots}}{\cos(dx\cdots}\right.\right.}{d\left(-1+\cos\left(dx+c\right)\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*(cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
))+(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))+cos(d*x+c)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)*a
```

maxima [B] time = 1.66, size = 997, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\cos(c + d\,x))^{3/2}}{\cos(c + d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(3/2), x)
```

3.210 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=81

$$\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+10/3*a^2*sin(d
*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2762, 21, 2771}

$$\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (1
0*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}-\frac{1}{3}(2a)\int \frac{-\frac{5a}{2}-\frac{5}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx\\ &= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{1}{3}(5a)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 52, normalized size = 0.64

$$\frac{2a(5\cos(c+dx)+1)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 5*Cos[c + d*x])*Tan[(c + d*x)/2])/(3*d
*Cos[c + d*x]^(3/2))
```

fricas [A] time = 5.66, size = 62, normalized size = 0.77

$$\frac{2\left(5\,a\cos\left(dx+c\right)+a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 55, normalized size = 0.68

$$-\frac{2\left(5\left(\cos^{2}\left(dx+c\right)\right)-4\cos\left(dx+c\right)-1\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,a}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(5*cos(d*x+c)^2-4*cos(d*x+c)-1)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/
cos(d*x+c)^(3/2)*a
```

maxima [A] time = 0.73, size = 125, normalized size = 1.54

$$\frac{4\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

mupad [B] time = 1.22, size = 89, normalized size = 1.10

$$\frac{2\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(5\,\sin\left(c+d\,x\right)+2\,\sin\left(2\,c+2\,d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*a*(a*(cos(c + d*x) + 1))^(1/2)*(5*sin(c + d*x) + 2*sin(2*c + 2*d*x) + 5*
sin(3*c + 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c + 2*
d*x) + cos(3*c + 3*d*x) + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(5/2), x)
```

3.211 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+6/5*a^2*sin(d*
x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+12/5*a^2*sin(d*x+c)/d/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2762, 21, 2772, 2771}

$$\frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (6
*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (12*
a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
```

```
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}-\frac{1}{5}(2a)\int \frac{-\frac{9a}{2}-\frac{9}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,d\\ &= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{1}{5}(9a)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{1}{5}(6\\ &= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{}{5d\sqrt{}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 62, normalized size = 0.51

$$\frac{2a(3\cos(c+dx)+3\cos(2(c+dx))+4)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*T
an[(c + d*x)/2])/(5*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 2.16, size = 73, normalized size = 0.60

$$\frac{2\left(6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 65, normalized size = 0.54

$$-\frac{2\left(6\left(\cos^{3}\left(dx+c\right)\right)-3\left(\cos^{2}\left(dx+c\right)\right)-2\cos\left(dx+c\right)-1\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\;a}{5d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d*(6*cos(d*x+c)^3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*(a*(1+cos(d*x+c)))^(1
/2)/sin(d*x+c)/cos(d*x+c)^(5/2)*a
```

maxima [B] time = 1.13, size = 217, normalized size = 1.79

$$\frac{4\left(\frac{5\,\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7\,\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{2\,\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 4/5*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

mupad [B] time = 2.10, size = 133, normalized size = 1.10

$$\frac{4\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(8\,\sin\left(c+d\,x\right)+6\,\sin\left(2\,c+2\,d\,x\right)+11\,\sin\left(3\,c+3\,d\,x\right)+3\,\sin\left(4\,c+4\,d\,x\right)+3\,\sin}{5\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (4*a*(a*(cos(c + d*x) + 1))^(1/2)*(8*sin(c + d*x) + 6*sin(2*c + 2*d*x) + 11
*sin(3*c + 3*d*x) + 3*sin(4*c + 4*d*x) + 3*sin(5*c + 5*d*x)))/(5*d*cos(c +
d*x)^(1/2)*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*c
os(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.212 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{104a^2\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+26/35*a^2*sin(
d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)/d/c
os(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)/d/cos(d*x+c)^
(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2762, 21, 2772, 2771}

$$\frac{104a^2\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
6*a^2*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (1
04*a^2*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) +
(208*a^2*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{1}{7}(2a)\int \frac{-\frac{13a}{2}-\frac{13}{2}a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}(13a)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{35}$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{10}$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{10}$$

Mathematica [A] time = 0.22, size = 72, normalized size = 0.45

$$\frac{2a(117\cos(c+dx)+26\cos(2(c+dx))+26\cos(3(c+dx))+41)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(41 + 117*Cos[c + d*x] + 26*Cos[2*(c + d*x)
] + 26*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

fricas [A] time = 0.98, size = 86, normalized size = 0.53

$$\frac{2\left(104\,a\cos\left(dx+c\right)^{3}+52\,a\cos\left(dx+c\right)^{2}+39\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*
a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)
^5 + d*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 75, normalized size = 0.47

$$-\frac{2\left(104\left(\cos^{4}\left(dx+c\right)\right)-52\left(\cos^{3}\left(dx+c\right)\right)-13\left(\cos^{2}\left(dx+c\right)\right)-24\cos\left(dx+c\right)-15\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,a}{105d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(104*cos(d*x+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-15
)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)*a
```

maxima [A] time = 1.20, size = 263, normalized size = 1.63

$$\frac{4\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right.}{105\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 4/105*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^
(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 1))
```

mupad [B] time = 4.59, size = 157, normalized size = 0.98

$$\frac{91\,a\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}-35\,a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}+26\,a\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,\sqrt{a+\cdots}}{\frac{315\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{315\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{105\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{105\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\cdots\right)}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2),x)
```

```
[Out] (91*a*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2) - 35*a*sin(c/2 +
(d*x)/2)*(a + a*cos(c + d*x))^(1/2) + 26*a*sin((7*c)/2 + (7*d*x)/2)*(a + a*
cos(c + d*x))^(1/2))/((315*d*cos(c + d*x)^(1/2)*cos(c/2 + (d*x)/2))/8 + (31
5*d*cos(c + d*x)^(1/2)*cos((3*c)/2 + (3*d*x)/2))/8 + (105*d*cos(c + d*x)^(1
/2)*cos((5*c)/2 + (5*d*x)/2))/8 + (105*d*cos(c + d*x)^(1/2)*cos((7*c)/2 + (
7*d*x)/2))/8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.213 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=200

$$\frac{163a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{163a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{163a^3\sin(c+dx)}{64d\sqrt{a\cos(c}$$

```
[Out] 163/64*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+163/96*a
^3*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+17/24*a^3*cos(d*x+c
)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+163/64*a^3*sin(d*x+c)*cos(d*x+c
)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+a*c
os(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.36, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2763, 2981, 2770, 2774, 216}

$$\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{163a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(64*d
) + (163*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]
]) + (163*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x
]]) + (17*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a$$

$$= \frac{17a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,\mathrm{si}}{4d}$$

$$= \frac{163a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{17a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\,\mathrm{c}}{}$$

$$= \frac{163a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{163a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{1}{}$$

$$= \frac{163a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{163a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{1}{}$$

$$= \frac{163a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{163a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{163}{}$$

Mathematica [C] time = 4.40, size = 182, normalized size = 0.91

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{5/2}\left(-6\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(-\frac{1}{2},\frac{3}{2},2;1,\frac{9}{2};2\sin^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] +
 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-3/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2
] - 24*(3 + Cos[c + d*x])*Hypergeometric2F1[-1/2, 3/2, 9/2, 2*Sin[(c + d*x)
/2]^2]*Sin[c + d*x]^2 - 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{-1/2, 3/2,
2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*
d)
```

fricas [A] time = 2.62, size = 137, normalized size = 0.68

$$\frac{\left(48\,a^2\cos(dx+c)^3 + 184\,a^2\cos(dx+c)^2 + 326\,a^2\cos(dx+c) + 489\,a^2\right)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(}{192\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*((48*a^2*cos(d*x + c)^3 + 184*a^2*cos(d*x + c)^2 + 326*a^2*cos(d*x +
c) + 489*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 48
9*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.24, size = 234, normalized size = 1.17

$$\frac{(-1+\cos(dx+c))^2\left(48\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+184\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^2*(48*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+184*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+
326*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+489*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+489*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4*a^2
```

maxima [B] time = 3.27, size = 7450, normalized size = 37.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/768*(10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
```

*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d
*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*co
s(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x + 4*c)^3
 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(2*a^2*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*
d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4
*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x +
 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d
*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3
2*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c)
 + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(16*a^2*
cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*cos(4*d*x + 4*c) +
3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a^2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 19*a^2
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*si
n(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 12*(4*a^2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a^2*sin(4*d*x
 + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 - 8
*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 14*a^2*cos(4*d*x +
4*c)^2 + 19*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d
*x + 4*c)^2 - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x
 + 4*c)^3 - 2*a^2*cos(4*d*x + 4*c)^2 - 13*a^2*cos(4*d*x + 4*c) + (3*a^2*cos
(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*
x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(
4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^
2 - 19*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))*sin(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
c))) + 4(3*a^2*cos(4*d*x + 4*c)^3 - 11*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*cos
(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*(2*a^2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)

```
 - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(3
*a^2*cos(4*d*x + 4*c) - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^
(1/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^
3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x +
4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d
*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4
*(3*a^2*sin(4*d*x + 4*c)^3 + 3*(a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x +
4*c) + a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
+ 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(3*a^2*sin(4*d*x + 4*c)^3 + 160*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c)^2 + 6*a^2*cos(
4*d*x + 4*c) + 43*a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 2*(6*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 6*(a^2*cos(4*d*x + 4*c)^2 - a
^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (320*a^2*cos(4*d*x + 4*c)^2 + 320*a
^2*sin(4*d*x + 4*c)^2 - 317*a^2*cos(4*d*x + 4*c) - 3*a^2)*sin(1/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) - 2*(20*a^2*cos(4*d*x + 4*c)^2 + 26*a^2*sin(4*d*x + 4*c)^2 -
 317*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x
 + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*(
10*a^2*cos(4*d*x + 4*c)^2 + 13*a^2*sin(4*d*x + 4*c)^2 - 160*a^2*sin(4*d*x +
 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 10*a^2*cos(4*d
*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 3*(a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (160*a^2*cos(4*d*x
+ 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*sin(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 + 120*a^2*cos(4*d*x + 4*c
)^2 - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*
x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 - 3*
a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 4*(3*a^2*cos(4*d*x + 4*c)^3 + 74*a^2*cos(4*d*x + 4*c)^2 - 197*a^2*cos(4*d*
x + 4*c) + (3*a^2*cos(4*d*x + 4*c) + 80*a^2)*sin(4*d*x + 4*c)^2 + 120*a^2 -
```

80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 + 126*a^2*cos(4*d*x + 4*c)^2 + 243*a^2*cos(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c)^2 + 120*a^2 - 40*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(6*a^2*cos(4*d*x + 4*c)^3 + 214*a^2*cos(4*d*x + 4*c)^2 - 3*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 240*a^2*cos(4*d*x + 4*c) + 2*(3*a^2*cos(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c)^2 - (160*a^2*cos(4*d*x + 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 - 157*a^2*cos(4*d*x + 4*c) - 3*a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (80*a^2*cos(4*d*x + 4*c)^2 + 80*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(320*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 157*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(80*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (3*a^2*cos(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 6*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x +

```
4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 -
2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(
4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c)
)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*
x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*cos(4*d*x + 4*c)
^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*
a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*
x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*
sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arc
tan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
```

```
4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d
*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4
*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c)
+ a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(
4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)) - 1))*sqrt(a))/((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*
d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + d\,x)^{3/2}\,(a + a\,\cos(c + d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.214 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{25a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{25a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos$$

```
[Out] 25/8*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+13/12*a^3*
cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+25/8*a^3*sin(d*x+c)*co
s(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^(3/2)*sin(d*x+c)
*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.30, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2763, 2981, 2770, 2774, 216}

$$\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{25a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (25*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (25*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) +
 (13*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
 (a^2*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}\,dx = \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\int \sqrt{\cos(c+dx)}$$

$$= \frac{13a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}{3d}$$

$$= \frac{25a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a}{}$$

$$= \frac{25a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a}{}$$

$$= \frac{25a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{25a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a}{}$$

Mathematica [C] time = 4.30, size = 182, normalized size = 1.14

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{5/2}\left(-2\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(\frac{1}{2},\frac{3}{2},2;1,\frac{9}{2};2\sin^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] +
 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2
] - 8*(3 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2
]^2]*Sin[c + d*x]^2 - 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{1/2, 3/2, 2},
 {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

fricas [A] time = 1.77, size = 124, normalized size = 0.78

$$\frac{\left(8\,a^2\cos\left(dx+c\right)^2+34\,a^2\cos\left(dx+c\right)+75\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-75\left(a^2\cos\left(dx+\right.\right.}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*((8*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 75*a^2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 75*(a^2*cos(d*x + c) + a^2)*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.20, size = 197, normalized size = 1.23

$$-\frac{(-1+\cos(dx+c))\left(8\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\sin(dx+c)+34\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+75\right.}{24d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))*(8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*s
in(d*x+c)+34*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+75*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+75*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2
)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*a^2
```

maxima [B] time = 2.23, size = 1964, normalized size = 12.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*
x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
```

```
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))
- 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
```

```
c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) - 1))*sqrt(a))/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+d\,x)}\ (a+a\ \cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.215 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=120

$$\frac{19a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

```
[Out] 19/4*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+9/4*a^3*si
n(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a^2*sin(d*x+c)*cos(d
*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2763, 2981, 2774, 216}

$$\frac{19a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (19*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (9*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) +
(a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{a^2\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{5a^2}{2}+\right.}{\sqrt{\cos(c+dx}}$$

$$= \frac{9a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d} +$$

$$= \frac{9a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d} -$$

$$= \frac{19a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{9a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{\cos(c+dx)}\,\sqrt{a}}{}$$

Mathematica [C] time = 4.17, size = 182, normalized size = 1.52

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{5/2}\left(2\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(\frac{3}{2},\frac{3}{2},2;1,\frac{9}{2};2\sin^2\left(\frac{1}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] +
 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2]
 + 8*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]
^2]*Sin[c + d*x]^2 + 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 3/2, 2},
{1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

fricas [A] time = 2.01, size = 111, normalized size = 0.92

$$\frac{\left(2\,a^2\cos\left(dx+c\right)+11\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-19\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\,\arctan\left(\right.}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2*cos(d*x + c) + 11*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c) - 19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 188, normalized size = 1.57

$$\frac{\left(2\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)+19\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+11\cos\left(dx+c\right)\sin\left(\right.\right.}{4d\sqrt{\cos\left(dx+c\right)}\,\left(1+\cos\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(2*cos(d*x+c)^2*sin(d*x+c)+19*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+11*co
s(d*x+c)*sin(d*x+c)+19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/cos
(d*x+c)^(1/2)/(1+cos(d*x+c))*a^2
```

maxima [B] time = 1.81, size = 1106, normalized size = 9.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
+ 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*x
 + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
 + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\cos(c + d\,x))^{5/2}}{\sqrt{\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.216 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{5a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 5*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d-a^3*sin(d*x+c
)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+2*a^2*sin(d*x+c)*(a+a*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2762, 2981, 2774, 216}

$$\frac{5a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (5*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a^
3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (2a)\int \frac{\left(-\frac{3a}{2}+\frac{1}{2}a\cos(c+dx)\right)\sqrt{a+a\cos(}}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(5a^2\right)\int$$

$$= -\frac{a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(5a^2\right)\text{Subs}}{}$$

$$= \frac{5a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+d}}{d\sqrt{\cos(c+}}$$

Mathematica [C] time = 4.24, size = 182, normalized size = 1.60

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{5/2}\left(6\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(\frac{3}{2},2,\frac{5}{2};1,\frac{9}{2};2\sin^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] +
 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2]
 + 24*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2
]^2]*Sin[c + d*x]^2 + 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 2, 5/2},
 {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

fricas [A] time = 1.03, size = 127, normalized size = 1.11

$$\frac{\left(a^2\cos\left(dx+c\right)+2\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5\left(a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\sqrt{a}\,a}{d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] ((a^2*cos(d*x + c) + 2*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) - 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x +
 c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.18, size = 269, normalized size = 2.36

$$-\frac{\left(5\left(\cos^2\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+10\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{c}{1}}}{\cos(dx}\right.\right.}{d\,(-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(5*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+10*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))+5*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
```

```
cos(d*x+c)))^(1/2)/cos(d*x+c))+cos(d*x+c)^2*sin(d*x+c)+2*cos(d*x+c)*sin(d*x
+c))*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
/cos(d*x+c)^(3/2)*a^2
```

maxima [B] time = 1.82, size = 973, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)
 + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\ \cos(c + d\,x))^{5/2}}{\cos(c + d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.217 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{2a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+14/3*a^3*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/3*a^2*sin(d*x+c)*(a+a*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A] time = 0.23, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2762, 2980, 2774, 216}

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (14
*a^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a
^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
```

```
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{3}(2a)\int \frac{\left(-\frac{7a}{2}-\frac{3}{2}a\cos(c+dx)\right)\sqrt{a+a\cos(}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + a^2\int$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(2a^2\right)}{}$$

$$= \frac{2a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos}{3d\,c}$$

Mathematica [C] time = 9.93, size = 356, normalized size = 3.02

$$\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)(a(\cos(c+dx)+1))^{5/2}\left(256\sin^6\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(\frac{3}{2},2,\frac{7}{2};1,\frac{9}{2};2\sin^2\left(\frac{c}{2}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^5*(25
6*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[c/2 +
 (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*
Sin[c/2 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6*(2 - 3*Sin[c/2 + (d*x)/2]^2 + Si
n[c/2 + (d*x)/2]^4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[c/2 + (d*x)/2]^2]
]*(15 - 10*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4))/Sqrt[Sin[c/2 + (
d*x)/2]^2] - 14*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(45 + 30*Sin[c/2 + (d*x)/2
]^2 - 31*Sin[c/2 + (d*x)/2]^4 + 12*Sin[c/2 + (d*x)/2]^6)))/(672*d)
```

fricas [A] time = 2.38, size = 131, normalized size = 1.11

$$\frac{2\left(\left(8\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(a^{2}\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}\right.}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*((8*a^2*cos(d*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
*sin(d*x + c) - 3*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*arctan(
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos
(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.19, size = 333, normalized size = 2.82

$$\frac{2\left(3\left(\cos^{3}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+9\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c}{\text{co}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3/d*(3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+9*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c))+9*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*(cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*cos(
d*x+c)^2*sin(d*x+c)+cos(d*x+c)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)^4/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)*a^2
```

maxima [B] time = 1.54, size = 1395, normalized size = 11.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
```

```
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \cos(c + d x))^{5/2}}{\cos(c + d x)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.218 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{86a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 22/15*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+86/15*a^3*si
n(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/5*a^2*sin(d*x+c)*(a+a*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

Rubi [A] time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2762, 2980, 2771}

$$\frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{86a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (22*a^3*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) +
(86*a^3*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) +
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{1}{5}(2a)\int \frac{\left(-\frac{11a}{2}-\frac{7}{2}a\cos(c+dx)\right)\sqrt{a+a\cos}}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{15}$$

$$= \frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{86a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.18, size = 64, normalized size = 0.53

$$\frac{a^2(28\cos(c+dx)+43\cos(2(c+dx))+49)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(49 + 28*Cos[c + d*x] + 43*Cos[2*(c + d*x)]
)*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 1.01, size = 81, normalized size = 0.67

$$\frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)
^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.17, size = 67, normalized size = 0.55

$$-\frac{2\left(43\left(\cos^{3}\left(dx+c\right)\right)-29\left(\cos^{2}\left(dx+c\right)\right)-11\cos\left(dx+c\right)-3\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,a^{2}}{15d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(43*cos(d*x+c)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*(a*(1+cos(d*x+c))
)^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)*a^2
```

maxima [A] time = 1.37, size = 151, normalized size = 1.25

$$\frac{8\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2))
```

mupad [B] time = 2.14, size = 135, normalized size = 1.12

$$\frac{2\,a^2\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(98\,\sin\left(c+d\,x\right)+56\,\sin\left(2\,c+2\,d\,x\right)+141\,\sin\left(3\,c+3\,d\,x\right)+28\,\sin\left(4\,c+4\,d\,x\right)+\right.}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*a^2*(a*(cos(c + d*x) + 1))^(1/2)*(98*sin(c + d*x) + 56*sin(2*c + 2*d*x)
+ 141*sin(3*c + 3*d*x) + 28*sin(4*c + 4*d*x) + 43*sin(5*c + 5*d*x)))/(15*d*
cos(c + d*x)^(1/2)*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*
x) + 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.219 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{92a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 6/7*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+46/21*a^3*sin(
d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+92/21*a^3*sin(d*x+c)/d/cos
(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/7*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1
/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A] time = 0.29, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2762, 2980, 2772, 2771}

$$\frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (6*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4
6*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (9
2*a^3*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2
*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
```

```
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{1}{7}(2a)\int \frac{\left(-\frac{15a}{2}-\frac{11}{2}a\cos(c+dx)\right)\sqrt{a+a\,\mathrm{c}}}{\cos^{\frac{7}{2}}(c+dx)} \\
&= \frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{7}\Big(2 \\
&= \frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2}{} \\
&= \frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{21d}
\end{aligned}$$

Mathematica [A] time = 5.26, size = 74, normalized size = 0.46

$$\frac{a^2(93\cos(c+dx)+23\cos(2(c+dx))+23\cos(3(c+dx))+29)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{21d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(29 + 93*Cos[c + d*x] + 23*Cos[2*(c + d*x)]
 + 23*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(21*d*Cos[c + d*x]^(7/2))
```

fricas [A] time = 3.81, size = 94, normalized size = 0.58

$$\frac{2\left(46\,a^{2}\cos\left(dx+c\right)^{3}+23\,a^{2}\cos\left(dx+c\right)^{2}+12\,a^{2}\cos\left(dx+c\right)+3\,a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c) +
 3*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x
 + c)^5 + d*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 77, normalized size = 0.48

$$-\frac{2\left(46\left(\cos^{4}\left(dx+c\right)\right)-23\left(\cos^{3}\left(dx+c\right)\right)-11\left(\cos^{2}\left(dx+c\right)\right)-9\cos\left(dx+c\right)-3\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,a^{2}}{21d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d*(46*cos(d*x+c)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3)*(a
*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)*a^2
```

maxima [A] time = 0.85, size = 243, normalized size = 1.51

$$\frac{8\left(\frac{21\sqrt{2}\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}\,a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{63\sqrt{2}\,a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{36\sqrt{2}\,a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{8\sqrt{2}\,a^{\frac{5}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{21\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 8/21*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

mupad [B] time = 4.46, size = 163, normalized size = 1.01

$$\frac{35\,a^2\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}-\frac{35\,a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos(c+d\,x)}}{2}+\frac{23\,a^2\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,\sqrt{a+a\,\cos(c+d\,x)}}{2}}{\frac{63\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{63\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{21\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{21\,d\,\sqrt{\cos(c+d\,x)}\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2),x)
```

```
[Out] (35*a^2*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2) - (35*a^2*sin(c
/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2))/2 + (23*a^2*sin((7*c)/2 + (7*d*x)
/2)*(a + a*cos(c + d*x))^(1/2))/2)/((63*d*cos(c + d*x)^(1/2)*cos(c/2 + (d*x
)/2))/8 + (63*d*cos(c + d*x)^(1/2)*cos((3*c)/2 + (3*d*x)/2))/8 + (21*d*cos(
c + d*x)^(1/2)*cos((5*c)/2 + (5*d*x)/2))/8 + (21*d*cos(c + d*x)^(1/2)*cos((
7*c)/2 + (7*d*x)/2))/8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.220 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{584a^3\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}$$

```
[Out] 38/63*a^3*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+146/105*a^3*
sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+584/315*a^3*sin(d*x+c)
/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1168/315*a^3*sin(d*x+c)/d/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/9*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(9/2)
```

Rubi [A] time = 0.35, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2762, 2980, 2772, 2771}

$$\frac{584a^3\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (38*a^3*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) +
(146*a^3*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]])
+ (584*a^3*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]
) + (1168*a^3*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*
x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2
))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} - \frac{1}{9}(2a)\int \frac{\left(-\frac{19a}{2}-\frac{15}{2}a\cos(c+dx)\right)\sqrt{a+a\cos}}{\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{1}{21}(7$$

$$= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{31}$$

$$= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{31}$$

Mathematica [A] time = 5.35, size = 84, normalized size = 0.42

$$\frac{a^2(698\cos(c+dx)+803\cos(2(c+dx))+146\cos(3(c+dx))+146\cos(4(c+dx))+727)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos}}{315d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(727 + 698*Cos[c + d*x] + 803*Cos[2*(c + d*
x)] + 146*Cos[3*(c + d*x)] + 146*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(315*d
*Cos[c + d*x]^(9/2))
```

fricas [A] time = 1.94, size = 107, normalized size = 0.53

$$\frac{2\left(584\,a^2\cos\left(dx+c\right)^4+292\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+130\,a^2\cos\left(dx+c\right)+35\,a^2\right)\sqrt{a\cos\left(dx+c\right)}}{315\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.16, size = 87, normalized size = 0.43

$$-\frac{2\left(584\left(\cos^{5}\left(dx+c\right)\right)-292\left(\cos^{4}\left(dx+c\right)\right)-73\left(\cos^{3}\left(dx+c\right)\right)-89\left(\cos^{2}\left(dx+c\right)\right)-95\cos\left(dx+c\right)-35\right)\sqrt{}}{315d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x)
[Out] -2/315/d*(584*cos(d*x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+c)^2
-95*cos(d*x+c)-35)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)*a^2
```

maxima [A] time = 1.16, size = 289, normalized size = 1.44

$$\frac{8\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{572\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{104\sqrt{2}a^{\frac{5}{2}}\mathrm{s}}{(\cos(dx+}\right.}{315\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
[Out] 8/315*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^
(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*s
qrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 1))
```

mupad [B] time = 6.41, size = 279, normalized size = 1.39

$$\frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\left(\frac{192\,a^2\,\mathrm{e}^{\frac{c\,9\mathrm{i}}{2}+\frac{d\,x\,9\mathrm{i}}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5\,d}-\frac{16\,a^2\,\mathrm{e}^{\frac{c\,9\mathrm{i}}{2}+\frac{d\,x\,9\mathrm{i}}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3}{}\right.}{3\,d}\right.}{12\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{e}^{\frac{c\,9\mathrm{i}}{2}+\frac{d\,x\,9\mathrm{i}}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{e}^{\frac{c\,9\mathrm{i}}{2}+\frac{d\,x\,9\mathrm{i}}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+8\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{e}^{\frac{c\,9\mathrm{i}}{2}+\frac{d\,x\,9\mathrm{i}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2),x)
```

```
[Out] ((a + a*cos(c + d*x))^(1/2)*((192*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin(c/2 +
(d*x)/2))/(5*d) - (16*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x)/
2))/(3*d) + (1168*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 + (5*d*x)/2))/
(35*d) + (2336*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((9*c)/2 + (9*d*x)/2))/(31
5*d)))/(12*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*x)/2)
 + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((3*c)/2 + (3*d*x)/2)
 + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((5*c)/2 + (5*d*x)/2)
 + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((7*c)/2 + (7*d*x)/2)
 + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (9*d*x)/2)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.221 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 4*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/4)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2762, 8}

$$\frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/4),x]
```

```
[Out] (4*a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(1/4)*Sqrt[a + a*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx = \frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} - (4a)\int 0\,dx$$
$$= \frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 1.34

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{3/2}}{d\sqrt[4]{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/4),x]
```

```
[Out] (2*(a*(1 + Cos[c + d*x]))^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(d*Cos
[c + d*x]^(1/4))
```

fricas [A] time = 1.10, size = 50, normalized size = 1.32

$$\frac{4\sqrt{a\cos(dx+c)+a}\,a\cos(dx+c)^{\frac{3}{4}}\sin(dx+c)}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4*sqrt(a*cos(d*x + c) + a)*a*cos(d*x + c)^(3/4)*sin(d*x + c)/(d*cos(d*x + c
)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x)
```

```
[Out] int((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x)
```

maxima [B] time = 1.13, size = 121, normalized size = 3.18

$$\frac{4\left(\frac{\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] 4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^(1/4))
```

mupad [B] time = 0.58, size = 42, normalized size = 1.11

$$\frac{4\,a\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{d\,{\cos\left(c+d\,x\right)}^{1/4}\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/4),x)
```

```
[Out] (4*a*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*cos(c + d*x)^(1/4)*(cos(
c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}{\cos^{\frac{5}{4}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/4),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(5/4), x)
```

3.222 $\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a\cos(e+fx)+a}}\right)}{f}$$

```
[Out] 2*arcsin(sin(f*x+e)*a^(1/2)/(a+a*cos(f*x+e))^(1/2))*a^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2774, 216}

$$\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a\cos(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[e + f*x]]/Sqrt[Cos[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + a*Cos[e + f*x]]])/f
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(e+fx)}{\sqrt{a+a\cos(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+a\cos(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 1.35

$$\frac{\sqrt{2}\,\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\cos(e+fx)+1)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[e + f*x]]/Sqrt[Cos[e + f*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sqrt[a*(1 + Cos[e + f*x])]*Sec[(e
 + f*x)/2])/f
```

fricas [A] time = 1.14, size = 119, normalized size = 3.22

$$\left[\frac{\sqrt{-a}\,\log\left(\frac{2\,a\cos\left(fx+e\right)^2-2\,\sqrt{a\cos\left(fx+e\right)+a}\,\sqrt{-a}\,\sqrt{\cos\left(fx+e\right)}\,\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)}{f}, -\frac{2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)+a}\,\sqrt{\cos\left(fx\right.}}{\sqrt{a}\,\sin\left(fx+e\right)}\right.}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(a*cos(f*x + e) + a)*sqrt(-a)*sqr
t(cos(f*x + e))*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))/f, -
2*sqrt(a)*arctan(sqrt(a*cos(f*x + e) + a)*sqrt(cos(f*x + e))/(sqrt(a)*sin(f
*x + e)))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\cos\left(fx+e\right)+a}}{\sqrt{\cos\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(f*x + e) + a)/sqrt(cos(f*x + e)), x)
```

maple [B] time = 0.10, size = 80, normalized size = 2.16

$$\frac{2\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{a\left(1+\cos\left(fx+e\right)\right)}\,\arctan\left(\frac{\sin\left(fx+e\right)\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}{\cos\left(fx+e\right)}\right)}{f\sqrt{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x)
```

```
[Out] 2/f/cos(f*x+e)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(a*(1+cos(f*x+e)))^(
1/2)*arctan(sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)/cos(f*x+e))
```

maxima [B] time = 1.22, size = 146, normalized size = 3.95

$$\frac{\sqrt{a}\,\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + si
n(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*
x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+a\,\cos\left(e+f\,x\right)}}{\sqrt{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))^(1/2)/cos(e + f*x)^(1/2),x)
```

```
[Out] int((a + a*cos(e + f*x))^(1/2)/cos(e + f*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(e+fx\right)+1\right)}}{\sqrt{\cos\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**(1/2)/cos(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(e + f*x) + 1))/sqrt(cos(e + f*x)), x)
```

3.223 $\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}$$

```
[Out] -2*arcsin(sin(f*x+e)*a^(1/2)/(a-a*cos(f*x+e))^(1/2))*a^(1/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2774, 216}

$$-\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[e + f*x]]/Sqrt[-Cos[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a - a*Cos[e + f*x]]])/f
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, \frac{a\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}$$
$$= -\frac{2\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}$$

Mathematica [C] time = 3.61, size = 188, normalized size = 4.95

$$\frac{\sqrt{\cos(e)-i\sin(e)}\,\sqrt{-\cos(e+fx)}\,\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sqrt{a-a\cos(e+fx)}\,\left(\tanh^{-1}\left(\frac{e^{ifx}}{\sqrt{\cos(e)-i\sin(e)}\,\sqrt{e^{2ifx}(\cos(e)+i\sin}}\right.\right.}{\sqrt{2}\,f\sqrt{\cos(e+fx)(\cos(fx)+i\sin(fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[e + f*x]]/Sqrt[-Cos[e + f*x]],x]
```

```
[Out] ((ArcTanh[E^(I*f*x)/(Sqrt[Cos[e] - I*Sin[e]]*Sqrt[Cos[e] + E^((2*I)*f*x)*(C
os[e] + I*Sin[e]) - I*Sin[e]])] + ArcTanh[Sqrt[Cos[e] + E^((2*I)*f*x)*(Cos[
e] + I*Sin[e]) - I*Sin[e]]/Sqrt[Cos[e] - I*Sin[e]]])*Sqrt[-Cos[e + f*x]]*Sq
rt[a - a*Cos[e + f*x]]*(I + Cot[(e + f*x)/2])*Sqrt[Cos[e] - I*Sin[e]])/(Sqr
t[2]*f*Sqrt[Cos[e + f*x]*(Cos[f*x] + I*Sin[f*x])])
```

fricas [A] time = 0.84, size = 164, normalized size = 4.32

$$\left[\frac{\sqrt{-a}\,\log\left(\frac{4\sqrt{-a\cos(fx+e)+a}\left(2\cos(fx+e)^2+3\cos(fx+e)+1\right)\sqrt{-a}\,\sqrt{-\cos(fx+e)}-\left(8\,a\cos(fx+e)^2+8\,a\cos(fx+e)+a\right)\sin(fx+e)}{\sin(fx+e)}\right)}{2\,f}, \frac{\sqrt{a}\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log((4*sqrt(-a*cos(f*x + e) + a)*(2*cos(f*x + e)^2 + 3*cos(f*
x + e) + 1)*sqrt(-a)*sqrt(-cos(f*x + e)) - (8*a*cos(f*x + e)^2 + 8*a*cos(f*
x + e) + a)*sin(f*x + e))/sin(f*x + e))/f, sqrt(a)*arctan(1/2*sqrt(-a*cos(f
*x + e) + a)*sqrt(-cos(f*x + e))*(2*cos(f*x + e) + 1)/(sqrt(a)*cos(f*x + e)
*sin(f*x + e)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-4*sqrt(2*a)*sign(sin(1/2*(f*x+exp(1
))))*atan((-sqrt(2)+2*(4*sqrt(2)-2*sqrt(-tan(1/2*(1/2*f*x+1/2*exp(1)))^4+6*
tan(1/2*(1/2*f*x+1/2*exp(1)))^2-1))/(-2*tan(1/2*(1/2*f*x+1/2*exp(1)))^2+6))
/sqrt(2))/sqrt(2)/f
```

maple [B] time = 0.17, size = 91, normalized size = 2.39

$$-\frac{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{-2a\left(-1+\cos\left(fx+e\right)\right)}\,\sin\left(fx+e\right)\arctan\left(\frac{\sqrt{-\frac{2\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{2}}{2}\right)}{f\sqrt{-\cos\left(fx+e\right)}\,\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-2*a*(-1+cos(f*x+e)))^(1/2)*sin(
f*x+e)*arctan(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*2^(1/2))/(-cos(f*x+e
))^(1/2)/(-1+cos(f*x+e))
```

maxima [B] time = 0.86, size = 420, normalized size = 11.05

$$\frac{\sqrt{-a}\left(\log\left(4\sqrt{\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1}\,\cos\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a)*(log(4*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2
+ 4*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + 8*(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan
```

```
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 4) - log(cos(f*x + e)^2 + sin(
f*x + e)^2 + sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) + 2*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(f*x + e)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x + e)*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a-a\,\cos\left(e+f\,x\right)}}{\sqrt{-\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(e + f*x))^(1/2)/(-cos(e + f*x))^(1/2),x)
[Out] int((a - a*cos(e + f*x))^(1/2)/(-cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\cos\left(e+fx\right)-1\right)}}{\sqrt{-\cos\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(f*x+e))**(1/2)/(-cos(f*x+e))**(1/2),x)
[Out] Integral(sqrt(-a*(cos(e + f*x) - 1))/sqrt(-cos(e + f*x)), x)
```

3.224 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=171

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+}}\right.}{\sqrt{a}\,d}$$

```
[Out] 7/4*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*
sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)
/d/a^(1/2)+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1/4*sin
(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2778, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+}}\right.}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*Sqrt[a]*d) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])]])/(Sqrt[a]*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*S
qrt[a + a*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a +
a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\sqrt{\cos(c+dx)}(-3a+a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}dx}{4a}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\frac{a^2}{2}-\frac{7}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} d}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{7\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx}{8a} - \int$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}dx, x, -\right.}{4ad}$$

$$= \frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{\cos(c+dx)}\sin}{4d\sqrt{a+a\cos(c}$$

Mathematica [C] time = 1.25, size = 289, normalized size = 1.69

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(-4\sqrt{1+e^{2i(c+dx)}}\sin\left(\frac{1}{2}(c+dx)\right)+2\sqrt{1+e^{2i(c+dx)}}\sin\left(\frac{3}{2}(c+dx)\right)-7\sin\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*((7*I)*ArcTanh[Sqrt[1 + E^((2*I)*(c +
d*x))]]*Cos[(c + d*x)/2] - 4*Sqrt[1 + E^((2*I)*(c + d*x))]*Sin[(c + d*x)/2]
 - 7*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]*Sin[(c + d*x)/2] + 7*ArcSinh[E^
(I*(c + d*x))]*((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 8*Sqrt[2]*ArcTa
nh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]*((-I)*Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Sin[(3*
(c + d*x))/2]))/(4*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a*(1 + Cos[c + d*x]
)])
```

fricas [A] time = 1.70, size = 155, normalized size = 0.91

$$\frac{\sqrt{a\cos(dx+c)+a}\,(2\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)-7\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}}{\sqrt{a}\sin(d}\right.}{4(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d
*x + c) - 7*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 4*sqrt(2)*(a*cos(d*x + c) + a)*arc
tan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.21, size = 196, normalized size = 1.15

$$-\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))^{3}\left(2\sin(dx+c)\cos(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\sqrt{2}\right.}{8d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*cos(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*sin(d
*x+c)*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*arcsin((-1+cos(d*x+c))/sin(d*x+c)))/(
cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6*2^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.225 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+arctan(1/2*sin
(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/
a^(1/2)+sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2778, 2982, 2782, 205, 2774, 216}

$$-\frac{\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -(ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]/(Sqrt[a]*d)) + (S
qrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a
 + a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{2a}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} - \frac{(2a)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{\cos(c+dx)}\,\text{si}}{d\sqrt{a+a\cos(}}$$

Mathematica [C] time = 1.29, size = 227, normalized size = 1.77

$$-\frac{ie^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(-\sqrt{2}\,e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right) - 4e^{i(c+dx)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right) + \sqrt{2}}{\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}\,\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-I)*(-(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))]) - 4*E^(I*(c + d*
x))*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
+ Sqrt[2]*((-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c +
 d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2]*Sqrt[Cos[c
 + d*x]])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt
[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.18, size = 143, normalized size = 1.12

$$\frac{\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right) - \frac{\sqrt{2}\,(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{\sqrt{a}} + \sqrt{a\cos\left(\right.}}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a)
+ sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.18, size = 159, normalized size = 1.24

$$-\frac{\left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{2}\left(-\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin\left(dx+c\right)+\sqrt{2}\,\arctan\left(\frac{\sin(dx}{\cdots}\right.\right.}{2d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin\left(dx+c\right)^{4}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(-2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2^(1/2)*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*arcsin((-1+cos(d*x+c))/sin(d*
x+c)))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4*2^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos{(c + d\,x)}^{3/2}}{\sqrt{a + a\,\cos{(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}{(c + dx)}}{\sqrt{a\,(\cos{(c + dx)} + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.226 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*si
n(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d
/a^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2777, 2774, 216, 2782, 205}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a} - \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\,s}{\sqrt{\cos(c+dx}}\right.}{d}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] time = 0.44, size = 161, normalized size = 1.69

$$\frac{i\left(1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right.\right.}{\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}\,\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (I*(1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(-
ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*
```

```
Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/
(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.10, size = 89, normalized size = 0.94

$$\frac{\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.18, size = 125, normalized size = 1.32

$$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{a\,(1+\cos(dx+c))}\,\,(-1+\cos(dx+c))\left(\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right)}{d\sin(dx+c)^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(2^(1/2)*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+arcsin((-1+cos
(d*x+c))/sin(d*x+c)))/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2
)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.227 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2782, 205}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 0.91

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2])/(d*Sqrt[a*
(1 + Cos[c + d*x])])
```

fricas [A] time = 1.03, size = 159, normalized size = 2.84

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{c}}{2\left(\cos(dx+c)^2+c\right.}\right.}{\sqrt{a}\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(-1/a)
*sqrt(cos(d*x + c))*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*
x + c))*sqrt(a)))/(sqrt(a)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 0.13, size = 69, normalized size = 1.23

$$-\frac{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}}{d\sqrt{\cos\left(dx+c\right)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c)))^
(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

3.228 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)+2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2779, 12, 2782, 205}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
```

```
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{a}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} - \int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+d$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{}{\sqrt{\cos}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+$$

Mathematica [C] time = 2.47, size = 180, normalized size = 1.94

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\cos(c+dx)(\cos(c+dx)+2)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{}\right.\right.}{d\,\mathrm{co}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc[
(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x
)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (Hypergeometric2F1[2,
 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/1
0))/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.12, size = 132, normalized size = 1.42

$$-\frac{\frac{\sqrt{2}\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{a}}\right)}{\sqrt{a}}-2\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{ad\cos(dx+c)^{2}+ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x +
 c))*sqrt(a)))/sqrt(a) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.17, size = 206, normalized size = 2.22

$$-\frac{\left(\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{2}(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+2\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+\text{arcs}\right.}{d\left(-1+\cos(dx+c)\right)\left(1+\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)+2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)+arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)+2^(1/2)*cos(d*x+c)*sin(d*x+c))*sin(d*x+c)^2*(a*(1+cos(d*x+c))
)^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\cos(c+dx)+1\right)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)
```

3.229 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2))*2^(1/2)/d/a^(1/2)+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1
/2)-2/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2779, 2984, 12, 2782, 205}

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])]])/(Sqrt[a]*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*
Sqrt[a + a*Cos[c + d*x]]) - (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
```

```
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c}$$

Mathematica [C] time = 7.67, size = 473, normalized size = 3.61

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)+12\left(3\sin^4\left(\frac{c}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(12*Cos[(c + d*x)/2]^4*Hypergeo
metricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*
x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin
[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c
/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2
))))/(63*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

fricas [A] time = 2.21, size = 145, normalized size = 1.11

$$-\frac{2\sqrt{a\cos\left(dx+c\right)+a}\left(\cos\left(dx+c\right)-1\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{3\sqrt{2}\left(a\cos(dx+c)^{3}+a\cos(dx+c)^{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)\cdots}}{2\left(\cos(dx+c\cdots\right.}\right.}{\sqrt{a}}}{3\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(
d*x + c) - 3*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*arctan(1/2*sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^
2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)
^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 0.18, size = 274, normalized size = 2.09

$$-\frac{\left(3\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{3}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+9\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+9\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*(3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)+9*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)+9*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)+cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-2^(1/2)*cos(d*
x+c)*sin(d*x+c))*sin(d*x+c)^4*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^2/(1
+cos(d*x+c))^3/cos(d*x+c)^(5/2)*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(5/2)), x)
```

3.230 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=169

$$-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)+2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(
1/2)-2/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+26/15*sin(d*
x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2779, 2984, 12, 2782, 205}

$$-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/
2)*Sqrt[a + a*Cos[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c}$$

Mathematica [C] time = 10.02, size = 1540, normalized size = 9.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48
825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (
d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11
/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^
10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 -
 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 22665
6*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c
/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[S
```

```
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*S
qrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh
[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/
2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c
+ d*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x
)/2]^2)))/(675*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7
/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

fricas [A] time = 2.01, size = 157, normalized size = 0.93

$$\frac{2\,\sqrt{a\cos\left(dx+c\right)+a}\left(13\,\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+3\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\frac{15\,\sqrt{2}\left(a\cos(dx+c)^{4}+a\cos(dx+\ldots}{\ldots}}{15\left(ad\cos\left(dx+c\right)^{4}+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqr
t(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(a*cos(d*x + c)^4 + a*cos(d*x + c
)^3)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x
 + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)
^4 + a*d*cos(d*x + c)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 0.27, size = 341, normalized size = 2.02

$$-\frac{\left(15\left(\cos^{4}\left(dx+c\right)\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}+60\left(\cos^{3}\left(dx+c\right)\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d*(15*cos(d*x+c)^4*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+
cos(d*x+c)))^(7/2)+60*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(
d*x+c)/(1+cos(d*x+c)))^(7/2)+90*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+60*cos(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+15*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+13*cos(d*x+c)^3*sin(d*x+c)*2^(1
/2)-cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+3*2^(1/2)*cos(d*x+c)*sin(d*x+c))*sin(d*
x+c)^6*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^3/(1+cos(d*x+c))^4/cos(d*x+
c)^(7/2)*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.231 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{7\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{4d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{\cos(c+dx)+1}}$$

```
[Out] 7/4*arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))/d-arcsin(sin(d*x+c)/(1+cos(d*x+
c)))*2^(1/2)/d+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)-1/4*s
in(d*x+c)*cos(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2778, 2983, 2982, 2781, 216, 2774}

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{7\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{4d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (7*ArcSin[Sin[c +
d*x]/Sqrt[1 + Cos[c + d*x]]])/(4*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*
d*Sqrt[1 + Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 +
 Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
```

```
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}} - \frac{1}{4}\int \frac{(-3+\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}} - \frac{1}{4}\int \frac{\frac{1}{2}-\frac{7}{2}\cos(c+d\ldots}{\sqrt{\cos(c+dx)}\,\sqrt{1+\cos\ldots}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}} + \frac{7}{8}\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx - \ldots$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\ldots}{\sqrt{1\ldots}}\right.}{4d}$$

$$= -\frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{7\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(\ldots}{2d\ldots}$$

Mathematica [C] time = 0.90, size = 286, normalized size = 2.27

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(-4\sqrt{1+e^{2i(c+dx)}}\,\sin\left(\frac{1}{2}(c+dx)\right)+2\sqrt{1+e^{2i(c+dx)}}\,\sin\left(\frac{3}{2}(c+dx)\right)-7\sin\left(\frac{1}{2}(c+dx\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((7*I)*ArcTanh[Sqrt
[1 + E^((2*I)*(c + d*x))]]*Cos[(c + d*x)/2] - 4*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Sin[(c + d*x)/2] - 7*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]*Sin[(c + d*x
)/2] + 7*ArcSinh[E^(I*(c + d*x))]*((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
) + 8*Sqrt[2]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
 d*x))])]*((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*Sqrt[1 + E^((2*I)*
(c + d*x))]*Sin[(3*(c + d*x))/2]))/(4*d*Sqrt[1 + E^((2*I)*(c + d*x))])
```

fricas [A] time = 1.99, size = 135, normalized size = 1.07

$$\frac{(2\cos(dx+c)-1)\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)+4\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{\cos(dx+c\ldots}}{\sin(\ldots}\right.}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

[Out] 1/4*((2*cos(d*x + c) - 1)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c) + 4*(sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - 7*(cos(d*x + c) + 1)*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/(d*cos(d*x + c) + d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)

maple [A] time = 0.16, size = 187, normalized size = 1.48

$$-\frac{\sqrt{2+2\cos(dx+c)}\,\left(\cos^{\frac{5}{2}}(dx+c)\right)(-1+\cos(dx+c))^{3}\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+4\sqrt{2}\,\operatorname{arcs}\right.}{8d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x)

[Out] -1/8/d*(2+2*cos(d*x+c))^(1/2)*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^3*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^{5/2}}{\sqrt{\cos(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(cos(c + d*x) + 1)^(1/2),x)
[Out] int(cos(c + d*x)^(5/2)/(cos(c + d*x) + 1)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.232 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

```
[Out] -arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))/d+arcsin(sin(d*x+c)/(1+cos(d*x+c))
)*2^(1/2)/d+sin(d*x+c)*cos(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2778, 2982, 2781, 216, 2774}

$$\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d - ArcSin[Sin[c + d*x]/S
qrt[1 + Cos[c + d*x]]]/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Co
s[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
```

```
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} - \frac{1}{2}\int \frac{-1+\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}\,dx \\
&= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} - \frac{1}{2}\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{1+\cos(c
\\
&= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-
\\
&= \frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} - \frac{\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.86, size = 224, normalized size = 2.64

$$-\frac{ie^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(-\sqrt{2}\,e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right) - 4e^{i(c+dx)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right) + \sqrt{2}\left(\sqrt{\right.}{\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] ((-I)*(-(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))]) - 4*E^(I*(c + d*
x))*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
+ Sqrt[2]*((-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c +
 d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2]*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*Sqrt[1 + E^((2*
I)*(c + d*x))])
```

fricas [A] time = 1.47, size = 125, normalized size = 1.47

$$-\frac{\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sq
rt(cos(d*x + c))/sin(d*x + c)) - (cos(d*x + c) + 1)*arctan(sqrt(cos(d*x + c
) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - sqrt(cos(d*x + c) + 1)*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```

maple [A] time = 0.14, size = 151, normalized size = 1.78

$$-\frac{\left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\sqrt{2+2\cos\left(dx+c\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{2}\left(\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin\left(dx+c\right.}{2d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*cos(d*x+c)^(3/2)*(2+2*cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(2^(1/2)*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)+arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(cos(c + d*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(cos(c + d*x) + 1), x)
```

3.233 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] 2*arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))/d-arcsin(sin(d*x+c)/(1+cos(d*x+c)
))*2^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2777, 2774, 216, 2781}

$$\frac{2\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*ArcSin[Sin[c +
d*x]/Sqrt[1 + Cos[c + d*x]]])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = -\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx + \int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} + \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}$$
$$= -\frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d}$$

Mathematica [C] time = 0.26, size = 135, normalized size = 2.50

$$-\frac{i\left(1+e^{i(c+dx)}\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] ((-I)*(1 + E^(I*(c + d*x)))*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(-1
 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ArcTanh[Sqrt
[1 + E^((2*I)*(c + d*x))]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])])/(d*Sqrt[
1 + E^((2*I)*(c + d*x))])
```

fricas [A] time = 0.88, size = 70, normalized size = 1.30

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x +
 c)) - 2*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(cos(d*x + c) + 1), x)
```

maple [B] time = 0.14, size = 124, normalized size = 2.30

$$-\frac{\sqrt{2+2\cos(dx+c)}\,\left(\sqrt{\cos}\,(dx+c)\right)(-1+\cos(dx+c))\left(\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+2\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right.\right.}{2d\sin(dx+c)^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(2+2*cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(2^(1/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))+2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c)))/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(cos(c + d*x) + 1)^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(cos(c + d*x) + 1), x)
```

3.234 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2781, 216}

$$\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx &= -\frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}\\ &= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 1.81

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2])/(d*Sqrt[1
+ Cos[c + d*x]])
```

fricas [B] time = 0.66, size = 54, normalized size = 2.00

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*
x + c)/(cos(d*x + c)^2 + cos(d*x + c)))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.09, size = 63, normalized size = 2.33

$$-\frac{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2+2\cos(dx+c)}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+2*cos(d*x+c))^(1
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(cos(c + d*x) + 1)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(cos(c + d*x) + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos\left(c+dx\right)+1}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))), x)
```

3.235 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] -arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d+2*sin(d*x+c)/d/cos(d*x+c)^(1/2
)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2779, 2781, 216}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
```

```
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} - \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} + \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}$$

$$= -\frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

Mathematica [C] time = 1.79, size = 178, normalized size = 2.87

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\cos(c+dx)(\cos(c+dx)+2)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-\cos(c+dx)+\cos(c+dx)\right.\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc[
(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x
)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (Hypergeometric2F1[2,
 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/1
0))/(d*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]])
```

fricas [B] time = 2.13, size = 121, normalized size = 1.95

$$-\frac{\left(\sqrt{2}\cos(dx+c)^2+\sqrt{2}\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)}\right)-2\sqrt{\cos(dx+c)+1}\sqrt{\cos(d}}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(c
os(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c)^2 + cos(d*x
```

```
+ c))) - 2*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 + d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.15, size = 210, normalized size = 3.39

$$-\frac{\left(\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{2}(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+2\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}}{2d\left(-1+\cos(dx+c)\right)\left(1+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)+2*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*cos(d*x+c)*sin(d*x+c))*sin(d*x+c)
^2*(2+2*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
*2^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(cos(c + d*x) + 1)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(cos(c + d*x) + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(c+dx)+1}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**(3/2)), x)
```

3.236 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}+\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

```
[Out] arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/
2)/(1+cos(d*x+c))^(1/2)-2/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1
/2)
```

Rubi [A] time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2779, 2984, 12, 2781, 216}

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}+\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d + (2*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) - (2*Sin[c + d*x])/(3*d*Sqrt[Co
s[c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
```

```
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx &= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}-\frac{1}{3}\int \frac{1-2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}\\
&= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\\
&= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\\
&= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\\
&= \frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}-\frac{}{3d\sqrt{\cos(c}}
\end{aligned}$$

Mathematica [C] time = 6.63, size = 471, normalized size = 4.81

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)+12\left(3\sin^4\left(\frac{c}{2}+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(12*Cos[(c + d*x)/2]^4*Hypergeo
metricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*
x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin
[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c
/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2
))))/(63*d*Sqrt[1 + Cos[c + d*x]]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

fricas [A] time = 3.61, size = 134, normalized size = 1.37

$$-\frac{2\sqrt{\cos(dx+c)+1}\,(\cos(dx+c)-1)\sqrt{\cos(dx+c)}\,\sin(dx+c)-3\left(\sqrt{2}\,\cos(dx+c)^3+\sqrt{2}\,\cos(dx+c)^2\right)\operatorname{arcta}}{3\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(cos(d*x + c) + 1)*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d*
x + c) - 3*(sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2)*arctan(1/2*sqr
t(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c)^2
 + cos(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\,\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(5/2)), x)

maple [B] time = 0.14, size = 278, normalized size = 2.84

$$-\frac{\left(3\sqrt{2}\ \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^3(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+9\sqrt{2}\ \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x)

[Out] -1/6/d*(3*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+9*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+9*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+3*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2*cos(d*x+c)^2*sin(d*x+c)-2*cos(d*x+c)*sin(d*x+c))*sin(d*x+c)^4*(2+2*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)*2^(1/2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^(5/2)*(cos(c + d*x) + 1)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(cos(c + d*x) + 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos\left(c+dx\right)+1}\ \cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**(5/2)), x)
```

3.237 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=134

$$-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26}{15d\sqrt{\cos(c+}}$$

```
[Out] -arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d+2/5*sin(d*x+c)/d/cos(d*x+c)^(5
/2)/(1+cos(d*x+c))^(1/2)-2/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^
(1/2)+26/15*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2779, 2984, 12, 2781, 216}

$$-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26}{15d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*Sin[c + d*x])/(
5*d*Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Cos
[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[
c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
```

```
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{1}{5}\int \frac{1-4\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}$$

$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{}{15d\cos}$$

Mathematica [C] time = 7.92, size = 1538, normalized size = 11.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48
825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (
d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11
/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^
10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 -
 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 22665
6*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c
/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/
```

```
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*S
qrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh
[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/
2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c
+ d*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x
)/2]^2)))/(675*d*Sqrt[1 + Cos[c + d*x]]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)*
(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

fricas [A] time = 2.24, size = 146, normalized size = 1.09

$$\frac{2\left(13\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+3\right)\sqrt{\cos\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-15\left(\sqrt{2}\cos\left(dx+c\right)^{4}+\sqrt{2}\right.}{15\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(cos(d*x + c) + 1)*sqrt(
cos(d*x + c))*sin(d*x + c) - 15*(sqrt(2)*cos(d*x + c)^4 + sqrt(2)*cos(d*x +
 c)^3)*arctan(1/2*sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x
 + c)/(cos(d*x + c)^2 + cos(d*x + c))))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^
3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos\left(dx+c\right)+1}\,\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 0.17, size = 344, normalized size = 2.57

$$-\frac{\left(15 \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right) \sqrt{2}\ \left(\cos^4(dx+c)\right) \left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}} + 60 \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right) \sqrt{2}\ \left(\cos^3(dx+c)\right) \left(\frac{\cos(}{1+\cos}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/30/d*(15*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^4*(cos(d*
x+c)/(1+cos(d*x+c)))^(7/2)+60*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*co
s(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+90*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*2^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+60*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7
/2)+15*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(7/2)+26*cos(d*x+c)^3*sin(d*x+c)-2*cos(d*x+c)^2*sin(d*x+c)+6*cos(d*x+c)*
sin(d*x+c))*sin(d*x+c)^6*(2+2*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))^3/(1+cos(d*
x+c))^4/cos(d*x+c)^(7/2)*2^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(cos(c + d*x) + 1)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(cos(c + d*x) + 1)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{3\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -3*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-1/2*cos(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+9/4*arctan(1/2*sin(d*x+c)*a^(1
/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+3/2*
sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2765, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{3\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
(9*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-3a\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-\frac{3a^2}{2}+3a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+}}}{2a^3}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{3\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a^2} +$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{9\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx,\right.}{}$$

$$= -\frac{3\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\,\text{si}}{2d(a+a\cos(c}$$

Mathematica [C] time = 5.36, size = 229, normalized size = 1.32

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\,\sec^2\left(\frac{1}{2}(c+dx)\right)}{d} + \frac{3ie^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(2\sinh^{-1}\left(e^{i(c+dx)}\right)+3\sqrt{2}\right.}{\sqrt{2}\,d\sqrt{1+e^{2i(}}}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((3*I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(2*ArcSinh[E^(I*(c + d*x))] + 3*Sqrt[2]*ArcTanh[(1 - E
^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 2*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (Sqr
t[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/
2]))/d))/(a*(1 + Cos[c + d*x]))^(3/2)
```

fricas [A] time = 1.74, size = 192, normalized size = 1.10

$$-\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-2\sqrt{a\cos(dx+c)+a}\,(2\cos}{4\left(a^2d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*s
qrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d*x + c
) - 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2
 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.18, size = 227, normalized size = 1.30

$$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(-1+\cos(dx+c))^3\sqrt{a(1+\cos(dx+c))}\left(2\left(\cos^2(dx+c)\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\cos(dx+c)\sqrt{2}\right.}{4d\left(\frac{\cos(dx}{1+\cos(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^3*(a*(1+cos(d*x+c)))^(1/2)*(2*cos(d*
x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+cos(d*x+c)*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+6*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+9*arcsin((-1+cos(d*x+c))/sin(d*x+c))*s
in(d*x+c)-3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+cos(d
*x+c)))^(5/2)/sin(d*x+c)^7*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan(1/
2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/
2)/d*2^(1/2)-1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.29, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2765, 2982, 2782, 205, 2774, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) - (
5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\frac{a}{2}-2a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a^2} - \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2d} -$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\,\sin(}{2d(a+a\cos(c+}$$

Mathematica [C] time = 3.79, size = 215, normalized size = 1.60

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{d} - \frac{ie^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(4\sinh^{-1}\left(e^{i(c+dx)}\right)+5\sqrt{2}\,\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+}}}\right.\right.}{\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)
))/E^(I*(c + d*x))]*(4*ArcSinh[E^(I*(c + d*x))] + 5*Sqrt[2]*ArcTanh[(1 - E^
(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 4*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]) - (Sqrt
[Cos[c + d*x]]*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/d))/(a*(1 + Cos[c + d*x])
)^(3/2)
```

fricas [A] time = 1.83, size = 182, normalized size = 1.36

$$\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right) - 8\left(\cos(dx+c)^2+2\cos(dx+\right.}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 8*(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)
*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
 + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.17, size = 195, normalized size = 1.46

$$\frac{\left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{2}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(4\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\sqrt{2}\,\sin\left(dx+c\right)+\cos\right.}{4d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^2*(a*(1+cos(d*x+c)))^(1/2)*(4*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c
)+cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*sin(d*x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.240 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 1/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2))/a^(3/2)/d*2^(1/2)+1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c)
)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2764, 12, 2782, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
+ d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a
 + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{a}{2\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}\\
&= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{4a}\\
&= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2d}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 118, normalized size = 1.22

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)+1}\left(\sqrt{\cos(c+dx)+1}\,\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\right)}{2d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[1 + Cos[c + d*x]]*(ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[
(c + d*x)/2]^2]]*Sqrt[1 + Cos[c + d*x]] + 2*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sin[(c + d*x)/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 3.33, size = 145, normalized size = 1.49

$$\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)+2\,\sqrt{a\cos\left(dx+c\right)+a}}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(
d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.15, size = 146, normalized size = 1.51

$$\frac{\left(\sqrt{\cos}\,\left(dx+c\right)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)\left(\cos\left(dx+c\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\right.}{4d\sin\left(dx+c\right)^{3}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(cos(d*x+c)
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/sin(d*x+c)^3/(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos{\left(c + dx \right)}}}{\left(a \left(\cos{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.241 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 3/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c)
)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2766, 12, 2782, 205}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx &= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\int \frac{3a}{2\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}\\
&= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{4a}\\
&= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx,x,-\frac{a\sin(c
\end{aligned}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.58, size = 106, normalized size = 1.09

$$-\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(3\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\,\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right.\right.}{2d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] -1/2*(Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*(2 + 3*ArcTanh[Sqrt[-(Sec[c + d*x
]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])*Sin[(c
 + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 1.14, size = 146, normalized size = 1.51

$$\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2\sqrt{a\cos(dx+c)+}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqr
t(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*co
s(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.14, size = 170, normalized size = 1.75

$$-\frac{\sqrt{a(1+\cos(dx+c))}\left(3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)\cos(dx+c)+3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\right.\right.}{4d(1+\cos(dx+c))\sqrt{\cos(dx+c)}\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-cos(d*x+c)^2*2^(1
/2)+cos(d*x+c)*2^(1/2))/(1+cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)*2^(1/2)/
a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \left(\cos{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}} \sqrt{\cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.242 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a}$$

```
[Out] -7/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)
)^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/cos(d*x+
c)^(1/2)+5/2*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2766, 2984, 12, 2782, 205}

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (-7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sqrt[Cos[c + d*x]]
*(a + a*Cos[c + d*x])^(3/2)) + (5*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
```

```
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{5a}{2}-a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,d}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\,c}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\,c}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\,c}}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c}$$

Mathematica [C] time = 7.31, size = 456, normalized size = 3.33

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{5}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{70\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-35} - \frac{1}{6}\left(1-2\sin^2\left(\frac{c}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]*((4*Cos[(c +
d*x)/2]^4*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(-35 + 70*Sin[c/2 + (d*x)
/2]^2) - (Csc[c/2 + (d*x)/2]^6*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3*ArcTanh[Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(-25 + 91*Sin[c/2 + (d*x)/2]^2 - 100
*Sin[c/2 + (d*x)/2]^4 + 34*Sin[c/2 + (d*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-75 + 298*Sin[c/2 + (d*x)/2]^2 - 350*Sin[
c/2 + (d*x)/2]^4 + 124*Sin[c/2 + (d*x)/2]^6)))/6))/(d*(a*(1 + Cos[c + d*x])
)^(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

fricas [A] time = 1.19, size = 171, normalized size = 1.25

$$-\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\,\sqrt{a\,\mathrm{co}}}{4\left(a^{2}d\cos\left(dx+c\right)^{3}+2\,a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(7*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(
d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*
(5*cos(d*x + c) + 4)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3
 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.18, size = 245, normalized size = 1.79

$$\frac{\left(-7\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-14\cos\left(dx+c\right)\sin\left(dx+c\right)\arcsin\left(\frac{-1+\cos(dx}{\sin(dx+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(-7*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)-14*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-7*sin(d*x+c)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+5*cos(d*x+c)^3*2^(1/2
)-cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/2))*sin(d*x+c)*(a*(1+cos(d*x+c)))^
(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\cos(c+dx)+1))^{\frac{3}{2}}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*cos(c + d*x)**(3/2)), x)
```

3.243 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{11\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+11/4*arctan(1/2*s
in(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)+7/6*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-19/6*s
in(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2766, 2984, 12, 2782, 205}

$$\frac{11\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (11*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)
*(a + a*Cos[c + d*x])^(3/2)) + (7*Sin[c + d*x])/(6*a*d*Cos[c + d*x]^(3/2)*S
qrt[a + a*Cos[c + d*x]]) - (19*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{7a}{2}-2a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= \frac{11\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+}$$

Mathematica [C] time = 9.24, size = 589, normalized size = 3.33

$$\cot^3\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-80\sin^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)\,{}_4F_3\left(2,2,2,\frac{7}{2};1,1,\frac{11}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{d}{2}\right.}{2\sin^2\left(\frac{c}{2}+\frac{d}{2}\right.}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^2*(-80*Cos[(c +
 d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 + 120*Cos[(c + d*
x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]
^2) + 21*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
```

```
2 + (d*x)/2]^2)]]*(-392 + 2347*Sin[c/2 + (d*x)/2]^2 - 5391*Sin[c/2 + (d*x)/
2]^4 + 5972*Sin[c/2 + (d*x)/2]^6 - 3232*Sin[c/2 + (d*x)/2]^8 + 696*Sin[c/2
+ (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(
-5880 + 37165*Sin[c/2 + (d*x)/2]^2 - 89856*Sin[c/2 + (d*x)/2]^4 + 103992*Si
n[c/2 + (d*x)/2]^6 - 58336*Sin[c/2 + (d*x)/2]^8 + 12960*Sin[c/2 + (d*x)/2]^
10))))/(945*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/
2))
```

fricas [A] time = 1.27, size = 185, normalized size = 1.05

$$\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^{4}+2\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)-2\sqrt{a}}{12\left(a^{2}d\cos\left(dx+c\right)^{4}+2a^{2}d\cos\left(dx+c\right)^{3}+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(33*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(
a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) +
a)*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 0.19, size = 313, normalized size = 1.77

$$-\frac{\left(33\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)+99\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\mathrm{co}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

[Out] -1/12/d*(33*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+99*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+99*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+33*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-19*cos(d*x+c)^4*2^(1/2)+7*cos(d*x+c)^3*2^(1/2)+16*cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/2))*sin(d*x+c)^3*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)*2^(1/2)/a^2

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{5/2}\,(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)), x)

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)

[Out] Timed out

3.244 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{5\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{35\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\cos}{4d(a\cos(c+dx}$$

```
[Out] -5*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*cos(d*x+
c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-15/16*cos(d*x+c)^(3/2)*sin(d*x
+c)/a/d/(a+a*cos(d*x+c))^(3/2)+115/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)
/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+35/16*sin(d*x+c
)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2765, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{5\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{115\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{\sin(c+dx)\cos}{4d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
(115*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)) - (15*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16
*a*d*(a + a*Cos[c + d*x])^(3/2)) + (35*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16
*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-5a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{45a^2}{4}-\frac{35}{2}a^2 \cdots\right.}{\sqrt{a+a\cos(c+dx}\cdots}}{8a^4}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\sqrt{\cos(c+dx)}\,\sin(\cdots}{16a^2d\sqrt{a+a\cos(c\cdots}}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\sqrt{\cos(c+dx)}\,\sin(\cdots}{16a^2d\sqrt{a+a\cos(c\cdots}}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\sqrt{\cos(c+dx)}\,\sin(\cdots}{16a^2d\sqrt{a+a\cos(c\cdots}}$$

$$= -\frac{5\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx)\cdots}{4d(a+a\cos\cdots}$$

Mathematica [C] time = 6.71, size = 385, normalized size = 1.80

$$\frac{\sqrt{\cos(c+dx)}\,\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{8\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}+\frac{8\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}-\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{\tan\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}+\frac{23\sec\left(\frac{c}{2}\right)\cdots}{\cdots}\right.}{(a(\cos(c+dx)+1))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((5*I)/2)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x
))]*(8*ArcSinh[E^(I*(c + d*x))] + (23*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 8*ArcTanh[Sqrt[1 + E^((2*I)*(c
 + d*x))]])*Cos[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*
(a*(1 + Cos[c + d*x]))^(5/2)) + (Cos[c/2 + (d*x)/2]^5*Sqrt[Cos[c + d*x]]*((
8*Cos[(d*x)/2]*Sin[c/2])/d + (8*Cos[c/2]*Sin[(d*x)/2])/d + (23*Sec[c/2]*Sec
[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(4*d) - (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*Sin[
(d*x)/2])/(2*d) + (23*Sec[c/2 + (d*x)/2]*Tan[c/2])/(4*d) - (Sec[c/2 + (d*x)
/2]^3*Tan[c/2])/(2*d)))/(a*(1 + Cos[c + d*x]))^(5/2)
```

fricas [A] time = 1.78, size = 236, normalized size = 1.10

$$-\frac{115\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(16*cos(d*x + c)^2 + 55*cos(d
*x + c) + 35)*sqrt(cos(d*x + c))*sin(d*x + c) - 160*(cos(d*x + c)^3 + 3*cos
(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*
cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.19, size = 344, normalized size = 1.61

$$\left(\cos^{\frac{7}{2}}(dx+c)\right)\sqrt{a\,(1+\cos(dx+c))}\;(-1+\cos(dx+c))^{5}\left(16\left(\cos^{3}(dx+c)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\left(\cos^{2}(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x)

[Out] 1/32/d*cos(d*x+c)^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*(16*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+80*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+115*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-20*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+80*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+115*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-35*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^11*2^(1/2)/a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{7/2}}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.245 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx$

Optimal. Leaf size=174

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sqrt{\cos(c}}{16ad(a\cos(c+dx)+}$$

```
[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*cos(d*x+c
)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-43/32*arctan(1/2*sin(d*x+c)*a^(
1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-11/
16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.44, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2765, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sqrt{\cos(c}}{16ad(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) - (
43*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) - (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-4a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\frac{11a^2}{4}-8a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c}}}{8a^4}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a^3} -$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx\right.}{a^3}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin}{4d(a+a\cos(c}$$

Mathematica [C] time = 6.64, size = 349, normalized size = 2.01

$$\frac{\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} + \frac{\tan\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} - \frac{15\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} - \frac{15\tan\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\right.}{4d}\right.}{(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((-1/4*I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)
)]*(32*ArcSinh[E^(I*(c + d*x))] + 43*Sqrt[2]*ArcTanh[(1 - E^(I*(c + d*x)))/
(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 32*ArcTanh[Sqrt[1 + E^((2*I)*(c
+ d*x))]])*Cos[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*(
a*(1 + Cos[c + d*x]))^(5/2)) + (Cos[c/2 + (d*x)/2]^5*Sqrt[Cos[c + d*x]]*((-
15*Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(4*d) + (Sec[c/2]*Sec[c/2 +
(d*x)/2]^4*Sin[(d*x)/2])/(2*d) - (15*Sec[c/2 + (d*x)/2]*Tan[c/2])/(4*d) + (
Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(2*d)))/(a*(1 + Cos[c + d*x]))^(5/2)
```

fricas [A] time = 2.03, size = 226, normalized size = 1.30

$$\frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - 2\sqrt{a\cos(}}{32\left(a^3d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(15*cos(d*x + c) + 11)*sqrt(cos
(d*x + c))*sin(d*x + c) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.17, size = 312, normalized size = 1.79

$$\frac{\left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{4}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(15\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+32\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^4*(a*(1+cos(d*x+c)))^(1/2)*(15*cos(
d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+32*cos(d*x+c)*sin(d*x+c)
*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+43
*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-4*cos(d*x+c)*2^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+32*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+43*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c)-11*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^9*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.246 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 3/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)
)^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c
))^(5/2)+7/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.26, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2765, 2978, 12, 2782, 205}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + (7*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*
d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\frac{a}{2}-3a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int -\frac{3a^2}{4\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(}}}{8a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c}}}{32a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx,\right.}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)}}{16ad(a+a\cos}$$

Mathematica [A] time = 0.82, size = 149, normalized size = 1.09

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\,\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right) + \sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\,\left(5-2\tan^2\left(\frac{1}{2}(c\right.\right.\right.}{4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\,(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(3*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sq
rt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[
(c + d*x)/2]*(5 - 2*Tan[(c + d*x)/2]^2)))/(4*d*Sqrt[Cos[(c + d*x)/2]^2]*(a*
(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 0.95, size = 180, normalized size = 1.31

$$\frac{3\sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\left(a\cos(dx+c)^2 + a\cos(dx+c)\right)}\right) + 2}{32\left(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + 3a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

[Out] 1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c)
 + a)*(7*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)

maple [A] time = 0.16, size = 214, normalized size = 1.56

$$\frac{\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))^3\left(\cos^{\frac{3}{2}}(dx+c)\right)\left(7\left(\cos^2(dx+c)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-4\cos(dx+c)\,\sqrt{}\right.}{32d\left(\frac{\mathrm{co}}{1+\mathrm{c}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)

[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*cos(d*x+c)^(3/2)*(7*cos(d
*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-4*cos(d*x+c)*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))-3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*sin(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c
)^7*2^(1/2)/a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)^{3/2}}{{\left(a+a\,\cos (c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
[Out] Integral(cos(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.247 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 5/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)
)^(1/2))/a^(5/2)/d*2^(1/2)+1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c
))^(5/2)+1/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2764, 2978, 12, 2782, 205}

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*
(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
```

```
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{a}{2}+a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{5a^2}{4\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx}}}{8a^4}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+d}}}{32a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x,\right.}{16}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)}\,\mathrm{s}}{16ad(a+a\cos}$$

Mathematica [A] time = 1.04, size = 122, normalized size = 0.89

$$\frac{\sin^3\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(6\csc^2\left(\frac{1}{2}(c+dx)\right)-5\cot^4\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh\right.}{8d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*(-2 + 6*Csc[(c + d*x)/2]^2 - 5*ArcTanh
[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]^4*Sqrt[2 - 2*Se
c[c + d*x]])*Sin[(c + d*x)/2]^3)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 1.23, size = 178, normalized size = 1.30

$$\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)+2}{32\left(a^3d\cos\left(dx+c\right)^3+3a^3d\cos\left(dx+c\right)^2+3a^3d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c)
 + a)*(cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.16, size = 213, normalized size = 1.55

$$-\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(-1+\cos\left(dx+c\right)\right)^2\left(\left(\cos^2\left(dx+c\right)\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+4\cos\left(dx+c\right)\right.}{32d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(cos(d*
x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*cos(d*x+c)*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+5*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))-5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*sin(d*x+c))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(5/2), x)
```

3.248 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 19/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+
c))^(5/2)-9/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.26, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2766, 2978, 12, 2782, 205}

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*d*(a + a*Cos[c + d*x])^(5/2)) - (9*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
```

```
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{7a}{2}-a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\ }{4\sqrt{\cos(\ldots}}}{\ldots}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{19\int \frac{\ }{\sqrt{\mathrm{co}\ldots}}}{\ldots}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{19\,\mathrm{Subst}\ldots}{\ldots}$$

$$= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \ldots$$

Mathematica [A] time = 1.28, size = 134, normalized size = 0.98

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)(9\cos(c+dx)+13)\sqrt{2-2\sec(c+dx)}-76\cos^4\left(\frac{1}{2}(c+dx)\right)\tan\right.}{32\sqrt{2}\,a^2d\sqrt{\cos(c+dx)-1}\,\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -1/32*(Sec[(c + d*x)/2]^2*(-76*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]
^2)]]*Cos[(c + d*x)/2]^4 + Cos[c + d*x]*(13 + 9*Cos[c + d*x])*Sqrt[2 - 2*Se
c[c + d*x]])*Tan[(c + d*x)/2])/(Sqrt[2]*a^2*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[
a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.12, size = 180, normalized size = 1.31

$$\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c
) + a)*(9*cos(d*x + c) + 13)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.17, size = 245, normalized size = 1.79

$$\frac{\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))\left(19\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)\sin(dx+c)+38\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(19*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+38*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+
c)*cos(d*x+c)-9*cos(d*x+c)^3*2^(1/2)+19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-4*cos(d*x+c)^2*2^(1/2)+13*cos(
d*x+c)*2^(1/2))/(1+cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)^3*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\cos(c+dx)+1))^{\frac{5}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((a*(cos(c + d*x) + 1))**(5/2)*sqrt(cos(c + d*x))), x)
```

3.249 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{75\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a\cos(c+a}$$

```
[Out] -75/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/cos(d*
x+c)^(1/2)-13/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+49/
16*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2766, 2978, 2984, 12, 2782, 205}

$$\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{75\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (-75*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Sqrt[Cos[c + d*x
]]*(a + a*Cos[c + d*x])^(5/2)) - (13*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x
]]*(a + a*Cos[c + d*x])^(3/2)) + (49*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{9a}{2}-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx}{16ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx}{16ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx}{16ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx}{16ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{75\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(}$$

Mathematica [C] time = 7.90, size = 506, normalized size = 2.86

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{8\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^6\left(\frac{1}{2}(c+dx)\right){}_4F_3\left(2,2,2,\frac{5}{2};1,1,\frac{11}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{315\left(2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}+\frac{1}{120}\left(1-2\sin\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((8*Cos[(c +
d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(315*(-1 + 2*Sin[c
/2 + (d*x)/2]^2)) + (Csc[c/2 + (d*x)/2]^8*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Si
n[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-343
 + 1465*Sin[c/2 + (d*x)/2]^2 - 2021*Sin[c/2 + (d*x)/2]^4 + 824*Sin[c/2 + (d
```

```
*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5145
 + 33980*Sin[c/2 + (d*x)/2]^2 - 87764*Sin[c/2 + (d*x)/2]^4 + 109737*Sin[c/2
 + (d*x)/2]^6 - 66122*Sin[c/2 + (d*x)/2]^8 + 15344*Sin[c/2 + (d*x)/2]^10)))
/120))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

fricas [A] time = 2.26, size = 205, normalized size = 1.16

$$-\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c\right.}\right.}{32\left(a^{3}d\cos\left(dx+c\right)^{4}+3a^{3}d\cos\left(dx+c\right)^{3}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(75*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + c
os(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sq
rt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt
(a*cos(d*x + c) + a)*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt(cos(d*
x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.19, size = 303, normalized size = 1.71

$$\frac{\left(75\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)+225\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)\arcsin\left(\frac{-1+c}{\sin}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*(75*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*cos(d*x+c)^3*sin(d*x+c)+225*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x
```

```
+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+225*cos(d*x+c)*sin(d*x+c
)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+75*s
in(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)-49*cos(d*x+c)^4*2^(1/2)-36*cos(d*x+c)^3*2^(1/2)+53*cos(d*x+c)^2*2^(1/2)
+32*cos(d*x+c)*2^(1/2))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(1+cos(d*x+c))^
2/cos(d*x+c)^(3/2)*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.250 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{163\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{95\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+d}}$$

```
[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-17/16*sin(d*x+c)/
a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+163/32*arctan(1/2*sin(d*x+c)*a^
(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+95
/48*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-299/48*sin(d*x
+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2766, 2978, 2984, 12, 2782, 205}

$$\frac{95\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{163\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (163*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/
2)*(a + a*Cos[c + d*x])^(5/2)) - (17*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/
2)*(a + a*Cos[c + d*x])^(3/2)) + (95*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(
3/2)*Sqrt[a + a*Cos[c + d*x]]) - (299*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{11a}{2}-3a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos$$

$$= \frac{163\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c$$

Mathematica [C] time = 10.79, size = 639, normalized size = 2.94

$$-\frac{\cot^5\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(640\sin^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^8\left(\frac{1}{2}(c+dx)\right)\,{}_5F_4\left(2,2,2,2,\frac{7}{2};1,1,1,\frac{13}{2};\frac{\sin}{2\sin}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -1/41580*(Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^4*(640
*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 13/2}, S
```

```
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 -
1280*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 13/2}, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6
+ 5*Sin[c/2 + (d*x)/2]^2) + 33*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-105*ArcTanh[Sqrt[Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-10935 + 72902
*Sin[c/2 + (d*x)/2]^2 - 188110*Sin[c/2 + (d*x)/2]^4 + 234156*Sin[c/2 + (d*x
)/2]^6 - 140732*Sin[c/2 + (d*x)/2]^8 + 33208*Sin[c/2 + (d*x)/2]^10) + Sqrt[
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-1148175 + 10333785*Si
n[c/2 + (d*x)/2]^2 - 38990350*Sin[c/2 + (d*x)/2]^4 + 79946462*Sin[c/2 + (d*
x)/2]^6 - 96281836*Sin[c/2 + (d*x)/2]^8 + 68243596*Sin[c/2 + (d*x)/2]^10 -
26448512*Sin[c/2 + (d*x)/2]^12 + 4344400*Sin[c/2 + (d*x)/2]^14))))/(d*(a*(1
 + Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

fricas [A] time = 1.78, size = 219, normalized size = 1.01

$$\frac{489\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\ldots}{2\left(a\cos(dx+c)^2+a\cos(d\ldots\right)}\right.\ldots}{96\left(a^3d\cos(dx+c)^5+3a^3d\cos(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(489*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + c
os(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*(2
99*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^
3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 0.19, size = 377, normalized size = 1.74

$$\left(489\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c)\right)\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+1956\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96/d*(489*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(5/2)+1956*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+2934*(cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)
+1956*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
cos(d*x+c)*sin(d*x+c)+489*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*sin(d*x+c)-299*2^(1/2)*cos(d*x+c)^5-204*cos(d*x+c)^4*2^
(1/2)+343*cos(d*x+c)^3*2^(1/2)+192*cos(d*x+c)^2*2^(1/2)-32*cos(d*x+c)*2^(1/
2))*sin(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^3/co
s(d*x+c)^(5/2)*2^(1/2)/a^3
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.251 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{637\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{189\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}-\frac{259\sin(c+dx}{192a^2d(a\cos(}$$

```
[Out] -7*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d-1/6*cos(d*x+
c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-7/16*cos(d*x+c)^(5/2)*sin(d*x+
c)/a/d/(a+a*cos(d*x+c))^(5/2)-259/192*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+
a*cos(d*x+c))^(3/2)+637/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c
)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+189/64*sin(d*x+c)*cos(d*x
+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2765, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{259\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{189\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}+\frac{637\tan^{-1}\left(\frac{\sqrt{a}\,\mathrm{si}}{\sqrt{2}\sqrt{\cos(c+dx}}\right)}{64\sqrt{2}a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) +
(637*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/
(6*d*(a + a*Cos[c + d*x])^(7/2)) - (7*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(16*
a*d*(a + a*Cos[c + d*x])^(5/2)) - (259*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(19
2*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + (189*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
```

```
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2}-7a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{105a^2}{4}-\frac{77}{2}a^2 c\right.}{(a+a\cos(c+dx))^{3/}}}{24a^4}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c}{192a^2d(a+a\cos(c+a}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c}{192a^2d(a+a\cos(c+a}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c}{192a^2d(a+a\cos(c+a}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c}{192a^2d(a+a\cos(c+a}$$

$$= -\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} + \frac{637\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{7}{2}}(c+dx}{6d(a+a\cos}$$

Mathematica [C] time = 6.75, size = 448, normalized size = 1.76

$$\frac{\sqrt{\cos(c+dx)}\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{16\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}+\frac{16\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}+\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}+\frac{\tan\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}-\frac{15\sec\left(\frac{c}{2}\right)\cdots}{\cdots}\right.}{(a(\cos(c+dx)+1))^{7/2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (((7*I)/8)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x
))]*(64*ArcSinh[E^(I*(c + d*x))] + 91*Sqrt[2]*ArcTanh[(1 - E^(I*(c + d*x)))
/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 64*ArcTanh[Sqrt[1 + E^((2*I)*(c
 + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*
(a*(1 + Cos[c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Cos[c + d*x]]*((
16*Cos[(d*x)/2]*Sin[c/2])/d + (16*Cos[c/2]*Sin[(d*x)/2])/d + (523*Sec[c/2]*
Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(24*d) - (15*Sec[c/2]*Sec[c/2 + (d*x)/2]
^4*Sin[(d*x)/2])/(4*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(3*d)
 + (523*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - (15*Sec[c/2 + (d*x)/2]^3*Tan[
c/2])/(4*d) + (Sec[c/2 + (d*x)/2]^5*Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x])
)^(7/2)
```

fricas [A] time = 3.85, size = 280, normalized size = 1.10

$$-\frac{1911\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos\cdots}}{\sqrt{a}\sin(dx+c)}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(1911*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2
+ 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(192*cos(d*x + c)^3 + 1099*cos(d*
x + c)^2 + 1442*cos(d*x + c) + 567)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2688*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c
)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x
+ c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

maple [B] time = 0.19, size = 464, normalized size = 1.83

$$\frac{\left(\cos^{\frac{9}{2}}\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{7}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(192\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^{4}\left(dx+c\right)\right)+907\left(\cos^{3}\left(d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*cos(d*x+c)^(9/2)*(-1+cos(d*x+c))^7*(a*(1+cos(d*x+c)))^(1/2)*(192*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+907*cos(d*x+c)^3*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1344*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+1911*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+343*cos(d*x+c)^2*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2688*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arct
an(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3822*cos(d*x+c)
*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-875*cos(d*x+c)*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+1344*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+1911*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*sin(d*x+c)-567*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c
)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^15*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{9/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.252 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{177\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{49\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(}{6d(a\cos(c+dx)}$$

```
[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d-1/6*cos(d*x+c
)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-17/48*cos(d*x+c)^(3/2)*sin(d*x+
c)/a/d/(a+a*cos(d*x+c))^(5/2)-177/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)
/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-49/64*sin(d*x+c
)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.60, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2765, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{49\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{177\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(}{6d(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) - (
177*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - (17*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(48*
a*d*(a + a*Cos[c + d*x])^(5/2)) - (49*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*
a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-6a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{51a^2}{4}-24a^2 \cdots\right.}{(a+a\cos(c+dx))^{3\cdots}}}{24a^4}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{49\sqrt{\cos(c+dx)}\,\sin(\cdots}{64a^2d(a+a\cos(c+\cdots}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{49\sqrt{\cos(c+dx)}\,\sin(\cdots}{64a^2d(a+a\cos(c+\cdots}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{49\sqrt{\cos(c+dx)}\,\sin(\cdots}{64a^2d(a+a\cos(c+\cdots}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} - \frac{177\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx)\,\mathrm{s}\cdots}{6d(a+a\cos(\cdots}$$

Mathematica [C] time = 6.74, size = 412, normalized size = 1.93

$$\frac{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\left(-\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} - \frac{\tan\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} + \frac{11\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} + \frac{11\tan\left(\frac{c}{2}\right)\sec^3\left(\cdots\right.}{4d}\cdots\right.}{(a(\cos(c+dx)+1))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((-1/4*I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)
)]*(64*ArcSinh[E^(I*(c + d*x))] + (177*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 64*ArcTanh[Sqrt[1 + E^((2*I)*
(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))
]*(a*(1 + Cos[c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Cos[c + d*x]]*
((-247*Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(24*d) + (11*Sec[c/2]*Se
```

```
c[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(4*d) - (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*Sin
[(d*x)/2])/(3*d) - (247*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) + (11*Sec[c/2 +
 (d*x)/2]^3*Tan[c/2])/(4*d) - (Sec[c/2 + (d*x)/2]^5*Tan[c/2])/(3*d)))/(a*(1
 + Cos[c + d*x]))^(7/2)
```

fricas [A] time = 3.50, size = 270, normalized size = 1.26

$$531\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx}}{\sqrt{a}\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(531*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(247*cos(d
*x + c)^2 + 362*cos(d*x + c) + 147)*sqrt(cos(d*x + c))*sin(d*x + c) - 768*(
cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
 + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
 c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

maple [B] time = 0.18, size = 432, normalized size = 2.02

$$\left(\cos^{\frac{7}{2}}\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{6}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(384\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\sqrt{2}\left(\cos^{2}\left(dx+c\right)\right)\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*cos(d*x+c)^(7/2)*(-1+cos(d*x+c))^6*(a*(1+cos(d*x+c)))^(1/2)*(384*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d
*x+c)^2*sin(d*x+c)+247*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+768*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))+115*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+531*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+38
4*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*s
in(d*x+c)-215*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1062*cos
(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-147*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)+531*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))
/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^13*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{7/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.253 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=177

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{67\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}-\frac{13\sin(c+dx)\sqrt{\cos\cdots}}{48ad(a\cos(c+dx\cdots}$$

```
[Out] -1/6*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)+5/128*arctan(1/2*
sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)
/d*2^(1/2)-13/48*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)+67/
192*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.40, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2765, 2977, 2978, 12, 2782, 205}

$$\frac{67\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{5\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}-\frac{13\sin(c+dx)\sqrt{\cos\cdots}}{48ad(a\cos(c+dx\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(6
*d*(a + a*Cos[c + d*x])^(7/2)) - (13*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a
*d*(a + a*Cos[c + d*x])^(5/2)) + (67*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*
a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-5a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}dx}{6a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\,\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\frac{13a^2}{4}-\frac{27}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx}}{24a^4}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\,\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{67\sqrt{\cos(c+dx)}\,\sin(c}{192a^2d(a+a\cos(c+}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\,\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{67\sqrt{\cos(c+dx)}\,\sin(c}{192a^2d(a+a\cos(c+}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\,\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{67\sqrt{\cos(c+dx)}\,\sin(c}{192a^2d(a+a\cos(c+}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}}{48ad(a+a\cos(}$$

Mathematica [A] time = 2.68, size = 176, normalized size = 0.99

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\,\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c}{\cos(c+}}}{24a^4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(\cos(c+dx)+1)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*Sqrt[a*(1 + Cos[c + d*x])]*(15*ArcSin[Sin[(c + d*x)/2]/
Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]*(33 - 26*Tan[(c + d*x)/2]^2 + 8*Ta
n[(c + d*x)/2]^4)))/(24*a^4*d*Sqrt[Cos[(c + d*x)/2]^2]*(1 + Cos[c + d*x])^4
)
```

fricas [A] time = 1.43, size = 214, normalized size = 1.21

$$\frac{15\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sqrt{\cos(}}{2\left(a\cos(dx+c)^2+a\cos\right.}\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4
*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) +
2*sqrt(a*cos(d*x + c) + a)*(67*cos(d*x + c)^2 + 50*cos(d*x + c) + 15)*sqrt(
cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3
+ 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

maple [A] time = 0.18, size = 280, normalized size = 1.58

$$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(-1+\cos(dx+c))^{5}\sqrt{a(1+\cos(dx+c))}\left(67\left(\cos^{3}(dx+c)\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+15\arcsin\left(\frac{-1\cdots}{\mathrm{s}\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^5*(a*(1+cos(d*x+c)))^(1/2)*(67*cos
(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-17*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+30*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d
*x+c))-35*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-15*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^11*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(7/2),x)
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.254 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=177

$$\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{17\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+d$$

```
[Out] 7/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+
c))^(7/2)+3/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)+17/19
2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.40, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2765, 2978, 12, 2782, 205}

$$\frac{17\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6
*d*(a + a*Cos[c + d*x])^(7/2)) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*
d*(a + a*Cos[c + d*x])^(5/2)) + (17*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a
^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\frac{a}{2}-4a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{a^2}{4}-\frac{9}{2}a^2\cos(c+dx}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+}}{24a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{17\sqrt{\cos(c+dx)}\,\sin(}{192a^2d(a+a\cos(c+}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{17\sqrt{\cos(c+dx)}\,\sin(}{192a^2d(a+a\cos(c+}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{17\sqrt{\cos(c+dx)}\,\sin(}{192a^2d(a+a\cos(c+}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx}}{16ad(a+a\cos}$$

Mathematica [A] time = 1.93, size = 148, normalized size = 0.84

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left((135\cos(c+dx)+140\cos(2(c+dx))+17\cos(3(c+dx))+140)\sqrt{2-2\sec(c}\right.}{3072\sqrt{2}\,a^3d\sqrt{\cos(c+dx)-1}\,\sqrt{a(\cos(c+d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(672*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*
Cos[(c + d*x)/2]^6 + (140 + 135*Cos[c + d*x] + 140*Cos[2*(c + d*x)] + 17*Co
s[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a
^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.15, size = 214, normalized size = 1.21

$$\frac{21\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{co}}{2\left(a\cos(dx+c)^2+a\,c\right.}\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(21*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4
*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) +
2*sqrt(a*cos(d*x + c) + a)*(17*cos(d*x + c)^2 + 70*cos(d*x + c) + 21)*sqrt(
cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3
+ 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (dx+c)^{\frac{3}{2}}}{(a\cos (dx+c)+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

maple [A] time = 0.18, size = 280, normalized size = 1.58

$$-\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a(1+\cos (dx+c))}\,(-1+\cos (dx+c))^4\left(17\left(\cos^3(dx+c)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+53\left(\cos^2(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(17*co
s(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+53*cos(d*x+c)^2*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*arcsin((-1+cos(d*x+c))/sin(d*x+c))*c
os(d*x+c)^2*sin(d*x+c)-49*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)+42*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-21*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^9*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (dx+c)^{\frac{3}{2}}}{(a\cos (dx+c)+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^{3/2}}{(a+a\cos(c+dx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(7/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.255 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=177

$$\frac{13\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}+\frac{\sin(c+dx)\sqrt{\cos}}{6d(a\cos(c+dx}$$

```
[Out] 13/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))/a^(7/2)/d*2^(1/2)+1/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x
+c))^(7/2)+1/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-5/19
2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.40, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2764, 2978, 12, 2782, 205}

$$-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{13\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}+\frac{\sin(c+dx)\sqrt{c}}{6d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (13*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d
*(a + a*Cos[c + d*x])^(5/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2
*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
```

```
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{a}{2}+2a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}dx}{6a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{11a^2}{4}+\frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3}}}{24a^4}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+}{192a^2d(a+a\cos(c+dx}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+}{192a^2d(a+a\cos(c+dx}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+}{192a^2d(a+a\cos(c+dx}$$

$$= \frac{13\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} + \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}}{16ad(a+a\cos}$$

Mathematica [A] time = 2.86, size = 149, normalized size = 0.84

$$\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\sqrt{a(\cos(c+dx)+1)}\,\left(4\cos(c+dx)-5\cos(2(c+dx))-156\cos^4\left(\frac{1}{2}\right.\right.}{192a^4d(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*(73 + 4*Cos
[c + d*x] - 5*Cos[2*(c + d*x)] - 156*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d
*x)/2]^2)]]*Cos[(c + d*x)/2]^4*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])
*Sin[(c + d*x)/2])/(192*a^4*d*(1 + Cos[c + d*x])^4)
```

fricas [A] time = 1.13, size = 214, normalized size = 1.21

$$\frac{39\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sqrt{\cos(}}{2\left(a\cos(dx+c)^2+a\cos\right.}\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] 1/384*(39*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c)^2 - 2*cos(d*x + c) - 39)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)

maple [A] time = 0.17, size = 280, normalized size = 1.58

$$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{a\,(1+\cos(dx+c))}\,\left(-1+\cos(dx+c)\right)^{3}\left(-5\left(\cos^{3}(dx+c)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\arcsin\left(\frac{-}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x)

[Out] 1/384/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(-5*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+7*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+78*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+37*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-39*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos \left(c+d\,x\right)}}{{\left(a+a\,\cos \left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.256 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=177

$$\frac{63\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{103\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)\sqrt{}}{6d(a\cos(c+}$$

```
[Out] 63/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x
+c))^(7/2)-5/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-103/
192*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.41, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2766, 2978, 12, 2782, 205}

$$-\frac{103\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{63\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)}{6d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] (63*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(5/2)) - (103*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192
*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{11a}{2}-2a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{7}{\sqrt{\cos(}}}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{103\sqrt{\mathrm{co}}}{192a^2d}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{103\sqrt{\mathrm{co}}}{192a^2d}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{103\sqrt{\mathrm{co}}}{192a^2d}$$

$$= \frac{63\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} -$$

Mathematica [A] time = 2.17, size = 148, normalized size = 0.84

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\Big((1089\cos(c+dx)+532\cos(2(c+dx))+103\cos(3(c+dx))+532)\sqrt{2-2\,\mathrm{s}}}{3072\sqrt{2}\,a^3d\sqrt{\cos(c+dx)-1}\,\sqrt{a(\cos(c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] -1/3072*(Sec[(c + d*x)/2]^4*(-6048*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x
)/2]^2)]]*Cos[(c + d*x)/2]^6 + (532 + 1089*Cos[c + d*x] + 532*Cos[2*(c + d*
x)] + 103*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(Sq
rt[2]*a^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.16, size = 214, normalized size = 1.21

$$\frac{189\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,\sqrt{\mathrm{c}}}{2\left(a\cos(dx+c)^2+a\right.}\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(189*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqr
t(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) -
 2*sqrt(a*cos(d*x + c) + a)*(103*cos(d*x + c)^2 + 266*cos(d*x + c) + 195)*s
qrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c
)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.17, size = 313, normalized size = 1.77

$$\frac{\sqrt{a(1+\cos(dx+c))}\;(-1+\cos(dx+c))^{2}\left(-189\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{3}(dx+c)\right)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(-189*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+
103*cos(d*x+c)^4*2^(1/2)-567*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+163*cos(d*x+c)^3*2^(1/2)-567
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)*cos(d*x+c)-71*cos(d*x+c)^2*2^(1/2)-189*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-195*cos(d*x+c)*2^(1/2))/(
1+cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)^5*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.257 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{363\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}\,(a\cos(c}$$

```
[Out] -363/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*
x+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/cos(
d*x+c)^(1/2)-19/48*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2)-1
99/192*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+691/192*sin
(d*x+c)/a^3/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2766, 2978, 2984, 12, 2782, 205}

$$\frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{363\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] (-363*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*Sqrt[Cos[c + d*
x]]*(a + a*Cos[c + d*x])^(7/2)) - (19*Sin[c + d*x])/(48*a*d*Sqrt[Cos[c + d*
x]]*(a + a*Cos[c + d*x])^(5/2)) - (199*Sin[c + d*x])/(192*a^2*d*Sqrt[Cos[c
+ d*x]]*(a + a*Cos[c + d*x])^(3/2)) + (691*Sin[c + d*x])/(192*a^3*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{13a}{2}-3a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,d}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{cc}}$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{cc}}$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{cc}}$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{cc}}$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{cc}}$$

$$= -\frac{363\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(}$$

Mathematica [C] time = 8.44, size = 559, normalized size = 2.58

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{16\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^8\left(\frac{1}{2}(c+dx)\right){}_5F_4\left(2,2,2,2,\frac{5}{2};1,1,1,\frac{13}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{3465\left(2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)} - \frac{\left(1-2\sin^2\left(\frac{c}{2}+\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((16*Cos[(c +
 d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2 +
```

```
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-1 +
 2*Sin[c/2 + (d*x)/2]^2)) - (Csc[c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*ArcTan
h[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2
]^6*(2187 - 12908*Sin[c/2 + (d*x)/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4 - 26380
*Sin[c/2 + (d*x)/2]^6 + 8752*Sin[c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-229635 + 2120790*Sin[c/2 + (d*x)/2]^2
- 8267707*Sin[c/2 + (d*x)/2]^4 + 17646926*Sin[c/2 + (d*x)/2]^6 - 22251094*S
in[c/2 + (d*x)/2]^8 + 16548816*Sin[c/2 + (d*x)/2]^10 - 6712984*Sin[c/2 + (d
*x)/2]^12 + 1144608*Sin[c/2 + (d*x)/2]^14)))/1680))/(d*(a*(1 + Cos[c + d*x]
))^(7/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

fricas [A] time = 1.00, size = 239, normalized size = 1.10

$$-\frac{1089\sqrt{2}\left(\cos(dx+c)^5+4\cos(dx+c)^4+6\cos(dx+c)^3+4\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}}{\cdots}\right.}{384\left(a^4d\cos(dx+c)^5+4a^4d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(1089*sqrt(2)*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3
+ 4*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*c
os(d*x + c))) - 2*(691*cos(d*x + c)^3 + 1874*cos(d*x + c)^2 + 1599*cos(d*x
+ c) + 384)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*
d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*
d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.21, size = 377, normalized size = 1.74

$$\frac{(-1+\cos(dx+c))\left(-1089\left(\cos^4(dx+c)\right)\sin(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-4356\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*(-1+cos(d*x+c))*(-1089*cos(d*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-4356*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-653
4*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)+691*2^(1/2)*cos(d*x+c)^5-4356*cos(d*x+c)*sin(d*x+c)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1183*cos(
d*x+c)^4*2^(1/2)-1089*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)-275*cos(d*x+c)^3*2^(1/2)-1215*cos(d*x+c)^2*2^(1/
2)-384*cos(d*x+c)*2^(1/2))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/(1+cos(d*x
+c))^2/cos(d*x+c)^(3/2)*2^(1/2)/a^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.258 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=257

$$\frac{1015\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{193\sin(c+dx)}{64a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{629\sin(c+dx)}{64a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c-}$$

```
[Out] -1/6*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2)-23/48*sin(d*x+c)/
a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-109/64*sin(d*x+c)/a^2/d/cos(d*x
+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1015/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(
1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+193/64*sin(
d*x+c)/a^3/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-629/64*sin(d*x+c)/a^3/
d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2766, 2978, 2984, 12, 2782, 205}

$$\frac{193\sin(c+dx)}{64a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{109\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{629\sin(c+dx)}{64a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] (1015*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*Cos[c + d*x]^(3
/2)*(a + a*Cos[c + d*x])^(7/2)) - (23*Sin[c + d*x])/(48*a*d*Cos[c + d*x]^(3
/2)*(a + a*Cos[c + d*x])^(5/2)) - (109*Sin[c + d*x])/(64*a^2*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + (193*Sin[c + d*x])/(64*a^3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (629*Sin[c + d*x])/(64*a^3*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{15a}{2}-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= \frac{1015\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(}$$

Mathematica [C] time = 8.43, size = 273, normalized size = 1.06

$$\frac{ie^{-\frac{3}{2}i(c+dx)}\cos^7\left(\frac{1}{2}(c+dx)\right)\left(3045\sqrt{2}\left(1+e^{i(c+dx)}\right)^6\left(1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)-2\left(8277e^{i(c+dx)}+14\right.\right.}{96d\left(1+e^{i(c+dx)}\right)^{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] ((I/96)*(-2*(1887 + 8277*E^(I*(c + d*x)) + 14388*E^((2*I)*(c + d*x)) + 1310
8*E^((3*I)*(c + d*x)) + 5622*E^((4*I)*(c + d*x)) - 5622*E^((5*I)*(c + d*x))
 - 13108*E^((6*I)*(c + d*x)) - 14388*E^((7*I)*(c + d*x)) - 8277*E^((8*I)*(c
 + d*x)) - 1887*E^((9*I)*(c + d*x))) + 3045*Sqrt[2]*(1 + E^(I*(c + d*x)))^6
*(1 + E^((2*I)*(c + d*x)))^(3/2)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])*Cos[(c + d*x)/2]^7)/(d*E^(((3*I)/2)*(c + d*x)
)*(1 + E^(I*(c + d*x)))^6*Cos[c + d*x]^(3/2)*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 0.98, size = 253, normalized size = 0.98

$$\frac{3045\sqrt{2}\left(\cos(dx+c)^6+4\cos(dx+c)^5+6\cos(dx+c)^4+4\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos}}{2}\right.}{384\left(a^4d\cos(dx+c)^6+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(3045*sqrt(2)*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 +
 4*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*
cos(d*x + c))) - 2*(1887*cos(d*x + c)^4 + 5082*cos(d*x + c)^3 + 4251*cos(d*
x + c)^2 + 896*cos(d*x + c) - 128)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*
cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 0.21, size = 435, normalized size = 1.69

$$\frac{\left(-3045\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^5(dx+c)\right)\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}-15225\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*(-3045*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^5*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-15225*arcsin((-1+cos(d*x+c))/sin(d*x+c))*c
os(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-30450*(cos(d*x+c)/
(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d
*x+c)-30450*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*cos(d*x+c)^2*sin(d*x+c)-15225*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+1887*2^(1/2)*cos(d*x+c)
^6-3045*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)+3195*2^(1/2)*cos(d*x+c)^5-831*cos(d*x+c)^4*2^(1/2)-3355*cos(d*
x+c)^3*2^(1/2)-1024*cos(d*x+c)^2*2^(1/2)+128*cos(d*x+c)*2^(1/2))*(a*(1+cos(
d*x+c)))^(1/2)/sin(d*x+c)/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)*2^(1/2)/a^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.259 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=217

$$\frac{35\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}a^{9/2}d}+\frac{853\sin(c+dx)\sqrt{\cos(c+dx)}}{3072a^3d(a\cos(c+dx)+a)^{3/2}}-\frac{187\sin(c+dx)\sqrt{\cos(c+dx)}}{768a^2d(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)}{8d(a\cos(c+}$$

```
[Out] -1/8*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)-19/96*cos(d*x+c)^
(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(7/2)+35/2048*arctan(1/2*sin(d*x+c)*a
^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(9/2)/d*2^(1/2)-1
87/768*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(5/2)+853/3072*si
n(d*x+c)*cos(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.56, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2765, 2977, 2978, 12, 2782, 205}

$$\frac{853\sin(c+dx)\sqrt{\cos(c+dx)}}{3072a^3d(a\cos(c+dx)+a)^{3/2}}-\frac{187\sin(c+dx)\sqrt{\cos(c+dx)}}{768a^2d(a\cos(c+dx)+a)^{5/2}}+\frac{35\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}a^{9/2}d}-\frac{\sin(c+dx)}{8d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

```
[Out] (35*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(1024*Sqrt[2]*a^(9/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(8*d*(a + a*Cos[c + d*x])^(9/2)) - (19*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*a*d*(a + a*Cos[c + d*x])^(7/2)) - (187*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
768*a^2*d*(a + a*Cos[c + d*x])^(5/2)) + (853*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3072*a^3*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-7a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{7/2}}dx}{8a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{57a^2}{4}-\frac{65}{2}a^2 \cos\right.}{(a+a\cos(c+dx))^{5/2}}}{48a^4}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\,\sin(c}{768a^2d(a+a\cos(c+d}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\,\sin(c}{768a^2d(a+a\cos(c+d}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\,\sin(c}{768a^2d(a+a\cos(c+d}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\,\sin(c}{768a^2d(a+a\cos(c+d}$$

$$= \frac{35\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{1024\sqrt{2}\,a^{9/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx}{96ad(a+a\cos}$$

Mathematica [A] time = 6.04, size = 347, normalized size = 1.60

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^9\left(\frac{c}{2}+\frac{dx}{2}\right)\left(1-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^{9/2}\left(\frac{1}{8}\left(\frac{1}{1-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}+\frac{7}{6\left(1-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec\right.}\right.\right.}{d\sqrt{\cos^2\left(\frac{1}{2}(c+dx\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^9*Sin[c/2 + (d*x)/2]*(1 - Sec[(c + d*x)/2]^2*Sin[c/2
+ (d*x)/2]^2)^(9/2)*((35*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]
]*Sqrt[Cos[(c + d*x)/2]^2]*Csc[c/2 + (d*x)/2])/(128*(1 - Sec[(c + d*x)/2]^2
```

```
*Sin[c/2 + (d*x)/2]^2)^(9/2)) + (35/(16*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (
d*x)/2]^2)^4) + 35/(24*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2)^3) + 7
/(6*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2)^2) + (1 - Sec[(c + d*x)/2
]^2*Sin[c/2 + (d*x)/2]^2)^(-1))/8))/(d*Sqrt[Cos[(c + d*x)/2]^2]*(a*(1 + Cos
[c + d*x]))^(9/2))
```

fricas [A] time = 2.03, size = 248, normalized size = 1.14

$$\frac{105\sqrt{2}\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\right.}{6144\left(a^5d\cos(dx+c)^5+5a^5d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/6144*(105*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a
*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^
2 + a*cos(d*x + c))) + 2*(853*cos(d*x + c)^3 + 819*cos(d*x + c)^2 + 455*cos
(d*x + c) + 105)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/
(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 +
10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

maple [A] time = 0.19, size = 346, normalized size = 1.59

$$\frac{\left(\cos^{\frac{7}{2}}(dx+c)\right)(-1+\cos(dx+c))^7\sqrt{a(1+\cos(dx+c))}\left(853\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^4(dx+c)\right)+105\arcsin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x)
```

[Out] 1/6144/d*cos(d*x+c)^(7/2)*(-1+cos(d*x+c))^7*(a*(1+cos(d*x+c)))^(1/2)*(853*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+105*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-34*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+315*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-364*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+315*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-350*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+105*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-105*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^15*2^(1/2)/a^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{7/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(9/2),x)

[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(9/2),x)

[Out] Timed out

3.260 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=217

$$\frac{45\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}\,a^{9/2}d}+\frac{73\sin(c+dx)\sqrt{\cos(c+dx)}}{1024a^3d(a\cos(c+dx)+a)^{3/2}}+\frac{33\sin(c+dx)\sqrt{\cos(c+dx)}}{256a^2d(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx}{8d(a\cos(c}$$

```
[Out] -1/8*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)+45/2048*arctan(1/
2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(9/
2)/d*2^(1/2)-5/32*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(7/2)+33
/256*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(5/2)+73/1024*sin(d
*x+c)*cos(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.57, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2765, 2977, 2978, 12, 2782, 205}

$$\frac{73\sin(c+dx)\sqrt{\cos(c+dx)}}{1024a^3d(a\cos(c+dx)+a)^{3/2}}+\frac{33\sin(c+dx)\sqrt{\cos(c+dx)}}{256a^2d(a\cos(c+dx)+a)^{5/2}}+\frac{45\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}\,a^{9/2}d}-\frac{\sin(c+dx}{8d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

```
[Out] (45*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(1024*Sqrt[2]*a^(9/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(8*d*(a + a*Cos[c + d*x])^(9/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(32
*a*d*(a + a*Cos[c + d*x])^(7/2)) + (33*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(25
6*a^2*d*(a + a*Cos[c + d*x])^(5/2)) + (73*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(1024*a^3*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-6a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{7/2}}\,dx}{8a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\frac{15a^2}{4}-21a^2\cos(c+dx}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+d}}{48a^4}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c}{256a^2d(a+a\cos(c+}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c}{256a^2d(a+a\cos(c+}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c}{256a^2d(a+a\cos(c+}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c}{256a^2d(a+a\cos(c+}$$

$$= \frac{45\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024\sqrt{2}\,a^{9/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx}}{32ad(a+a\cos}$$

Mathematica [A] time = 2.21, size = 158, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left((2466\cos(c+dx)+1072\cos(2(c+dx))+702\cos(3(c+dx))+73\cos(4(c+\right.}{65536\sqrt{2}\,a^4d\sqrt{\cos(c+dx)-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^6*(5760*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]
*Cos[(c + d*x)/2]^8 + (999 + 2466*Cos[c + d*x] + 1072*Cos[2*(c + d*x)] + 70
2*Cos[3*(c + d*x)] + 73*Cos[4*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c
+ d*x)/2])/(65536*Sqrt[2]*a^4*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c +
 d*x])])
```

fricas [A] time = 1.14, size = 248, normalized size = 1.14

$$\frac{45\sqrt{2}\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}}{}\right.}{2048\left(a^5 d\cos(dx+c)^5+5a^5 d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/2048*(45*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 +
 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 + a*cos(d*x + c))) + 2*(73*cos(d*x + c)^3 + 351*cos(d*x + c)^2 + 195*cos(d
*x + c) + 45)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^
5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*
a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

maple [A] time = 0.19, size = 346, normalized size = 1.59

$$-\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(-1+\cos(dx+c))^6\sqrt{a(1+\cos(dx+c))}\left(73\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^4(dx+c)\right)+278\left(\cos^3(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x)
```

```
[Out] -1/2048/d*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^6*(a*(1+cos(d*x+c)))^(1/2)*(73*2
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+278*cos(d*x+c)^3*2^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)^3*sin(d*x+c)-156*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
```

```
))^(1/2)+135*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-150
*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+135*cos(d*x+c)*sin(d*
x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-45*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)/sin(d*x+c)^13*2^(1/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{5/2}}{(a+a\,\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(9/2),x)
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(9/2),x)
[Out] Timed out
```

3.261 $\int \frac{1}{\sqrt{\cos(x)}\,\sqrt{1+\cos(x)}}\,dx$

Optimal. Leaf size=16

$$\sqrt{2}\,\sin^{-1}\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[Out] arcsin(sin(x)/(1+cos(x)))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2781, 216}

$$\sqrt{2}\,\sin^{-1}\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[x]]*Sqrt[1 + Cos[x]]),x]
```

```
[Out] Sqrt[2]*ArcSin[Sin[x]/(1 + Cos[x])]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(x)}\,\sqrt{1+\cos(x)}}\,dx &= -\left(\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(x)}{1+\cos(x)}\right)\right)\\ &= \sqrt{2}\,\sin^{-1}\left(\frac{\sin(x)}{1+\cos(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.88

$$\frac{2\cos\left(\frac{x}{2}\right)\tan^{-1}\left(\frac{\sin\left(\frac{x}{2}\right)}{\sqrt{\cos(x)}}\right)}{\sqrt{\cos(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[x]]*Sqrt[1 + Cos[x]]),x]
```

```
[Out] (2*ArcTan[Sin[x/2]/Sqrt[Cos[x]]]*Cos[x/2])/Sqrt[1 + Cos[x]]
```

fricas [B] time = 0.81, size = 31, normalized size = 1.94

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(x)+1}\sqrt{\cos(x)}\sin(x)}{2\left(\cos(x)^2+\cos(x)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(x) + 1)*sqrt(cos(x))*sin(x)/(cos(x)^2 +
 cos(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(x)+1}\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(x) + 1)*sqrt(cos(x))), x)
```

maple [B] time = 0.05, size = 36, normalized size = 2.25

$$-\frac{\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sqrt{2+2\cos(x)}\,\arcsin\left(\frac{-1+\cos(x)}{\sin(x)}\right)}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^(1/2)/(cos(x)+1)^(1/2),x)
```

```
[Out] -1/cos(x)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*(2+2*cos(x))^(1/2)*arcsin((-1+cos
(x))/sin(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{\cos(x)}\ \sqrt{\cos(x)+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^(1/2)*(cos(x) + 1)^(1/2)),x)
```

```
[Out] int(1/(cos(x)^(1/2)*(cos(x) + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos{\left(x \right)} + 1}\, \sqrt{\cos{\left(x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**(1/2)/(1+cos(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(x) + 1)*sqrt(cos(x))), x)
```

3.262 $\int \frac{1}{\sqrt{\cos(x)}\,\sqrt{a+a\cos(x)}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{\cos(x)}\,\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

```
[Out] arctan(1/2*sin(x)*a^(1/2)*2^(1/2)/cos(x)^(1/2)/(a+a*cos(x))^(1/2))*2^(1/2)/
a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2782, 205}

$$\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{\cos(x)}\,\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]])]
)/Sqrt[a]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx = -\left((2a)\operatorname{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(x)}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\right)\right)$$
$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.78

$$\frac{2\cos\left(\frac{x}{2}\right)\tan^{-1}\left(\frac{\sin\left(\frac{x}{2}\right)}{\sqrt{\cos(x)}}\right)}{\sqrt{a(\cos(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]]),x]
```

```
[Out] (2*ArcTan[Sin[x/2]/Sqrt[Cos[x]]]*Cos[x/2])/Sqrt[a*(1 + Cos[x])]
```

fricas [A] time = 0.97, size = 105, normalized size = 2.56

$$\left[\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(x)}\sin(x)-3\cos(x)^2-2\cos(x)+1}{\cos(x)^2+2\cos(x)+1}\right), \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(x)}}{2(\cos(x)}\right.}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(-1/a)*sqrt(
cos(x))*sin(x) - 3*cos(x)^2 - 2*cos(x) + 1)/(cos(x)^2 + 2*cos(x) + 1)), sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(cos(x))*sin(x)/((cos(x)^2 +
 cos(x))*sqrt(a)))/sqrt(a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos(x)+a}\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(x) + a)*sqrt(cos(x))), x)
```

maple [A] time = 0.06, size = 42, normalized size = 1.02

$$-\frac{\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sqrt{a(\cos(x)+1)}\,\arcsin\left(\frac{-1+\cos(x)}{\sin(x)}\right)\sqrt{2}}{\sqrt{\cos(x)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x)
```

```
[Out] -1/cos(x)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*(a*(cos(x)+1))^(1/2)*arcsin((-1+c
os(x))/sin(x))*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos(x)}\,\sqrt{a+a\,\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^(1/2)*(a + a*cos(x))^(1/2)),x)
```

```
[Out] int(1/(cos(x)^(1/2)*(a + a*cos(x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\cos\left(x\right)+1\right)}\,\sqrt{\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**(1/2)/(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(x) + 1))*sqrt(cos(x))), x)
```

3.263 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx$

Optimal. Leaf size=129

$$-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4d}$$

```
[Out] -3/4*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))*a^
(1/2)/d-1/2*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a-a*cos(d*x+c))^(1/2)+3/4*a*si
n(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2770, 2775, 207}

$$-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*C
os[c + d*x]])])/(4*d) + (3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a -
 a*Cos[c + d*x]]) - (a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a - a*Cos
[c + d*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx = -\frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} - \frac{3}{4}\int \sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}\,d$$

$$= \frac{3a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} - \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} + \frac{3}{8}\int \frac{\sqrt{a-}}{\sqrt{}}$$

$$= \frac{3a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} - \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} + \frac{(3a)\,\mathrm{Subs}}{}$$

$$= -\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{4d} + \frac{3a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}}$$

Mathematica [C] time = 4.22, size = 289, normalized size = 2.24

$$-\frac{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}\,\left(2\sqrt{2}\,\left(\cos\left(\frac{3}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] -1/8*(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]*(3*ArcTanh[E^(I*d*x)/(Sqr
t[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Si
n[c]])]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + 3*ArcTanh[Sqrt[Cos
[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]
]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + 2*Sqrt[2]*(-2*Cos[(c + d
*x)/2] + Cos[(3*(c + d*x))/2])*Csc[(c + d*x)/2]*Sqrt[Cos[c + d*x]*(Cos[d*x]
 + I*Sin[d*x])]))/(d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x
))*Sin[c]])
```

fricas [A] time = 0.61, size = 155, normalized size = 1.20

$$\frac{3\sqrt{a}\log\left(\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-4}{16d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(a)*log((4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(
d*x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*a*cos(d*
x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*sqrt(-a*cos(d*x +
c) + a)*(2*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt(cos(d*x + c)))/(d*sin(d*
x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 165, normalized size = 1.28

$$\frac{(-1+\cos(dx+c))\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\,\mathrm{arctanh}\left(\sqrt{\right.}\right.}{8d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+3*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(-2*a*(-1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [B] time = 1.09, size = 1063, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(-a) + 3*sqrt(-a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) + arctan2((cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\,\cos^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a-a*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))*cos(c + d*x)**(3/2), x)
```

3.264 $\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))*a^(1/2)
/d-a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2770, 2775, 207}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[
c + d*x]])])/d - (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a - a*Cos[c +
d*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
```

```
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx = -\frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{1}{2}\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= -\frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{a\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos}}\right.}{d}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}}$$

Mathematica [C] time = 0.75, size = 264, normalized size = 3.11

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\left(-2\sqrt{2}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+i\sin(dx))} + \sqrt{\cos(c)-i\sin(c)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c
] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])]
*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + ArcTanh[Sqrt[Cos[c] + E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]]*(I + Co
t[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] - 2*Sqrt[2]*Cot[(c + d*x)/2]*Sqrt[C
os[c + d*x]*(Cos[d*x] + I*Sin[d*x])]))/(2*d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c]
 + I*(-1 + E^((2*I)*d*x))*Sin[c]])
```

fricas [A] time = 0.94, size = 142, normalized size = 1.67

$$\frac{\sqrt{a}\log\left(-\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}+\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c) - 4}{4d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a)*log(-(4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) + (8*a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*sqrt(-a*cos(d*x + c)
 + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*sin(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.15, size = 94, normalized size = 1.11

$$-\frac{(1+\cos(dx+c))\left(-\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\cos(dx+c)\right)\sqrt{-2a(-1+\cos(dx+c))}\,\sqrt{2}}{2d\sqrt{\cos(dx+c)}\,\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(1+cos(d*x+c))*(-arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)*(-2*a*(-1
+cos(d*x+c)))^(1/2)*2^(1/2)
```

maxima [B] time = 0.98, size = 795, normalized size = 9.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c
) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)))*sqrt(-a) + sqrt(-a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
```

```
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\,\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a-a*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))*sqrt(cos(c + d*x)), x)
```

3.265 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{d}$$

```
[Out] -2*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))*a^(1
/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2775, 207}

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*C
os[c + d*x]])])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{d}$$
$$= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{d}$$

Mathematica [C] time = 0.54, size = 278, normalized size = 5.79

$$\frac{2e^{idx}\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\sqrt{\cos(c)-i\sin(c)}\,\sqrt{a-a\cos(c+dx)}\,\sqrt{e^{-idx}\left(i\sin(c)\left(-1+e^{2idx}\right)+\cos(c)\left(1+e^{2idx}\right)\right)}\,\left(\right.}{d\left(i\cos\left(\frac{c}{2}\right)\left(-1+e^{idx}\right)-\sin\left(\frac{c}{2}\right)\left(1+e^{idx}\right)\right)\sqrt{2i\sin(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*E^(I*d*x)*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((
2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])] + ArcTanh[Sqrt[Cos[c] + E^((2*I
)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]])*Sqrt[a - a
*Cos[c + d*x]]*(Cos[c/2] + I*Sin[c/2])*Sqrt[Cos[c] - I*Sin[c]]*Sqrt[((1 + E
^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)])/(d*(I*(-1
 + E^(I*d*x))*Cos[c/2] - (1 + E^(I*d*x))*Sin[c/2])*Sqrt[2*(1 + E^((2*I)*d*x
))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]])
```

fricas [A] time = 0.95, size = 155, normalized size = 3.23

$$\left[\frac{\sqrt{a}\,\log\left(\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\,\sqrt{\cos(dx+c)}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)}{2d}, \frac{\sqrt{-a}\,\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*a*cos(d*x +
 c) + a)*sin(d*x + c))/sin(d*x + c))/d, sqrt(-a)*arctan(1/2*sqrt(-a*cos(d*x
 + c) + a)*sqrt(-a)*(2*cos(d*x + c) + 1)/(a*sqrt(cos(d*x + c))*sin(d*x + c)
))/d]
```

giac [B] time = 1.05, size = 122, normalized size = 2.54

$$\frac{2\sqrt{a}\log\left(\frac{2\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+2\sqrt{2}-\sqrt{\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1}+1\right)}{\left|-2\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+4\sqrt{2}+2\sqrt{\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1}-2\right|}\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)*log(2*(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(tan(1/4*d*x + 1/
4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/abs(-2*tan(1/4*d*x + 1/4*c)^2 +
 4*sqrt(2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)
- 2))*sgn(sin(1/2*d*x + 1/2*c))/d
```

maple [B] time = 0.12, size = 84, normalized size = 1.75

$$\frac{\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-2a\left(-1+\cos\left(dx+c\right)\right)}\sin\left(dx+c\right)\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{d\sqrt{\cos\left(dx+c\right)}\left(-1+\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*a*(-1+cos(d*x+c)))^(1/2)*
sin(d*x+c)*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(-1+
cos(d*x+c))
```

maxima [B] time = 1.05, size = 148, normalized size = 3.08

$$\sqrt{-a}\arctan\left(\left(\cos\left(2dx+2c\right)^2+\sin\left(2dx+2c\right)^2+2\cos\left(2dx+2c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2dx+2c\right),\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + s
in(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d
*x + c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a-a\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
[Out] int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))/sqrt(cos(c + d*x)), x)
```

3.266 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2771}

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 1.08

$$\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a - a*Cos[c + d*x]]*Cot[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

fricas [A] time = 2.32, size = 42, normalized size = 1.14

$$\frac{2\,\sqrt{-a\cos\left(dx+c\right)+a}\,\left(\cos\left(dx+c\right)+1\right)}{d\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)/(d*sqrt(cos(d*x + c))*sin(d*
x + c))
```

giac [A] time = 1.08, size = 62, normalized size = 1.68

$$-\frac{2\,\sqrt{2}\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-1\right)\sqrt{a}\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(tan(1/4*d*x + 1/4*c)^2 - 1)*sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))/(
sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)
```

maple [A] time = 0.12, size = 46, normalized size = 1.24

$$-\frac{\sqrt{-2a\left(-1+\cos\left(dx+c\right)\right)}\,\sin\left(dx+c\right)\sqrt{2}}{d\left(-1+\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(-2*a*(-1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(-1+cos(d*x+c))/cos(d*x+c)^(1/
2)*2^(1/2)
```

maxima [B] time = 0.63, size = 82, normalized size = 2.22

$$\frac{2\left(\sqrt{2}\,\sqrt{a}-\frac{\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a) - sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(
d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(3/2))
```

mupad [B] time = 0.79, size = 42, normalized size = 1.14

$$-\frac{2\,\sin\left(c+d\,x\right)\,\sqrt{-a\,\left(\cos\left(c+d\,x\right)-1\right)}}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] -(2*sin(c + d*x)*(-a*(cos(c + d*x) - 1))^(1/2))/(d*cos(c + d*x)^(1/2)*(cos(
c + d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))/cos(c + d*x)**(3/2), x)
```

3.267 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}$$

```
[Out] 2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2)-4/3*a*sin(d*x+c)
/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2772, 2771}

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) - (4*a
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} - \frac{2}{3}\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} - \frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Mathematica [A] time = 0.12, size = 52, normalized size = 0.66

$$-\frac{2(2\cos(c+dx)-1)\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(-1 + 2*Cos[c + d*x])*Sqrt[a - a*Cos[c + d*x]]*Cot[(c + d*x)/2])/(3*d*C
os[c + d*x]^(3/2))
```

fricas [A] time = 2.98, size = 52, normalized size = 0.66

$$-\frac{2\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+\cos(dx+c)-1\right)}{3\,d\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + cos(d*x + c) - 1)/(d*cos
(d*x + c)^(3/2)*sin(d*x + c))
```

giac [A] time = 0.96, size = 90, normalized size = 1.14

$$\frac{2\sqrt{2}\left(\left(\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-15\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+15\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-1\right)\sqrt{a}\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{3\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(((tan(1/4*d*x + 1/4*c)^2 - 15)*tan(1/4*d*x + 1/4*c)^2 + 15)*ta
n(1/4*d*x + 1/4*c)^2 - 1)*sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x +
 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

maple [A] time = 0.13, size = 56, normalized size = 0.71

$$\frac{\left(-1+2\cos\left(dx+c\right)\right)\sqrt{-2a\left(-1+\cos\left(dx+c\right)\right)}\ \sin\left(dx+c\right)\sqrt{2}}{3d\left(-1+\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/3/d*(-1+2*cos(d*x+c))*(-2*a*(-1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(-1+cos(d*x
+c))/cos(d*x+c)^(3/2)*2^(1/2)
```

maxima [B] time = 0.85, size = 174, normalized size = 2.20

$$-\frac{2\left(\sqrt{2}\sqrt{a}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(2)*sqrt(a) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 3*sqrt(2)*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

mupad [B] time = 1.39, size = 85, normalized size = 1.08

$$\frac{4\sqrt{-a\,\left(\cos\left(c+d\,x\right)-1\right)}\,\left(\sin\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)-2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (4*(-a*(cos(c + d*x) - 1))^(1/2)*(sin(c + d*x) - sin(2*c + 2*d*x) + sin(3*c
 + 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2*cos(2*c + 2*d*x) +
cos(3*c + 3*d*x) - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}}{\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))/cos(c + d*x)**(5/2), x)
```

3.268 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$-\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

```
[Out] 2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2)-8/15*a*sin(d*x+c
)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)/d/cos(d*x+c)
^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2772, 2771}

$$-\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]) - (8*a
*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) + (16*a*S
in[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} - \frac{4}{5}\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} - \frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{8}{15}\int$$
$$= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} - \frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{}{15d\sqrt{}}$$

Mathematica [A] time = 0.15, size = 62, normalized size = 0.53

$$\frac{2(-4\cos(c+dx)+4\cos(2(c+dx))+7)\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cos(c+dx)}}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[a - a*Cos[c + d*x]]*(7 - 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Cot[(
c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 0.69, size = 64, normalized size = 0.54

$$\frac{2\left(8\cos(dx+c)^{3}+4\cos(dx+c)^{2}-\cos(dx+c)+3\right)\sqrt{-a\cos(dx+c)+a}}{15\,d\cos(dx+c)^{\frac{5}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(-a*cos(d
*x + c) + a)/(d*cos(d*x + c)^(5/2)*sin(d*x + c))
```

giac [A] time = 2.30, size = 120, normalized size = 1.02

$$-\frac{2\sqrt{2}\left(\left(\left(\left(\left(7\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-75\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+430\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-430\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+75\right)}{15\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{4}-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+1\right)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*sqrt(2)*(((((7*tan(1/4*d*x + 1/4*c)^2 - 75)*tan(1/4*d*x + 1/4*c)^2 +
430)*tan(1/4*d*x + 1/4*c)^2 - 430)*tan(1/4*d*x + 1/4*c)^2 + 75)*tan(1/4*d*x
 + 1/4*c)^2 - 7)*sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x + 1/4*c)^4
 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(5/2)*d)
```

maple [A] time = 0.12, size = 66, normalized size = 0.56

$$-\frac{\left(8\left(\cos^2(dx+c)\right)-4\cos(dx+c)+3\right)\sqrt{-2a\left(-1+\cos(dx+c)\right)}\,\sin(dx+c)\sqrt{2}}{15d\left(-1+\cos(dx+c)\right)\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/15/d*(8*cos(d*x+c)^2-4*cos(d*x+c)+3)*(-2*a*(-1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/(-1+cos(d*x+c))/cos(d*x+c)^(5/2)*2^(1/2)
```

maxima [B] time = 0.98, size = 221, normalized size = 1.87

$$\frac{2\left(7\sqrt{2}\sqrt{a}-\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(7*sqrt(2)*sqrt(a) - 17*sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 25*sqrt(2)*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 15*sqrt(2)
*sqrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 1))
```

mupad [B] time = 2.89, size = 158, normalized size = 1.34

$$\frac{8\sqrt{2\,a\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\ \left(7\,\sin\left(c+d\,x\right)-4\,\sin\left(2\,c+2\,d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)-2\,\sin\left(4\,c+4\,d\,x\right)+2\,\sin(5\right.}{15\,d\sqrt{1-2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\ \left(-16\sin\left(c+d\,x\right)^2-4\sin\left(2\,c+2\,d\,x\right)^2+20\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+10\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+2\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (8*(2*a*sin(c/2 + (d*x)/2)^2)^(1/2)*(7*sin(c + d*x) - 4*sin(2*c + 2*d*x) +
9*sin(3*c + 3*d*x) - 2*sin(4*c + 4*d*x) + 2*sin(5*c + 5*d*x)))/(15*d*(1 - 2
*sin(c/2 + (d*x)/2)^2)^(1/2)*(20*sin(c/2 + (d*x)/2)^2 - 4*sin(2*c + 2*d*x)^
2 + 10*sin((3*c)/2 + (3*d*x)/2)^2 + 2*sin((5*c)/2 + (5*d*x)/2)^2 - 16*sin(c
 + d*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.269 $\int \sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=114

$$-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}-\frac{3\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}$$

```
[Out] -3/4*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-1/2*cos(d*
x+c)^(3/2)*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)+3/4*sin(d*x+c)*cos(d*x+c)^(1/2
)/d/(1-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2770, 2775, 207}

$$-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}-\frac{3\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/(4*d
) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x]]) - (Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 - Cos[c + d*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} - \frac{3}{4}\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$$

$$= \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} + \frac{3}{8}\int \frac{\sqrt{1-\cos}}{\sqrt{\cos(c}}$$

$$= \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1}\right.}{}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} - \frac{\cos^{\frac{3}{2}}}{2d}$$

Mathematica [C] time = 0.48, size = 284, normalized size = 2.49

$$-\frac{\sqrt{-((\cos(c+dx)-1)\cos(c+dx))}\left(2\sqrt{2}\left(\cos\left(\frac{3}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2),x]
```

```
[Out] -1/8*(Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])]*(3*ArcTanh[E^(I*d*x)/(Sqrt[
Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[
c]])]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + 3*ArcTanh[Sqrt[Cos[c
] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]]*
(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + 2*Sqrt[2]*(-2*Cos[(c + d*x
)/2] + Cos[(3*(c + d*x))/2])*Csc[(c + d*x)/2]*Sqrt[Cos[c + d*x]*(Cos[d*x] +
 I*Sin[d*x])]))/(d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))
*Sin[c]])
```

fricas [A] time = 0.86, size = 124, normalized size = 1.09

$$-\frac{2\left(2\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-3\right)\sqrt{-\cos\left(dx+c\right)+1}\,\sqrt{\cos\left(dx+c\right)}-3\log\left(-\frac{2\left(\cos(dx+c)+1\right)\sqrt{-\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\cdots}{\sin(dx+c)\cdots}\right.}{8\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt(-cos(d*x + c) + 1)*sqrt(
cos(d*x + c)) - 3*log(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(c
os(d*x + c)) - (2*cos(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c
))/(d*sin(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.15, size = 164, normalized size = 1.44

$$\frac{\left(-1+\cos\left(dx+c\right)\right)\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{2}\left(dx+c\right)\right)-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos\left(dx+c\right)-3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\,\mathrm{arctanh}\left(\sqrt{\cdots}\right.\right.}{8d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+3*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(2-2*cos(d*x+c))^(1/2)*cos
(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [B] time = 0.75, size = 1305, normalized size = 11.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - (cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))) + 3*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 3*log(sqrt(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) + 3*log(((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 2*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 3*lo
g(((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))^2 + (cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) - 2*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,\sqrt{1-\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)*cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(3/2), x)
```

3.270 $\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-sin(d*x+c)*cos(
d*x+c)^(1/2)/d/(1-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2770, 2775, 207}

$$\frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]],x]
```

```
[Out] ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
```

```
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx &= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}-\frac{1}{2}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx \\
&= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right.}{d} \\
&= \frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.73, size = 252, normalized size = 3.50

$$\frac{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)}\left(-2\sqrt{2}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+i\sin(dx))}+\sqrt{\cos(c)-i\sin(c)}\left(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(C
os[c] + I*Sin[c]) - I*Sin[c]])]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[
c]] + ArcTanh[Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/S
qrt[Cos[c] - I*Sin[c]]]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] - 2*
Sqrt[2]*Cot[(c + d*x)/2]*Sqrt[Cos[c + d*x]*(Cos[d*x] + I*Sin[d*x])])*Sqrt[C
os[c + d*x]*Sin[(c + d*x)/2]^2])/(2*d*Sqrt[Cos[c + d*x]*(Cos[d*x] + I*Sin[d
*x])])
```

fricas [A] time = 0.79, size = 111, normalized size = 1.54

$$-\frac{2\left(\cos\left(dx+c\right)+1\right)\sqrt{-\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}-\log\left(-\frac{2\left(\cos(dx+c)+1\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}+(2\cos(dx+c)+1)}{\sin(dx+c)}\right.}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - log
(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*cos
(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.13, size = 94, normalized size = 1.31

$$\frac{(1+\cos(dx+c))\left(\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\cos(dx+c)\right)\sqrt{2-2\cos(dx+c)}\,\sqrt{2}}{2d\sqrt{\cos(dx+c)}\,\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(1+cos(d*x+c))*(arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)-cos(d*x+c))*(2-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
/sin(d*x+c)*2^(1/2)
```

maxima [B] time = 1.03, size = 966, normalized size = 13.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((cos(d*x + c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))) - log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
```

```
s(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - log(
((cos(d*x + c)^2 + sin(d*x + c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))^2 + (cos(d*x + c)^2 + sin(d*x + c)^2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))) + 1) + log(((cos(d*x + c)^2 + sin(d*x + c)^
2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(d*x +
c)^2 + sin(d*x + c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))^2)*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))
+ 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+dx)}\,\sqrt{1-\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)*cos(d*x+c)**(1/2),x)
[Out] Integral(sqrt(1 - cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.271 $\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] -2*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2775, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d}$$

Mathematica [C] time = 0.51, size = 277, normalized size = 7.49

$$\frac{2e^{idx}\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\sqrt{\cos(c)-i\sin(c)}\,\sqrt{1-\cos(c+dx)}\,\sqrt{e^{-idx}\left(i\sin(c)\left(-1+e^{2idx}\right)+\cos(c)\left(1+e^{2idx}\right)\right)}\,\Big(\mathrm{ta}}{d\left(i\cos\left(\frac{c}{2}\right)\left(-1+e^{idx}\right)-\sin\left(\frac{c}{2}\right)\left(1+e^{idx}\right)\right)\sqrt{2i\sin(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*E^(I*d*x)*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((
2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])] + ArcTanh[Sqrt[Cos[c] + E^((2*I
)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]])*Sqrt[1 - C
os[c + d*x]]*(Cos[c/2] + I*Sin[c/2])*Sqrt[Cos[c] - I*Sin[c]]*Sqrt[((1 + E^(
(2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)])/(d*(I*(-1 +
 E^(I*d*x))*Cos[c/2] - (1 + E^(I*d*x))*Sin[c/2])*Sqrt[2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]])
```

fricas [A] time = 0.87, size = 64, normalized size = 1.73

$$\frac{\log\left(-\frac{2\,(\cos(dx+c)+1)\sqrt{-\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}-(2\,\cos(dx+c)+1)\sin(dx+c)}{\sin(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (2*
cos(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))/d
```

giac [B] time = 1.40, size = 119, normalized size = 3.22

$$\frac{2\,\log\left(\frac{2\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2+2\,\sqrt{2}-\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^4-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2+1}+1\right)}{\left|-2\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2+4\,\sqrt{2}+2\,\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^4-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^2+1}-2\right|}\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(2*(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(tan(1/4*d*x + 1/4*c)^4 -
 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/abs(-2*tan(1/4*d*x + 1/4*c)^2 + 4*sqrt(
2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 2))*sg
n(sin(1/2*d*x + 1/2*c))/d
```

maple [B] time = 0.09, size = 83, normalized size = 2.24

$$\frac{\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2-2\cos\left(dx+c\right)}\,\sin\left(dx+c\right)\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{d\sqrt{\cos\left(dx+c\right)}\,\left(-1+\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-2*cos(d*x+c))^(1/2)*sin(d*
x+c)*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(-1+cos(d*
x+c))
```

maxima [B] time = 1.07, size = 221, normalized size = 5.97

$$2\ \operatorname{arsinh}(1)+\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+\sqrt{\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\ \cos\left(2\,dx+2\,c\right)+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arcsinh(1) + log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))^2) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\cos\left(c+dx\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.272 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

```
[Out] 2*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2771}

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 1.11

$$\frac{2\sqrt{1-\cos(c+dx)}\cot\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
[Out] (2*Sqrt[1 - Cos[c + d*x]]*Cot[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

fricas [A] time = 0.74, size = 41, normalized size = 1.17

$$\frac{2\left(\cos\left(dx+c\right)+1\right)\sqrt{-\cos\left(dx+c\right)+1}}{d\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] 2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)/(d*sqrt(cos(d*x + c))*sin(d*x
+ c))
```

giac [A] time = 0.64, size = 59, normalized size = 1.69

$$-\frac{2\sqrt{2}\left(\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}-1\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
[Out] -2*sqrt(2)*(tan(1/4*d*x + 1/4*c)^2 - 1)*sgn(sin(1/2*d*x + 1/2*c))/(sqrt(tan
(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)
```

maple [A] time = 0.10, size = 45, normalized size = 1.29

$$-\frac{\sin\left(dx+c\right)\sqrt{2-2\cos\left(dx+c\right)}\,\sqrt{2}}{d\left(-1+\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
[Out] -1/d*sin(d*x+c)*(2-2*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)*2^(
1/2)
```

maxima [B] time = 0.78, size = 75, normalized size = 2.14

$$\frac{2\left(\sqrt{2}-\frac{\sqrt{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2) - sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(d*(sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

mupad [B] time = 0.87, size = 31, normalized size = 0.89

$$\frac{2\,\sin\left(c+d\,x\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{1-\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*sin(c + d*x))/(d*cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\cos\left(c+dx\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(1 - cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.273 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}-\frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}$$

```
[Out] 2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2)-4/3*sin(d*x+c)/d/(1-
cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2772, 2771}

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}-\frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) - (4*Sin[c
 + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)} - \frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.68

$$-\frac{2\sqrt{1-\cos(c+dx)}\,(2\cos(c+dx)-1)\cot\left(\frac{1}{2}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 - Cos[c + d*x]]*(-1 + 2*Cos[c + d*x])*Cot[(c + d*x)/2])/(3*d*Cos
[c + d*x]^(3/2))
```

fricas [A] time = 2.05, size = 51, normalized size = 0.68

$$-\frac{2\left(2\cos(dx+c)^2+\cos(dx+c)-1\right)\sqrt{-\cos(dx+c)+1}}{3\,d\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*cos(d*x + c)^2 + cos(d*x + c) - 1)*sqrt(-cos(d*x + c) + 1)/(d*cos(d
*x + c)^(3/2)*sin(d*x + c))
```

giac [A] time = 1.41, size = 87, normalized size = 1.16

$$\frac{2\sqrt{2}\left(\left(\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-15\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+15\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-1\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{3\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(((tan(1/4*d*x + 1/4*c)^2 - 15)*tan(1/4*d*x + 1/4*c)^2 + 15)*ta
n(1/4*d*x + 1/4*c)^2 - 1)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x + 1/4*c)^
4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

maple [A] time = 0.11, size = 55, normalized size = 0.73

$$\frac{(-1+2\cos(dx+c))\sin(dx+c)\sqrt{2-2\cos(dx+c)}\ \sqrt{2}}{3d(-1+\cos(dx+c))\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/3/d*(-1+2*cos(d*x+c))*sin(d*x+c)*(2-2*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))/c
os(d*x+c)^(3/2)*2^(1/2)
```

maxima [B] time = 0.89, size = 164, normalized size = 2.19

$$-\frac{2\left(\sqrt{2}-\frac{4\sqrt{2}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sqrt{2}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(2) - 4*sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sqrt(2)*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 1))
```

mupad [B] time = 1.55, size = 84, normalized size = 1.12

$$\frac{4\sqrt{1-\cos(c+d\,x)}\ (\sin(c+d\,x)-\sin(2\,c+2\,d\,x)+\sin(3\,c+3\,d\,x))}{3\,d\sqrt{\cos(c+d\,x)}\ (3\,\cos(c+d\,x)-2\,\cos(2\,c+2\,d\,x)+\cos(3\,c+3\,d\,x)-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (4*(1 - cos(c + d*x))^(1/2)*(sin(c + d*x) - sin(2*c + 2*d*x) + sin(3*c + 3*
d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2*cos(2*c + 2*d*x) + cos(3
*c + 3*d*x) - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(1 - cos(c + d*x))/cos(c + d*x)**(5/2), x)
```

3.274 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}+\frac{16\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

```
[Out] 2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2)-8/15*sin(d*x+c)/d/co
s(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2)+16/15*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)
/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2772, 2771}

$$-\frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}+\frac{16\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*Sin[c + d*x])/(5*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)) - (8*Sin[c
 + d*x])/(15*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) + (16*Sin[c + d*x
])/(15*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{5}\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)} - \frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)} + \frac{8}{15}\int \frac{\sqrt{1-}}{\mathrm{co}}$$
$$= \frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)} - \frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)} + \frac{}{15d\sqrt{1-\mathrm{c}}}$$

Mathematica [A] time = 0.12, size = 61, normalized size = 0.54

$$\frac{2\sqrt{1-\cos(c+dx)}\,\left(8\cos^2(c+dx)-4\cos(c+dx)+3\right)\cot\left(\frac{1}{2}(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[1 - Cos[c + d*x]]*(3 - 4*Cos[c + d*x] + 8*Cos[c + d*x]^2)*Cot[(c +
d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 1.19, size = 63, normalized size = 0.56

$$\frac{2\left(8\cos\left(dx+c\right)^3+4\cos\left(dx+c\right)^2-\cos\left(dx+c\right)+3\right)\sqrt{-\cos\left(dx+c\right)+1}}{15\,d\cos\left(dx+c\right)^{\frac{5}{2}}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(-cos(d*x
 + c) + 1)/(d*cos(d*x + c)^(5/2)*sin(d*x + c))
```

giac [A] time = 0.82, size = 117, normalized size = 1.04

$$-\frac{2\sqrt{2}\left(\left(\left(\left(\left(7\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-75\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+430\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-430\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+75\right)}{15\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*sqrt(2)*(((((7*tan(1/4*d*x + 1/4*c)^2 - 75)*tan(1/4*d*x + 1/4*c)^2 +
430)*tan(1/4*d*x + 1/4*c)^2 - 430)*tan(1/4*d*x + 1/4*c)^2 + 75)*tan(1/4*d*x
 + 1/4*c)^2 - 7)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x + 1/4*c)^4 - 6*tan
(1/4*d*x + 1/4*c)^2 + 1)^(5/2)*d)
```

maple [A] time = 0.11, size = 65, normalized size = 0.58

$$-\frac{\left(8\left(\cos^{2}(dx+c)\right)-4\cos(dx+c)+3\right)\sqrt{2-2\cos(dx+c)}\ \sin(dx+c)\sqrt{2}}{15d\left(-1+\cos(dx+c)\right)\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/15/d*(8*cos(d*x+c)^2-4*cos(d*x+c)+3)*(2-2*cos(d*x+c))^(1/2)*sin(d*x+c)/(
-1+cos(d*x+c))/cos(d*x+c)^(5/2)*2^(1/2)
```

maxima [B] time = 0.66, size = 209, normalized size = 1.87

$$\frac{2\left(7\sqrt{2}-\frac{17\sqrt{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{25\sqrt{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{15\sqrt{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(7*sqrt(2) - 17*sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(
2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 15*sqrt(2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

mupad [B] time = 2.00, size = 156, normalized size = 1.39

$$\frac{8\sqrt{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}\ \left(7\sin(c+d\,x)-4\sin(2\,c+2\,d\,x)+9\sin(3\,c+3\,d\,x)-2\sin(4\,c+4\,d\,x)+2\sin(5\right.}{15\,d\sqrt{1-2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}\ \left(-16\sin(c+d\,x)^{2}-4\sin(2\,c+2\,d\,x)^{2}+20\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+10\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^{2}+2\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (8*(2*sin(c/2 + (d*x)/2)^2)^(1/2)*(7*sin(c + d*x) - 4*sin(2*c + 2*d*x) + 9*
sin(3*c + 3*d*x) - 2*sin(4*c + 4*d*x) + 2*sin(5*c + 5*d*x)))/(15*d*(1 - 2*s
in(c/2 + (d*x)/2)^2)^(1/2)*(20*sin(c/2 + (d*x)/2)^2 - 4*sin(2*c + 2*d*x)^2
+ 10*sin((3*c)/2 + (3*d*x)/2)^2 + 2*sin((5*c)/2 + (5*d*x)/2)^2 - 16*sin(c +
 d*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.275 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=185

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 7/4*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))/d/a
^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x
+c))^(1/2))*2^(1/2)/d/a^(1/2)+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a-a*cos(d*
x+c))^(1/2)+1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2778, 2983, 2982, 2782, 208, 2775, 207}

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*
x]])])/(4*Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*Sqrt[a - a*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*d*Sqrt[a - a*Cos[c + d*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\,(3a+a\cos(c+dx))}{\sqrt{a-a\cos(c+dx)}}\,dx}{4a}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} - \frac{\int \frac{-\frac{a^2}{2}-\frac{7}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\,dx}{4a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} - \frac{7\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{8a} + \int \frac{}{\sqrt{}}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{}{\sqrt{\cos(}}\right.}{4d}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{\cos}}{4d\sqrt{}}$$

Mathematica [C] time = 1.15, size = 256, normalized size = 1.38

$$-\frac{ie^{-2i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\left(7\sqrt{2}\,e^{2i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-16e^{2i(c+dx)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\,\left(\right.\right.}{8\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}\,\sqrt{a-a\cos(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] ((-1/8*I)*(-1 + E^(I*(c + d*x)))*(7*Sqrt[2]*E^((2*I)*(c + d*x))*ArcSinh[E^(
I*(c + d*x))] - 16*E^((2*I)*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(Sqrt[1 + E^((2*I)*(c + d*x))]
*(1 + 2*E^(I*(c + d*x)) + 2*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x))) + 7*
E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt[Cos[c + d
*x]])/(Sqrt[2]*d*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a -
 a*Cos[c + d*x]])
```

fricas [A] time = 0.80, size = 226, normalized size = 1.22

$$\frac{4\sqrt{2}\sqrt{a}\,\log\left(-\frac{\frac{2\sqrt{2}\,\sqrt{-a\cos(dx+c)+a}\,(\cos(dx+c)+1)\,\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+7\sqrt{a}\,\log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x +
 c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((
cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 7*sqrt(a)*log(-(2*sqrt(-a*c
os(d*x + c) + a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d
*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*sqrt(-a*cos(d*x +
 c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*s
in(d*x + c))
```

giac [A] time = 4.30, size = 157, normalized size = 0.85

$$-\frac{\sqrt{2}\left(\frac{7\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{8|a|\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{2\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}|a|+2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\cdots}\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{2}a}\cdots\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(7*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a) - 8*abs(a)*arctan(sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a) - 2*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3
/2)*abs(a) + 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*abs(a))/((a*tan(1/2*d*
x + 1/2*c)^2 + a)^2*a))/d
```

maple [A] time = 0.16, size = 197, normalized size = 1.06

$$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(-1+\cos(dx+c))^{3}\left(-2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^{2}(dx+c)\right)+4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)-3\sqrt{\frac{\cos(\cdots}{1+\cos\cdots}}\cdots\right)}{4d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{-2a(-1+\cos(dx+c))}\,\sin(d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^3*(-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*cos(d*x+c)^2+4*2^(1/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^
```

```
(1/2))-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-7*arctanh((cos(d*x+c)
/(1+cos(d*x+c)))^(1/2))-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)/(-2*a*(-1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{-a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(-a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a - a*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(5/2)/(a - a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a-a*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.276 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))/d/a^(1/
2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))
^(1/2))*2^(1/2)/d/a^(1/2)+sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))^(1
/2)
```

Rubi [A] time = 0.30, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2778, 2982, 2782, 208, 2775, 207}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]
)]/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[
c + d*x]]*Sqrt[a - a*Cos[c + d*x]])]])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[a - a*Cos[c + d*x]])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
```

```
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\,dx}{2a}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+ \ldots}}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{(2a)\,\text{S}\ldots}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{\cos\ldots}}{d\sqrt{\ldots}}$$

Mathematica [C] time = 0.89, size = 228, normalized size = 1.62

$$-\frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\left(\sqrt{2}\,e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right) - 4e^{i(c+dx)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right) + \sqrt{2}\left(\sqrt{1\ldots}\right.\right.}{2\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}\,\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] ((-1/2*I)*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c +
 d*x))] - 4*E^(I*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 +
 E^((2*I)*(c + d*x))])] + Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*
(c + d*x))] + E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt
[Cos[c + d*x]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sq
rt[a - a*Cos[c + d*x]])
```

fricas [A] time = 0.77, size = 212, normalized size = 1.50

$$\frac{\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\frac{2\sqrt{2}\,\sqrt{-a\cos(dx+c)+a}\,(\cos(dx+c)+1)\,\sqrt{\cos(dx+c)}}{\sqrt{a}} - (3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c) + \sqrt{a}\,\log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}\,\sqrt{a\ldots}}{}\right.}{2\,ad\sin\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c
) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((co
```

```
s(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + sqrt(a)*log(-(2*sqrt(-a*cos(d
*x + c) + a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d*x +
 c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*sqrt(-a*cos(d*x + c)
+ a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*sin(d*x + c))
```

giac [A] time = 2.60, size = 131, normalized size = 0.93

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,|a|\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{2\,|a|\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,|a|}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)a}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a)/sqrt(-a))/(sqrt(-a)*a) - 2*abs(a)*arctan(sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a) - 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a
bs(a)/((a*tan(1/2*d*x + 1/2*c)^2 + a)*a))/d
```

maple [A] time = 0.15, size = 167, normalized size = 1.18

$$-\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)(-1+\cos(dx+c))^{2}\left(\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \cos(dx+c)-\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.\right.}{d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{-2a(-1+\cos(dx+c))}\ \sin(dx+c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^2*(2^(1/2)*arctanh(1/2*2^(1/2)/(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-a
rctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(-2*a*(-1+cos(d*x+c)))^(1/2)/sin(d*x+c)
^3*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{-a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(-a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a - a*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a - a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a-a*cos(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(-a*(cos(c + d*x) - 1)), x)
```

3.277 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))/d/a^(
1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c
))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2777, 2775, 207, 2782, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*
x]])])/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx = -\frac{\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a} + \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx$$
$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, \frac{}{\sqrt{\cos}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] time = 0.38, size = 161, normalized size = 1.50

$$-\frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx}}\right.\right.}{\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] ((-I)*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))
]*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2
```

```
]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])
)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])
```

fricas [A] time = 0.92, size = 162, normalized size = 1.51

$$\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)+2\sqrt{a}\log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)+1)}{\sin}\right.}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c
) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((co
s(d*x + c) - 1)*sin(d*x + c))) + 2*sqrt(a)*log(-(2*sqrt(-a*cos(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d*x + c) + a)*si
n(d*x + c))/sin(d*x + c)))/(a*d)
```

giac [A] time = 1.93, size = 86, normalized size = 0.80

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{a\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)|a|}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/
sqrt(-a))/sqrt(-a) - a*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))
/sqrt(-a))*abs(a)/(a^2*d)
```

maple [A] time = 0.11, size = 118, normalized size = 1.10

$$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)(-1+\cos(dx+c))\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)+2\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{2}}{d\sqrt{-2a(-1+\cos(dx+c))}\,\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(-2^(1/2)*arctanh(1/2*2^(1/2)/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2))+2*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/(
-2*a*(-1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{a-a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a - a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a - a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-a(\cos(c+dx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(-a*(cos(c + d*x) - 1)), x)
```

3.278 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2782, 208}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] time = 0.33, size = 118, normalized size = 2.03

$$\frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] (I*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*A
rcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(d*S
qrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])
```

fricas [A] time = 1.33, size = 144, normalized size = 2.48

$$\left[\frac{\sqrt{2}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2\sqrt{a}\,d}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(\right.}{\sin(dx+c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)*s
qrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x +
c) - 1)*sin(d*x + c)))/(sqrt(a)*d), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(
-a*cos(d*x + c) + a)*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))/d]
```

giac [B] time = 0.76, size = 137, normalized size = 2.36

$$-\frac{\sqrt{2}\left(\frac{a^2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}\right)}{|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\left(a\arctan\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{-a}}\right)-a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\sqrt{-a}\,|a|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(a^2*(arctan(sqrt(2)*sqrt(a)/sqrt(-a))/(sqrt(-a)*a) - arctan(sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a))/(abs(a)*sgn(tan(1/2*
d*x + 1/2*c))) - (a*arctan(sqrt(2)*sqrt(a)/sqrt(-a)) - a*arctan(sqrt(a)/sqr
t(-a)))*sgn(tan(1/2*d*x + 1/2*c))/(sqrt(-a)*abs(a)))/d
```

maple [A] time = 0.14, size = 77, normalized size = 1.33

$$-\frac{2\sin\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\;\mathrm{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)}{d\sqrt{-2a\left(-1+\cos\left(dx+c\right)\right)}\;\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)/(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2))/(-2*a*(-1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)
```

maxima [C] time = 1.36, size = 209, normalized size = 3.60

$$-\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{2}\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c)+1)\right)}{\sqrt{a}\left|e^{(i\,dx+i\,c)}-1\right|},\frac{2\left(\sqrt{2}\left(\cos(2\,dx+2\,c)^2+\mathrm{s}\right.\right.}{}\right.}{\sqrt{-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*abs(e^(I*d*x + I*c) -
1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1)))/(sqrt(-a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*sqrt(cos(c + d*x))), x)
```

3.279 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1
/2)
```

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2779, 12, 2782, 208}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[a - a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
```

```
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{a}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}} + \int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, \frac{}{\sqrt{\cos(}}\right.}{d}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos}}$$

Mathematica [C] time = 0.38, size = 157, normalized size = 1.65

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right) - \frac{e^{-\frac{1}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\,\sqrt{\cos(c+dx)}\,\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] (2*(-(((1 + E^((2*I)*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])/(Sqrt[2]*E^((I/2)*(c + d*x)))) + 2*Sqrt[1 + E
^((2*I)*(c + d*x))]*Cos[(c + d*x)/2])*Sin[(c + d*x)/2])/(d*Sqrt[1 + E^((2*I
)*(c + d*x))]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

fricas [A] time = 0.58, size = 152, normalized size = 1.60

$$\frac{\sqrt{2}\sqrt{a}\cos(dx+c)\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\sqrt{-a\cos(dx+}}{2\,ad\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*cos(d*x + c)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)
*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d
*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*sqrt(-a*cos(d*
x + c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)*sin(d*
x + c))
```

giac [A] time = 0.77, size = 68, normalized size = 0.72

$$\frac{\sqrt{2}\,a\left(\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,|a|}+\frac{2}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,|a|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*a*(arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a
bs(a)) + 2/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*abs(a)))/d
```

maple [A] time = 0.14, size = 160, normalized size = 1.68

$$\frac{\left(\sin^3(dx+c)\right)\left(\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\cos(dx+c)+\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right.\right.}{d\sqrt{-2a\,(-1+\cos(dx+c))}\;\cos(dx+c)^{\frac{3}{2}}\left(\cos^2(dx+c)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*sin(d*x+c)^3*(2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*2^(
1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)+2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2*
cos(d*x+c))/(-2*a*(-1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/(cos(d*x+c)^2-1)*
2^(1/2)
```

maxima [C] time = 1.00, size = 351, normalized size = 3.69

$$2\cos\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)\sin\left(dx+c\right)-2\left(\cos\left(dx+c\right)+1\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] (2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
2*(cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*abs(e^(I*d*x + I*c)
- 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1))))/((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(-a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a-a*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*cos(c + d*x)**(3/2)), x)
```

3.280 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^
(1/2)+2/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2779, 2984, 12, 2782, 208}

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3
/2)*Sqrt[a - a*Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sq
rt[a - a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
```

```
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{a+2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+}}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+}}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos}}$$

Mathematica [C] time = 0.32, size = 171, normalized size = 1.27

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+1)-\frac{3e^{-\frac{3}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{2\sqrt{2}}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] (2*((-3*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))]])])/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))) + 2*Sq
rt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x]))*Sin[(c + d
*x)/2])/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^(3/2)*Sqrt[a - a*Co
s[c + d*x]])
```

fricas [A] time = 1.03, size = 165, normalized size = 1.22

$$\frac{3\sqrt{2}\sqrt{a}\cos(dx+c)^2\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\sqrt{-a\cos(d}}{6ad\cos(dx+c)^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*sqrt(a)*cos(d*x + c)^2*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c)
+ a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*s
in(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*sqrt(-a*co
s(d*x + c) + a)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(
a*d*cos(d*x + c)^2*sin(d*x + c))
```

giac [A] time = 0.73, size = 90, normalized size = 0.67

$$\frac{\sqrt{2}\,a\left(\frac{3\,\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,|a|}-\frac{4\,a}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,|a|}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*a*(3*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*abs(a)) - 4*a/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)*abs(a)))/d
```

maple [A] time = 0.17, size = 171, normalized size = 1.27

$$-\frac{\left(\sin^{5}(dx+c)\right)\left(3\sqrt{2}\,\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)+3\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(d}{1+\cos(}}}\right.\right.}{3d\left(-1+\cos(dx+c)\right)^{2}\sqrt{-2a\left(-1+\cos(dx+c)\right)}\,\left(1+\cos(dx+c)\right)\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*sin(d*x+c)^5*(3*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))-2*cos(d*x+c))/(-1+cos(d*x+c))^2/(-2*a*(-1+cos(d*x+c)))^(1/2)/(1+cos(d*
x+c))/cos(d*x+c)^(5/2)*2^(1/2)
```

maxima [C] time = 1.19, size = 504, normalized size = 3.73

$$-\frac{3\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)^{2}+\sqrt{2}\sin\left(2\,dx+2\,c\right)^{2}+2\,\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\right)\arctan\left(\frac{2\,\sqrt{2}\left(\cos(2\,dx+2\,c)^{2}+\sin(2\,dx+\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sq
rt(a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*a
bs(e^(I*d*x + I*c) - 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1))) - 2*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x
 + c) + 3)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 4*(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d
*x + c) - 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))))/((c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(-a)*d
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\,\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a-a*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*cos(c + d*x)**(5/2)), x)
```

3.281 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=173

$$\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^
(1/2)+2/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2)+26/15*sin(d
*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2779, 2984, 12, 2782, 208}

$$\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5
/2)*Sqrt[a - a*Cos[c + d*x]]) + (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*S
qrt[a - a*Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{a+4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos}$$

Mathematica [C] time = 0.66, size = 218, normalized size = 1.26

$$\frac{e^{-\frac{5}{2}i(c+dx)}\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\left(15e^{i(c+dx)}+40e^{2i(c+dx)}+40e^{3i(c+dx)}+15e^{4i(c+dx)}+13e^{5i(c+dx)}+13\right)-1\right.}{60d\sqrt{1+e^{2i(c+dx)}}\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] ((2*Sqrt[1 + E^((2*I)*(c + d*x))]*(13 + 15*E^(I*(c + d*x)) + 40*E^((2*I)*(c
 + d*x)) + 40*E^((3*I)*(c + d*x)) + 15*E^((4*I)*(c + d*x)) + 13*E^((5*I)*(c
 + d*x))) - 15*Sqrt[2]*(1 + E^((2*I)*(c + d*x)))^3*ArcTanh[(1 + E^(I*(c + d
*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])*Sin[(c + d*x)/2])/(60*d*E^(
((5*I)/2)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^(5/2)*Sqrt[
a - a*Cos[c + d*x]])
```

fricas [A] time = 0.91, size = 177, normalized size = 1.02

$$\frac{15\sqrt{2}\sqrt{a}\cos(dx+c)^3\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\left(13\cos(dx\right.}{30\,ad\cos(dx+c)^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*sqrt(a)*cos(d*x + c)^3*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c
) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)
*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(13*cos(
d*x + c)^3 + 14*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt(-a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3*sin(d*x + c))
```

giac [A] time = 1.28, size = 136, normalized size = 0.79

$$\frac{\sqrt{2}\,a\left(\frac{15\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,|a|}+\frac{2\left(15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2+10\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+12\,a^2\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,|a|}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*a*(15*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sq
rt(-a)*abs(a)) + 2*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2 + 10*(a*tan(1/2*d*x
 + 1/2*c)^2 - a)*a + 12*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*abs(a)))/d
```

maple [B] time = 0.20, size = 305, normalized size = 1.76

$$\frac{\left(\sin^7(dx+c)\right)\left(15\sqrt{2}\left(\cos^3(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)+45\sqrt{2}\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*sin(d*x+c)^7*(15*2^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7
/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+45*2^(1/2)*cos(d
*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2))+45*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/
2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+15*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))-26*cos(d*x+c)^3-2*cos(d*x+c)^2-6*cos(d*x+c))/(-1+cos(d*x+c))^3/(-2*
a*(-1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/cos(d*x+c)^(7/2)*2^(1/2)
```

maxima [C] time = 0.98, size = 692, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sq
rt(a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*a
bs(e^(I*d*x + I*c) - 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1))) - 26*(co
s(2*d*x + 2*c)^2*sin(d*x + c) + sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*cos(2*d
*x + 2*c)*sin(d*x + c) + sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + 24*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*sin(d*x + c) - 24*(cos(d*x + c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + 2*((13*cos(d*x + c) + 15)*cos(2*d*x + 2*c)^2 +
 (13*cos(d*x + c) + 15)*sin(2*d*x + 2*c)^2 + 2*(13*cos(d*x + c) + 15)*cos(2
*d*x + 2*c) + 13*cos(d*x + c) + 15)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) - 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*(7*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (7*cos(d*x + c) + 5)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(5/4)*sqrt(-a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a-a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.282 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}+\frac{7\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin}{\sqrt{2}\sqrt{1-\cos(c}}\right.}{d}$$

```
[Out] 7/4*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-arctanh(1/2
*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1/2)/d+1/2*co
s(d*x+c)^(3/2)*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)+1/4*sin(d*x+c)*cos(d*x+c)^
(1/2)/d/(1-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2778, 2983, 2982, 2782, 206, 2775, 207}

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}+\frac{7\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin}{\sqrt{2}\sqrt{1-\cos(c}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/(4*d)
 - (Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
 + d*x]])])/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x
]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 - Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} + \frac{1}{4}\int \frac{\sqrt{\cos(c+dx)}\,(3+\cos(c+dx))}{\sqrt{1-\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} - \frac{1}{4}\int \frac{-\frac{1}{2}-\frac{7}{2}\cos(c+d}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos}}$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} - \frac{7}{8}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx +$$

$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{}{\sqrt{1-\mathrm{co}}}\right.}{4d}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\,\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c}}{4d\sqrt{1}}$$

Mathematica [C] time = 0.19, size = 255, normalized size = 1.58

$$-\frac{ie^{-2i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\left(7\sqrt{2}\,e^{2i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-16e^{2i(c+dx)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}}{8\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}\,\sqrt{1-\cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] ((-1/8*I)*(-1 + E^(I*(c + d*x)))*(7*Sqrt[2]*E^((2*I)*(c + d*x))*ArcSinh[E^(
I*(c + d*x))] - 16*E^((2*I)*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(Sqrt[1 + E^((2*I)*(c + d*x))]
*(1 + 2*E^(I*(c + d*x)) + 2*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x))) + 7*
E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt[Cos[c + d
*x]])/(Sqrt[2]*d*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 -
 Cos[c + d*x]])
```

fricas [A] time = 0.93, size = 239, normalized size = 1.48

$$\frac{4\sqrt{2}\,\log\left(-\frac{2\left(\sqrt{2}\,\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}-(3\,\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+2\left(2\,\cos(dx+c)^2+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c)
+ 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c)
 - 1)*sin(d*x + c)))*sin(d*x + c) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + 7*log(2*(sqrt(-cos(d*x + c)
 + 1)*sqrt(cos(d*x + c)) + sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 7*log
(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - sin(d*x + c))/sin(d*x + c)
)*sin(d*x + c))/(d*sin(d*x + c))
```

giac [A] time = 1.81, size = 162, normalized size = 1.01

$$-\frac{\sqrt{2}\left(7\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1}}{\sqrt{2}+\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1}}\right)-\frac{4\left(\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{\frac{3}{2}}+2\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1}\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{2}}+8\log\left(\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}\right.\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(7*sqrt(2)*log((sqrt(2) - sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))/
(sqrt(2) + sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))) - 4*((-tan(1/2*d*x + 1/2*c)^
2 + 1)^(3/2) + 2*sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))/(tan(1/2*d*x + 1/2*c)^2
 + 1)^2 + 8*log(sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1) - 8*log(-sqrt(-tan(1
/2*d*x + 1/2*c)^2 + 1) + 1))/d
```

maple [A] time = 0.15, size = 194, normalized size = 1.20

$$-\frac{\left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{3}\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{2}\left(dx+c\right)\right)+3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos\left(dx+c\right)-4\sqrt{2}\,\operatorname{arctar}\right.}{4d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin\left(dx+c\right)^{5}\sqrt{2-2\cos\left(dx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^3*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*cos(d*x+c)^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*2^(1/2)*a
rctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+7*arctanh((cos(d*x+c)
/(1+cos(d*x+c)))^(1/2))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)/sin(d*x+c)^5/(2-2*cos(d*x+c))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{-\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(-cos(d*x + c) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(1 - cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(5/2)/(1 - cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(1-cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.283 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-arctanh(1/2*sin
(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1/2)/d+sin(d*x+c)
*cos(d*x+c)^(1/2)/d/(1-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2778, 2982, 2782, 206, 2775, 207}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt
[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]
)])/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
```

```
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} + \frac{1}{2}\int \frac{1+\cos(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{1}{2}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c}$$
$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d} - \frac{2\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+}}{d\sqrt{1-}}$$

Mathematica [C] time = 0.21, size = 227, normalized size = 1.92

$$-\frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\left(\sqrt{2}e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-4e^{i(c+dx)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left(\sqrt{1+\right.\right.}{2\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] ((-1/2*I)*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c +
 d*x))] - 4*E^(I*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 +
 E^((2*I)*(c + d*x))])] + Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*
(c + d*x))] + E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt
[Cos[c + d*x]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sq
rt[1 - Cos[c + d*x]])
```

fricas [B] time = 1.29, size = 225, normalized size = 1.91

$$\frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+2(\cos(dx+c)+1)\sqrt{-\cos}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) +
1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) -
 1)*sin(d*x + c)))*sin(d*x + c) + 2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) +
```

```
 1)*sqrt(cos(d*x + c)) + log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c))
+ sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - log(2*(sqrt(-cos(d*x + c) + 1)
*sqrt(cos(d*x + c)) - sin(d*x + c))/sin(d*x + c))*sin(d*x + c))/(d*sin(d*x
+ c))
```

giac [A] time = 1.75, size = 141, normalized size = 1.19

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{\sqrt{2}+\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}\right)-\frac{4\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+2\log\left(\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+1\right)-2\log\left(-\cdots\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))/(sq
rt(2) + sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))) - 4*sqrt(-tan(1/2*d*x + 1/2*c)^
2 + 1)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*log(sqrt(-tan(1/2*d*x + 1/2*c)^2 +
1) + 1) - 2*log(-sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1))/d
```

maple [A] time = 0.13, size = 166, normalized size = 1.41

$$-\frac{\left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{2}\left(\sqrt{2}\;\mathrm{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\;\cos\left(dx+c\right)-\mathrm{arctanh}\left(\sqrt{\cdots}\right.\right.}{d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{2-2\cos\left(dx+c\right)}\;\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^2*(2^(1/2)*arctanh(1/2*2^(1/2)/(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2))-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-a
rctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(2-2*cos(d*x+c))^(1/2)/sin(d*x+c)^3*2^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{-\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(-cos(d*x + c) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c + d\,x)^{3/2}}{\sqrt{1 - \cos (c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(1 - cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)/(1 - cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{1 - \cos (c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(1-cos(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(1 - cos(c + d*x)), x)
```

3.284 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-arctanh(1/2*s
in(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2777, 2775, 207, 2782, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d - (
Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d
*x]])])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx &= \int \frac{1}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\,dx - \int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx\\ &= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{c}}\right.}{d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\,\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [C] time = 0.12, size = 160, normalized size = 1.88

$$-\frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right.}{\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}\,\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] ((-I)*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))
]*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2
```

```
]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])
)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]])
```

fricas [B] time = 1.20, size = 170, normalized size = 2.00

$$\frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)+2\log\left(\frac{2\left(\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}+\sin(\right.}{\sin(dx+c)}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) +
1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) -
 1)*sin(d*x + c))) + 2*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) +
sin(d*x + c))/sin(d*x + c)) - 2*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x
 + c)) - sin(d*x + c))/sin(d*x + c)))/d
```

giac [A] time = 1.80, size = 105, normalized size = 1.24

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{\sqrt{2}+\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}\right)+\log\left(\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}+1\right)-\log\left(-\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))/(sq
rt(2) + sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))) + log(sqrt(-tan(1/2*d*x + 1/2*c
)^2 + 1) + 1) - log(-sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1))/d
```

maple [A] time = 0.10, size = 117, normalized size = 1.38

$$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)(-1+\cos(dx+c))\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)+2\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{2}}{d\sin(dx+c)\sqrt{2-2\cos(dx+c)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(-2^(1/2)*arctanh(1/2*2^(1/2)/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2))+2*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/s
in(d*x+c)/(2-2*cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{-\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-cos(d*x + c) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(1 - cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(1/2)/(1 - cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{1-\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(1-cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(cos(c + d*x))/sqrt(1 - cos(c + d*x)), x)
```

3.285 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1
/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2782, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Mathematica [C] time = 0.14, size = 110, normalized size = 2.34

$$\frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}\,d\sqrt{-((\cos(c+dx)-1)\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (I*(-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 + E^(I*(
c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(Sqrt[2]*d*E^(I*(c + d
*x))*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])
```

fricas [B] time = 0.82, size = 84, normalized size = 1.79

$$\frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) + 1
)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) -
1)*sin(d*x + c)))/d
```

giac [A] time = 0.62, size = 79, normalized size = 1.68

$$\frac{\sqrt{2}\left(\log\left(\sqrt{2}+1\right)-\log\left(\sqrt{2}-1\right)-\log\left(\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+1\right)+\log\left(-\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+1\right)\right)}{2\,d\,\text{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(log(sqrt(2) + 1) - log(sqrt(2) - 1) - log(sqrt(-tan(1/2*d*x +
1/2*c)^2 + 1) + 1) + log(-sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1))/(d*sgn(ta
n(1/2*d*x + 1/2*c)))
```

maple [B] time = 0.13, size = 84, normalized size = 1.79

$$\frac{4\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(-1+\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)}{d\sqrt{\cos\left(dx+c\right)}\,\left(2-2\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 4/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*sin(d*x+c)*arctanh(1/
2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(2-2*cos(d*x+
c))^(3/2)
```

maxima [C] time = 0.99, size = 248, normalized size = 5.28

$$\sqrt{2}\log\left(\frac{4\left(\left|ie^{(idx+ic)}-i\right|^{2}+2\sqrt{\cos(2dx+2c)^{2}+\sin(2dx+2c)^{2}+2\cos(2dx+2c)+1}\left(\cos\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)+1)\right)^{2}+\sin\left(\frac{1}{2}\arctan\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(4*(abs(I*e^(I*d*x + I*c) - I)^2 + 2*sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x + I*c) - I)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*sqrt(2)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(I*d*x + I*c) - I)^2)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2)), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\cos\left(c+dx\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(1 - cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.286 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1
/2)/d+2*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2779, 2782, 206}

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d) + (2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x
]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} + \int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 0.17, size = 152, normalized size = 1.83

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right) - \frac{e^{-\frac{1}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{-((\cos(c+dx)-1)\cos(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (2*(-(((1 + E^((2*I)*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))]])])/(Sqrt[2]*E^((I/2)*(c + d*x)))) + 2*Sqrt[1 + E
^((2*I)*(c + d*x))]*Cos[(c + d*x)/2])*Sin[(c + d*x)/2])/(d*Sqrt[1 + E^((2*I
)*(c + d*x))]*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])
```

fricas [A] time = 0.84, size = 144, normalized size = 1.73

$$\frac{\sqrt{2}\cos(dx+c)\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c) + 4(\cos(dx+}{2d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*cos(d*x + c)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-co
s(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((c
os(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(cos(d*x + c) + 1)*sqrt(-c
os(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)*sin(d*x + c))
```

giac [A] time = 0.64, size = 72, normalized size = 0.87

$$\frac{\sqrt{2}\left(\frac{4}{\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}-\log\left(\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+1\right)+\log\left(-\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(4/sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) - log(sqrt(-tan(1/2*d*x +
1/2*c)^2 + 1) + 1) + log(-sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1))/d
```

maple [B] time = 0.13, size = 159, normalized size = 1.92

$$\frac{\left(\sin^{3}\left(dx+c\right)\right)\left(\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\cos\left(dx+c\right)+\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}}\right.\right.}{d\sqrt{2-2\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{\frac{3}{2}}\left(\cos^{2}\left(dx+c\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*sin(d*x+c)^3*(2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*2^(
1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)+2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-2*
cos(d*x+c))/(2-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)/(cos(d*x+c)^2-1)*2^(1/2
)
```

maxima [C] time = 1.02, size = 400, normalized size = 4.82

$$-\frac{\sqrt{2}\left(2\,\sqrt{2}\,\sin\left(dx+c\right)\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)+2\left(\sqrt{2}\,\cos\left(dx+c\right)+\sqrt{2}\right)\cos\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(2)*sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + 2*(sqrt(2)*cos(d*x + c) + sqrt(2))*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*log(4*(abs(I*e^(I*d*x + I*c) - I)^2 +
 2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x +
 I*c) - I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*sqr
t(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(
I*d*x + I*c) - I)^2))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,\sqrt{1-\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.287 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1
/2)/d+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2)+2/3*sin(d*x+c)
/d/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2779, 2984, 12, 2782, 206}

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d) + (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(
3/2)) + (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
```

```
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\int \frac{1+2\cos(c+dx)}{\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}} -$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}} +$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}} -$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\,\sqrt{1-\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+$$

Mathematica [C] time = 0.28, size = 170, normalized size = 1.39

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+1)-\frac{3e^{-\frac{3}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{2\sqrt{2}}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)),x]
```

```
[Out] (2*((-3*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))])])/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))) + 2*Sq
rt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x]))*Sin[(c + d
*x)/2])/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]]*Cos[c + d
*x]^(3/2))
```

fricas [A] time = 0.89, size = 157, normalized size = 1.29

$$\frac{3\sqrt{2}\cos(dx+c)^2\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\left(\cos(d\right.}{6\,d\cos(dx+c)^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*cos(d*x + c)^2*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt
(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))
/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)
^2*sin(d*x + c))
```

giac [A] time = 0.79, size = 89, normalized size = 0.73

$$-\frac{\sqrt{2}\left(\frac{8}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}+3\log\left(\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}+1\right)-3\log\left(-\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(8/((tan(1/2*d*x + 1/2*c)^2 - 1)*sqrt(-tan(1/2*d*x + 1/2*c)^2
+ 1)) + 3*log(sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1) - 3*log(-sqrt(-tan(1/2
*d*x + 1/2*c)^2 + 1) + 1))/d
```

maple [A] time = 0.15, size = 170, normalized size = 1.39

$$-\frac{\left(\sin^{5}\left(dx+c\right)\right)\left(3\sqrt{2}\ \cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)+3\sqrt{2}\ \left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(d}{1+\cos(}}}\right.\right.}{3d\left(-1+\cos\left(dx+c\right)\right)^{2}\sqrt{2-2\cos\left(dx+c\right)}\ \left(1+\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*sin(d*x+c)^5*(3*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))-2*cos(d*x+c))/(-1+cos(d*x+c))^2/(2-2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/
cos(d*x+c)^(5/2)*2^(1/2)
```

maxima [C] time = 1.01, size = 563, normalized size = 4.61

$$\frac{3\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)\log\left(\frac{4\left(\left|i\,e^{(i\,dx+i\,c)}-i\right|^{2}+2\,\sqrt{\cos(2\,dx+2\,c)^{2}+\sin(2\,dx+2\,c)^{2}+2\,\cos}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(4*(abs(I*e^(I*d*x + I*c) - I)^2 + 2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x + I*c) - I)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(I*d*x + I*c) - I)^2) - 2*(sqrt(2)*sin(d
*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)
*cos(d*x + c) + 3*sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4) - 4*(sqrt(2)*sin(d*x + c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
```

```
*x + 2*c) + 1)) + (sqrt(2)*cos(d*x + c) - sqrt(2))*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4))/((sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin
(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\cos\left(c+dx\right)}\,\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
[Out] Integral(1/(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(5/2)), x)
```

3.288 $\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{2^{5/6}\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,F_1\left(\frac{1}{2};-\frac{4}{3},\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}$$

```
[Out] 2^(5/6)*AppellF1(1/2,-4/3,1/6,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*(a+a*cos
(d*x+c))^(1/3)*sin(d*x+c)/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2787, 2785, 133}

$$\frac{2^{5/6}\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,F_1\left(\frac{1}{2};-\frac{4}{3},\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (2^(5/6)*AppellF1[1/2, -4/3, 1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*x])
/2]*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(5/6))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
```

```
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx = \frac{\sqrt[3]{a+a\cos(c+dx)}\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{1+\cos(c+dx)}\,dx}{\sqrt[3]{1+\cos(c+dx)}}$$

$$= \frac{\left(\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{4/3}}{\sqrt[6]{2-x}\sqrt{x}}\,dx, x, 1-\cos(c+\right.}{d\sqrt{1-\cos(c+dx)}\,(1+\cos(c+dx))^{5/6}}$$

$$= \frac{2^{5/6}F_1\left(\frac{1}{2};-\frac{4}{3},\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)\sqrt[3]{a+a\cos(c+}}{d(1+\cos(c+dx))^{5/6}}$$

Mathematica [F] time = 15.61, size = 0, normalized size = 0.00

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3),x]
[Out] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3), x]
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+a)^{\frac{1}{3}}\cos(dx+c)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{4}{3}}(dx+c)\right)(a+a\cos(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x)
[Out] int(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^{\frac{1}{3}}\cos(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{4/3}\,(a+a\,\cos(c+d\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(4/3)*(a+a*cos(d*x+c))**(1/3),x)
[Out] Timed out
```

3.289 $\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[6]{2}\,\sin(c+dx)(a\cos(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{4}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

```
[Out] 2*2^(1/6)*AppellF1(1/2,-4/3,-1/6,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*(a+a*
cos(d*x+c))^(2/3)*sin(d*x+c)/d/(1+cos(d*x+c))^(7/6)
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2787, 2785, 133}

$$\frac{2\sqrt[6]{2}\,\sin(c+dx)(a\cos(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{4}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -4/3, -1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*
x])/2]*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/6)
)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
```

```
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx = \frac{(a+a\cos(c+dx))^{2/3}\int \cos^{\frac{4}{3}}(c+dx)(1+\cos(c+dx))^{2/3}\,dx}{(1+\cos(c+dx))^{2/3}}$$

$$= \frac{\left((a+a\cos(c+dx))^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{4/3}\sqrt[6]{2-x}}{\sqrt{x}}\,dx,x,1-\cos\right.}{d\sqrt{1-\cos(c+dx)}\,(1+\cos(c+dx))^{7/6}}$$

$$= \frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};-\frac{4}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)(a+a\cos(c}{d(1+\cos(c+dx))^{7/6}}$$

Mathematica [F] time = 3.43, size = 0, normalized size = 0.00

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3),x]
[Out] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{4}{3}}(dx+c)\right)(a+a\cos(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x)
[Out] int(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{4/3}\,(a+a\,\cos(c+d\,x))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(2/3),x)
[Out] int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(4/3)*(a+a*cos(d*x+c))**(2/3),x)
[Out] Timed out
```

3.290 $\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[6]{2}\,\sin(c+dx)(a\cos(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{5}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

```
[Out] 2*2^(1/6)*AppellF1(1/2,-5/3,-1/6,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*(a+a*
cos(d*x+c))^(2/3)*sin(d*x+c)/d/(1+cos(d*x+c))^(7/6)
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2787, 2785, 133}

$$\frac{2\sqrt[6]{2}\,\sin(c+dx)(a\cos(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{5}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -5/3, -1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*
x])/2]*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/6)
)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
```

```
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx = \frac{(a+a\cos(c+dx))^{2/3}\int \cos^{\frac{5}{3}}(c+dx)(1+\cos(c+dx))^{2/3}\,dx}{(1+\cos(c+dx))^{2/3}}$$
$$= \frac{\left((a+a\cos(c+dx))^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{5/3}\sqrt[6]{2-x}}{\sqrt{x}}\,dx, x, 1-\text{co}\right.}{d\sqrt{1-\cos(c+dx)}\,(1+\cos(c+dx))^{7/6}}$$
$$= \frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};-\frac{5}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)(a+a\cos(}{d(1+\cos(c+dx))^{7/6}}$$

Mathematica [F] time = 2.70, size = 0, normalized size = 0.00

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3),x]
[Out] Integrate[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3), x]
```

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)^{\frac{5}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{5}{3}}\left(dx+c\right)\right)\left(a+a\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\cos\left(dx+c\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/3}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/3)*(a + a*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)^(5/3)*(a + a*cos(c + d*x))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/3)*(a+a*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

3.291 $\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=151

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}}{3}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/5
*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3768, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= a\int \sec^{\frac{5}{2}}(c+dx)\,dx + a\int \sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \sqrt{\sec(}$$
$$= \frac{6a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(c+}{}$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6a\sqrt{\sec(c+dx)}\,\sin(c+}{5d}$$
$$= -\frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}$$

Mathematica [C] time = 1.63, size = 268, normalized size = 1.77

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/E^(I*(c +
d*x)) + (1 - E^((2*I)*c))*Sqrt[Sec[c + d*x]]*(9*Cos[d*x]*Csc[c] + (5 + 3*Se
c[c + d*x])*Tan[c + d*x])))/(15*(d - d*E^((2*I)*c)))
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

maple [B] time = 0.79, size = 384, normalized size = 2.54

$$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{40\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\right.}{5\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\,\cos(c+d\,x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.292 $\int (a + a\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx$

Optimal. Leaf size=123

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\sqrt{}}{}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= a\int \sec^{\frac{3}{2}}(c+dx)\,dx + a\int \sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}a\int \sqrt{\sec$$
$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\Big(a\sqrt{\cos(c$$
$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\Big(\frac{1}{2}(}{}$$

Mathematica [C] time = 1.15, size = 255, normalized size = 2.07

$$a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\Bigg(i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/E^(I*(c + d*
x)) - (-1 + E^((2*I)*c))*Sqrt[Sec[c + d*x]]*(3*Cos[d*x]*Csc[c] + Tan[c + d*
x])))/(3*(d - d*E^((2*I)*c)))
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 0.98, size = 369, normalized size = 3.00

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.293 $\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\right.}{d}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))\,dx$$
$$= a\int \sqrt{\sec(c+dx)}\,dx + a\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(a\sqrt{\cos(c+dx)}\,\sqrt{\right.$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+}{d}$$
$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [C] time = 1.35, size = 124, normalized size = 1.28

$$-\frac{2iae^{-i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)-1\right)\sqrt{\sec(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((-2*I)*a*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*x]])
/(d*E^(I*(c + d*x)))
```

fricas [F] time = 2.62, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.61, size = 146, normalized size = 1.51

$$-\frac{2a\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*a*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.294 $\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3238, 3787, 3771, 2639, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\\ &= a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a\int \sqrt{\sec(c+dx)}\,dx\\ &= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\right.\\ &= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{}\end{aligned}$$

Mathematica [C] time = 1.11, size = 141, normalized size = 1.88

$$-\frac{2ia\left(-2\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+2e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)+e^{2i(c+dx)}+1\right)}{d\left(1+e^{2i(c+dx)}\right)\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*a*(1 + E^((2*I)*(c + d*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))]))/(d*(1 + E^((2*I)*(c + d*x)))*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left((a\cos(dx+c)+a)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.51, size = 150, normalized size = 2.00

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int\sqrt{\sec\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x
)), x))
```

3.295 $\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx &= \int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx \\
&= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c
\\
&= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)
\\
&= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{3d}
\end{aligned}$$

Mathematica [C] time = 1.29, size = 140, normalized size = 1.39

$$\frac{ae^{-2ic}(\sin(2c)-i\cos(2c))\left(-\frac{12\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+2i\sin(c+\right.}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*((-I)*Cos[2*c] + Sin[2*c])*(6 - (12*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*
x] + (2*I)*Sin[c + d*x]))/(3*d*E^((2*I)*c)*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 1.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\cos\left(dx+c\right)+a}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\cos\left(dx+c\right)+a}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.51, size = 225, normalized size = 2.23

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
```

```
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos\left(dx+c\right)+a}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{1}{\sqrt{\sec\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
[Out] a*(Integral(cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d
*x)), x))
```

3.296 $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}{5d}$$

```
[Out] 2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5
*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}} \\
&= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx}} \\
&= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{se}}{3d}
\end{aligned}$$

Mathematica [C] time = 0.94, size = 224, normalized size = 1.76

$$-\frac{iae^{-3i(c+dx)}\left(-72e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+40e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((-1/120*I)*a*(1 + Cos[c + d*x])*(-3 - 10*E^(I*(c + d*x)) + 33*E^((2*I)*(c
+ d*x)) + 39*E^((4*I)*(c + d*x)) + 10*E^((5*I)*(c + d*x)) + 3*E^((6*I)*(c +
 d*x)) - 72*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 40*E^((3*I)*(c + d*x))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)])*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x)))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.49, size = 219, normalized size = 1.72

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-28\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\cdots}\right.}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\cos(c+d\,x)}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(
-3/2), x))
```

3.297 $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a\sqrt{\cos}}{}$$

```
[Out] 2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10/
21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}(5a)\int \frac{1}{\sec^{\frac{3}{2}}(c+d \\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5a)\int \sqrt{\sec(c+dx)}\,dx + \\
&= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \\
&= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{21d}
\end{aligned}$$

Mathematica [C] time = 2.16, size = 198, normalized size = 1.31

$$\frac{ae^{-4i(c+dx)}\sqrt{\sec(c+dx)}\,(\cos(4(c+dx))+i\sin(4(c+dx)))\left(504ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])*((-504*I)*Cos
[c + d*x] + ((504*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4,
1/2, 3/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) - (200*I)*Sqrt[1 + E^((2*I
)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + 42*S
in[c + d*x] + 130*Sin[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*
x)]))/(420*d*E^((4*I)*(c + d*x)))
```

fricas [F] time = 4.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\cos\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a\cos\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

maple [A] time = 0.55, size = 270, normalized size = 1.79

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\cos\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.298 $\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{16a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{}}{}$$

```
[Out] 4/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+16/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{16a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (16*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \left(2a^2\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \int \sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2a^2\right)$$
$$= \frac{16a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\,\mathrm{se}}{}$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{16a^2\sqrt{\sec(c+dx)}\,\mathrm{s}}{5d}$$
$$= -\frac{16a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 1.87, size = 261, normalized size = 1.62

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\,(24\csc(c)\cos(dx)+\tan(c+dx)(3\sec(c+dx)+10)) - \frac{2i\sqrt{2}}{}\right.}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))
/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(24*Cos[d*x]*Csc
[c] + (10 + 3*Sec[c + d*x])*Tan[c + d*x])))/(30*d)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2\cos\left(dx+c\right)^2 + 2\,a^2\cos\left(dx+c\right) + a^2\right)\sec\left(dx+c\right)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(7/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

maple [B] time = 1.02, size = 386, normalized size = 2.40

$$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-\frac{4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}+\frac{17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}\cdots\right.\right.}}{30\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\cdots\right.\right.}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-4/5*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d
*x+1/2*c)^2)^(1/2)+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2} (a+a\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.299 $\int (a + a\cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)\,dx$

Optimal. Leaf size=131

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2}{}$$

```
[Out] 2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (4*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\,dx$$
$$= \left(2a^2\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\sec(c+dx)}\,\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{4a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\right)$$
$$= \frac{4a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\sqrt{\right.}$$
$$= -\frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F}{}$$

Mathematica [C] time = 1.36, size = 250, normalized size = 1.91

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\,(\tan(c+dx)+6\csc(c)\cos(dx)) - \frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(-1+\right.\right.}{}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E
^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(
E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c]
 + Tan[c + d*x])))/(6*d)
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2\cos(dx+c)^2 + 2\,a^2\cos(dx+c) + a^2\right)\sec(dx+c)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

maple [B] time = 0.98, size = 371, normalized size = 2.83

$$\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}\,{\left(a+a\,\cos(c+d\,x)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.300 $\int (a+a\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)\, dx$

Optimal. Leaf size=64

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3788, 3771, 2641, 4043}

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4043

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\begin{aligned}
\int (a + a\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)\, dx &= \int \frac{(a + a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\, dx \\
&= \left(2a^2\right)\int \sqrt{\sec(c+dx)}\, dx + \int \frac{a^2 + a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\, dx \\
&= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \left(2a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{c}} \\
&= \frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\,\sin}{d}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 48, normalized size = 0.75

$$\frac{2a^2\sqrt{\sec(c+dx)}\left(\sin(c+dx) + 2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Sin[c + d*x]))/d
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos(dx+c)^2 + 2\,a^2\cos(dx+c) + a^2\right)\sec(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.55, size = 104, normalized size = 1.62

$$-\frac{4a^2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x)
```

```
[Out] -4*a^2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.301 $\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{d}$$

```
[Out] 2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3788, 3771, 2639, 4045, 2641}

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(2a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(4a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{s\right.$$

$$= \frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}$$

$$= \frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

Mathematica [C] time = 1.05, size = 127, normalized size = 1.19

$$\frac{a^2\left(\frac{24i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-4i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+\sin(c+dx)-6i\right)\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*(((24*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt
[1 + E^((2*I)*(c + d*x))] + 2*(-6*I - (4*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c
+ d*x])))/(3*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 2.13, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^{2} \cos\left(dx+c\right)^{2} + 2\,a^{2} \cos\left(dx+c\right) + a^{2}\right)\sqrt{\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \cos\left(dx+c\right) + a\right)^{2} \sqrt{\sec\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.47, size = 228, normalized size = 2.13

$$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^{2}\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}\right.\right.}\right.}{3\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)
```

```
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int \cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx+\int \sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x
)**2*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

3.302 $\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c}{5d}$$

```
[Out] 2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(2a^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}\left(8a^2\right)\int \frac{}{\sqrt{s}}$$

$$= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos}}$$

$$= \frac{16a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [C] time = 1.51, size = 136, normalized size = 1.01

$$\frac{a^2\left(\frac{192i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 40i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 40\sin(c+dx) + 6\sin(2(c+d\right.}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*Sin
[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\cos(dx+c)^2 + 2a^2\cos(dx+c) + a^2}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/sqrt(sec(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.75, size = 250, normalized size = 1.85

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-13*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\cos(c+d\,x))^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x
)**2/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

3.303 $\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2\sqrt{\cos}}{}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+8/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]
])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(2a^2\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(6a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}\left(12a^2\right)\int \frac{}{\sec}$$

$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(4a^2\right)\int \sqrt{\sec(c+dx)}\,dx$$

$$= \frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+d}{5d\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{7d}$$

Mathematica [C] time = 1.71, size = 149, normalized size = 0.93

$$\frac{a^2\left(\frac{672i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2\left(-80i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 85\sin(c+dx) + 28\sin(2(\right.\right.}{140d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 85
*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)])))/(140*d*Sqrt[Sec
[c + d*x]])
```

fricas [F] time = 2.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\cos\left(dx+c\right)^2 + 2\,a^2\cos\left(dx+c\right) + a^2}{\sec\left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/sec(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.60, size = 272, normalized size = 1.69

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\cos(c+d\,x))^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(cos(c + d*
x)**2/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

3.304 $\int (a+a\cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=187

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)\sqrt{s}}{5d}$$

```
[Out] 52/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)
/d+2/7*a^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+28/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3768, 3771, 2639, 2641}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)\sqrt{s}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

```
[Out] (-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a + a\cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx)\,dx = \int \sec^{\frac{3}{2}}(c + dx)(a + a\sec(c + dx))^3\,dx$$
$$= \int \left(a^3 \sec^{\frac{3}{2}}(c + dx) + 3a^3 \sec^{\frac{5}{2}}(c + dx) + 3a^3 \sec^{\frac{7}{2}}(c + dx) + a^3 \sec^{\frac{9}{2}}(c\right.$$
$$= a^3 \int \sec^{\frac{3}{2}}(c + dx)\,dx + a^3 \int \sec^{\frac{9}{2}}(c + dx)\,dx + \left(3a^3\right) \int \sec^{\frac{5}{2}}(c + dx)\,$$
$$= \frac{2a^3\sqrt{\sec(c + dx)}\,\sin(c + dx)}{d} + \frac{2a^3 \sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{d} + \frac{6a^3 \sec}{}$$
$$= \frac{28a^3\sqrt{\sec(c + dx)}\,\sin(c + dx)}{5d} + \frac{52a^3 \sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{21d} + \frac{6a^3}{}$$
$$= -\frac{2a^3\sqrt{\cos(c + dx)}\,E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c + dx)}}{d} + \frac{2a^3\sqrt{\cos(c + dx)}\,I}{}$$
$$= -\frac{28a^3\sqrt{\cos(c + dx)}\,E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c + dx)}}{5d} + \frac{52a^3\sqrt{\cos(c + dx}}{}$$

Mathematica [C] time = 2.75, size = 279, normalized size = 1.49

$$\frac{a^3(\cos(c + dx) + 1)^3 \sec^6\left(\frac{1}{2}(c + dx)\right)\left(\sqrt{\sec(c + dx)}\,\left(294\csc(c)\cos(dx) + (63\cos(c + dx) + 65\cos(2(c + dx)) +\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(294*Cos[d*x]
*Csc[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c
 + d*x])))/(420*d)
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3 \cos(dx + c)^3 + 3\,a^3 \cos(dx + c)^2 + 3\,a^3 \cos(dx + c) + a^3\right)\sec(dx + c)^{\frac{9}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

maple [B] time = 1.05, size = 439, normalized size = 2.35

$$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(-\frac{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{160\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\cdots}{10\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-3/160*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^3-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-
2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+53/105*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/20*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/448*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-13/
168*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2} (a+a\,\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.305 $\int (a + a\cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)\, dx$

Optimal. Leaf size=157

$$\frac{2a^3 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{2a^3 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{d} + \frac{36a^3 \sin(c + dx)\sqrt{\sec(c + dx)}}{5d} + \frac{4a^3 \sqrt{\cos(c + dx)}\,\sqrt{s}}{}$$

```
[Out] 2*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d+3
6/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3771, 2641, 3768, 2639}

$$\frac{2a^3 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{2a^3 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{d} + \frac{36a^3 \sin(c + dx)\sqrt{\sec(c + dx)}}{5d} + \frac{4a^3 \sqrt{\cos(c + dx)}\,\sqrt{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx=\int\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3\,dx$$

$$=\int\left(a^3\sqrt{\sec(c+dx)}+3a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+a^3\sec^{\frac{7}{2}}(c$$

$$=a^3\int\sqrt{\sec(c+dx)}\,dx+a^3\int\sec^{\frac{7}{2}}(c+dx)\,dx+\left(3a^3\right)\int\sec^{\frac{3}{2}}(c+dx)$$

$$=\frac{6a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}+\frac{2a^3\operatorname{sec}}{}$$

$$=\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{36a^3\sqrt{\sec(c+dx)}\,\mathrm{s}}{5d}$$

$$=-\frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^3\sqrt{\cos(c+dx)}\,I}{}$$

$$=-\frac{36a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 1.90, size = 259, normalized size = 1.65

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\,(18\csc(c)\cos(dx)+\tan(c+dx)(\sec(c+dx)+5))-\frac{2i\sqrt{2}e^{-i(c+}}{}\right.}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E
^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(
E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c
] + (5 + Sec[c + d*x])*Tan[c + d*x])))/(20*d)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

maple [B] time = 0.95, size = 386, normalized size = 2.46

$$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{10\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{\cos\left(\frac{dx}{2}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2} (a+a\,\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.306 $\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a}{}$$

[Out] 2/3*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.18, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3771, 2639, 2641, 3768}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a}{}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]

[Out] (-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx=\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$

$$=\int \left(\frac{a^3}{\sqrt{\sec(c+dx)}}+3a^3\sqrt{\sec(c+dx)}+3a^3\sec^{\frac{3}{2}}(c+dx)+a^3\sec^{\frac{5}{2}}(c$$

$$=a^3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+a^3\int \sec^{\frac{5}{2}}(c+dx)\,dx+\left(3a^3\right)\int \sqrt{\sec(c+dx}$$

$$=\frac{6a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}a^3\int$$

$$=\frac{2a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,F}{}$$

$$=-\frac{4a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 0.97, size = 157, normalized size = 1.20

$$-\frac{ia^3\sec^{\frac{3}{2}}(c+dx)\left(6e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+20\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)c}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

```
[Out] ((-1/3*I)*a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*I
)*(c + d*x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
)/E^((2*I)*(c + d*x)) + 20*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hyper
geometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x] + (9
*I)*Sin[2*(c + d*x)]))/d
```

fricas [F] time = 1.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

maple [B] time = 0.88, size = 371, normalized size = 2.83

$$\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ a^3\left(10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(10*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2-18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2} (a+a\,\cos(c+d\,x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.307 $\int (a + a\cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)\, dx$

Optimal. Leaf size=131

$$\frac{2a^3 \sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{20a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^3\sqrt{\cos(c+ $$

```
[Out] 2/3*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d+4
*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{2a^3 \sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{20a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^3\sqrt{\cos(c+ $$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)} + \frac{3a^3}{\sqrt{\sec(c+dx)}} + 3a^3\sqrt{\sec(c+dx)} + a^3\sec^{\frac{3}{2}}(c+dx\right.$$
$$= a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a^3\int \sec^{\frac{3}{2}}(c+dx)\,dx + \left(3a^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}$$
$$= \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{3}a^3\int \sqrt{\sec(c+dx}$$
$$= \frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$
$$= \frac{4a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

Mathematica [C] time = 1.31, size = 135, normalized size = 1.03

$$\frac{a^3\left(\frac{24i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2\left(-10i\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + \sin(c+dx) + 3\tan(c+dx)\right.\right.}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(((24*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt
[1 + E^((2*I)*(c + d*x))] + 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c
 + d*x] + 3*Tan[c + d*x])))/(3*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 2.24, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^3\cos(dx+c)^3 + 3\,a^3\cos(dx+c)^2 + 3\,a^3\cos(dx+c) + a^3\right)\sec(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.76, size = 172, normalized size = 1.31

$$-\frac{4a^3\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(co\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.308 $\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{5d}$$

```
[Out] 2/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+3
6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2639, 2641}

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])
/(d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)} + \frac{3a^3}{\sqrt{\sec(c+dx)}} + a^3\sqrt{\sec(c+dx)}\right)$$
$$= a^3\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a^3\int \sqrt{\sec(c+dx)}\,dx + \left(3a^3\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx}$$
$$= \frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{5}\left(3a^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^3$$
$$= \frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F}{}$$
$$= \frac{36a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 1.28, size = 137, normalized size = 1.05

$$\frac{a^3\left(\frac{144i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2\left(-20i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 10\sin(c+dx) + \sin(2(c\right.\right.}{10d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*(((144*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 10*
Sin[c + d*x] + Sin[2*(c + d*x)])))/(10*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^3\cos(dx+c)^3 + 3\,a^3\cos(dx+c)^2 + 3\,a^3\cos(dx+c) + a^3\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.56, size = 250, normalized size = 1.91

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{5\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cos(c+dx)\sqrt{\sec(c+dx)}\,dx + \int 3\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx + \int \cos^3(c+dx)\sqrt{\sec(c+dx)}\,dx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(3*cos(c + d
*x)**2*sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**3*sqrt(sec(c + d*x))
, x) + Integral(sqrt(sec(c + d*x)), x))
```

3.309 $\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3\sqrt{c}}{}$$

```
[Out] 2/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)+6/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+52/21*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2641, 2639}

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \int \left(\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)} + \frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$= a^3\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a^3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(3a^3\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(3a^3\right)$$

$$= \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^3\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx}{5d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{21d}$$

Mathematica [C] time = 1.80, size = 146, normalized size = 0.91

$$\frac{a^3\left(\frac{4704i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 1040i\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 1070\sin(c+dx) + 252\sin(\right.}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1
070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt
[Sec[c + d*x]])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^3\cos(dx+c)^3 + 3\,a^3\cos(dx+c)^2 + 3\,a^3\cos(dx+c) + a^3}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.71, size = 272, normalized size = 1.69

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^6+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+65*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\ \cos(c + d\,x))^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{3\cos(c + dx)}{\sqrt{\sec(c + dx)}}\,dx + \int \frac{3\cos^2(c + dx)}{\sqrt{\sec(c + dx)}}\,dx + \int \frac{\cos^3(c + dx)}{\sqrt{\sec(c + dx)}}\,dx + \int \frac{1}{\sqrt{\sec(c + dx)}}\,dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(3*cos(c + d
*x)**2/sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**3/sqrt(sec(c + d*x))
, x) + Integral(1/sqrt(sec(c + d*x)), x))
```

3.310 $\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(\right.}{21d}$$

```
[Out] 2/9*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)+6/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+68/45*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+44/21*a^3*sin(d*x+c)/d/sec(d*x+c)^
(1/2)+68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+44/21*a^3*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2639, 2641}

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
5*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]
^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}\right) dx$$
$$= a^3 \int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^3 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right) \int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(3a^3\right)$$
$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a^3 \int$$
$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{15}($$
$$= \frac{18a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{3d}$$
$$= \frac{68a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{44a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{21d}$$

Mathematica [C] time = 2.29, size = 156, normalized size = 0.83

$$\frac{a^3\left(\frac{22848i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 5280i\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 5820\sin(c+dx) + 2044\right.}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*Sin[
4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^3\cos\left(dx+c\right)^3 + 3\,a^3\cos\left(dx+c\right)^2 + 3\,a^3\cos\left(dx+c\right) + a^3}{\sec\left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3)/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

maple [A] time = 0.60, size = 260, normalized size = 1.39

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)

[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")

```
[Out] integrate((a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\cos(c+dx))^3}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{3\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{3\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{\cos^3(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(3*cos(c +
d*x)**2/sec(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**3/sec(c + d*x)**(3
/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

3.311 $\int (a+a\cos(c+dx))^4 \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=187

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{\mathrm{se}}}{5d}$$

[Out] 94/21*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+8/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+64/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.25, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3771, 2641, 3768, 2639}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{\mathrm{se}}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(9/2),x]

[Out] (-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^4\,dx$$
$$= \int \left(a^4\sqrt{\sec(c+dx)} + 4a^4\sec^{\frac{3}{2}}(c+dx) + 6a^4\sec^{\frac{5}{2}}(c+dx) + 4a^4\sec^{\frac{7}{2}}(c$$
$$= a^4\int \sqrt{\sec(c+dx)}\,dx + a^4\int \sec^{\frac{9}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sec^{\frac{3}{2}}(c+dx)\,d$$
$$= \frac{8a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{4a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{5}{2}}}{}$$
$$= \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{64a^4\sqrt{\sec(c+dx)}\,\sin}{5d}$$
$$= -\frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^4\sqrt{\cos(c+dx)}\,F}{}$$
$$= -\frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 2.03, size = 271, normalized size = 1.45

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\,\left(672\csc(c)\cos(dx)+\tan(c+dx)\left(15\sec^2(c+dx)+84\sec(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(168*(1 + E^((2*I)*(c + d*x))) + 168*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 85*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(672*Cos[d*x]
*Csc[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x])))/(840*
d)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4\cos(dx+c)^4 + 4\,a^4\cos(dx+c)^3 + 6\,a^4\cos(dx+c)^2 + 4\,a^4\cos(dx+c) + a^4\right)\sec(dx+c)^{\frac{9}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2 + 4*a^4*cos(d*x + c) + a^4)*sec(d*x + c)^(9/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2), x)

maple [B] time = 1.02, size = 439, normalized size = 2.35

$$-\frac{32\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^4 \left(\frac{253\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{420\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} - \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x)

[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(253/420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/896*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-47/672*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^4,x)
```

```
[Out] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.312 $\int (a+a\cos(c+dx))^4 \sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}}{}$$

```
[Out] 8/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+66/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3771, 2639, 2641, 3768}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \int \left(\frac{a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)} + 6a^4\sec^{\frac{3}{2}}(c+dx) + 4a^4\sec^{\frac{5}{2}}(c$$
$$= a^4\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^4\int \sec^{\frac{7}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sqrt{\sec(c+dx}$$
$$= \frac{12a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^4\,\text{se}}{}$$
$$= \frac{2a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}\,F}{}$$
$$= -\frac{10a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{32a^4\sqrt{\cos(c+dx}}{}$$
$$= -\frac{56a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 2.49, size = 278, normalized size = 1.73

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\,(30\cos(c)\sin(dx)-3(5\cos(2c)-61)\csc(c)\cos(dx)+2\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 20*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
)/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(-3*(-61 + 5*Co
s[2*c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x])*Tan[
c + d*x])))/(240*d)
```

fricas [F] time = 1.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4\cos\left(dx+c\right)^4 + 4\,a^4\cos\left(dx+c\right)^3 + 6\,a^4\cos\left(dx+c\right)^2 + 4\,a^4\cos\left(dx+c\right) + a^4\right)\sec\left(dx+c\right)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2), x)
```

maple [B] time = 1.08, size = 386, normalized size = 2.40

$$-\frac{32\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(-\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\mathrm{Elliptic}\cdots\right.}{20\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(-7/20*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+41/60*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/320*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^3-33/40*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1
/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/24*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^4,x)
[Out] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.313 $\int (a+a\cos(c+dx))^4 \sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=118

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F}{3d}$$

```
[Out] 2/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+8*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+40/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(5/2),x]
```

```
[Out] (40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \int \left(\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\sec(c+dx)}} + 6a^4\sqrt{\sec(c+dx)} + 4a^4\sec^{\frac{3}{2}}(c+dx\right.$$

$$= a^4\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a^4\int \sec^{\frac{5}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{8a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^4\sec^{\frac{3}{2}}(c+dx)\sin}{3d}$$

$$= \frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{12a^4\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

$$= \frac{40a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{8}{}$$

Mathematica [A] time = 0.34, size = 70, normalized size = 0.59

$$\frac{a^4\sec^{\frac{3}{2}}(c+dx)\left(5\sin(c+dx)+24\sin(2(c+dx))+\sin(3(c+dx))+80\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d)
```

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2), x)
```

maple [B] time = 0.82, size = 292, normalized size = 2.47

$$8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x)
```

```
[Out] 8/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^2-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+7
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^4,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.314 $\int (a+a\cos(c+dx))^4 \sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=159

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{3d}$$

```
[Out] 2/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3238, 3791, 3769, 3771, 2639, 2641, 3768}

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(3/2),x]
```

```
[Out] (56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \int \left(\frac{a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{6a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)}\right.$$
$$= a^4 \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a^4 \int \sec^{\frac{3}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}$$
$$= \frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} +$$
$$= \frac{12a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}\,F}{}$$
$$= \frac{56a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 1.50, size = 150, normalized size = 0.94

$$\frac{a^4\left(\frac{672i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 320i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 80\sin(c+dx) + 63\tan(c+\right.}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^4*(-336*I + ((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*
x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 80*S
in[c + d*x] + 3*Sec[c + d*x]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d*Sqr
t[Sec[c + d*x]])
```

fricas [F] time = 1.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4\cos(dx+c)^4 + 4\,a^4\cos(dx+c)^3 + 6\,a^4\cos(dx+c)^2 + 4\,a^4\cos(dx+c) + a^4\right)\sec(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.68, size = 194, normalized size = 1.22

$$-\frac{8a^4\left(-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+26\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{15\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x)
```

```
[Out] -8/15*a^4*(-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+26*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2} (a+a\,\cos(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^4,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.315 $\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4\sqrt{}}{}$$

```
[Out] 2/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+8/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+94/21*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/d
```

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2641, 2639}

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \int\left(\frac{a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\sec(c+dx)}} + a^4\right.$$
$$= a^4\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a^4\int\sqrt{\sec(c+dx)}\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}$$
$$= \frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^4\right)\int \frac{}{\sec}$$
$$= \frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$
$$= \frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{6a^4\sqrt{\cos(c+dx)}\,F}{}$$
$$= \frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 1.56, size = 146, normalized size = 0.91

$$\frac{a^4\left(\frac{10752i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2720i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 1910\sin(c+dx) + 336\sin\right.}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^4*(-5376*I + ((10752*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
 d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
1910*Sin[c + d*x] + 336*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqr
t[Sec[c + d*x]])
```

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4\cos(dx+c)^4 + 4\,a^4\cos(dx+c)^3 + 6\,a^4\cos(dx+c)^2 + 4\,a^4\cos(dx+c) + a^4\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.54, size = 272, normalized size = 1.69

$$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-258\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x)
```

```
[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^4,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int 6\cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx+\int 4\cos^3(c+dx)\sqrt{\sec(c+dx)}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(1/2),x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(6*cos(c + d
*x)**2*sqrt(sec(c + d*x)), x) + Integral(4*cos(c + d*x)**3*sqrt(sec(c + d*x
)), x) + Integral(cos(c + d*x)**4*sqrt(sec(c + d*x)), x) + Integral(sqrt(se
c(c + d*x)), x))
```

3.316 $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=187

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(\right.}{7d}$$

```
[Out] 2/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2)+8/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+122/45*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+32/7*a^4*sin(d*x+c)/d/sec(d*x+c)^
(1/2)+152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+32/7*a^4*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2639, 2641}

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(\right.}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x
]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \int \left(\frac{a^4}{\sec^{\frac{9}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{a^4}{\sqrt{\sec(c+dx)}}\right.$$
$$= a^4\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^4\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(4a^4\right)$$
$$= \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{12a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{9}\left(7a\right.$$
$$= \frac{2a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+d}{7d\sec^{\frac{5}{2}}(c+d}$$
$$= \frac{46a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{3d}$$
$$= \frac{152a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{7d}$$

Mathematica [C] time = 2.12, size = 156, normalized size = 0.83

$$\frac{a^4\left(\frac{51072i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 11520i\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 12240\sin(c+dx) + 355\right.}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
+ 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 70*Si
n[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\cos(dx+c)^4 + 4\,a^4\cos(dx+c)^3 + 6\,a^4\cos(dx+c)^2 + 4\,a^4\cos(dx+c) + a^4}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{4}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.58, size = 260, normalized size = 1.39

$$-\frac{8\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^{4}\left(280\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+34\left(\cos^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x)
```

```
[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos
(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(
1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{4}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\cos(c+d\,x))^4}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \frac{4\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{6\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{4\cos^3(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{\cos^4(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{1}{\sqrt{\sec(c+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(6*cos(c + d
*x)**2/sqrt(sec(c + d*x)), x) + Integral(4*cos(c + d*x)**3/sqrt(sec(c + d*x
)), x) + Integral(cos(c + d*x)**4/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(
sec(c + d*x)), x))
```

3.317 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=164

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{3ad}$$

```
[Out] 5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(
d*x+c))-3*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.17, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3818, 3787, 3768, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d
) - (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (5*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*a*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{5}{2}a\sec(c+dx)\right)dx}{a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{5\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a}$$

$$= -\frac{3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} +$$

$$= -\frac{3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} +$$

$$= \frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3ad}$$

Mathematica [C] time = 3.29, size = 285, normalized size = 1.74

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\sqrt{\sec(c+dx)}\left(18\csc(c)\cos(dx)+\sec(c+dx)\left(\tan\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+\right.\right.\right.\right.}{3ad(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5
*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((
2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec[c + d*x]*(-5*Sec[(
c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2]))))/(3*a*d*(1 + Cos[c +
 d*x]))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a},\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

maple [B] time = 0.90, size = 413, normalized size = 2.52

$$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \left(10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*x
+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d
*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.318 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3\sqrt{\cos(c+dx)}$$

```
[Out] -sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))+3*sin(d*x+c)*sec(d*x+c)^(1/
2)/a/d-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [**A**] time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3818, 3787, 3771, 2641, 3768, 2639}

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3\sqrt{\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{3}{2}a\sec(c+dx)\right)dx}{a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a}$$

$$= \frac{3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} - \frac{\left(\sqrt{\cos(c\right.}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c}{d(a}$$

$$= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{ad}$$

Mathematica [C] time = 1.96, size = 256, normalized size = 1.88

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{\sec(c+dx)}\left(6\csc(c)\cos(dx)-2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+}\right.\right.}{(-1+}\right)}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] -
E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^(
(2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*Tan[(c + d*x)/2]))/
d))/(a*(1 + Cos[c + d*x]))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.66, size = 253, normalized size = 1.86

$$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\cdots\right)\right)\right)}{a\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\cdots\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(cos(c + d*x) + 1), x)/a
```

3.319 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{ad}$$

```
[Out] -sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3818, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{\left(\sqrt{\cos\right.}{}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{ad}$$

Mathematica [C] time = 1.02, size = 180, normalized size = 1.64

$$-\frac{4i\left(-\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e\right.}{ad\left(1+e^{i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x)))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*
(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c +
 d*x]])/(a*d*(1 + E^(I*(c + d*x)))^3)
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.52, size = 200, normalized size = 1.82

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\bigl(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\bigl(\mathrm{EllipticF}\bigl(\mathrm{co}}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\;\sin\bigl(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
```

```
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x)),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x)/a
```

3.320 $\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=110

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{ad}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3820, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d))
+ (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx &= \int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx\\ &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{2a^2}\\ &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}\\ &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{}\end{aligned}$$

Mathematica [C] time = 0.96, size = 181, normalized size = 1.65

$$-\frac{4i\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+\right.}{ad\left(1+e^{i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x))
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)
*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c
+ d*x]])/(a*d*(1 + E^(I*(c + d*x)))^3)
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.60, size = 198, normalized size = 1.80

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\operatorname{EllipticF}\left(\cos\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))),x)
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}+\sqrt{\sec\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
[Out] Integral(1/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x)/a
```

3.321 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+a\right.}{ad}$$

```
[Out] -sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3819, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+a\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+d}}}{2a}$$

$$= \frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{a}$$

Mathematica [C] time = 1.69, size = 311, normalized size = 2.78

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+2\cos\left(\frac{1}{2}(c+3dx)\right)+\cos\left(\frac{1}{2}(5c+3dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{2d}+\frac{2i\sqrt{2}e^{-i(}}{}\right.}{a(\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E
^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*d*x)/2
] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d
*x]])/(2*d)))/(a*(1 + Cos[c + d*x]))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.53, size = 199, normalized size = 1.78

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\text{EllipticF}\left(\cos\left(\right.\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\,\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(3/2)), x)/a
```

3.322 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}+\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3\sqrt{\cos(c+}}{}$$

```
[Out] 5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-sin(d*x+c)/d/(a+a*sec(d*x+c))/sec(d*x+c
)^(1/2)-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3819, 3787, 3769, 3771, 2641, 2639}

$$\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}+\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
+ (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*
d) + (5*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Sec
[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} + \frac{5\int \sqrt{\sec(c+}}{6a}$$

$$= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - $$

$$= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3}$$

Mathematica [C] time = 4.12, size = 312, normalized size = 2.23

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{\sec(c+dx)}\,\left(\sin(2c)\cos(2dx) - 6\cos(c)\sin(dx) + \cos(2c)\sin(2dx) + 3(\cos(2c)+2)\csc(c)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] +
5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]]))/(E^(I*(c + d*x))*(-1 + E^(
(2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[d*x]*Csc[c] + Cos[2
*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 6*Cos[c]*Sin[d*
x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2])))/(3*a*d*(1 + Cos[c + d*x]))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 0.83, size = 215, normalized size = 1.54

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(9\,\mathrm{EllipticE}\left(\right.\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(9*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-
8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.323 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=168

$$-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3ad}$$

```
[Out] 7/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(
d*x+c))-5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+21/5*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.17, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3819, 3787, 3769, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (21*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a*d) + (7*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d
*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{7\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}}{2a}$$

$$= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+a}$$

$$= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+a}$$

$$= \frac{21\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}$$

Mathematica [C] time = 2.69, size = 341, normalized size = 2.03

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\sqrt{\sec(c+dx)}\left(18(11\cos(2c)+17)\csc(c)\cos(dx)+4\left(10\sin(2c)\cos(2dx)-3\sin(3c)\cos(3a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] +
 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E
^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])*Cos[d*x]*Csc[c] +
4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*Sec[c/2]*Sec[(c + d*
x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*Sin[2*d*x] - 3*Cos[3*
c]*Sin[3*d*x] - 30*Tan[c/2]))))/(60*a*d*(1 + Cos[c + d*x]))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

maple [A] time = 0.59, size = 229, normalized size = 1.36

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(25\,\text{EllipticF}\right.\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4
+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))),x)
[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.324 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=202

$$-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{3a^2d}$$

```
[Out] 10/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d-7/3*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/
d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-7*sin
(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.27, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) - (7*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (10*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a^2*d) - (7*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(
1 + Sec[c + d*x])) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

```
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{21a^2}{2} - 15a\right.}{3a^4}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{7\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a^2} + \frac{5\int}{}$$

$$= -\frac{7\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+}{3a^2d(1+\sec(c+dx}$$

$$= -\frac{7\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+}{3a^2d(1+\sec(c+dx}$$

$$= \frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{3a^2d}$$

Mathematica [C] time = 2.44, size = 287, normalized size = 1.42

$$-\frac{\left(-1+e^{ic}\right)\csc\left(\frac{c}{2}\right)e^{-\frac{1}{2}i(4c+3dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(7e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(1+e^{i(c+dx)}\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -1/12*((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x)) -
 65*E^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x)) -
 53*E^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c + d*
x)))^3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] + 7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)))^(3
/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*
```

```
x]])/(a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + Cos[c +
d*x])^2)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

maple [A] time = 0.99, size = 413, normalized size = 2.04

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
```

```
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+16*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.325 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx$

Optimal. Leaf size=176

$$-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4\sqrt{\cos}}{}$$

```
[Out] -5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(5/2)*
sin(d*x+c)/d/(a+a*sec(d*x+c))^2+4*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d-4*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.25, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - (5*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{5a^2}{2} - 6\right.}{3a^4}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int \sqrt{\sec(c+dx)}\,dx}{6a^2} + $$

$$= \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} - $$

$$= -\frac{4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}$$

Mathematica [C] time = 1.30, size = 252, normalized size = 1.43

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-4ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+\cdots}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -1/6*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-4*I)*(1 + E^(I*(c + d*x)))^3*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 + 50*C
os[c + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*(c +
d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^2*d*E^(I*d*x)*(1 +
Cos[c + d*x])^2)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

maple [A] time = 0.75, size = 405, normalized size = 2.30

$$-\frac{2\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(5\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**
2
```

3.326 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+a\right.}{a^2d}$$

```
[Out] -1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-sin(d*x+c)*sec(d*x+c)
^(1/2)/a^2/d/(1+sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.24, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3816, 4019, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+a\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) +
 (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2
*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{5}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2} + \frac{\int \frac{}{\sqrt{s}}}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}\right.}{3a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3a^2d}$$

Mathematica [C] time = 1.21, size = 242, normalized size = 1.62

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c + d
*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin
[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 1.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

maple [A] time = 0.65, size = 257, normalized size = 1.72

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{Ell}}{6a^2\cos\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec\left(c+dx\right)}}{\cos^2\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.327 $\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] 1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3815, 21, 3771, 2641}

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{6a^2}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{3d(a+a\sec(c+dx))^2}$$

Mathematica [A] time = 0.37, size = 98, normalized size = 1.27

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+4\sqrt{\cos(c+dx)}\cos^3\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{2}(c+dx\right.\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3
*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sqrt(sec(d*x +
c))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

maple [B] time = 0.56, size = 188, normalized size = 2.44

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\;\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\;\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2),x)
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^2\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}+2\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}+\sqrt{\sec\left(c+dx\right)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
[Out] Integral(1/(cos(c + d*x)**2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c
+ d*x)) + sqrt(sec(c + d*x))), x)/a**2
```

3.328 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+a\right.}{a^2d}$$

```
[Out] -1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+sin(d*x+c)*sec(d*x+c)
^(1/2)/a^2/d/(1+sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.24, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3817, 4019, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$$

$$=-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{5a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}dx}{3a^2}$$

$$=\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{\frac{3a^2}{2}-a^2\sec(c+}{\sqrt{\sec(c+dx)}}}{3a^4}$$

$$=\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \sqrt{\sec(c+d}}{3a^2}$$

$$=\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\left(\sqrt{\cos(c+dx}\right.}{}$$

$$=-\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}$$

Mathematica [C] time = 1.39, size = 239, normalized size = 1.60

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\left(1+e^{i(c+dx)}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(
-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c
+ 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(3
/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.69, size = 257, normalized size = 1.72

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{Ell}}{6a^{2}\sqrt{-2\left(\sin\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x+1/2
*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{2}(c+dx)\sec^{\frac{3}{2}}(c+dx)+2\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(cos(c + d*x)**2*sec(c + d*x)**(3/2) + 2*cos(c + d*x)*sec(c + d*
x)**(3/2) + sec(c + d*x)**(3/2)), x)/a**2
```

3.329 $\int \frac{1}{(a+a\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=152

$$-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c\right.}{a^2d}$$

```
[Out] -5/3*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*x+c)*sec(d*
x+c)^(1/2)/d/(a+a*sec(d*x+c))^2+4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.24, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a
^2*d) - (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{7a}{2}+\frac{3}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-6a^2+\frac{5}{2}a^2 s}{\sqrt{\sec(c}}}{3a}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int\sqrt{\sec(c}}{6a}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(5\sqrt{\cos(c+\right.}{}$$

$$= \frac{4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3a}$$

Mathematica [C] time = 1.99, size = 259, normalized size = 1.70

$$-\frac{\sin(c)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(2ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\left(1+e^{i(c}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -1/6*(Cos[(c + d*x)/2]*Csc[c/2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c]*(Cos[d*x
] + I*Sin[d*x])*((-24*I)*Cos[(c + d*x)/2] - (18*I)*Cos[(3*(c + d*x))/2] - (
6*I)*Cos[(5*(c + d*x))/2] + 20*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2] + ((2*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c
 + d*x)) + Sin[(c + d*x)/2] + 2*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/
2]))/(a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(5
/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \cos\left(dx+c\right)+a\right)^{2} \sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 0.65, size = 257, normalized size = 1.69

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{E}\cdots\right.}{6a^{2}\sqrt{-2\left(\mathrm{s}\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x
+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d*x+1
/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \cos\left(dx+c\right)+a\right)^{2} \sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.330 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}+\frac{10\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7\sqrt{\cos}}{}$$

```
[Out] 10/3*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)-7/3*sin(d*x+c)/a^2/d/(1+sec(d*x+c))/
sec(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-7*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.26, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}+\frac{10\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (-7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (7*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{9a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+$$

$$= \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{}{3d\sqrt{\sec(c}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\,F\Big(}{}$$

Mathematica [C] time = 1.81, size = 257, normalized size = 1.44

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(7ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\left(1+e^{i(c+dx)}\right)^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-84*I)*Cos[(
c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] + 8
0*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*I)*
(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*x)/2]
 + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]
))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(7
/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)
```

maple [A] time = 0.64, size = 270, normalized size = 1.52

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(16\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d*
x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x
+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*d*x+
1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.331 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=200

$$-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{a^2d}$$

```
[Out] 56/15*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)-3*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)
/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-5*sin(
d*x+c)/a^2/d/sec(d*x+c)^(1/2)+56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.28, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3238, 3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)),x]
```

```
[Out] (56*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^2
*d) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (56*Sin[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])
/(a^2*d*Sqrt[Sec[c + d*x]]) - (3*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{11a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx}$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+}$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+}$$

$$= \frac{56\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{}$$

Mathematica [C] time = 1.84, size = 271, normalized size = 1.36

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(-112ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\left(1+e^{i(c+dx)}\right)^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((1344*I)*Cos[
(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/2]
 - 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (
(112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) - 34*Sin[(c
```

```
+ d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*Sin[(7
*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + Cos[c +
d*x])^2)
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(9
/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2)), x)
```

maple [A] time = 0.67, size = 283, normalized size = 1.42

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d
*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,(a+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.332 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49\cdots}{}$$

```
[Out] -1/5*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*sec(d*x+c)^(5/2)
*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-13/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+
a^3*sec(d*x+c))+49/10*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d-49/10*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.37, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3238, 3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(6*a^3*d) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) - (Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
```

Q[n, 0]

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{11}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(12a^2-\frac{41}{2}a^2\sec(c\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+}{6d\left(a^3+a^3\sec(c+dx\right)}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+}{6d\left(a^3+a^3\sec(c+dx\right)}$$

$$= \frac{49\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+d}{15ad(a+a\sec(c+dx)}$$

$$= -\frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{49\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} -$$

$$= -\frac{49\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{6a^3d}$$

Mathematica [C] time = 2.31, size = 363, normalized size = 1.64

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}(c+3dx)\right)+3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 1243
*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2] +
 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[
(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

maple [B] time = 0.91, size = 555, normalized size = 2.51

$$-\,{-2\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(65\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\dots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*EllipticF(cos(1/2*d*x
```

```
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.333 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{10a^3d}$$

```
[Out] -1/5*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-2/5*sec(d*x+c)^(3/2)*
sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-9/10*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a
^3*sec(d*x+c))+9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/2*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.36, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3816, 4019, 3787, 3771, 2639, 2641}

$$-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3
*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*
a^3*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (9*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(3a^2-\frac{21}{2}a^2\sec(c+a\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx}{10d\left(a^3+a^3\sec(c+dx)\right)}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx}{10d\left(a^3+a^3\sec(c+dx)\right)}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx}{10d\left(a^3+a^3\sec(c+dx)\right)}$$

$$= \frac{9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{2a^3d}$$

Mathematica [C] time = 2.35, size = 274, normalized size = 1.41

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-3ie^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-3*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*(34 +
69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin[c +
d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2
] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

maple [A] time = 0.56, size = 268, normalized size = 1.37

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{El}\right.}{20a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec\left(c+dx\right)}}{\cos^3\left(c+dx\right)+3\cos^2\left(c+dx\right)+3\cos\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
+ d*x) + 1), x)/a**3
```

3.334 $\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{10a^3d}$$

```
[Out] -1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a
^3*sec(d*x+c))+1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3816, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d
) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^
3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (4*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-2a^2-}{\sqrt{\sec(c+d}}}{}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+}}{6d\left(a^3+\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+}}{6d\left(a^3+\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+}}{6d\left(a^3+\right.}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{6a}$$

Mathematica [C] time = 2.09, size = 363, normalized size = 1.86

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+7\cos\left(\frac{1}{2}(c+3dx)\right)+26\cos\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
```

```
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] -
 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^
((2*I)*c))) - ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 7*Cos[(c + 3*d
*x)/2] + 26*Cos[(5*c + 3*d*x)/2] + 10*Cos[(3*c + 5*d*x)/2] + 5*Cos[(7*c + 5
*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqr
t[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sqrt(sec(d*x + c))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^3\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.63, size = 270, normalized size = 1.38

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{Elli}}{60a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
```

```
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^3(c+dx)\sqrt{\sec(c+dx)}+3\cos^2(c+dx)\sqrt{\sec(c+dx)}+3\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(cos(c + d*x)**3*sqrt(sec(c + d*x)) + 3*cos(c + d*x)**2*sqrt(sec
(c + d*x)) + 3*cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x)/a*
*3
```

3.335 $\int \frac{1}{(a+a\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{10a^3d}$$

```
[Out] 1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)*sec(d*
x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^
3*sec(d*x+c))-1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3815, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*
d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a
^3*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{a^2}{2}+3a^2\sec}{\sqrt{\sec(c+dx)}\,(a+}}{15a}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,s}{6d\left(a^3+a^3\sec\right.}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,s}{6d\left(a^3+a^3\sec\right.}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,s}{6d\left(a^3+a^3\sec\right.}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{6a}$$

Mathematica [C] time = 2.00, size = 363, normalized size = 1.86

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+17\cos\left(\frac{1}{2}(c+3dx)\right)+16\cos\left(\frac{1}{2}(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^
```

```
((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
+ 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E
^((2*I)*c))) + ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 17*Cos[(c + 3
*d*x)/2] + 16*Cos[(5*c + 3*d*x)/2] + 20*Cos[(3*c + 5*d*x)/2] - 5*Cos[(7*c +
 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*S
qrt[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.64, size = 270, normalized size = 1.38

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{I}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.336 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+a\right.}{10a^3d}$$

```
[Out] -1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+2/5*sin(d*x+c)*sec(d*
x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^
3*sec(d*x+c))-9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/2*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.36, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3817, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+a\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (-9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2
*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$$

$$=-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{9a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$=-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}-\frac{\int \frac{3a^2-\frac{9}{2}}{\sqrt{\sec(c+dx}}}{}$$

$$=-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+}}{2d\left(a^3+\right.}$$

$$=-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+}}{2d\left(a^3+\right.}$$

$$=-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+}}{2d\left(a^3+\right.}$$

$$=-\frac{9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{}$$

Mathematica [C] time = 3.04, size = 272, normalized size = 1.39

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(3e^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+d}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*
(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3*(1
 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
```

```
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d*x] +
 (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*
Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sec(d*x + c)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 0.66, size = 270, normalized size = 1.38

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{E}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2*d*x
+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos{(dx+c)}+a)^3\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.337 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=195

$$-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{13\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\right.}{10a^3d}$$

```
[Out] -1/5*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-13/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+
a^3*sec(d*x+c))+49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13
/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.36, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{13\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(6*a^3*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3)
 - (8*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{11a}{2}+\frac{5}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{41a^2}{2}+1}{\sqrt{\sec(c+dx)}}}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+}}{6d\left(a^3+\right.}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+}}{6d\left(a^3+\right.}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+}}{6d\left(a^3+\right.}$$

$$= \frac{49\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [C] time = 2.15, size = 378, normalized size = 1.94

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1134\cos\left(\frac{1}{2}(c-dx)\right)+1071\cos\left(\frac{1}{2}(3c+dx)\right)+923\cos\left(\frac{1}{2}(c+3dx)\right)+6\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2] + 923*
Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x)/2] +
```

```
265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c + 7*d*x)/
2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32))/(15*a^3*d
*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sec(d*x + c)^(7/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)), x)
```

maple [A] time = 0.89, size = 270, normalized size = 1.38

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(348\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d
*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2
*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3),x)
[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.338 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=221

$$\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119}{}$$

[Out] 11/2*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2)-2/3*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-119/30*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/sec(d*x+c)^(1/2)-119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d

Rubi [A] time = 0.38, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119}{}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)),x]

[Out] (-119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{13a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec(c$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec(c$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec(c$$

$$= \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{}{3ad\sqrt{\sec$$

$$= -\frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx$$

$$= -\frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\,F$$

Mathematica [C] time = 2.32, size = 285, normalized size = 1.29

$$e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(119ie^{-\frac{3}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\left(1+e^{i(c+dx)}\right)^5\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Cos
[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x))/
```

```
2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x))/2]
+ 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c + d*x)
)/2]))/(120*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sec(d*x + c)^(9/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2)), x)
```

maple [A] time = 0.78, size = 283, normalized size = 1.28

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(160\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*
d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
```

```
^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(
1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3),x)
[Out] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.339 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=153

$$\frac{2a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{12a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{32a\sin(c+dx)\sqrt{\sec(c}}{35d\sqrt{a\cos(c+dx)}}$$

```
[Out] 16/35*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+12/35*a*sec(d*
x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*sec(d*x+c)^(7/2)*sin(d
*x+c)/d/(a+a*cos(d*x+c))^(1/2)+32/35*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*c
os(d*x+c))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4222, 2772, 2771}

$$\frac{2a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{12a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{32a\sin(c+dx)\sqrt{\sec(c}}{35d\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2),x]
```

```
[Out] (32*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (1
6*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (12*
a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*S
ec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}\left(6\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a}}{}$$

$$= \frac{12a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{35}\left(24\sqrt{}\right.$$

$$= \frac{16a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{12a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{7}{2}}}{7d\sqrt{}}$$

$$= \frac{32a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{16a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{12a\,\mathrm{se}}{35d}$$

Mathematica [A] time = 0.21, size = 71, normalized size = 0.46

$$\frac{2(18\cos(c+dx)+4\cos(2(c+dx))+4\cos(3(c+dx))+9)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(9 + 18*Cos[c + d*x] + 4*Cos[2*(c + d*x)] + 4
*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(35*d)
```

fricas [A] time = 0.98, size = 81, normalized size = 0.53

$$\frac{2\left(16\cos(dx+c)^3+8\cos(dx+c)^2+6\cos(dx+c)+5\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{35\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos
(d*x + c)))
```

giac [A] time = 1.16, size = 143, normalized size = 0.93

$$\frac{4\sqrt{2}\left(\left(\left(\left(7\left(5\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-10\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+267\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-3684\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+186\right.\right.\right.}{35\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 4/35*sqrt(2)*((((7*(5*(tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2
+ 267)*tan(1/4*d*x + 1/4*c)^2 - 3684)*tan(1/4*d*x + 1/4*c)^2 + 1869)*tan(1/
4*d*x + 1/4*c)^2 - 350)*tan(1/4*d*x + 1/4*c)^2 + 35)*sqrt(a)*sgn(cos(1/2*d*
x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
 1/4*c)^2 + 1)^(7/2)*d)
```

maple [A] time = 0.22, size = 82, normalized size = 0.54

$$-\frac{2\left(16\left(\cos^4(dx+c)\right)-8\left(\cos^3(dx+c)\right)-2\left(\cos^2(dx+c)\right)-\cos(dx+c)-5\right)\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{9}{2}}\sqrt{a(1+\right.}}{35d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/35/d*(16*cos(d*x+c)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)*cos(d*
x+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

maxima [B] time = 1.02, size = 283, normalized size = 1.85

$$\frac{2\left(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{58\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right.}{35d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^4/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

mupad [B] time = 5.19, size = 163, normalized size = 1.07

$$\frac{14\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)\sqrt{a+a\cos\left(c+dx\right)}\sqrt{\frac{2\,e^{c\,1i+d\,x\,1i}}{e^{c\,2i+d\,x\,2i}+1}}+4\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)\sqrt{a+a\cos\left(c+dx\right)}\sqrt{\frac{2\,e^{c\,1i+d\,x\,1i}}{e^{c\,2i+d\,x\,2i}+1}}}{\frac{105\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}+\frac{105\,d\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{8}+\frac{35\,d\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{8}+\frac{35\,d\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (14*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x*
1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) + 4*sin((7*c)/2 + (7*d*x)/2)*(a + a*co
s(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2))/
((105*d*cos(c/2 + (d*x)/2))/8 + (105*d*cos((3*c)/2 + (3*d*x)/2))/8 + (35*d*
cos((5*c)/2 + (5*d*x)/2))/8 + (35*d*cos((7*c)/2 + (7*d*x)/2))/8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.340 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{8a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 8/15*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a*sec(d*x+c
)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)*sec(d*x+c)^(
1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4222, 2772, 2771}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{8a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

```
[Out] (16*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (8
*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\ \sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a}}{}$$

$$= \frac{8a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{15}\left(8\sqrt{\text{co}}\right.$$

$$= \frac{16a\sqrt{\sec(c+dx)}\ \sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{8a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{5}{2}}}{5d\sqrt{}}$$

Mathematica [A] time = 0.13, size = 61, normalized size = 0.53

$$\frac{2(4\cos(c+dx)+4\cos(2(c+dx))+7)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7 + 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Sec
[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

fricas [A] time = 1.03, size = 71, normalized size = 0.62

$$\frac{2\sqrt{a\cos(dx+c)+a}\left(8\cos(dx+c)^2+4\cos(dx+c)+3\right)\sin(dx+c)}{15\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sin(d
*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))
```

giac [A] time = 0.59, size = 116, normalized size = 1.01

$$\frac{4\sqrt{2}\left(\left(\left(5\left(3\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-20\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+282\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}-100\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+15\right)\sqrt{a}}{15\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{4}-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^{2}+1\right)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*sqrt(2)*(((5*(3*tan(1/4*d*x + 1/4*c)^2 - 20)*tan(1/4*d*x + 1/4*c)^2 +
282)*tan(1/4*d*x + 1/4*c)^2 - 100)*tan(1/4*d*x + 1/4*c)^2 + 15)*sqrt(a)*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6*ta
n(1/4*d*x + 1/4*c)^2 + 1)^(5/2)*d)
```

maple [A] time = 0.21, size = 72, normalized size = 0.63

$$-\frac{2\left(8\left(\cos^{3}\left(dx+c\right)\right)-4\left(\cos^{2}\left(dx+c\right)\right)-\cos\left(dx+c\right)-3\right)\cos\left(dx+c\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{15d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(8*cos(d*x+c)^3-4*cos(d*x+c)^2-cos(d*x+c)-3)*cos(d*x+c)*(1/cos(d*x+
c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

maxima [B] time = 0.98, size = 237, normalized size = 2.06

$$\frac{2\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{15d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
```

```
+ c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

mupad [**B**] time = 1.84, size = 134, normalized size = 1.17

$$\frac{8\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(7\,\sin\left(c+d\,x\right)+4\,\sin\left(2\,c+2\,d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+2\,\sin\left(4\,c+4\,d\,x\right)\right.}{15\,d\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c+5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (8*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(7*sin(c + d*x) + 4*
sin(2*c + 2*d*x) + 9*sin(3*c + 3*d*x) + 2*sin(4*c + 4*d*x) + 2*sin(5*c + 5*
d*x)))/(15*d*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2
*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.341 $\int \sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=77

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/3*a*sin(d*x+c)
*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4222, 2772, 2771}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

```
[Out] (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a}}{}$$

$$= \frac{4a\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.66

$$\frac{2(2\cos(c+dx)+1)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 2*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Tan[(
c + d*x)/2])/(3*d)
```

fricas [A] time = 0.79, size = 59, normalized size = 0.77

$$\frac{2\sqrt{a\cos(dx+c)+a}\,(2\cos(dx+c)+1)\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 1)*sin(d*x + c)/((d*cos(d*x
+ c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

giac [A] time = 0.62, size = 87, normalized size = 1.13

$$\frac{4\sqrt{2}\left(\left(3\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-10\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+3\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)}{3\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*((3*tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2 + 3)*sq
rt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)
^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

maple [A] time = 0.20, size = 62, normalized size = 0.81

$$-\frac{2\left(2\left(\cos^{2}\left(dx+c\right)\right)-\cos\left(dx+c\right)-1\right)\cos\left(dx+c\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(2*cos(d*x+c)^2-cos(d*x+c)-1)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+
cos(d*x+c)))^(1/2)/sin(d*x+c)
```

maxima [B] time = 1.12, size = 190, normalized size = 2.47

$$\frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2
)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 1))
```

mupad [B] time = 0.78, size = 84, normalized size = 1.09

$$\frac{4\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (4*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(
2*c + 2*d*x) + sin(3*c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x)
 + cos(3*c + 3*d*x) + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.342 $\int \sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4222, 2771}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 39, normalized size = 1.08

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/d
```

fricas [A] time = 1.00, size = 40, normalized size = 1.11

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x
+ c)))
```

giac [A] time = 0.94, size = 58, normalized size = 1.61

$$\frac{4\sqrt{2}\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)}{\sqrt{\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{4}-6\,\tan\left(\frac{1}{4}\,dx+\frac{1}{4}\,c\right)^{2}+1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/(sqrt(tan(
1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)
```

maple [A] time = 0.20, size = 50, normalized size = 1.39

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\cos\left(dx+c\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(-1+cos(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/
2)/sin(d*x+c)
```

maxima [B] time = 1.24, size = 98, normalized size = 2.72

$$\frac{2\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

mupad [B] time = 0.29, size = 43, normalized size = 1.19

$$\frac{2\sin(c+d\,x)\,\sqrt{a\,(\cos(c+d\,x)+1)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{d\,(\cos(c+d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2))/(d*(co
s(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.343 $\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4222, 2774, 216}

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.09, size = 70, normalized size = 1.23

$$\frac{\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Co
s[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/d
```

fricas [A] time = 1.20, size = 119, normalized size = 2.09

$$\left[\frac{\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{a\cos(dx+c)+a}\sqrt{-a}\sqrt{\cos(dx+c)}\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d}, -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sqr
t(cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))/d, -
2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cos(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [B] time = 0.23, size = 100, normalized size = 1.75

$$-\frac{2\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^2\left(dx+c\right)-1\right)}{d\sin\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/s
in(d*x+c)^2*(cos(d*x+c)^2-1)
```

maxima [B] time = 1.46, size = 146, normalized size = 2.56

$$\frac{\sqrt{a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\cos \left(c + dx\right) + 1\right)} \sqrt{\sec \left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x)), x)
```

3.344 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{a}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

```
[Out] a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+arcsin(sin(d*x+c)*a^
(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4222, 2770, 2774, 216}

$$\frac{\sqrt{a}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(}}{\sqrt{\cos(c-}}$$

$$= \frac{a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\right.}{d}$$

$$= \frac{\sqrt{a}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c}}$$

Mathematica [A] time = 0.13, size = 97, normalized size = 1.05

$$\frac{\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+d\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Si
n[(c + d*x)/2]))/(2*d)
```

fricas [A] time = 0.75, size = 88, normalized size = 0.96

$$-\frac{\sqrt{a}\,(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.22, size = 132, normalized size = 1.43

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\left(-1+\cos\left(dx+c\right)\right)^{2}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin\left(dx+c\right)+\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\right)}{d\sqrt{\frac{1}{\cos(dx+c)}}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*((cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)+arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c)))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/s
in(d*x+c)^4
```

maxima [B] time = 1.63, size = 791, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
```

```
1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cos(c+dx)+1)}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sqrt(sec(c + d*x)), x)
```

3.345 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=136

$$\frac{a\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c}{4d\sqrt{\sec(c+dx)}\,\sqrt{}}$$

```
[Out] 1/2*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+3/4*a*sin(d*x+c)
/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+3/4*arcsin(sin(d*x+c)*a^(1/2)/(a
+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4222, 2770, 2774, 216}

$$\frac{a\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c}{4d\sqrt{\sec(c+dx)}\,\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (3*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos
[c + d*x]]*Sec[c + d*x]^(3/2)) + (3*a*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+$$

$$= \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{8}\left(3\sqrt{c\right.$$

$$\left(3\sqrt{\cos\right.$$

$$= \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} -$$

$$= \frac{3\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}}$$

Mathematica [A] time = 0.26, size = 111, normalized size = 0.82

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(2\sin\left(\frac{1}{2}(c\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*
(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(8*d)
```

fricas [A] time = 1.22, size = 108, normalized size = 0.79

$$-\frac{3\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}\left(2\,\cos(dx+c)^2+3\,\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*(2*cos(d*x +
c)^2 + 3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.24, size = 169, normalized size = 1.24

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\left(-1+\cos\left(dx+c\right)\right)^{3}\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos\left(dx+c\right)\sin\left(dx+c\right)+3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{4d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(2*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
```

```
d*x+c)))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^
6
```

maxima [B] time = 1.64, size = 1059, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

3.346 $\int (a + a\cos(c + dx))^{3/2} \sec^{\frac{9}{2}}(c + dx)\, dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{26a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{208a^2\sin(c+dx)\cdot}{105d\sqrt{a\cos(c+}}$$

```
[Out] 104/105*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+26/35*a^2*
sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a^2*sec(d*x+c)^(7/
2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)*sec(d*x+c)^(1
/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2762, 21, 2772, 2771}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{26a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{208a^2\sin(c+dx)\cdot}{105d\sqrt{a\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (208*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]])
+ (104*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]
) + (26*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]
) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{1}{7}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -$$

$$= \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}\left(13a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{26a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{35}(52$$

$$= \frac{104a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2}{7}$$

$$= \frac{208a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{104a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{26}{}$$

Mathematica [A] time = 0.28, size = 72, normalized size = 0.45

$$\frac{2a(117\cos(c+dx)+26\cos(2(c+dx))+26\cos(3(c+dx))+41)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(41 + 117*Cos[c + d*x] + 26*Cos[2*(c + d*x)
] + 26*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

fricas [A] time = 0.66, size = 86, normalized size = 0.53

$$\frac{2\left(104\,a\cos\left(dx+c\right)^{3}+52\,a\cos\left(dx+c\right)^{2}+39\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*
a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c
)^3)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 83, normalized size = 0.52

$$-\frac{2\left(104\left(\cos^4(dx+c)\right)-52\left(\cos^3(dx+c)\right)-13\left(\cos^2(dx+c)\right)-24\cos(dx+c)-15\right)\cos(dx+c)\sqrt{a\left(1+\cos\right.}}{105d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(104*cos(d*x+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-15
)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)*a
```

maxima [A] time = 1.25, size = 263, normalized size = 1.63

$$\frac{4\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right.}{105d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 4/105*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^
(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 1))
```

mupad [B] time = 4.14, size = 221, normalized size = 1.37

$$-\frac{-35\,a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{2\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}+91\,a\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{2\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}+\dots}{\frac{315\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{315\,d\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{105\,d\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{105\,d\,\cos\left(\frac{7\,c}{2}+\dots\right)}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] (91*a*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*
x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) - 35*a*sin(c/2 + (d*x)/2)*(a + a*cos
(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) +
26*a*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x
*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2))/((315*d*cos(c/2 + (d*x)/2))/8 + (315
*d*cos((3*c)/2 + (3*d*x)/2))/8 + (105*d*cos((5*c)/2 + (5*d*x)/2))/8 + (105*
d*cos((7*c)/2 + (7*d*x)/2))/8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.347 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=121

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{6a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{12a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 6/5*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a^2*sec(d*
x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+12/5*a^2*sin(d*x+c)*sec(d*x+
c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2762, 21, 2772, 2771}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{6a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{12a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (12*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (
6*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{1}{5}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots$$

$$= \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\left(9a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots$$

$$= \frac{6a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\left(6a\sqrt{\cdots}\right.$$

$$= \frac{12a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{6a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cdots}{5d\cdots}$$

Mathematica [A] time = 0.19, size = 62, normalized size = 0.51

$$\frac{2a(3\cos(c+dx)+3\cos(2(c+dx))+4)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*S
ec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(5*d)
```

fricas [A] time = 0.69, size = 73, normalized size = 0.60

$$\frac{2\left(6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 73, normalized size = 0.60

$$-\frac{2\left(6\left(\cos^{3}\left(dx+c\right)\right)-3\left(\cos^{2}\left(dx+c\right)\right)-2\cos\left(dx+c\right)-1\right)\cos\left(dx+c\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}a}{5d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d*(6*cos(d*x+c)^3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*cos(d*x+c)*(a*(1+cos(
d*x+c)))^(1/2)*(1/cos(d*x+c))^(7/2)/sin(d*x+c)*a
```

maxima [B] time = 0.96, size = 217, normalized size = 1.79

$$\frac{4\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{5d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 4/5*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

mupad [B] time = 1.63, size = 135, normalized size = 1.12

$$\frac{4\,a\,\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(8\,\sin\left(c+d\,x\right)+6\,\sin\left(2\,c+2\,d\,x\right)+11\,\sin\left(3\,c+3\,d\,x\right)+3\,\sin\left(4\,c+4\,d\,x\right)\dots\right.}{5\,d\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c+5\,d\dots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] (4*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(8*sin(c + d*x) +
6*sin(2*c + 2*d*x) + 11*sin(3*c + 3*d*x) + 3*sin(4*c + 4*d*x) + 3*sin(5*c +
 5*d*x)))/(5*d*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) +
 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.348 $\int (a + a\cos(c + dx))^{3/2} \sec^{\frac{5}{2}}(c + dx)\, dx$

Optimal. Leaf size=81

$$\frac{2a^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d\sqrt{a\cos(c + dx) + a}} + \frac{10a^2 \sin(c + dx)\sqrt{\sec(c + dx)}}{3d\sqrt{a\cos(c + dx) + a}}$$

```
[Out] 2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+10/3*a^2*sin(d
*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4222, 2762, 21, 2771}

$$\frac{2a^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d\sqrt{a\cos(c + dx) + a}} + \frac{10a^2 \sin(c + dx)\sqrt{\sec(c + dx)}}{3d\sqrt{a\cos(c + dx) + a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (10*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (
2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}-\frac{1}{3}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -\\
&= \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{1}{3}\left(5a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \\
&= \frac{10a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 52, normalized size = 0.64

$$\frac{2a(5\cos(c+dx)+1)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 5*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Tan
[(c + d*x)/2])/(3*d)
```

fricas [A] time = 0.90, size = 60, normalized size = 0.74

$$\frac{2\,(5\,a\cos\,(dx+c)+a)\sqrt{a\cos\,(dx+c)+a}\,\sin\,(dx+c)}{3\left(d\cos\,(dx+c)^2+d\cos\,(dx+c)\right)\sqrt{\cos\,(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*
x + c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.18, size = 63, normalized size = 0.78

$$-\frac{2\left(5\left(\cos^{2}\left(dx+c\right)\right)-4\cos\left(dx+c\right)-1\right)\cos\left(dx+c\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}a}{3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(5*cos(d*x+c)^2-4*cos(d*x+c)-1)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*
(1/cos(d*x+c))^(5/2)/sin(d*x+c)*a
```

maxima [**A**] time = 1.01, size = 125, normalized size = 1.54

$$\frac{4\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

mupad [B] time = 0.79, size = 91, normalized size = 1.12

$$\frac{2\,a\,\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(5\,\sin\left(c+d\,x\right)+2\,\sin\left(2\,c+2\,d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)\right)}{3\,d\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] (2*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(5*sin(c + d*x) +
2*sin(2*c + 2*d*x) + 5*sin(3*c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*c
+ 2*d*x) + cos(3*c + 3*d*x) + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.349 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(
1/2)
```

Rubi [A] time = 0.19, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2762, 21, 2774, 216}

$$\frac{2a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{c}}$$

$$= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a}}{\sqrt{}}$$

$$= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}}{d}$$

$$= \frac{2a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+}}{d\sqrt{a+a}}$$

Mathematica [A] time = 0.18, size = 85, normalized size = 0.89

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/
d
```

fricas [A] time = 1.17, size = 91, normalized size = 0.95

$$-\frac{2\left((a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}\,a\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*a*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.22, size = 168, normalized size = 1.75

$$\frac{2\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+\sin(dx\right.}{d(1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x)
```

```
[Out] 2/d*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+sin(d*x+c))*c
os(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))*a
```

maxima [B] time = 1.35, size = 997, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos (c+d\,x)}\right)^{3/2}\,{\left(a+a\,\cos (c+d\,x)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.350 $\int (a + a\cos(c + dx))^{3/2}\sqrt{\sec(c + dx)}\, dx$

Optimal. Leaf size=95

$$\frac{3a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

```
[Out] a^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+3*a^(3/2)*arcsin(s
in(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/d
```

Rubi [A] time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2763, 21, 2774, 216}

$$\frac{3a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
```

```
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{}{d\sqrt{a+a\,\mathrm{co}}}$$

Mathematica [A] time = 0.15, size = 99, normalized size = 1.04

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]
]*Sin[(c + d*x)/2]))/(2*d)
```

fricas [A] time = 1.07, size = 90, normalized size = 0.95

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}\,a\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - 3*(a*cos(d*x
+ c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.22, size = 130, normalized size = 1.37

$$-\frac{\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin\left(dx+c\right)+3\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\right)\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos\right.}{d\sin\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d
*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-
1)*a
```

maxima [B] time = 1.26, size = 803, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos (c+d\,x)}}\,\left(a+a\,\cos (c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.351 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{7a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{7a^2\sin(}{4d\sqrt{\sec(c+dx)}\,\sqrt{}}$$

[Out] 1/2*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+7/4*a^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+7/4*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.24, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2763, 21, 2770, 2774, 216}

$$\frac{a^2\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(}{4d\sqrt{\sec(c+dx)}\,\sqrt{}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]

[Out] (7*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (7*a^2*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c}}{}$$

$$= \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(7a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos}$$

$$= \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{8}\left(7\right.$$

$$= \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(7a\right.}{}$$

$$= \frac{7a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a^2\sin(c+dx}{2d\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}$$

Mathematica [A] time = 0.25, size = 111, normalized size = 0.79

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(-5\sin\left(\frac{1}{2}(c+dx)\right)+6\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(7*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] - 5*Sin[(c + d*x)/2]
+ 6*Sin[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(8*d)
```

fricas [A] time = 1.14, size = 112, normalized size = 0.80

$$-\frac{7\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right) - \frac{\left(2\,a\cos(dx+c)^2+7\,a\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^2 + 7*a*cos(d*x +
c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 170, normalized size = 1.21

$$\frac{(-1+\cos(dx+c))^2 \left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \cos(dx+c)\sin(dx+c)+7\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)+7\arctan\left(\frac{\sin(dx+c)\sqrt{\ }}{\cos(d}\right.\right.}{4d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{1}{\cos(dx+c)}}\ \sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin
(d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c))
)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
*a
```

maxima [B] time = 1.57, size = 1080, normalized size = 7.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
```

```
*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\cos(c + d\,x))^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.352 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{11a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{11a^2\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin}{3d\sec^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+11/12*a^2*sin(
d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+11/8*a^2*sin(d*x+c)/d/(a+a
*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+11/8*a^(3/2)*arcsin(sin(d*x+c)*a^(1/2)/
(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.31, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2763, 21, 2770, 2774, 216}

$$\frac{11a^2\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)}{3d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*
Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (11*a^2*Sin[c + d*x])/(12*d*Sqrt[a + a*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (11*a^2*Sin[c + d*x])/(8*d*Sqrt[a + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c}{}$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{6}\left(11a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{8}($$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{}{8d}$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{}{8d}$$

$$= \frac{11a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d} + \frac{a^2\sin(c+d}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.55, size = 126, normalized size = 0.70

$$\frac{a\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\sqrt{a(\cos(c+dx)+1)}\,\left(33\sqrt{2}\,\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(26\sin\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(33*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x
]]*(26*Sin[(c + d*x)/2] + 9*Sin[(3*(c + d*x))/2] + 2*Sin[(5*(c + d*x))/2]))
)/(48*d)
```

fricas [A] time = 0.80, size = 123, normalized size = 0.68

$$-\frac{33\,(a\cos(dx+c)+a)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(8\,a\cos(dx+c)^3+22\,a\cos(dx+c)^2+33\,a\cos(dx+c)\right)\sqrt{a\cos(dx+c)+}}{\sqrt{\cos(dx+c)}}}{24\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a*cos(d*x + c)^3 + 22*a*cos(d*x
 + c)^2 + 33*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.26, size = 205, normalized size = 1.14

$$-\frac{(-1+\cos(dx+c))^3\left(8\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\sin(dx+c)+22\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+33\right.}{24d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\left(\frac{1}{\cos(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2
*sin(d*x+c)+22*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+33*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+33*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(c
os(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c)^6*a
```

maxima [B] time = 1.86, size = 1942, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x
```

```
+ 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
1*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a)
 + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - a
*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
 cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arctan2
((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
```

```
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^
(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
 cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))/
d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\cos(c + d\,x))^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.353 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=201

$$\frac{38a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{146a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{584a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{1168a^3\sin(c+$$

```
[Out] 584/315*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+146/105*a^
3*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+38/63*a^3*sec(d*x+c)
^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*a^2*sec(d*x+c)^(9/2)*sin(d*x
+c)*(a+a*cos(d*x+c))^(1/2)/d+1168/315*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+
a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2762, 2980, 2772, 2771}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}+\frac{38a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{146a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (1168*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]])
 + (584*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]
]) + (146*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*
x]]) + (38*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*
d)
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} - \frac{1}{9}\left(2a\sqrt{\cos(c+dx)}\right.$$

$$= \frac{38a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)}{9d}$$

$$= \frac{146a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{38a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + $$

$$= \frac{584a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + $$

$$= \frac{1168a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{584a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + $$

Mathematica [A] time = 5.38, size = 84, normalized size = 0.42

$$\frac{a^2(698\cos(c+dx)+803\cos(2(c+dx))+146\cos(3(c+dx))+146\cos(4(c+dx))+727)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(727 + 698*Cos[c + d*x] + 803*Cos[2*(c + d*
x)] + 146*Cos[3*(c + d*x)] + 146*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)*Tan[(
c + d*x)/2])/(315*d)
```

fricas [A] time = 0.83, size = 107, normalized size = 0.53

$$\frac{2\left(584\,a^2\cos(dx+c)^4+292\,a^2\cos(dx+c)^3+219\,a^2\cos(dx+c)^2+130\,a^2\cos(dx+c)+35\,a^2\right)\sqrt{a\cos(dx+}}{315\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x + c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) /((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x + c)))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.21, size = 95, normalized size = 0.47

$$-\frac{2\left(584\left(\cos^{5}\left(dx+c\right)\right)-292\left(\cos^{4}\left(dx+c\right)\right)-73\left(\cos^{3}\left(dx+c\right)\right)-89\left(\cos^{2}\left(dx+c\right)\right)-95\cos\left(dx+c\right)-35\right)\cos}{315d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x)

[Out] -2/315/d*(584*cos(d*x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+c)^2 -95*cos(d*x+c)-35)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(11/2)/sin(d*x+c)*a^2

maxima [A] time = 1.02, size = 289, normalized size = 1.44

$$\frac{8\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{572\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{104\sqrt{2}a^{\frac{5}{2}}\sin}{(\cos(dx+c)}\right.}{315\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="maxima")

[Out] 8/315*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^ (5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*s qrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) +

```
1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 1))
```

mupad [B] time = 4.85, size = 306, normalized size = 1.52

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{192\,a^2\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a+a\,\cos(c+d\,x)}}{5\,d}-\frac{16\,a^2\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\sqrt{a+a\,\cos(c+d\,x)}}{3\,d}+\frac{1168\,a^2\,e^{\frac{c\,9i}{2}}\cdots}{\cdots}\right.}{12\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+8\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+2\,e^{\frac{c}{}\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((192*a^2*exp((c
*9i)/2 + (d*x*9i)/2)*sin(c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(5*d) -
 (16*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c +
 d*x))^(1/2))/(3*d) + (1168*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 + (5
*d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(35*d) + (2336*a^2*exp((c*9i)/2 + (d*x
*9i)/2)*sin((9*c)/2 + (9*d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(315*d)))/(12*
exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*x)/2) + 8*exp((c*9i)/2 + (d*x*9i)/2
)*cos((3*c)/2 + (3*d*x)/2) + 8*exp((c*9i)/2 + (d*x*9i)/2)*cos((5*c)/2 + (5*
d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*cos((7*c)/2 + (7*d*x)/2) + 2*exp((c*
9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (9*d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.354 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{46a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d\sqrt{a\cos(c+dx)+a}}+\frac{92a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sec$$

```
[Out] 46/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+6/7*a^3*sec(
d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a^2*sec(d*x+c)^(7/2)*s
in(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+92/21*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/
(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2762, 2980, 2772, 2771}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{46a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d\sqrt{a\cos(c+dx)+a}}+\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (92*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) +
(46*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) +
(6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (2
*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
```

```
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} - \frac{1}{7}\left(2a\sqrt{\cos(c+dx)}\right.$$

$$= \frac{6a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin}{7d}$$

$$= \frac{46a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{6a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a}}{}$$

$$= \frac{92a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{6a^3}{7}$$

Mathematica [A] time = 5.36, size = 74, normalized size = 0.46

$$\frac{a^2(93\cos(c+dx)+23\cos(2(c+dx))+23\cos(3(c+dx))+29)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(29 + 93*Cos[c + d*x] + 23*Cos[2*(c + d*x)]
 + 23*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(21*d)
```

fricas [A] time = 1.05, size = 94, normalized size = 0.58

$$\frac{2\left(46\,a^2\cos\left(dx+c\right)^3+23\,a^2\cos\left(dx+c\right)^2+12\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c) +
 3*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*
x + c)^3)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.20, size = 85, normalized size = 0.53

$$-\frac{2\left(46\left(\cos^{4}\left(dx+c\right)\right)-23\left(\cos^{3}\left(dx+c\right)\right)-11\left(\cos^{2}\left(dx+c\right)\right)-9\cos\left(dx+c\right)-3\right)\cos\left(dx+c\right)\sqrt{a\left(1+\cos\left(dx\right.\right.}}{21d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d*(46*cos(d*x+c)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3)*co
s(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)*a^2
```

maxima [A] time = 1.04, size = 243, normalized size = 1.51

$$\frac{8\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{21d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 8/21*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

mupad [B] time = 4.21, size = 227, normalized size = 1.41

$$\frac{-\frac{35a^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a+a\cos(c+dx)}\sqrt{\frac{2e^{c1i+dx1i}}{e^{c2i+dx2i}+1}}}{2}+35a^{2}\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)\sqrt{a+a\cos\left(c+dx\right)}\sqrt{\frac{2e^{c1i+dx1i}}{e^{c2i+dx2i}+1}}+\frac{23a^{2}\sin\left(\frac{7c}{2}\right.}{}}{\frac{63d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}+\frac{63d\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{8}+\frac{21d\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{8}+\frac{21d\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] (35*a^2*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i +
d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) - (35*a^2*sin(c/2 + (d*x)/2)*(a +
a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/
2))/2 + (23*a^2*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp
(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2))/2)/((63*d*cos(c/2 + (d*x)
/2))/8 + (63*d*cos((3*c)/2 + (3*d*x)/2))/8 + (21*d*cos((5*c)/2 + (5*d*x)/2)
)/8 + (21*d*cos((7*c)/2 + (7*d*x)/2))/8)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.355 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=121

$$\frac{22a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{86a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}$$

```
[Out] 22/15*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a^2*sec(
d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+86/15*a^3*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4222, 2762, 2980, 2771}

$$\frac{22a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{86a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (86*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) +
(22*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\left(2a\sqrt{\cos(c+dx)}\right.$$

$$= \frac{22a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{5d}$$

$$= \frac{86a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{22a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2}{}$$

Mathematica [A] time = 0.28, size = 64, normalized size = 0.53

$$\frac{a^2(28\cos(c+dx)+43\cos(2(c+dx))+49)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(49 + 28*Cos[c + d*x] + 43*Cos[2*(c + d*x)]
)*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

fricas [A] time = 1.03, size = 81, normalized size = 0.67

$$\frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x +
 c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.19, size = 75, normalized size = 0.62

$$-\frac{2\left(43\left(\cos^3\left(dx+c\right)\right)-29\left(\cos^2\left(dx+c\right)\right)-11\cos\left(dx+c\right)-3\right)\cos\left(dx+c\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}}{15d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(43*cos(d*x+c)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*cos(d*x+c)*(a*(1+
cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(7/2)/sin(d*x+c)*a^2
```

maxima [A] time = 0.82, size = 151, normalized size = 1.25

$$\frac{8\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2))
```

mupad [B] time = 1.62, size = 137, normalized size = 1.13

$$\frac{2\,a^2\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(98\,\sin\left(c+d\,x\right)+56\,\sin\left(2\,c+2\,d\,x\right)+141\,\sin\left(3\,c+3\,d\,x\right)+28\,\sin\left(4\,c\right.\right.}{15\,d\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(5\,c+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] (2*a^2*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(98*sin(c + d*x)
 + 56*sin(2*c + 2*d*x) + 141*sin(3*c + 3*d*x) + 28*sin(4*c + 4*d*x) + 43*si
n(5*c + 5*d*x)))/(15*d*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c +
3*d*x) + 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.356 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{2a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+\cdots}{3\cdots}$$

```
[Out] 2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2*a^(5/2)*arcs
in(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/d+14/3*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2762, 2980, 2774, 216}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^3\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (14*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$=\frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}-\frac{1}{3}\left(2a\sqrt{\cos(c+dx)}\right.$$

$$=\frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,s}{3d}$$

$$=\frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,s}{3d}$$

$$=\frac{2a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{14a^3\sqrt{\sec(c}}{3d\sqrt{a+}}$$

Mathematica [C] time = 6.32, size = 404, normalized size = 2.93

$$\frac{\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)(a(\cos(c+dx)+1))^{5/2}\left(256\sin^6\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^5*Sqr
t[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(256*
Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[c/2 + (
d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Si
n[c/2 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6*(2 - 3*Sin[c/2 + (d*x)/2]^2 + Sin[
c/2 + (d*x)/2]^4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[c/2 + (d*x)/2]^2]]*
(15 - 10*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4))/Sqrt[Sin[c/2 + (d*
x)/2]^2] - 14*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(45 + 30*Sin[c/2 + (d*x)/2]^
2 - 31*Sin[c/2 + (d*x)/2]^4 + 12*Sin[c/2 + (d*x)/2]^6)))/(672*d)
```

fricas [A] time = 1.05, size = 128, normalized size = 0.93

$$-\frac{2\left(3\left(a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8a^2\cos(dx+c)+a^2\right)\sqrt{a\cos(dx+c)+a}\sin(d}{\sqrt{\cos(dx+c)}}\right.}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a^2*cos(d*x +
c) + a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.22, size = 268, normalized size = 1.94

$$-\frac{2\left(3\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+6\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{}{1}}}{\cos(dx}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(3*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+6*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*cos(d*x+c)*sin(d*x+c)+sin(d*x+c))*cos(d*x
+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+
c))/(1+cos(d*x+c))^2*a^2
```

maxima [B] time = 1.81, size = 1395, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
```

```
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a+a\,\cos(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.357 $\int (a + a\cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx)\, dx$

Optimal. Leaf size=134

$$\frac{5a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{\sec}}{}$$

```
[Out] -a^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+5*a^(5/2)*arcsin(
sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d+2*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.28, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2762, 2981, 2774, 216}

$$\frac{5a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (5*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - \left(2a\sqrt{\cos(c+dx)}\right.$$

$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(}}{d}$$

$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(}}{d}$$

$$= \frac{5a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d} - \frac{a}{d\sqrt{a+a\,\mathrm{co}}}$$

Mathematica [C] time = 3.00, size = 202, normalized size = 1.51

$$\frac{\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(a(\cos(c+dx)+1))^{5/2}\left(6\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Se
c[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F
1[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 24*(3 + Cos[c + d*x])*Hypergeometr
ic2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 6*Csc[(c + d*x)
/2]^2*HypergeometricPFQ[{3/2, 2, 5/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[
c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

fricas [A] time = 0.79, size = 111, normalized size = 0.83

$$-\frac{5\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(a^2\cos(dx+c)+2\,a^2\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.22, size = 186, normalized size = 1.39

$$\frac{\left(5\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+\cos(dx+c)\sin(dx+c)+5\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin}{\ }\right.\right.}{d\left(1+\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(5*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+cos(d*x+c)*sin(d*x+c)+5*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c))+2*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)
))^(1/2)/(1+cos(d*x+c))*a^2
```

maxima [B] time = 1.57, size = 973, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)
 + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2} (a+a\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.358 $\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{19a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a}}{2d\sqrt{\sec}}$$

```
[Out] 9/4*a^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/2*a^2*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+19/4*a^(5/2)*arcsin(sin(d*x+
c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2763, 2981, 2774, 216}

$$\frac{19a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a}}{2d\sqrt{\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (19*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (9*a^3*Sin[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+a}{2d\sqrt{\sec(c+dx)}}$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+a}{2d\sqrt{\sec(c+dx)}}$$

$$= \frac{19a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{9}{4d\sqrt{a+a\,c}}$$

Mathematica [C] time = 3.01, size = 202, normalized size = 1.44

$$\frac{\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(a(\cos(c+dx)+1))^{5/2}\left(2\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+d\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Se
c[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F
1[1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 8*(3 + Cos[c + d*x])*Hypergeometri
c2F1[3/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 2*Csc[(c + d*x)/
2]^2*HypergeometricPFQ[{3/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c
 + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

fricas [A] time = 1.00, size = 120, normalized size = 0.86

$$-\frac{19\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2a^2\cos(dx+c)^2+11a^2\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*a^2*cos(d*x + c)^2 + 11*a^2*
cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*
cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 166, normalized size = 1.19

$$-\frac{\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+11\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+19\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\right)\sqrt{a\,(}}{4d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+11*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+19*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a^2
```

maxima [B] time = 1.74, size = 1106, normalized size = 7.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
+ 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*x
 + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
 + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
```

```
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos(c + d\,x)}}\,\left(a + a\,\cos(c + d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(1/2),x)
[Out] Timed out
```

3.359 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=180

$$\frac{25a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{13a^3\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{25a^3\sin}{8d\sqrt{\sec(c+dx)}\sqrt{}}$$

```
[Out] 13/12*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/3*a^2*sin(
d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+25/8*a^3*sin(d*x+c)/d/(a+a
*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+25/8*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/
(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.36, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2763, 2981, 2770, 2774, 216}

$$\frac{13a^3\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\sec^{\frac{3}{2}}(c+dx)}+\frac{25a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (25*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (13*a^3*Sin[c + d*x])/(12*d*Sqrt[a
+ a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c
 + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (25*a^3*Sin[c + d*x])/(8*d*Sqrt[a + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
```

```
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+d}$$

$$= \frac{13a^3\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{8}\left(25a\right.$$

$$= \frac{13a^3\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{}{8d\sqrt{a}}$$

$$= \frac{13a^3\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{}{8d\sqrt{a}}$$

$$= \frac{25a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{13a^3\sin(c+d}{12d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 3.12, size = 202, normalized size = 1.12

$$\frac{\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(a(\cos(c+dx)+1))^{5/2}\left(-2\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+d\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Se
c[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F
1[-1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 8*(3 + Cos[c + d*x])*Hypergeometr
ic2F1[1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 2*Csc[(c + d*x)
/2]^2*HypergeometricPFQ[{1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[
c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

fricas [A] time = 0.82, size = 133, normalized size = 0.74

$$-\frac{75\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right) - \frac{\left(8a^2\cos(dx+c)^3+34a^2\cos(dx+c)^2+75a^2\cos(dx+c)\right)\sqrt{a\cos(dx+}}{\sqrt{\cos(dx+c)}}}{24\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(75*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*
sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a^2
*cos(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.25, size = 207, normalized size = 1.15

$$\frac{(-1+\cos(dx+c))^2\left(8\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\sin(dx+c)+34\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+75\right.}{24d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{1}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*
sin(d*x+c)+34*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+75*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+75*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4*a^2
```

maxima [B] time = 1.71, size = 1964, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*
```

```
x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))
- 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
```

```
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) - 1))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.360 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{163a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{163a^3\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{17a^3\,\mathrm{s}}{24d\sec^{\frac{5}{2}}(c+dx}$$

```
[Out] 17/24*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+163/96*a^3*s
in(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*sin(d*x+c)*(a+a
*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+163/64*a^3*sin(d*x+c)/d/(a+a*cos(d*x+
c))^(1/2)/sec(d*x+c)^(1/2)+163/64*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*co
s(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.42, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2763, 2981, 2770, 2774, 216}

$$\frac{163a^3\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{17a^3\sin(c+dx)}{24d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{4d\sec^{\frac{5}{2}}(c+dx)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (163*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (17*a^3*Sin[c + d*x])/(24*d*Sqrt[
a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (163*a^3*Sin[c + d*x])/(96*d*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (163*a^3*Sin[c + d*x])/(64*d*Sqrt[a
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
```

```
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+d$$

$$= \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{48}\left(16\right.$$

$$= \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{}{96d\sqrt{a}}$$

$$= \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{}{96d\sqrt{a}}$$

$$= \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{}{96d\sqrt{a}}$$

$$= \frac{163a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{17a^3\sin(c+}{24d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 3.10, size = 202, normalized size = 0.92

$$\frac{\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(a(\cos(c+dx)+1))^{5/2}\left(-6\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+d\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Se
c[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F
1[-3/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 24*(3 + Cos[c + d*x])*Hypergeomet
ric2F1[-1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 6*Csc[(c + d*
x)/2]^2*HypergeometricPFQ[{-1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*S
in[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

fricas [A] time = 0.78, size = 146, normalized size = 0.66

$$-\frac{489\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(48\,a^2\cos(dx+c)^4+184\,a^2\cos(dx+c)^3+326\,a^2\cos(dx+c)^2+48\right.}{\sqrt{\cos(dx+c)}}}{192\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/192*(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c)^4 + 184
*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.27, size = 242, normalized size = 1.10

$$-\frac{(-1+\cos(dx+c))^3\left(48\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+184\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\sin(dx+\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))^3*(48*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+184*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)
+326*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+489*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+489*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c)^6*a^2
```

maxima [B] time = 2.14, size = 7450, normalized size = 33.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/768*(10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d
*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*co
s(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x + 4*c)^3
 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(2*a^2*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*
d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4
*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x +
 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d
*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3
2*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c)
 + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(16*a^2*
cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*cos(4*d*x + 4*c) +
3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a^2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 19*a^2
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*si
n(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 12*(4*a^2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a^2*sin(4*d*x
 + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 - 8
*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 14*a^2*cos(4*d*x +
4*c)^2 + 19*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d
*x + 4*c)^2 - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x
 + 4*c)^3 - 2*a^2*cos(4*d*x + 4*c)^2 - 13*a^2*cos(4*d*x + 4*c) + (3*a^2*cos
(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*
x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(
4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^
2 - 19*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
```

```
c)))*sin(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 11*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*cos
(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*(2*a^2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)
 - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(3
*a^2*cos(4*d*x + 4*c) - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^
(1/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^
3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x +
4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d
*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4
*(3*a^2*sin(4*d*x + 4*c)^3 + 3*(a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x +
4*c) + a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
+ 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(3*a^2*sin(4*d*x + 4*c)^3 + 160*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c)^2 + 6*a^2*cos(
4*d*x + 4*c) + 43*a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 2*(6*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 6*(a^2*cos(4*d*x + 4*c)^2 - a
^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (320*a^2*cos(4*d*x + 4*c)^2 + 320*a
^2*sin(4*d*x + 4*c)^2 - 317*a^2*cos(4*d*x + 4*c) - 3*a^2)*sin(1/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) - 2*(20*a^2*cos(4*d*x + 4*c)^2 + 26*a^2*sin(4*d*x + 4*c)^2 -
 317*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x
 + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*(
10*a^2*cos(4*d*x + 4*c)^2 + 13*a^2*sin(4*d*x + 4*c)^2 - 160*a^2*sin(4*d*x +
 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 10*a^2*cos(4*d
*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 3*(a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (160*a^2*cos(4*d*x
+ 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*sin(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
```

```
(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 + 120*a^2*cos(4*d*x + 4*c
)^2 - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*
x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 - 3*
a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 4*(3*a^2*cos(4*d*x + 4*c)^3 + 74*a^2*cos(4*d*x + 4*c)^2 - 197*a^2*cos(4*d*
x + 4*c) + (3*a^2*cos(4*d*x + 4*c) + 80*a^2)*sin(4*d*x + 4*c)^2 + 120*a^2 -
 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*
c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(a^2*cos(4*d*x + 4*c) + 40*
a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 + 126*a^2*cos(4*d*x +
 4*c)^2 + 243*a^2*cos(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(
4*d*x + 4*c)^2 + 120*a^2 - 40*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c
)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2
*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(6*a^2*cos(4
*d*x + 4*c)^3 + 214*a^2*cos(4*d*x + 4*c)^2 - 3*a^2*sin(4*d*x + 4*c)*sin(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 240*a^2*cos(4*d*x + 4*c) +
2*(3*a^2*cos(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c)^2 - (160*a^2*cos(4*d*
x + 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 - 157*a^2*cos(4*d*x + 4*c) - 3*a^2)
*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) - (80*a^2*cos(4*d*x + 4*c)^2 + 80*a^2*sin(
4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 2*(320*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2*sin(4*d*x + 4*c) + 157*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(80*a^2*cos(1/4*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (3*a^2*cos(4*d*x + 4*c) + 110*a^
2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 6*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*
c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d
*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x +
4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2
*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))
```

```
*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 -
2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(
4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c)
)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*
x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*cos(4*d*x + 4*c)
^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*
a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*
x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*
sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arc
tan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
```

```
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d
*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4
*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c)
+ a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(
4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)) - 1))*sqrt(a))/((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*
d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\cos(c + dx))^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.361 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=154

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{\cos(c+dx)+1}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26\sin}{15}$$

```
[Out] -2/15*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)+2/5*sec(d*x+c)^(5/
2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)-arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1
/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+26/15*sin(d*x+c)*sec(d*x+c)^(1/2)/d
/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4222, 2779, 2984, 12, 2781, 216}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{\cos(c+dx)+1}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26\sin}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[1 + Cos
[c + d*x]]) - (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[1 + Cos[c + d*
x]]) + (2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[1 + Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}} - \frac{1}{5}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1-4\cos(c+d}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}} - \frac{1}{15}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx}{5d\sqrt{1+\cos(c+dx)}}$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx}{5d\sqrt{1+\cos(c+dx)}}$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx}{5d\sqrt{1+\cos(c+dx)}}$$

$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} -$$

<u>Mathematica</u> [C] time = 7.81, size = 1540, normalized size = 10.00

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(
-1))^(7/2)*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105
*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x
)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hyperg
eometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1
770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hy
pergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeome
tric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*S
```

```
in[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
 - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*
Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2,
 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Si
n[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*d*Sqrt[1 + Cos[c +
 d*x]]*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

fricas [A] time = 1.25, size = 130, normalized size = 0.84

$$\frac{15\left(\sqrt{2}\cos\left(dx+c\right)^{3}+\sqrt{2}\cos\left(dx+c\right)^{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\left(13\cos(dx+c)^{2}-\cos(dx+c)+3\right)\sqrt{\cos(dx+c)+1}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*(sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2)*arctan(sqrt(2)*s
qrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(13*cos(d*x + c)
^2 - cos(d*x + c) + 3)*sqrt(cos(d*x + c) + 1)*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(cos(d*x + c) + 1), x)
```

maple [B] time = 0.22, size = 294, normalized size = 1.91

$$\frac{\left(15\sqrt{2}\ \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^3(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+45\sqrt{2}\ \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx+c}{1+\cos(dx+}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/30/d*(15*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)+45*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+45*2^(1/2)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15*2^(1/2)*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+26*cos(d*x
+c)^2*sin(d*x+c)-2*cos(d*x+c)*sin(d*x+c)+6*sin(d*x+c))*cos(d*x+c)*(2+2*cos(
d*x+c))^(1/2)*sin(d*x+c)^4*(1/cos(d*x+c))^(7/2)/(-1+cos(d*x+c))^2/(1+cos(d*
x+c))^3*2^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(cos(c + d*x) + 1)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.362 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{\cos(c+dx)+1}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\cos(c+dx)+1}}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)+arcsin(sin(d*x+c)/(1
+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d-2/3*sin(d*x+c)*se
c(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4222, 2779, 2984, 12, 2781, 216}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{\cos(c+dx)+1}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/d - (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[1 + Cos[c +
d*x]]) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[1 + Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
```

```
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} - \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1-2\cos(c+d}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos}}$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} - \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} - \frac{\left(\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\,\mathrm{s}}{}$$

Mathematica [C] time = 6.61, size = 473, normalized size = 4.01

$$-\frac{2\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(
-1))^(7/2)*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 +
12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (
d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*
x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]
*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[1 + Cos[c + d*x]]
)
```

fricas [A] time = 1.10, size = 114, normalized size = 0.97

$$-\frac{3\left(\sqrt{2}\cos\left(dx+c\right)^{2}+\sqrt{2}\cos\left(dx+c\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\sqrt{\cos(dx+c)+1}\,(\cos(dx+c)-1)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*arctan(sqrt(2)*sqrt
(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*sqrt(cos(d*x + c) +
 1)*(cos(d*x + c) - 1)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)
```

maple [B] time = 0.22, size = 228, normalized size = 1.93

$$\frac{\left(3\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+6\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*(3*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)+6*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*2^(1/2)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*cos(d*x+c)*sin(d*x+c)-2*sin(d*
x+c))*cos(d*x+c)*(2+2*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*sin(d*x+c)^2/(
-1+cos(d*x+c))/(1+cos(d*x+c))^2*2^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(cos(c + d*x) + 1)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.363 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] -arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d+2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4222, 2779, 2781, 216}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Cos[c +
 d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} - \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{1+\cos}}$$
$$= \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} + \frac{\left(\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \right.}{d}$$
$$= -\frac{\sqrt{2}\,\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}$$

Mathematica [C] time = 1.95, size = 178, normalized size = 2.17

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}\cos(c+dx)(\cos(c+dx)+2)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-\cos(c+dx)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 +
 Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c
+ d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (H
ypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d
*x]*Tan[c + d*x])/10))/(d*Sqrt[1 + Cos[c + d*x]])
```

fricas [A] time = 1.69, size = 86, normalized size = 1.05

$$\frac{\left(\sqrt{2}\,\cos\left(dx+c\right)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\,\sqrt{\cos(dx+c)+1}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] ((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqr
t(cos(d*x + c))/sin(d*x + c)) + 2*sqrt(cos(d*x + c) + 1)*sin(d*x + c)/sqrt(
cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```

maple [A] time = 0.18, size = 144, normalized size = 1.76

$$\frac{\left(\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\,\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+2\sin\left(dx+c\right)\right)c}{2d\left(1+\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+2*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(2+2*co
s(d*x+c))^(1/2)/(1+cos(d*x+c))*2^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(cos(c + d*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(cos(c + d*x) + 1), x)
```

3.364 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/d
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4222, 2781, 216}

$$\frac{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx$$
$$= -\frac{\left(\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}$$
$$= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.11, size = 68, normalized size = 1.45

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])/d
```

fricas [A] time = 1.04, size = 39, normalized size = 0.83

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x +
 c))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] integrate(sqrt(sec(d*x + c))/sqrt(cos(d*x + c) + 1), x)

maple [A] time = 0.19, size = 82, normalized size = 1.74

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{2+2\cos\left(dx+c\right)}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{2}\left(dx+c\right)-1\right)}{d\sin\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*(2+2*cos(d*x+c))^(1/2)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\cos\left(c+d\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{\cos\left(c+dx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(cos(c + d*x) + 1), x)
```

3.365 $\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] 2*arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/d-arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4222, 2777, 2774, 216, 2781}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (2*ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$$
$$= -\left(\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx\right.$$
$$= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,-\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2\sin^{-1}\left(\frac{\sin(c+\ldots}{\sqrt{1+\cos(\ldots}}\right.}{}$$

Mathematica [C] time = 0.60, size = 171, normalized size = 1.82

$$\frac{i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)\left(-\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh\ldots\right.}{d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c +
d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)
```

```
*(c + d*x))]])*Cos[(c + d*x)/2])/(d*E^((I/2)*(c + d*x))*Sqrt[1 + Cos[c + d*
x]])
```

fricas [A] time = 1.09, size = 70, normalized size = 0.74

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x +
 c)) - 2*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.21, size = 134, normalized size = 1.43

$$\frac{\sqrt{2+2\cos(dx+c)}\,\cos(dx+c)\left(-1+\cos(dx+c)\right)^{2}\left(\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+2\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\right)}{2d\sqrt{\frac{1}{\cos(dx+c)}}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2+2*cos(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(2^(1/2)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))+2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c)))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/s
in(d*x+c)^4*2^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)+1}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos\left(c+dx\right)+1}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*sqrt(sec(c + d*x))), x)
```

3.366 $\int \frac{1}{\sqrt{1+\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}+\frac{\text{s}}{d\sqrt{\cos(c+}}$$

[Out] sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] time = 0.23, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4222, 2778, 2982, 2781, 216, 2774}

$$\frac{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}+\frac{\text{s}}{d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]

[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + Sin[c + d*x]/(d*Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
```

```
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$$
$$= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$
$$= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$
$$= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}}{d}$$
$$= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{}$$

Mathematica [C] time = 0.88, size = 257, normalized size = 2.06

$$\frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(-e^{i(c+dx)}+e^{2i(c+dx)}-e^{3i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+2\sqrt{2}\right.}{4d\sqrt{\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(1 - E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) - E
^((3*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[
E^(I*(c + d*x))] + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - E^
(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c +
d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[1 + Cos[c + d*x]])
```

fricas [A] time = 0.93, size = 125, normalized size = 1.00

$$-\frac{\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sq
rt(cos(d*x + c))/sin(d*x + c)) - (cos(d*x + c) + 1)*arctan(sqrt(cos(d*x + c
```

```
) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - sqrt(cos(d*x + c) + 1)*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.22, size = 159, normalized size = 1.27

$$\frac{\sqrt{2+2\cos(dx+c)}\,\cos(dx+c)\,(-1+\cos(dx+c))^{3}\left(\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+a\right.}{2d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin(dx+c)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2+2*cos(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(2^(1/2)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/(1/cos(d*x+c)
)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c+d\,x)+1}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos(c+dx)+1}\,\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*sec(c + d*x)**(3/2)), x)
```

3.367 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=189

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c}{}$$

```
[Out] -2/15*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*sec(d*x+c)^(
5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1
/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d/a^(1/2)+26/15*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^
(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2779, 2984, 12, 2782, 205}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
(26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (2*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}}{5a}$$

$$= -\frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{26\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{26\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d\sqrt{a+a\cos(c+dx}}$$

$$= \frac{26\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5d\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} + \frac{26\sqrt{\sec(c+}{15d\sqrt{a+a}}$$

Mathematica [C] time = 7.76, size = 1542, normalized size = 8.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(
-1))^(7/2)*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105
*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x
)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hyperg
eometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1
770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hy
pergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
```

```
2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeome
tric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*S
in[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
 - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*
Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2,
 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Si
n[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*d*Sqrt[a*(1 + Cos[
c + d*x])]*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

fricas [A] time = 1.23, size = 141, normalized size = 0.75

$$\frac{\frac{15\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}\left(13\cos(dx+c)^2-\cos(dx+c)+3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*arctan(sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*
sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.25, size = 294, normalized size = 1.56

$$\frac{\left(15 \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{3}(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+45 \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{2}(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*(15*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(
cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+13*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-2^(1/2)
*cos(d*x+c)*sin(d*x+c)+3*sin(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)^4*(1/cos
(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3*
2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.368 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,c}}\right.}{\sqrt{a}\,d}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+arctan(1/2*sin(d*x
+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+
a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2779, 2984, 12, 2782, 205}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\,c}}\right.}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}}{3a}$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} - \frac{2\sqrt{\sec(c+dx)}}{3d\sqrt{a+a\,\mathrm{co}}}$$

Mathematica [C] time = 6.63, size = 475, normalized size = 3.15

$$-\frac{2\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(
-1))^(7/2)*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 +
12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (
d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*
x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]
*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[a*(1 + Cos[c + d*
x])])
```

fricas [A] time = 0.72, size = 125, normalized size = 0.83

$$-\frac{\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}\,(\cos(dx+c)-1)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqr
t(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sin(d*x + c)/sqrt(cos(d*x + c)))/(
a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.23, size = 227, normalized size = 1.50

$$\frac{\left(3\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+6\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3\,\mathrm{arcs}}{3d\,(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*(3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)+6*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)+3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)+2^(1/2)*cos(d*x+c)*sin(d*x+c)-sin(d*x+c)*2^(1/2))*cos(d*
x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x
+c))/(1+cos(d*x+c))^2*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.369 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2779, 12, 2782, 205}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
```

```
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+d}}}{a}$$

$$= \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

$$= \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx,\right.}{d}$$

$$= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} + \frac{2\sqrt{\sec(c+d}}{d\sqrt{a+a\,c}}$$

Mathematica [C] time = 1.87, size = 180, normalized size = 1.59

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}\cos(c+dx)(\cos(c+dx)+2)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-\cos(c+dx)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 +
 Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c
+ d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (H
ypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d
*x]*Tan[c + d*x])/10))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.88, size = 98, normalized size = 0.87

$$\frac{\frac{\sqrt{2}\,(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\,\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.21, size = 142, normalized size = 1.26

$$\frac{\left(\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+\sin\left(dx+c\right)\sqrt{2}+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right)\cos\left(dx\right.}{d\left(1+\cos\left(dx+c\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))+sin(d*x+c)*2^(1/2)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+c
```

```
os(d*x+c))/sin(d*x+c)))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^
(1/2)/(1+cos(d*x+c))*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.370 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctan(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/
2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4222, 2782, 205}

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= -\frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.09, size = 71, normalized size = 1.27

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.18, size = 144, normalized size = 2.57

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\cdots}{\sqrt{a}\sin(dx+\cdots}\right.}{\sqrt{a}\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(-1/a)
*sqrt(cos(d*x + c))*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, -sqrt(2)*arctan(sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/(sqrt(a)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.21, size = 88, normalized size = 1.57

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{2}\left(dx+c\right)-1\right)\sqrt{2}}{d\sin\left(dx+c\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2
^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.371 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=105

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctan(sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1
/2)-arctan(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*cos(d*x+c))
^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2777, 2774, 216, 2782, 205}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*
Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= -\left(\left(\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right.\right.$$

$$= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx}}\right.}{ad}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{}$$

Mathematica [C] time = 0.25, size = 173, normalized size = 1.65

$$\frac{i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)\left(-\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c +
d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)
*(c + d*x))]])*Cos[(c + d*x)/2])/(d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c +
 d*x])])
```

fricas [A] time = 1.16, size = 89, normalized size = 0.85

$$\frac{\sqrt{2}\,\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}\,\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.22, size = 134, normalized size = 1.28

$$\frac{\sqrt{a\,(1+\cos(dx+c))}\,\cos(dx+c)\,(-1+\cos(dx+c))^2\left(\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right)}{d\sqrt{\frac{1}{\cos(dx+c)}}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)^4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(2^(1/2)*arctan(s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+arcsin((-1+cos(d*x+
c))/sin(d*x+c)))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin
(d*x+c)^4*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x))), x)
```

3.372 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=168

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(}}{}$$

```
[Out] sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-arcsin(sin(d*x+c)*a^(1
/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2778, 2982, 2782, 205, 2774, 216}

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x]
)/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(d*Sqrt[a + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int -}{2a}$$

$$= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\text{Sub}}{}$$

$$= -\frac{\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} + \frac{\sqrt{2}\,\tan^{-1}\left(\frac{}{\sqrt{2}\,\sqrt{\text{co}}}\right.}{}$$

Mathematica [C] time = 0.45, size = 259, normalized size = 1.54

$$\frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(-e^{i(c+dx)}+e^{2i(c+dx)}-e^{3i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\sinh^{-1}\left(e^{i(c+dx)}\right)+2\sqrt{2}\right.}{4d\sqrt{a(\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(1 - E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) - E
^((3*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[
E^(I*(c + d*x))] + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - E^
(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c +
d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x
])])
```

fricas [A] time = 0.94, size = 143, normalized size = 0.85

$$\frac{\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\sqrt{2}\,(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{\sqrt{a}}+\sqrt{a}\cos\left(dx\right.}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a)
+ sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.22, size = 167, normalized size = 0.99

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\left(-1+\cos\left(dx+c\right)\right)^{3}\left(-\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin\left(dx+c\right)+\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{}}{\cos(}\right.\right.}{2d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin\left(dx+c\right)^{6}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*arcsin((-1+cos(d*x+c))/sin(d*x+c)))/
(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6*2^(1/2)
/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**(3/2)), x)
```

3.373 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{11\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(\ldots}{2d(a\cos(c+dx)\ldots}$$

```
[Out] -1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+7/6*sec(d*x+c)^(3
/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+11/4*arctan(1/2*sin(d*x+c)*a^(1/2
)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^(3/2)/d*2^(1/2)-19/6*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d
*x+c))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2766, 2984, 12, 2782, 205}

$$\frac{11\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(\ldots}{2d(a\cos(c+dx)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (11*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]]) - (
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (7*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{7a}{2}-2a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+}}}{2a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.}{}$$

$$= -\frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c-}{6ad\sqrt{a+a\cos(c+}}$$

$$= -\frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c-}{6ad\sqrt{a+a\cos(c+}}$$

$$= -\frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c-}{6ad\sqrt{a+a\cos(c+}}$$

$$= \frac{11\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d} - \frac{19\sqrt{\sec(c+d}}{6ad\sqrt{a+a}}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 1.07, size = 161, normalized size = 0.82

$$-\frac{33\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{a\cos(dx+c)+a}\,(}{12\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(19*cos(d*x + c)^2 + 12*cos(d*x + c)
 - 4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(
d*x + c)^2 + a^2*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{(a\cos (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.22, size = 258, normalized size = 1.31

$$\frac{\left(33\left(\cos^2(dx+c)\right)\sin(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+66\cos(dx+c)\sin(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d*(33*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)+66*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+33*sin(d*x+c)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-19*cos(d*x+c)^3*2^(
1/2)+7*cos(d*x+c)^2*2^(1/2)+16*cos(d*x+c)*2^(1/2)-4*2^(1/2))*cos(d*x+c)*sin
(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+co
s(d*x+c))^2*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{(a\cos (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.374 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(3/2)-7/4*arctan(1/2*si
n(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+5/2*sin(d*x+c)*sec(d*x+c)^(1/2)/
a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2766, 2984, 12, 2782, 205}

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (5*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{5a}{2}-a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+d}}}{2a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\sec(c+dx)}\ \mathrm{s}}{2d(a+a\cos(}$$

Mathematica [C] time = 6.51, size = 458, normalized size = 2.92

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{5}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{70\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-35} - \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c
/2 + (d*x)/2]^2)^(-1))^(3/2)*((4*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2
, 5/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c
/2 + (d*x)/2]^2)/(-35 + 70*Sin[c/2 + (d*x)/2]^2) - (Csc[c/2 + (d*x)/2]^6*(1
 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*(-3*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*(-25 + 91*Sin[c/2 + (d*x)/2]^2 - 100*Sin[c/2 + (d*x)/2]^4 + 34*Sin[c/
2 + (d*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*
(-75 + 298*Sin[c/2 + (d*x)/2]^2 - 350*Sin[c/2 + (d*x)/2]^4 + 124*Sin[c/2 +
(d*x)/2]^6)))/6))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 1.12, size = 136, normalized size = 0.87

$$\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)+\frac{2\,\sqrt{a\cos(dx+c)+a}\,(5\,\cos(dx+c)+4)\sin}{\sqrt{\cos(dx+c)}}}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*sq
rt(a*cos(d*x + c) + a)*(5*cos(d*x + c) + 4)*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.20, size = 184, normalized size = 1.17

$$-\frac{\left(-7\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin\left(dx+c\right)\cos\left(dx+c\right)+5\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\,-7\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin}{4d\sin\left(dx+c\right)(1+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*sin(d*x+c)*cos(d*x+c)+5*cos(d*x+c)^2*2^(1/2)-7*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-cos(d*x+c)*2^(1/2)
-4*2^(1/2))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*
x+c)/(1+cos(d*x+c))*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.375 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+3/4*arctan(1/2*si
n(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2766, 12, 2782, 205}

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) -
 Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
```

```
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{2\sqrt{\cos(c+}}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c+}}}{4a}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int\right.}{2a}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin}{2d(a+a\cos(c+}$$

Mathematica [A] time = 0.52, size = 99, normalized size = 0.85

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(3\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx))}\right)+2\right)}{4ad\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -1/4*((2 + 3*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x
)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(a*d*Sqrt[a*(1 + Cos[c +
 d*x])]*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 1.00, size = 126, normalized size = 1.08

$$-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos}}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.20, size = 151, normalized size = 1.29

$$-\frac{\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\cos\left(dx+c\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-3\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin\left(dx+c\right)-\sqrt{2}\sqrt{}\right.}{4d\sin\left(dx+c\right)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*
```

```
x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.376 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/4*arctan(1/2*sin
(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2764, 12, 2782, 205}

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + S
in[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)\right)}}{2}$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{4}$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d} + \frac{}{2d(a}$$

Mathematica [A] time = 0.46, size = 140, normalized size = 1.20

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\left(\sqrt{\cos(c+dx)+1}\,\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+}{\cos(c+d}}\right.}{2ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]*
(ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[1 + Cos[c + d*x]] +
 2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]))/(2*a*d*Sqrt[a*(
1 + Cos[c + d*x])])
```

fricas [A] time = 1.34, size = 125, normalized size = 1.07

$$-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-2\,\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(d\right.}}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.21, size = 156, normalized size = 1.33

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\left(-1+\cos\left(dx+c\right)\right)^2\left(\cos\left(dx+c\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\mathrm{ar}\right.}{4d\sqrt{\frac{1}{\cos(dx+c)}}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin\left(dx+c\right)^5 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(cos(d*x+c)*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(1/2)/
(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\cos(c+dx)+1))^{\frac{3}{2}}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)
```

3.377 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}$$

```
[Out] -1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*arcsin(sin(d*x+
c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2
)/d-5/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x
+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2765, 2982, 2782, 205, 2774, 216}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x]
)^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}}{}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}}{a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}}{}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{5\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{c}}\right.}{}$$

Mathematica [C] time = 6.54, size = 316, normalized size = 1.82

$$\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{2\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d} - \frac{2\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} + \frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} + \frac{\tan\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{(a(\cos(c+dx)+1))^{3/2}} - \frac{i\sqrt{2}e^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*(2*ArcSinh[E^(I*(c + d*x))] + (5*ArcTanh[(1 - E^(I*(c + d*
x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 2*ArcTanh[Sqrt[1 +
 E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1
+ Cos[c + d*x]))^(3/2)) + (Cos[c/2 + (d*x)/2]^3*Sqrt[Sec[c + d*x]]*((-2*Cos
[(d*x)/2]*Sin[c/2])/d - (2*Cos[c/2]*Sin[(d*x)/2])/d + (Sec[c/2]*Sec[c/2 + (
d*x)/2]^2*Sin[(d*x)/2])/d + (Sec[c/2 + (d*x)/2]*Tan[c/2])/d))/(a*(1 + Cos[c
 + d*x]))^(3/2)
```

fricas [A] time = 1.42, size = 182, normalized size = 1.05

$$\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - 8\left(\cos(dx+c)^2+2\cos(dx+\right.}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 8*(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)
*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
 + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.21, size = 203, normalized size = 1.17

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{3}\cos\left(dx+c\right)\left(4\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\sqrt{2}\,\sin\left(dx+c\right)+\cos\left(d\right.\right.}{4d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*cos(d*x+c)*(4*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+cos
(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*
x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^7*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.378 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$-\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)}}\right.}{2\sqrt{2}\,a^{3/2}d}$$

```
[Out] -1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+3/2*sin(d*x+c)/a/
d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-3*arcsin(sin(d*x+c)*a^(1/2)/(a+a*
cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d+9/4*arctan(1
/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2765, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)}}\right.}{2\sqrt{2}\,a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (-3*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + (9*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x
])^(3/2)*Sec[c + d*x]^(3/2)) + (3*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\text{se}}}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\text{se}}}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\text{se}}}$$

$$= -\frac{3\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{9\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{\text{c}}}\right.}{}$$

Mathematica [C] time = 6.57, size = 316, normalized size = 1.48

$$\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sin\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d}+\frac{2\cos\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d}-\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{\tan\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{(a(\cos(c+dx)+1))^{3/2}} + \frac{3i\sqrt{2}\,e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] + (3*ArcTanh[(1 - E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - ArcTanh[Sqrt[1 + E^
((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + C
os[c + d*x]))^(3/2)) + (Cos[c/2 + (d*x)/2]^3*Sqrt[Sec[c + d*x]]*((2*Cos[(3*
d*x)/2]*Sin[(3*c)/2])/d - (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/d +
(2*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d - (Sec[c/2 + (d*x)/2]*Tan[c/2])/d))/(a*(1
 + Cos[c + d*x]))^(3/2)
```

fricas [A] time = 1.21, size = 201, normalized size = 0.94

$$-\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-12\left(\cos\left(dx+c\right)^{2}+2\cos\left(d\right.\right.}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 12*
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*
(2*cos(d*x + c)^2 + 3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d
*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 0.22, size = 235, normalized size = 1.10

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{4}\cos\left(dx+c\right)\left(2\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\cos\left(dx+c\right)\sqrt{2}\right.}{4d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*cos(d*x+c)*(2*cos(d*x+c)^
2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+cos(d*x+c)*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+6*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c))*2^(1/2)*sin(d*x+c)+9*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*
x+c)-3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(5/2)/(cos
(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^9*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \cos\left(dx + c\right) + a\right)^{\frac{3}{2}} \sec\left(dx + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.379 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{163\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{95\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{a\cos}}$$

```
[Out] -1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-17/16*sec(d*x+c)^
(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+95/48*sec(d*x+c)^(3/2)*sin(d*x+
c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+163/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2
)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^(5/2)/d*2^(1/2)-299/48*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c
))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2766, 2978, 2984, 12, 2782, 205}

$$\frac{95\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{163\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\,\sqrt{\cos(c+dx)}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (163*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - (299*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]
]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (
17*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (
95*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{11a}{2}-3a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx)}}{4a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(}\right.}{}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{95\sec^{\frac{3}{2}}(c+dx)\sin(c}{48a^2d\sqrt{a+a\cos(c+}}$$

$$= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+}$$

$$= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+}$$

$$= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(}{16ad(a+a\cos(c+}$$

$$= \frac{163\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{299\sqrt{\sec(c-}}{48a^2d\sqrt{a-}}$$

Mathematica [C] time = 8.11, size = 641, normalized size = 2.70

$$-\frac{\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\cot^5\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(640\sin^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^8\left(\frac{1}{2}(c+dx)\right)\,{}_5F_4\left(2,2,2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -1/41580*(Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^4*((1
- 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(7/2)*(640*Cos[(c + d*x)/2]^8*Hypergeometri
cPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 - 1280*Cos[(c + d*x)/2]^6*Hypergeom
etricPFQ[{2, 2, 2, 7/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2) + 33*(1
 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*(-105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]]*Cos[(c + d*x)/2]^4*(-10935 + 72902*Sin[c/2 + (d*x)/2]^2 - 188110*Si
n[c/2 + (d*x)/2]^4 + 234156*Sin[c/2 + (d*x)/2]^6 - 140732*Sin[c/2 + (d*x)/2
]^8 + 33208*Sin[c/2 + (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*(-1148175 + 10333785*Sin[c/2 + (d*x)/2]^2 - 38990350*Sin
[c/2 + (d*x)/2]^4 + 79946462*Sin[c/2 + (d*x)/2]^6 - 96281836*Sin[c/2 + (d*x
)/2]^8 + 68243596*Sin[c/2 + (d*x)/2]^10 - 26448512*Sin[c/2 + (d*x)/2]^12 +
4344400*Sin[c/2 + (d*x)/2]^14))))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 0.98, size = 195, normalized size = 0.82

$$-\frac{489 \sqrt{2} \left(\cos(dx+c)^4 + 3\cos(dx+c)^3 + 3\cos(dx+c)^2 + \cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{96\left(a^3d\cos(dx+c)^4 + 3a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/96*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2
+ 160*cos(d*x + c) - 32)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 + a^3*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.24, size = 316, normalized size = 1.33

$$-\frac{\left(489\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^3(dx+c)\right)\sin(dx+c)+1467\left(\cos^2(dx+c)\right)\sin(dx+c)\arcsin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/96/d*(489*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*cos(d*x+c)^3*sin(d*x+c)+1467*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1467*cos(d*x+c)*sin(d
*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+
489*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)-299*cos(d*x+c)^4*2^(1/2)-204*cos(d*x+c)^3*2^(1/2)+343*cos(d*x+c)^2
*2^(1/2)+192*cos(d*x+c)*2^(1/2)-32*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)
*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(1+cos(d*x+c))^2*2^(1/2)/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.380 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{75\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{13\sin(c+dx)\sqrt{\ldots}}{16ad(a\cos(c+\ldots}$$

```
[Out] -1/4*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(5/2)-13/16*sin(d*x+c)*
sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)-75/32*arctan(1/2*sin(d*x+c)*a^(
1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+49/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a
*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2766, 2978, 2984, 12, 2782, 205}

$$\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{75\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{13\sin(c+dx)\sqrt{\mathrm{se}\ldots}}{16ad(a\cos(c+d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (4
9*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{9a}{2}-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx}}{4a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}\sin(c}{16a^2d\sqrt{a+a\cos(c+}}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}\sin(c}{16a^2d\sqrt{a+a\cos(c+}}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}\sin(c}{16a^2d\sqrt{a+a\cos(c+}}$$

$$= -\frac{75\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d} - \frac{\sqrt{\sec(c+dx)}}{4d(a+a\cos}$$

Mathematica [C] time = 6.81, size = 508, normalized size = 2.58

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{8\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^6\left(\frac{1}{2}(c+dx)\right){}_4F_3\left(2,2,2,\frac{5}{2};1,1,\frac{11}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{315\left(2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c
/2 + (d*x)/2]^2)^(-1))^(3/2)*((8*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2
```

```
, 2, 5/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]*Sin[c/2 + (d*x)/2]^2)/(315*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (Csc[c/2 + (d
*x)/2]^8*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-343 + 1465*Sin[c/2 + (d*x)/2]^2 - 20
21*Sin[c/2 + (d*x)/2]^4 + 824*Sin[c/2 + (d*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5145 + 33980*Sin[c/2 + (d*x)/2]^2 - 8
7764*Sin[c/2 + (d*x)/2]^4 + 109737*Sin[c/2 + (d*x)/2]^6 - 66122*Sin[c/2 + (
d*x)/2]^8 + 15344*Sin[c/2 + (d*x)/2]^10)))/120))/(d*(a*(1 + Cos[c + d*x]))^
(5/2))
```

fricas [A] time = 0.89, size = 170, normalized size = 0.86

$$\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\sqrt{a\cos(dx+}}{}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(49*cos(d*x + c)^2 + 85*cos(d*x
 + c) + 32)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.22, size = 258, normalized size = 1.31

$$-\frac{\left(-1+\cos\left(dx+c\right)\right)\left(75\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)+150\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{arcs}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(-1+cos(d*x+c))*(75*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+150*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-49*cos(d*
x+c)^3*2^(1/2)+75*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c)-36*cos(d*x+c)^2*2^(1/2)+53*cos(d*x+c)*2^(1/2)+32*2^(
1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3
/(1+cos(d*x+c))*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx + c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.381 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{19\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d\sqrt{}}$$

```
[Out] -1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-9/16*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+19/32*arctan(1/2*sin(d*x+c)*a^(1
/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2766, 2978, 12, 2782, 205}

$$\frac{19\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (9*Si
n[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
```

```
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c+}}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}$$

$$= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{s}{4d(a+a\cos(}$$

Mathematica [A] time = 0.94, size = 131, normalized size = 0.83

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(76\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx))}\right)-\cos(c+dx)(9\cos(c+dx)+13)\sec}{64\sqrt{2}a^2d\sqrt{1-\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((76*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]] - Cos[c + d*x]*(13 +
 9*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[2 - 2*Sec[c + d*x]])*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(64*Sqrt[2]*a^2*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[1 - S
ec[c + d*x]])
```

fricas [A] time = 1.10, size = 169, normalized size = 1.08

$$-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\sqrt{a\cos(dx}}{}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(9*cos(d*x + c)^2 + 13*cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos
(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

maple [A] time = 0.21, size = 222, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\left(-1+\cos\left(dx+c\right)\right)^{2}\left(9\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-19\cos\left(d\right.\right.}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x
+c))^2*(9*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-19*cos(d*x
+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+4*cos(d*x+c)*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)-19*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x
+c)-13*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.382 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=157

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{}{4d\sqrt{\mathrm{se}}}$$

```
[Out] 1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/16*sin(d*x+c)/a/
d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+5/32*arctan(1/2*sin(d*x+c)*a^(1/2
)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2764, 2978, 12, 2782, 205}

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{}{4d\sqrt{\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
+ Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + Sin[c
+ d*x]/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \frac{}{4d}$$

Mathematica [A] time = 0.69, size = 122, normalized size = 0.78

$$\frac{-2\tan^3\left(\frac{1}{2}(c+dx)\right) + 48\sin^4\left(\frac{1}{2}(c+dx)\right)\csc^3(c+dx) - 5\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(\right.}\right.}{32a^2d\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-5*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]*Sqrt
[2 - 2*Sec[c + d*x]] + 48*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 2*Tan[(c + d*
x)/2]^3)/(32*a^2*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 1.20, size = 167, normalized size = 1.06

$$-\frac{5\sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \frac{2\sqrt{a\cos(dx+c)}}{}}{32\left(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + 3a^3d\cos(dx+c) + a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c)^2 + 5*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
 + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.23, size = 221, normalized size = 1.41

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\left(-1+\cos\left(dx+c\right)\right)^{3}\left(\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+4\cos\left(dx+c\right)\sqrt{2}\,\sqrt{\cdots}\right.}{32d\sqrt{\frac{1}{\cos(dx+c)}}\,\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(cos(d*x+c)^2*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+5*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*
x+c))-5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)/sin(d*x+c)^7*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)),x)
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
[Out] Timed out
```

3.383 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{7\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d\sqrt{s}}$$

```
[Out] -1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+7/16*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+3/32*arctan(1/2*sin(d*x+c)*a^(1/
2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2765, 2978, 12, 2782, 205}

$$\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{7\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
- Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (7*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d} - \frac{}{4}$$

Mathematica [A] time = 0.70, size = 164, normalized size = 1.04

$$\frac{\sqrt{\cos(c+dx)}(\cos(c+dx)+1)^{3/2}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(6\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)+1}\sin^{-1}\left(\frac{\text{si}}{\sqrt{\text{c}}}\right.\right.}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]*Sqrt[Sec[c +
d*x]]*(6*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]
^2*Sqrt[1 + Cos[c + d*x]] - Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(Sin[(c +
 d*x)/2] - 7*Sin[(3*(c + d*x))/2])))/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 1.04, size = 169, normalized size = 1.08

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \frac{2\sqrt{a\cos(dx+}}{}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(7*cos(d*x + c)^2 + 3*cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.22, size = 222, normalized size = 1.41

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{4}\cos\left(dx+c\right)\left(7\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-4\cos\left(dx+c\right)\sqrt{2}\right.}{32d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*cos(d*x+c)*(7*cos(d*x+c)
^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-4*cos(d*x+c)*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)+3*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))-3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)/sin(d*x+c)^9*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.384 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] -1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)-11/16*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*arcsin(sin(d*x+c)*a^(1/2)/(a+
a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d-43/32*arct
an(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2765, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - (43*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*
x])^(5/2)*Sec[c + d*x]^(3/2)) - (11*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*
x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{43\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{c}}\right.}{}$$

Mathematica [C] time = 2.25, size = 373, normalized size = 1.74

$$\frac{e^{-\frac{1}{2}i(c+dx)}\left(\frac{1}{16}ie^{-2i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\left(-7e^{i(c+dx)}-8e^{2i(c+dx)}+8e^{3i(c+dx)}+7e^{4i(c+dx)}+15e^{5i(c+dx)}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (((I/16)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-43*(1 + E^(I*(c
+ d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(-15 - 7*E^(I*(c + d*x)) - 8
*E^((2*I)*(c + d*x)) + 8*E^((3*I)*(c + d*x)) + 7*E^((4*I)*(c + d*x)) + 15*E
^((5*I)*(c + d*x)) + 16*(1 + E^(I*(c + d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x)
)]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2])/E^((2*I)*(c +
 d*x)) - (16*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqr
t[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2]^5)/(4*
d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 1.54, size = 235, normalized size = 1.10

$$\frac{43\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-64\left(\cos\left(d\right.\right.}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(15*cos(d*x + c)^2 + 11*cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 0.22, size = 320, normalized size = 1.50

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{5}\cos\left(dx+c\right)\left(15\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+32\cos\left(dx+c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(15*cos(d*x+c
)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+32*cos(d*x+c)*sin(d*x+c)*2^(1
/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+43*cos(
d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-4*cos(d*x+c)*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+32*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+43*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*sin(d*x+c)-11*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+
c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^11*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.385 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=254

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] -1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)-15/16*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+35/16*sin(d*x+c)/a^2/d/(a+a*cos
(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-5*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c)
)^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d+115/32*arctan(1/2*sin(
d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4222, 2765, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{115\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (-5*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + (115*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c +
d*x])^(5/2)*Sec[c + d*x]^(5/2)) - (15*Sin[c + d*x])/(16*a*d*(a + a*Cos[c +
d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (35*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
```

```
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\,\mathrm{se}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\,\mathrm{se}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\,\mathrm{se}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\,\mathrm{se}}$$

$$= -\frac{5\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{115\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{}$$

Mathematica [C] time = 3.23, size = 412, normalized size = 1.62

$$\frac{e^{-\frac{1}{2}i(c+dx)}\left(40i\sqrt{2}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^5\left(\frac{1}{2}(c+dx)\right)\sinh^{-1}\left(e^{i(c+dx)}\right)+115i\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((40*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2]^5 + (115*I)*Sq
rt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]
```

```
*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]*Cos
[(c + d*x)/2]^5 - ((I/16)*(-8 - 47*E^(I*(c + d*x)) - 39*E^((2*I)*(c + d*x))
 - 16*E^((3*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 39*E^((5*I)*(c + d*x))
 + 47*E^((6*I)*(c + d*x)) + 8*E^((7*I)*(c + d*x)) + 40*E^(I*(c + d*x))*(1 +
 E^(I*(c + d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)
*(c + d*x))]])*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/E^((3*I)*(c + d*x)))/(4
*d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 1.80, size = 245, normalized size = 0.96

$$-\frac{115\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-160\left(\cos\right.}{32\left(a^{3}d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c)
+ 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 35*cos(d*x + c))*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2)), x)
```

maple [A] time = 0.24, size = 352, normalized size = 1.39

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{6}\cos\left(dx+c\right)\left(16\left(\cos^{3}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\left(\cos^{2}\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^6*cos(d*x+c)*(16*cos(d*x+c
)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*cos(d*x+c)^2*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+80*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+115*cos(d*x+c)*sin(d*x+c
)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-20*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+80*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))*2^(1/2)*sin(d*x+c)+115*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x
+c)-35*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(7/2)/(cos
(d*x+c)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^13*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}\,{\left(a+a\,\cos(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.386 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{1015\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{193\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^3d\sqrt{a\cos(c+dx)+a}}-\frac{629\sin(c+}{64a^3d\sqrt{a\,\mathrm{c}}}$$

```
[Out] -1/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-23/48*sec(d*x+c)^
(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-109/64*sec(d*x+c)^(3/2)*sin(d*x
+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)+193/64*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/d/(
a+a*cos(d*x+c))^(1/2)+1015/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2
)/d*2^(1/2)-629/64*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2766, 2978, 2984, 12, 2782, 205}

$$\frac{193\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^3d\sqrt{a\cos(c+dx)+a}}-\frac{109\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{629\sin(c+dx)\sqrt{\sec(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}+\frac{1015\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (1015*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*
d) - (629*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x
]]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) -
(23*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) -
(109*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2))
 + (193*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{15a}{2}-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx)}}{6a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(}{64a^2d(a+a\cos(c+}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(}{64a^2d(a+a\cos(c+}$$

$$= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(}{48ad(a+a\cos(c+}$$

$$= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(}{48ad(a+a\cos(c+}$$

$$= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(}{48ad(a+a\cos(c+}$$

$$= \frac{1015\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{629\sqrt{\sec(c}{64a^3d\sqrt{a}}$$

Mathematica [C] time = 8.40, size = 696, normalized size = 2.51

$$\frac{\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\cot^7\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-7680\sin^{14}\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^{10}\left(\frac{1}{2}(c+dx)\right)\,{}_6F_5\left(2,2,2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Cot[c/2 + (d*x)/2]^7*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^6*((1 - 2*Sin[c
/2 + (d*x)/2]^2)^(-1))^(7/2)*(-7680*Cos[(c + d*x)/2]^10*HypergeometricPFQ[{
2, 2, 2, 2, 2, 7/2}, {1, 1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 + 19200*Cos[(c + d*x)/2]^8*Hypergeo
metricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14*(-7 + 6*Sin[c/2 + (d*x)/2]^2)
+ 143*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*(315*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(351384 - 2928877*Sin[c/2 + (d*x)/2]^2 +
9953934*Sin[c/2 + (d*x)/2]^4 - 17629526*Sin[c/2 + (d*x)/2]^6 + 17139064*Sin
[c/2 + (d*x)/2]^8 - 8670660*Sin[c/2 + (d*x)/2]^10 + 1793816*Sin[c/2 + (d*x)
/2]^12) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-110685
960 + 1291549455*Sin[c/2 + (d*x)/2]^2 - 6601900452*Sin[c/2 + (d*x)/2]^4 + 1
9406027859*Sin[c/2 + (d*x)/2]^6 - 36160322412*Sin[c/2 + (d*x)/2]^8 + 443132
22590*Sin[c/2 + (d*x)/2]^10 - 35736693140*Sin[c/2 + (d*x)/2]^12 + 183052542
12*Sin[c/2 + (d*x)/2]^14 - 5410719584*Sin[c/2 + (d*x)/2]^16 + 704274992*Sin
[c/2 + (d*x)/2]^18))))/(3243240*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 1.25, size = 229, normalized size = 0.83

$$-\frac{3045\sqrt{2}\left(\cos(dx+c)^5+4\cos(dx+c)^4+6\cos(dx+c)^3+4\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos}}{}\right.}{384\left(a^4d\cos(dx+c)^5+4a^4d\cos(dx+c)^4+6a^4d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(3045*sqrt(2)*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3
+ 4*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x +
c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(1887*cos(d*x + c)^4
 + 5082*cos(d*x + c)^3 + 4251*cos(d*x + c)^2 + 896*cos(d*x + c) - 128)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^5
 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2
 + a^4*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

maple [A] time = 0.25, size = 390, normalized size = 1.41

$$\frac{\left(-1+\cos\left(dx+c\right)\right)\left(3045\left(\cos^{4}\left(dx+c\right)\right)\sin\left(dx+c\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+12180\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*(-1+cos(d*x+c))*(3045*cos(d*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+12180*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+182
70*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)+12180*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-1887*2^(1/2)*cos(d*x+c)^5+3045*s
in(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)-3195*cos(d*x+c)^4*2^(1/2)+831*cos(d*x+c)^3*2^(1/2)+3355*cos(d*x+c)^2*2^
(1/2)+1024*cos(d*x+c)*2^(1/2)-128*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*
(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/(1+cos(d*x+c))^2*2^(1/2)/a^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.387 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{363\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{691\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{199\sin(c+}{192a^2d(a\cos}$$

```
[Out] -1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(7/2)-19/48*sin(d*x+c)*
sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-199/192*sin(d*x+c)*sec(d*x+c)^(
1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)-363/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(
1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^(7/2)/d*2^(1/2)+691/192*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*cos(d
*x+c))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2766, 2978, 2984, 12, 2782, 205}

$$\frac{691\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{199\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{363\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{2}\sqrt{\cos(c+d}}\right.}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-363*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*
d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - (
19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - (
199*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
 + (691*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{13a}{2}-3a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+}}{6a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec\right.}}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{199\sqrt{\sec(c+dx)}\sin}{192a^2d(a+a\cos(c+}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{199\sqrt{\sec(c+dx)}\sin}{192a^2d(a+a\cos(c+}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{199\sqrt{\sec(c+dx)}\sin}{192a^2d(a+a\cos(c+}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{199\sqrt{\sec(c+dx)}\sin}{192a^2d(a+a\cos(c+}$$

$$= -\frac{363\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{\sqrt{\sec(c+}}{6d(a+a\,c}$$

Mathematica [C] time = 6.92, size = 561, normalized size = 2.37

$$2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{16\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^8\left(\frac{1}{2}(c+dx)\right){}_5F_4\left(2,2,2,2,\frac{5}{2};1,1,1,\frac{13}{2};\frac{\sin^2\left(\frac{c}{2}\right.}{2\sin^2\left(\frac{c}{2}+\right.}\right.}{3465\left(2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c
/2 + (d*x)/2]^2)^(-1))^(3/2)*((16*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2,
2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) - (Csc[
c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(2187 - 12908*Sin[c/2 + (d*x)
/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4 - 26380*Sin[c/2 + (d*x)/2]^6 + 8752*Sin[
c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]*(-229635 + 2120790*Sin[c/2 + (d*x)/2]^2 - 8267707*Sin[c/2 + (d*x)/2]^4 +
17646926*Sin[c/2 + (d*x)/2]^6 - 22251094*Sin[c/2 + (d*x)/2]^8 + 16548816*Si
n[c/2 + (d*x)/2]^10 - 6712984*Sin[c/2 + (d*x)/2]^12 + 1144608*Sin[c/2 + (d*
x)/2]^14)))/1680))/(d*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 1.07, size = 204, normalized size = 0.86

$$\frac{1089\,\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(\cdots}}{\sqrt{a}\,\sin(dx+c)}\cdots\right.}{384\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(1089*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(691*cos(d*x + c)^3 + 1874*cos(d*x
 + c)^2 + 1599*cos(d*x + c) + 384)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sq
rt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*
cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

maple [A] time = 0.24, size = 326, normalized size = 1.38

$$-\frac{(-1+\cos(dx+c))^2 \left(-1089 \arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right) \left(\cos^3(dx+c)\right) \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 691 \left(\cos^4(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*(-1+cos(d*x+c))^2*(-1089*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*
x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+691*cos(d*x+c)^4*2^(1/2
)-3267*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)^2*sin(d*x+c)+1183*cos(d*x+c)^3*2^(1/2)-3267*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-275
*cos(d*x+c)^2*2^(1/2)-1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*sin(d*x+c)-1215*cos(d*x+c)*2^(1/2)-384*2^(1/2))*cos(d*
x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/(1+cos(d*x+
c))*2^(1/2)/a^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+a\,\cos(c+d\,x)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.388 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=197

$$\frac{63\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{103\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{1\ldots}{16\ldots}$$

```
[Out] -1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)-5/16*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-103/192*sin(d*x+c)/a^2/d/(a+a*co
s(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+63/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/
2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2766, 2978, 12, 2782, 205}

$$-\frac{103\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}+\frac{63\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (63*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
 - Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) - (5*Si
n[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (103*S
in[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}$$

$$= \frac{63\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}\,a^{7/2}d} - \frac{}{6d(a+a\cos(}$$

Mathematica [A] time = 3.77, size = 153, normalized size = 0.78

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(6048\cos^6\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx))}\right.\right.}{3072\sqrt{2}\,a^3d\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}\sqrt{a(\cos(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(-2*(493 + 532*Cos[c + d*x] + 103*Cos[2*(c + d*x)])*Sqr
t[2 - 2*Sec[c + d*x]] + 6048*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2
)]]*Cos[(c + d*x)/2]^6*Sec[c + d*x])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3*d*
Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])
```

fricas [A] time = 1.08, size = 203, normalized size = 1.03

$$-\frac{189\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos}}{\sqrt{a}\sin(dx+c)}\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(189*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(103*cos(d*x + c)^3 + 266*cos(d*x
+ c)^2 + 195*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x
+ c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

maple [A] time = 0.22, size = 288, normalized size = 1.46

$$-\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\left(-1+\cos\left(dx+c\right)\right)^{3}\left(103\left(\cos^{3}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-189\,\mathrm{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d
*x+c))^3*(103*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-189*ar
csin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+163*cos(d*x+c)^2*2
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-378*cos(d*x+c)*sin(d*x+c)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))-71*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)-189*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-195*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.389 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=197

$$\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{5\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{}{16a}$$

```
[Out] 1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+1/16*sin(d*x+c)/a/
d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-5/192*sin(d*x+c)/a^2/d/(a+a*cos(d
*x+c))^(3/2)/sec(d*x+c)^(1/2)+13/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/
cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2764, 2978, 12, 2782, 205}

$$-\frac{5\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{}{16}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (13*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
 + Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + Sin[c
 + d*x]/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (5*Sin[c +
 d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
```

```
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{13\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} + \frac{}{6}$$

Mathematica [A] time = 1.18, size = 125, normalized size = 0.63

$$\frac{\sin(c+dx)(4\cos(c+dx)-5\cos(2(c+dx))+73)\sec^6\left(\frac{1}{2}(c+dx)\right)-312\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh}{3072a^3d\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-312*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]*Sq
rt[2 - 2*Sec[c + d*x]] + (73 + 4*Cos[c + d*x] - 5*Cos[2*(c + d*x)])*Sec[(c
+ d*x)/2]^6*Sin[c + d*x])/(3072*a^3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c
 + d*x]])
```

fricas [A] time = 1.26, size = 203, normalized size = 1.03

$$-\frac{39\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(d}}{\sqrt{a}\sin(dx+c)}\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/384*(39*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(5*cos(d*x + c)^3 - 2*cos(d*x + c)^
2 - 39*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c
)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.22, size = 288, normalized size = 1.46

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\cos\left(dx+c\right)\left(-1+\cos\left(dx+c\right)\right)^{4}\left(5\left(\cos^{3}\left(dx+c\right)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-39\arcsin\left(\frac{-1+\cos(d}{\sin(dx+}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^4*(5*cos(d*x+c)
^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-39*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*cos(d*x+c)^2*sin(d*x+c)-7*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)-78*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-3
7*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-39*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)+39*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/
(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^9*2^(1/2)
/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)),x)
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(1/2),x)
[Out] Timed out
```

3.390 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{17\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}+\frac{}{16}$$

```
[Out] -1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+3/16*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+17/192*sin(d*x+c)/a^2/d/(a+a*cos
(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+7/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)
/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2765, 2978, 12, 2782, 205}

$$\frac{17\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}+\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{}{16}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
- Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + (3*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (17*Sin
[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}}{}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}\,a^{7/2}d} - \frac{}{6}$$

Mathematica [A] time = 3.69, size = 153, normalized size = 0.78

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(2(140\cos(c+dx)+17\cos(2(c+dx))+59)\sqrt{2-2\sec(c+dx)}+672\cos^6\left(\frac{1}{2}\right.\right.}{3072\sqrt{2}\,a^3d\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}\sqrt{a(\cos(c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(2*(59 + 140*Cos[c + d*x] + 17*Cos[2*(c + d*x)])*Sqrt[2
 - 2*Sec[c + d*x]] + 672*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*
Cos[(c + d*x)/2]^6*Sec[c + d*x])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3*d*Sqrt
[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])
```

fricas [A] time = 1.90, size = 203, normalized size = 1.03

$$-\frac{21\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(}}{\sqrt{a}\sin(dx+c)}\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/384*(21*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(17*cos(d*x + c)^3 + 70*cos(d*x + c
)^2 + 21*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.22, size = 288, normalized size = 1.46

$$\frac{\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))^5\cos(dx+c)\left(17\left(\cos^3(dx+c)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+53\left(\cos^2(dx+c)\right)\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(17*cos(d*x+c
)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+53*cos(d*x+c)^2*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+21*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)^2*sin(d*x+c)-49*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+42*
cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-21*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+21*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)
)/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^11*2^(1
/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.391 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{67\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{6d\,\mathrm{s}}$$

```
[Out] -1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2)-13/48*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+67/192*sin(d*x+c)/a^2/d/(a+a*co
s(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+5/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2
)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2765, 2977, 2978, 12, 2782, 205}

$$\frac{67\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{}{6d\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
- Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)) - (13*Si
n[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (67*Si
n[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}\,a^{7/2}d} - \frac{}{6d(}$$

Mathematica [A] time = 4.39, size = 196, normalized size = 0.99

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\right.}{24a^4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[
c + d*x]]*(15*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c
 + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)
/2]*(33 - 26*Tan[(c + d*x)/2]^2 + 8*Tan[(c + d*x)/2]^4)))/(24*a^4*d*Sqrt[Co
s[(c + d*x)/2]^2]*(1 + Cos[c + d*x])^4)
```

fricas [A] time = 1.23, size = 203, normalized size = 1.03

$$-\frac{15\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos}}{\sqrt{a}\sin(dx+c)}\right.}{384\left(a^{4}d\cos\left(dx+c\right)^{4}+4a^{4}d\cos\left(dx+c\right)^{3}+6a^{4}d\cos\left(dx+c\right)^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(15*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(67*cos(d*x + c)^3 + 50*cos(d*x + c
)^2 + 15*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 0.21, size = 288, normalized size = 1.46

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{6}\cos\left(dx+c\right)\left(67\left(\cos^{3}\left(dx+c\right)\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+15\arcsin\left(\frac{-1+\mathrm{c}}{\sin(}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^6*cos(d*x+c)*(67*cos(d*x+
c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*arcsin((-1+cos(d*x+c))/si
```

```
n(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-17*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+30*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)-35*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*sin(d*x+c)-15*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^13*2^(
1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.392 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=254

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{177\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

```
[Out] -1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2)-17/48*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)-49/64*sin(d*x+c)/a^2/d/(a+a*cos
(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c)
)^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d-177/128*arctan(1/2*sin
(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2765, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{49\sin(c+dx)}{64a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{177\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) - (177*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*(a + a*Cos[c + d
*x])^(7/2)*Sec[c + d*x]^(5/2)) - (17*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d
*x])^(5/2)*Sec[c + d*x]^(3/2)) - (49*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c +
 d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
```

```
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}}{}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} - \frac{177\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{}$$

Mathematica [C] time = 6.79, size = 454, normalized size = 1.79

$$\frac{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{247\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{12d}-\frac{247\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{12d}+\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}+\frac{\tan\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}-41\sec\cdots\right.}{(a(\cos(c+dx)+1))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((-1/4*I)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*(64*ArcSinh[E^(I*(c + d*x))] + (177*ArcTanh[(1 - E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 64*ArcTanh[Sqrt[1 +
 E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(Sqrt[2]*d*E^((I/2)*(c + d*x)
)*(a*(1 + Cos[c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*
((-247*Cos[(d*x)/2]*Sin[c/2])/(12*d) - (247*Cos[c/2]*Sin[(d*x)/2])/(12*d) +
 (379*Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(24*d) - (41*Sec[c/2]*Sec
[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(12*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*Sin
[(d*x)/2])/(3*d) + (379*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - (41*Sec[c/2 +
 (d*x)/2]^3*Tan[c/2])/(12*d) + (Sec[c/2 + (d*x)/2]^5*Tan[c/2])/(3*d)))/(a*(
1 + Cos[c + d*x]))^(7/2)
```

fricas [A] time = 2.29, size = 279, normalized size = 1.10

$$\frac{531\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx\cdots}}{\sqrt{a}\sin(dx+c)}\right.}{384\left(a^4d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(531*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 768*(cos(d*x + c)^4 + 4*cos(d*x + c)^
3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x +
c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(247*cos(d*x + c)^3
+ 362*cos(d*x + c)^2 + 147*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*
a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7/2)), x)
```

maple [B] time = 0.26, size = 440, normalized size = 1.73

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{7}\cos\left(dx+c\right)\left(384\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\sqrt{2}\,\left(\cos^{2}\left(dx+c\right)\right)\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*cos(d*x+c)*(384*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)
^2*sin(d*x+c)+247*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+76
8*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/cos(d*x+c))+115*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)+531*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+384*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*
x+c)-215*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1062*cos(d*x+
c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-147*2^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+531*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/c
os(d*x+c))^(7/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^15*2^(1/2)/a^
4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.393 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{637\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c}}\right.}{64\sqrt{2}a^{7/2}d}$$

```
[Out] -1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2)-7/16*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)-259/192*sin(d*x+c)/a^2/d/(a+a*co
s(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+189/64*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(
1/2)/sec(d*x+c)^(1/2)-7*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d+637/128*arctan(1/2*sin(d*x+c)*a^
(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4222, 2765, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{259\sin(c+dx)}{192a^2d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{18}{64a^3d\sqrt{\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(9/2)),x]
```

```
[Out] (-7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) + (637*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*(a + a*Cos[c +
d*x])^(7/2)*Sec[c + d*x]^(7/2)) - (7*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d
*x])^(5/2)*Sec[c + d*x]^(5/2)) - (259*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
 + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (189*Sin[c + d*x])/(64*a^3*d*Sqrt[a +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
```

```
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\mathrm{s}}$$

$$= -\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} + \frac{637\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{}$$

Mathematica [C] time = 3.48, size = 460, normalized size = 1.56

$$\frac{e^{-\frac{1}{2}i(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(672i\sqrt{2}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^7\left(\frac{1}{2}(c+dx)\right)\sinh^{-1}\left(e^{i(c+dx)}\right) - \frac{1}{64}ie^{-4i(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(9/2)),x]
```

```
[Out] ((((-1/64*I)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-1911*E^(I*(c
 + d*x))*(1 + E^(I*(c + d*x)))^6*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 -
 E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(-96 -
 1003*E^(I*(c + d*x)) - 2169*E^((2*I)*(c + d*x)) - 2297*E^((3*I)*(c + d*x))
 - 779*E^((4*I)*(c + d*x)) + 779*E^((5*I)*(c + d*x)) + 2297*E^((6*I)*(c + d
*x)) + 2169*E^((7*I)*(c + d*x)) + 1003*E^((8*I)*(c + d*x)) + 96*E^((9*I)*(c
 + d*x)) + 672*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^6*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2])/E^((4*
I)*(c + d*x)) + (672*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2
]^7)*Sqrt[a*(1 + Cos[c + d*x]])/(24*a^4*d*E^((I/2)*(c + d*x))*(1 + Cos[c +
 d*x])^4)
```

fricas [A] time = 2.38, size = 289, normalized size = 0.98

$$-\frac{1911 \sqrt{2} \left(\cos (dx+c)^4 + 4 \cos (dx+c)^3 + 6 \cos (dx+c)^2 + 4 \cos (dx+c) + 1\right) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx+c)+a} \sqrt{c}}{\sqrt{a} \sin(dx+c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/384*(1911*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2
+ 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2688*(cos(d*x + c)^4 + 4*cos(d*x +
c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(192*cos(d*x + c)
^4 + 1099*cos(d*x + c)^3 + 1442*cos(d*x + c)^2 + 567*cos(d*x + c))*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos (dx+c)+a)^{\frac{7}{2}} \sec (dx+c)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(9/2)), x)
```

maple [A] time = 0.26, size = 472, normalized size = 1.61

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{8}\cos\left(dx+c\right)\left(192\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{4}\left(dx+c\right)\right)+907\left(\cos^{3}\left(dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^8*cos(d*x+c)*(192*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+907*cos(d*x+c)^3*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)+1344*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+1911*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+343*cos(d*x+c)^2*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+2688*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3822*cos(d*x+c)*sin(
d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-875*cos(d*x+c)*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)+1344*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+1911*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)-567*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^
(9/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(11/2)/sin(d*x+c)^17*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.394 $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{45\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}\,a^{9/2}d}+\frac{73\sin(c+dx)}{1024a^3d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}+\frac{}{2}$$

```
[Out] -1/8*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(3/2)-5/32*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+33/256*sin(d*x+c)/a^2/d/(a+a*cos
(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+73/1024*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(
3/2)/sec(d*x+c)^(1/2)+45/2048*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x
+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(9/2)
/d*2^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2765, 2977, 2978, 12, 2782, 205}

$$\frac{73\sin(c+dx)}{1024a^3d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}+\frac{33\sin(c+dx)}{256a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{5/2}}+\frac{45\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (45*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1024*Sqrt[2]*a^(9/2)*
d) - Sin[c + d*x]/(8*d*(a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(3/2)) - (5*
Sin[c + d*x])/(32*a*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + (33*
Sin[c + d*x])/(256*a^2*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (
73*Sin[c + d*x])/(1024*a^3*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{}}$$

$$= \frac{45\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{1024\sqrt{2}a^{9/2}d} - \frac{}{8a}$$

Mathematica [A] time = 4.35, size = 163, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2(999\cos(c+dx)+702\cos(2(c+dx))+73\cos(3(c+dx))+882)\sqrt{2-2\sec(c\right.}{65536\sqrt{2}a^4d\sqrt{-((\sec(c+dx)-1)\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^6*(2*(882 + 999*Cos[c + d*x] + 702*Cos[2*(c + d*x)] + 73*
Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]] + 5760*ArcTanh[Sqrt[-(Sec[c + d*
x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^8*Sec[c + d*x])*Tan[(c + d*x)/2])
/(65536*Sqrt[2]*a^4*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])
*Sec[c + d*x])])
```

fricas [A] time = 1.03, size = 237, normalized size = 1.00

$$-\frac{45\sqrt{2}\left(\cos\left(dx+c\right)^{5}+5\cos\left(dx+c\right)^{4}+10\cos\left(dx+c\right)^{3}+10\cos\left(dx+c\right)^{2}+5\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\right.}{2048\left(a^{5}d\cos\left(dx+c\right)^{5}+5a^{5}d\cos\left(dx+c\right)^{4}+10a^{5}d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2048*(45*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(73*cos(d*x +
 c)^4 + 351*cos(d*x + c)^3 + 195*cos(d*x + c)^2 + 45*cos(d*x + c))*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^5*d*cos(d*x + c)^5 + 5
*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 +
 5*a^5*d*cos(d*x + c) + a^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{9}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 0.25, size = 354, normalized size = 1.49

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{7}\cos\left(dx+c\right)\left(73\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{4}\left(dx+c\right)\right)+278\left(\cos^{3}\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/2048/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*cos(d*x+c)*(73*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+278*cos(d*x+c)^3*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*
x+c)^3*sin(d*x+c)-156*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+135*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-150*cos(d*
x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+135*cos(d*x+c)*sin(d*x+c)*ar
csin((-1+cos(d*x+c))/sin(d*x+c))-45*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c))^(5/2)/(
cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^15*2^(1/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{9}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(9/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(9/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.395 $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{35\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}\,a^{9/2}d}+\frac{853\sin(c+dx)}{3072a^3d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-$$

```
[Out] -1/8*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2)-19/96*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2)-187/768*sin(d*x+c)/a^2/d/(a+a*c
os(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+853/3072*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c)
)^(3/2)/sec(d*x+c)^(1/2)+35/2048*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(
d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(9
/2)/d*2^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2765, 2977, 2978, 12, 2782, 205}

$$\frac{853\sin(c+dx)}{3072a^3d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}-\frac{187\sin(c+dx)}{768a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{5/2}}+\frac{35\sqrt{\cos(c+dx)}\,\sqrt{\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (35*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1024*Sqrt[2]*a^(9/2)*
d) - Sin[c + d*x]/(8*d*(a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)) - (19
*Sin[c + d*x])/(96*a*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)) - (18
7*Sin[c + d*x])/(768*a^2*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) +
 (853*Sin[c + d*x])/(3072*a^3*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}}{}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}}$$

$$= \frac{35\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{1024\sqrt{2}\,a^{9/2}d} -$$

Mathematica [A] time = 6.10, size = 395, normalized size = 1.67

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\cos^9\left(\frac{c}{2}+\frac{dx}{2}\right)\left(1-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^{9/2}\left(\frac{1}{8}\left(\frac{}{1-\sin}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^9*Sin[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2
)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 +
(d*x)/2]^2)^(9/2)*((35*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*
Sqrt[Cos[(c + d*x)/2]^2]*Csc[c/2 + (d*x)/2])/(128*(1 - Sec[(c + d*x)/2]^2*S
in[c/2 + (d*x)/2]^2)^(9/2)) + (35/(16*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*
x)/2]^2)^4) + 35/(24*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2)^3) + 7/(
6*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2)^2) + (1 - Sec[(c + d*x)/2]^
2*Sin[c/2 + (d*x)/2]^2)^(-1))/8))/(d*Sqrt[Cos[(c + d*x)/2]^2]*(a*(1 + Cos[c
 + d*x]))^(9/2))
```

fricas [A] time = 1.16, size = 237, normalized size = 1.00

$$-\frac{105\sqrt{2}\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\right.}{6144\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/6144*(105*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(853*cos(d*x
 + c)^4 + 819*cos(d*x + c)^3 + 455*cos(d*x + c)^2 + 105*cos(d*x + c))*sqrt(
a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^5*d*cos(d*x + c)^5
+ 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^
2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{9}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(7/2)), x)
```

maple [A] time = 0.25, size = 354, normalized size = 1.49

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{8}\cos\left(dx+c\right)\left(853\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{4}\left(dx+c\right)\right)+105\arcsin\left(\frac{-1}{s}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/6144/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^8*cos(d*x+c)*(853*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+105*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-34*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+315*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin
(d*x+c)-364*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+315*cos(
d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-350*cos(d*x+c)*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+105*arcsin((-1+cos(d*x+c))/sin(d*x+c))*s
in(d*x+c)-105*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(7/
2)/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^17*2^(1/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{9}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(9/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(9/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.396 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{4a^2\sin(c+dx)\sqrt[4]{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 4*a^2*sec(d*x+c)^(1/4)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4222, 2762, 8}

$$\frac{4a^2\sin(c+dx)\sqrt[4]{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/4),x]
```

```
[Out] (4*a^2*Sec[c + d*x]^(1/4)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx = \left(\sqrt[4]{\cos(c+dx)}\,\sqrt[4]{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx$$
$$= \frac{4a^2\sqrt[4]{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left(4a\sqrt[4]{\cos(c+dx)}\,\sqrt[4]{\sec(c+dx)}\right)\int 0\,d$$
$$= \frac{4a^2\sqrt[4]{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.12, size = 51, normalized size = 1.34

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt[4]{\sec(c+dx)}\,(a(\cos(c+dx)+1))^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/4),x]
```

```
[Out] (2*(a*(1 + Cos[c + d*x]))^(3/2)*Sec[(c + d*x)/2]^2*Sec[c + d*x]^(1/4)*Tan[(
c + d*x)/2])/d
```

fricas [A] time = 1.09, size = 41, normalized size = 1.08

$$\frac{4\sqrt{a\cos(dx+c)+a}\,a\sin(dx+c)}{(d\cos(dx+c)+d)\cos(dx+c)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4*sqrt(a*cos(d*x + c) + a)*a*sin(d*x + c)/((d*cos(d*x + c) + d)*cos(d*x + c
)^(1/4))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int (a + a\cos(dx + c))^{\frac{3}{2}} \left(\sec^{\frac{5}{4}}(dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x)
```

```
[Out] int((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x)
```

maxima [B] time = 1.01, size = 121, normalized size = 3.18

$$\frac{4\left(\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] 4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^(1/4))
```

mupad [B] time = 0.74, size = 44, normalized size = 1.16

$$\frac{4\,a\,\sin(c+d\,x)\,\sqrt{a\,(\cos(c+d\,x)+1)}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{1/4}}{d\,(\cos(c+d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/4)*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] (4*a*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/4))/(d*(
cos(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

3.397 $\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=302

$$-\frac{a^4\left(8m^2+40m+35\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}-\frac{4a^4(2m+5)\sin(c+dx)\cos^m}{d(m+2)(}$$

```
[Out] a^4*(4*m^2+29*m+55)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+m)/(m^2+5*m+6)+cos(d*x
+c)^(1+m)*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d/(4+m)+2*(5+m)*cos(d*x+c)^(1+m
)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d/(3+m)/(4+m)-a^4*(8*m^2+40*m+35)*cos(d*x
+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d
/(m^3+7*m^2+14*m+8)/(sin(d*x+c)^2)^(1/2)-4*a^4*(5+2*m)*cos(d*x+c)^(2+m)*hyp
ergeom([1/2, 1+1/2*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(sin
(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2763, 2976, 2968, 3023, 2748, 2643}

$$-\frac{a^4\left(8m^2+40m+35\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}-\frac{4a^4(2m+5)\sin(c+dx)\cos^m}{d(m+2)(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(55 + 29*m + 4*m^2)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 +
 m)*(4 + m)) + (Cos[c + d*x]^(1 + m)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x
])/(d*(4 + m)) + (2*(5 + m)*Cos[c + d*x]^(1 + m)*(a^4 + a^4*Cos[c + d*x])*S
in[c + d*x])/(d*(3 + m)*(4 + m)) - (a^4*(35 + 40*m + 8*m^2)*Cos[c + d*x]^(1
 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c +
d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2]) - (4*a^4*(5 + 2*m)*C
os[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx = \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{d(4+m)} + \frac{\int \cos^m(c+dx)(a-$$

$$= \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{d(4+m)} + \frac{2(5+m)\cos^{1+m}(c$$

$$= \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{d(4+m)} + \frac{2(5+m)\cos^{1+m}(c$$

$$= \frac{a^4\left(55+29m+4m^2\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)\left(12+7m+m^2\right)} + \frac{\cos^{1+m}(c+dx)\left(a^2\right.$$

$$= \frac{a^4\left(55+29m+4m^2\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)\left(12+7m+m^2\right)} + \frac{\cos^{1+m}(c+dx)\left(a^2\right.$$

$$= \frac{a^4\left(55+29m+4m^2\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)\left(12+7m+m^2\right)} + \frac{\cos^{1+m}(c+dx)\left(a^2\right.$$

Mathematica [F] time = 3.06, size = 0, normalized size = 0.00

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4, x]
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4\cos(dx+c)^4+4a^4\cos(dx+c)^3+6a^4\cos(dx+c)^2+4a^4\cos(dx+c)+a^4\right)\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

maple [F] time = 3.82, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))(a+a\cos(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x)
[Out] int(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^m\,(a+a\,\cos(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^4,x)
[Out] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**4,x)
[Out] Timed out
```

3.398 $\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=232

$$-\frac{a^3(4m+5)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{a^3(4m+11)\sin(c+dx)\cos^{m+2}(c+dx)}{d(m+2)(m+3)\sqrt{s}}$$

```
[Out] a^3*(7+2*m)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(3+m)+cos(d*x+c)^(1+m)*(a^3
+a^3*cos(d*x+c))*sin(d*x+c)/d/(3+m)-a^3*(5+4*m)*cos(d*x+c)^(1+m)*hypergeom(
[1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)/(2+m)/(sin(d*
x+c)^2)^(1/2)-a^3*(11+4*m)*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2
*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2763, 2968, 3023, 2748, 2643}

$$-\frac{a^3(4m+5)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{a^3(4m+11)\sin(c+dx)\cos^{m+2}(c+dx)}{d(m+2)(m+3)\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (Co
s[c + d*x]^(1 + m)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) - (a^
3*(5 + 4*m)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) -
 (a^3*(11 + 4*m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4
+ m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^
2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{d(3+m)} + \frac{\int \cos^m(c+dx)(a+$$

$$= \frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{d(3+m)} + \frac{\int \cos^m(c+dx)\left(2a^3$$

$$= \frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c\right.}{d(3+m)}$$

$$= \frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c\right.}{d(3+m)}$$

$$= \frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c\right.}{d(3+m)}$$

Mathematica [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3, x]
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\cos\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

maple [F] time = 2.71, size = 0, normalized size = 0.00

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(a+a\cos\left(dx+c\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x)
[Out] int(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{m}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

3.399 $\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=173

$$-\frac{a^2(2m+3)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2a^2\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\right.}{d(m+2)\sqrt{\sin^2(c+}}$$

```
[Out] a^2*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)-a^2*(3+2*m)*cos(d*x+c)^(1+m)*hyperg
eom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)/(2+m)/(si
n(d*x+c)^2)^(1/2)-2*a^2*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m]
,cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2763, 2748, 2643}

$$-\frac{a^2(2m+3)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2a^2\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\right.}{d(m+2)\sqrt{\sin^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - (a^2*(3 + 2*m)*Cos[c
+ d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a^2*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rubi steps

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx = \frac{a^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} + \frac{\int \cos^m(c+dx)\left(a^2(3+2m)+2a^2(2\right.}{2+m}$$
$$= \frac{a^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} + \left(2a^2\right)\int \cos^{1+m}(c+dx)\,dx + \frac{\left(a^2(3+\right.}{}$$
$$= \frac{a^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} - \frac{a^2(3+2m)\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+}{2}\right.}{d(1+m)(2+m)\sqrt{}}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2, x]
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\cos\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

maple [F] time = 3.57, size = 0, normalized size = 0.00

$$\int (\cos^{m}(dx+c))(a+a\cos(dx+c))^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)^{2}\cos(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{m}\,(a+a\,\cos(c+d\,x))^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\cos(c+dx)\cos^{m}(c+dx)\,dx+\int \cos^{2}(c+dx)\cos^{m}(c+dx)\,dx+\int \cos^{m}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**
2*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**m, x))
```

3.400 $\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=131

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{a\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\text{co}\right.}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -a*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*si
n(d*x+c)/d/(1+m)/(sin(d*x+c)^2)^(1/2)-a*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+
1/2*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2748, 2643}

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{a\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\text{co}\right.}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (a*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^m(c+dx)\,dx + a\int \cos^{1+m}(c+dx)\,dx$$
$$= -\frac{a\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)\sqrt{\sin^2(c+dx)}} - \frac{a\cos^{2+}}{}$$

Mathematica [C] time = 1.02, size = 208, normalized size = 1.59

$$\frac{ia2^{-m-2}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{m+1}(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left((m-1)m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{1-m}{2};-e^{2i(c+dx)}\right)+(m\right.}{d(m-1)m(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x]),x]
```

```
[Out] (I*2^(-2 - m)*a*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(1 + m)*(1 + Co
s[c + d*x])*((-1 + m)*m*Hypergeometric2F1[1, (1 + m)/2, (1 - m)/2, -E^((2*I
)*(c + d*x))] + E^(I*(c + d*x))*(1 + m)*(2*(-1 + m)*Hypergeometric2F1[1, (2
 + m)/2, 1 - m/2, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*m*Hypergeometric2
F1[1, (3 + m)/2, (3 - m)/2, -E^((2*I)*(c + d*x))]))*Sec[(c + d*x)/2]^2)/(d*
(-1 + m)*m*(1 + m))
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((a\cos(dx+c)+a)\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))(a+a\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+a*cos(d*x+c)),x)
[Out] int(cos(d*x+c)^m*(a+a*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+a)\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^m\,(a+a\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a + a*cos(c + d*x)),x)
[Out] int(cos(c + d*x)^m*(a + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos(c+dx)\cos^m(c+dx)\,dx+\int \cos^m(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c)),x)
[Out] a*(Integral(cos(c + d*x)*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**m, x)
)
```

3.401 $\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{m\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{ad(m+1)\sqrt{\sin^2(c+dx)}}-\frac{\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\cos^2(c+dx)\right)}{ad\sqrt{\sin^2(c+dx)}}$$

```
[Out] cos(d*x+c)^m*sin(d*x+c)/d/(a+a*cos(d*x+c))-cos(d*x+c)^m*hypergeom([1/2, 1/2
*m],[1+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/a/d/(sin(d*x+c)^2)^(1/2)+m*cos(d*x+c
)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/a/d
/(1+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2769, 2748, 2643}

$$-\frac{\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\cos^2(c+dx)\right)}{ad\sqrt{\sin^2(c+dx)}}+\frac{m\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{ad(m+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^m*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - (Cos[c + d*x]^m*Hy
pergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*Sqr
t[Sin[c + d*x]^2]) + (m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*(1 + m)*Sqrt[Sin[c + d*x]
^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2769

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a
*f*(a + b*Sin[e + f*x])), x] + Dist[(d*n)/(a*b), Int[(c + d*Sin[e + f*x])^(
n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n
] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{m\int \cos^{-1+m}(c+dx)(a-a\cos(c+dx))\,dx}{a^2}\\ &= \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{m\int \cos^{-1+m}(c+dx)\,dx}{a}-\frac{m\int \cos^m(c+dx)\,dx}{a}\\ &= \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{ad\sqrt{\sin^2(c+dx)}}+\end{aligned}$$

Mathematica [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x]),x]
[Out] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x]), x]
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^m}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="fricas")
[Out] integral(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,1,0]%%%} / %%%{2,[0,0,1]%%%} Error: Bad Argument Value
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\cos^{m}\left(dx+c\right)}{a+a\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a+a*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m/(a+a*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{m}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^m}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(a + a*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^m/(a + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{m}{\left(c+dx\right)}}{\cos{\left(c+dx\right)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**m/(cos(c + d*x) + 1), x)/a
```

3.402 $\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{(1-2m)m\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{3a^2d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{2(1-m)(m+1)\sin(c+dx)\cos^{m+2}(c+}{3a^2d(m+2)\sqrt{\mathrm{si}}}$$

```
[Out] -2/3*(1-m)*cos(d*x+c)^(1+m)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^
(1+m)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*(1-2*m)*m*cos(d*x+c)^(1+m)*hyperg
eom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(1+m)/(sin(
d*x+c)^2)^(1/2)-2/3*(1-m)*(1+m)*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m],[
2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2766, 2978, 2748, 2643}

$$\frac{(1-2m)m\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{3a^2d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{2(1-m)(m+1)\sin(c+dx)\cos^{m+2}(c+}{3a^2d(m+2)\sqrt{\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*(1 - m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x]))
 - (Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((1 -
 2*m)*m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(3*a^2*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) - (2*(1
 - m)*(1 + m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(3*a^2*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{\cos^m(c+dx)(a(2-m)+am\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}\\ &= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \cos^m(c+d\\ &= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{((1-2m)m)\\ &= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{(1-2m)m\,\mathrm{co}\end{aligned}$$

Mathematica [F] time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2, x]
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{m}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,1,2,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%} Error: Bad
 Argument Value
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\cos^{m}\left(dx+c\right)}{\left(a+a\cos\left(dx+c\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{m}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^m}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(a + a*cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^m/(a + a*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{m}{\left(c + d x \right)}}{\cos^{2}{\left(c + d x \right)} + 2 \cos{\left(c + d x \right)} + 1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a+a*cos(d*x+c))**2,x)
[Out] Integral(cos(c + d*x)**m/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

3.403 $\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=150

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7b\sin(c+dx)\cos}{48d}$$

```
[Out] 35/128*b*x+a*sin(d*x+c)/d+35/128*b*cos(d*x+c)*sin(d*x+c)/d+35/192*b*cos(d*x
+c)^3*sin(d*x+c)/d+7/48*b*cos(d*x+c)^5*sin(d*x+c)/d+1/8*b*cos(d*x+c)^7*sin(
d*x+c)/d-a*sin(d*x+c)^3/d+3/5*a*sin(d*x+c)^5/d-1/7*a*sin(d*x+c)^7/d
```

Rubi [**A**] time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2633, 2635, 8}

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7b\sin(c+dx)\cos}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Cos[c + d*x]),x]
```

```
[Out] (35*b*x)/128 + (a*Sin[c + d*x])/d + (35*b*Cos[c + d*x]*Sin[c + d*x])/(128*d
) + (35*b*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (7*b*Cos[c + d*x]^5*Sin[c
+ d*x])/(48*d) + (b*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)
/d + (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^7(c+dx)\,dx + b\int \cos^8(c+dx)\,dx$$
$$= \frac{b\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}(7b)\int \cos^6(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1-3\right.\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{b\cos^7(c+dx)\sin(c+dx)}{8d}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{35b\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{7b\cos^5(c+dx)\sin(c+d}{48d}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{35b\cos(c+dx)\sin(c+dx)}{128d} + \frac{35b\cos^3(c+dx)\sin(c+d}{192d}$$
$$= \frac{35bx}{128} + \frac{a\sin(c+dx)}{d} + \frac{35b\cos(c+dx)\sin(c+dx)}{128d} + \frac{35b\cos^3(c+dx)\,\text{s}}{192d}$$

Mathematica [A] time = 0.20, size = 135, normalized size = 0.90

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{35b(c+dx)}{128d}+\frac{7b\sin(2(c+dx))}{32d}+\frac{7b\sin(4(c+d}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Cos[c + d*x]),x]
```

```
[Out] (35*b*(c + d*x))/(128*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*a
*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d) + (7*b*Sin[2*(c + d*x)])/
(32*d) + (7*b*Sin[4*(c + d*x)])/(128*d) + (b*Sin[6*(c + d*x)])/(96*d) + (b*
Sin[8*(c + d*x)])/(1024*d)
```

fricas [A] time = 1.12, size = 97, normalized size = 0.65

$$\frac{3675\,bdx + \left(1680\,b\cos(dx+c)^7 + 1920\,a\cos(dx+c)^6 + 1960\,b\cos(dx+c)^5 + 2304\,a\cos(dx+c)^4 + 2450\,b\,\text{co}\right.}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/13440*(3675*b*d*x + (1680*b*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 + 1960
*b*cos(d*x + c)^5 + 2304*a*cos(d*x + c)^4 + 2450*b*cos(d*x + c)^3 + 3072*a*
cos(d*x + c)^2 + 3675*b*cos(d*x + c) + 6144*a)*sin(d*x + c))/d
```

giac [A] time = 1.29, size = 122, normalized size = 0.81

$$\frac{35}{128}\,bx+\frac{b\sin(8\,dx+8\,c)}{1024\,d}+\frac{a\sin(7\,dx+7\,c)}{448\,d}+\frac{b\sin(6\,dx+6\,c)}{96\,d}+\frac{7\,a\sin(5\,dx+5\,c)}{320\,d}+\frac{7\,b\sin(4\,dx+4\,c)}{128\,d}+\frac{7\,a\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 35/128*b*x + 1/1024*b*sin(8*d*x + 8*c)/d + 1/448*a*sin(7*d*x + 7*c)/d + 1/9
6*b*sin(6*d*x + 6*c)/d + 7/320*a*sin(5*d*x + 5*c)/d + 7/128*b*sin(4*d*x + 4
*c)/d + 7/64*a*sin(3*d*x + 3*c)/d + 7/32*b*sin(2*d*x + 2*c)/d + 35/64*a*sin
(d*x + c)/d
```

maple [A] time = 0.04, size = 100, normalized size = 0.67

$$\frac{b\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(b*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x
+c))*sin(d*x+c)+35/128*d*x+35/128*c)+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c
)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.36, size = 105, normalized size = 0.70

$$-\frac{3072\left(5\,\sin(dx+c)^7-21\,\sin(dx+c)^5+35\,\sin(dx+c)^3-35\,\sin(dx+c)\right)a+35\left(128\,\sin(2\,dx+2\,c)^3-84\right.}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/107520*(3072*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 -
 35*sin(d*x + c))*a + 35*(128*sin(2*d*x + 2*c)^3 - 840*d*x - 840*c - 3*sin(
8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*b)/d
```

mupad [B] time = 3.23, size = 175, normalized size = 1.17

$$\frac{35\,b\,x}{128}+\frac{\left(2\,a-\frac{93\,b}{64}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+\left(6\,a-\frac{91\,b}{192}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(\frac{106\,a}{5}-\frac{1799\,b}{192}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{1026\,a}{35}+\frac{1085}{192}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*cos(c + d*x)),x)
```

```
[Out] (35*b*x)/128 + (tan(c/2 + (d*x)/2)*(2*a + (93*b)/64) + tan(c/2 + (d*x)/2)^1
5*(2*a - (93*b)/64) + tan(c/2 + (d*x)/2)^3*(6*a + (91*b)/192) + tan(c/2 + (
d*x)/2)^13*(6*a - (91*b)/192) + tan(c/2 + (d*x)/2)^5*((106*a)/5 + (1799*b)/
192) + tan(c/2 + (d*x)/2)^11*((106*a)/5 - (1799*b)/192) + tan(c/2 + (d*x)/2
)^7*((1026*a)/35 - (1085*b)/192) + tan(c/2 + (d*x)/2)^9*((1026*a)/35 + (108
5*b)/192))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

sympy [A] time = 8.96, size = 286, normalized size = 1.91

$$\begin{cases}\frac{16a\sin^7(c+dx)}{35d}+\frac{8a\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a\sin(c+dx)\cos^6(c+dx)}{d}+\frac{35bx\sin^8(c+dx)}{128}+\frac{35bx\sin^6(c+dx)\cos}{32} \\ x\left(a+b\cos(c)\right)\cos^7(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**7/(35*d) + 8*a*sin(c + d*x)**5*cos(c + d*x)**
2/(5*d) + 2*a*sin(c + d*x)**3*cos(c + d*x)**4/d + a*sin(c + d*x)*cos(c + d*
x)**6/d + 35*b*x*sin(c + d*x)**8/128 + 35*b*x*sin(c + d*x)**6*cos(c + d*x)*
*2/32 + 105*b*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*b*x*sin(c + d*x)**2
*cos(c + d*x)**6/32 + 35*b*x*cos(c + d*x)**8/128 + 35*b*sin(c + d*x)**7*cos
(c + d*x)/(128*d) + 385*b*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 511*b*s
in(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*b*sin(c + d*x)*cos(c + d*x)**7/
(128*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**7, True))
```

3.404 $\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=128

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^7(c+dx)}{7d}+\frac{3b\sin}{}$$

```
[Out] 5/16*a*x+b*sin(d*x+c)/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3*
sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d-b*sin(d*x+c)^3/d+3/5*b*sin(d*x
+c)^5/d-1/7*b*sin(d*x+c)^7/d
```

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2635, 8, 2633}

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^7(c+dx)}{7d}+\frac{3b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Cos[c + d*x]),x]
```

```
[Out] (5*a*x)/16 + (b*Sin[c + d*x])/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (b*Sin[c + d*x]^3)/d + (3*b*Sin[c + d*x]^5)/(5*d) - (b*Sin[c + d*x]
^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^6(c+dx)\,dx + b\int \cos^7(c+dx)\,dx$$
$$= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+dx)\,dx - \frac{b\,\mathrm{Subst}\left(\int \left(1-\right.\right.}{}$$
$$= \frac{b\sin(c+dx)}{d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$
$$= \frac{b\sin(c+dx)}{d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}$$
$$= \frac{5ax}{16} + \frac{b\sin(c+dx)}{d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(}{24d}$$

Mathematica [A] time = 0.19, size = 89, normalized size = 0.70

$$\frac{35a(45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))+60c+60dx)-960b\sin^7(c+dx)+4032b\sin^5(c+dx}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Cos[c + d*x]),x]
```

```
[Out] (6720*b*Sin[c + d*x] - 6720*b*Sin[c + d*x]^3 + 4032*b*Sin[c + d*x]^5 - 960*
b*Sin[c + d*x]^7 + 35*a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c +
 d*x)] + Sin[6*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.91, size = 86, normalized size = 0.67

$$\frac{525\,adx + \left(240\,b\cos\left(dx+c\right)^6 + 280\,a\cos\left(dx+c\right)^5 + 288\,b\cos\left(dx+c\right)^4 + 350\,a\cos\left(dx+c\right)^3 + 384\,b\cos\left(dx+\right.\right.}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(525*a*d*x + (240*b*cos(d*x + c)^6 + 280*a*cos(d*x + c)^5 + 288*b*co
s(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 384*b*cos(d*x + c)^2 + 525*a*cos(d*x
+ c) + 768*b)*sin(d*x + c))/d
```

giac [A] time = 0.60, size = 107, normalized size = 0.84

$$\frac{5}{16}\,ax+\frac{b\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{7\,b\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{7\,b\sin\left(3\,dx+3\,c\right)}{64\,d}+\frac{15\,a\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x + 1/448*b*sin(7*d*x + 7*c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 7/320*
b*sin(5*d*x + 5*c)/d + 3/64*a*sin(4*d*x + 4*c)/d + 7/64*b*sin(3*d*x + 3*c)/
d + 15/64*a*sin(2*d*x + 2*c)/d + 35/64*b*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 90, normalized size = 0.70

$$\frac{\frac{b\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+a\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/7*b*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x
+5/16*c))
```

maxima [A] time = 0.46, size = 94, normalized size = 0.73

$$-\frac{35\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a+192\left(5\,\sin\left(dx+c\right)^{7}-21\,\mathrm{si}\right.}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6720*(35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48
*sin(2*d*x + 2*c))*a + 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d
*x + c)^3 - 35*sin(d*x + c))*b)/d
```

mupad [B] time = 3.25, size = 154, normalized size = 1.20

$$\frac{5\,a\,x}{16}+\frac{\left(2\,b-\frac{11\,a}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(4\,b-\frac{7\,a}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{86\,b}{5}-\frac{85\,a}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\frac{424\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b*cos(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + (tan(c/2 + (d*x)/2)*((11*a)/8 + 2*b) + tan(c/2 + (d*x)/2)^3*((
7*a)/6 + 4*b) - tan(c/2 + (d*x)/2)^11*((7*a)/6 - 4*b) - tan(c/2 + (d*x)/2)^
13*((11*a)/8 - 2*b) + tan(c/2 + (d*x)/2)^5*((85*a)/24 + (86*b)/5) - tan(c/2
 + (d*x)/2)^9*((85*a)/24 - (86*b)/5) + (424*b*tan(c/2 + (d*x)/2)^7)/35)/(d*
(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 5.40, size = 238, normalized size = 1.86

$$\begin{cases} \frac{5ax\sin^6(c+dx)}{16} + \frac{15ax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15ax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5ax\cos^6(c+dx)}{16} + \frac{5a\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5a\sin^3(c+dx)}{6d} \\ x(a+b\cos(c))\cos^6(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 16*b*sin(c + d*x)*
*7/(35*d) + 8*b*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*b*sin(c + d*x)**3
*cos(c + d*x)**4/d + b*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(a + b
*cos(c))*cos(c)**6, True))
```

3.405 $\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=114

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5b\sin$$

```
[Out] 5/16*b*x+a*sin(d*x+c)/d+5/16*b*cos(d*x+c)*sin(d*x+c)/d+5/24*b*cos(d*x+c)^3*
sin(d*x+c)/d+1/6*b*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin
(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2633, 2635, 8}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5b\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Cos[c + d*x]),x]
```

```
[Out] (5*b*x)/16 + (a*Sin[c + d*x])/d + (5*b*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*b*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^5(c+dx)\,dx + b\int \cos^6(c+dx)\,dx$$
$$= \frac{b\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5b)\int \cos^4(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1-2\right.\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{5b\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{b\cos^5(c+dx)\sin(c+dx)}{6d}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{5b\cos(c+dx)\sin(c+dx)}{16d} + \frac{5b\cos^3(c+dx)\sin(c+dx)}{24d}$$
$$= \frac{5bx}{16} + \frac{a\sin(c+dx)}{d} + \frac{5b\cos(c+dx)\sin(c+dx)}{16d} + \frac{5b\cos^3(c+dx)\sin(c}{24d}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.68

$$\frac{192a\sin^5(c+dx) - 640a\sin^3(c+dx) + 960a\sin(c+dx) + 5b(45\sin(2(c+dx)) + 9\sin(4(c+dx)) + \sin(6(c+d}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Cos[c + d*x]),x]
```

```
[Out] (960*a*Sin[c + d*x] - 640*a*Sin[c + d*x]^3 + 192*a*Sin[c + d*x]^5 + 5*b*(60
*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)]))
/(960*d)
```

fricas [A] time = 0.99, size = 75, normalized size = 0.66

$$\frac{75\,bdx + \left(40\,b\cos(dx+c)^5 + 48\,a\cos(dx+c)^4 + 50\,b\cos(dx+c)^3 + 64\,a\cos(dx+c)^2 + 75\,b\cos(dx+c) + 12\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(75*b*d*x + (40*b*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 + 50*b*cos(d*x
 + c)^3 + 64*a*cos(d*x + c)^2 + 75*b*cos(d*x + c) + 128*a)*sin(d*x + c))/d
```

giac [A] time = 0.50, size = 92, normalized size = 0.81

$$\frac{5}{16}\,bx + \frac{b\sin(6\,dx+6\,c)}{192\,d} + \frac{a\sin(5\,dx+5\,c)}{80\,d} + \frac{3\,b\sin(4\,dx+4\,c)}{64\,d} + \frac{5\,a\sin(3\,dx+3\,c)}{48\,d} + \frac{15\,b\sin(2\,dx+2\,c)}{64\,d} + \frac{5\,a\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*b*x + 1/192*b*sin(6*d*x + 6*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 3/64*b*
sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 15/64*b*sin(2*d*x + 2*c)/d
 + 5/8*a*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 80, normalized size = 0.70

$$\frac{b\left(\frac{\left(\cos^5(dx+c)+\frac{5(\cos^3(dx+c))}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(b*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.58, size = 84, normalized size = 0.74

$$\frac{64\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4\right.}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a - 5*(4
*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2
*c))*b)/d
```

mupad [B] time = 0.66, size = 115, normalized size = 1.01

$$\frac{5bx}{16}+\frac{8a\sin(c+dx)}{15d}+\frac{5b\cos(c+dx)\sin(c+dx)}{16d}+\frac{4a\cos(c+dx)^2\sin(c+dx)}{15d}+\frac{a\cos(c+dx)^4\sin(c+}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*cos(c + d*x)),x)
```

```
[Out] (5*b*x)/16 + (8*a*sin(c + d*x))/(15*d) + (5*b*cos(c + d*x)*sin(c + d*x))/(1
6*d) + (4*a*cos(c + d*x)^2*sin(c + d*x))/(15*d) + (a*cos(c + d*x)^4*sin(c +
 d*x))/(5*d) + (5*b*cos(c + d*x)^3*sin(c + d*x))/(24*d) + (b*cos(c + d*x)^5
*sin(c + d*x))/(6*d)
```

sympy [A] time = 3.27, size = 216, normalized size = 1.89

$$\begin{cases} \frac{8a\sin^5(c+dx)}{15d} + \frac{4a\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^4(c+dx)}{d} + \frac{5bx\sin^6(c+dx)}{16} + \frac{15bx\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15bx\sin^2(c+dx)\cos}{16} \\ x(a+b\cos(c))\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d + 5*b*x*sin(c + d*x)**6/16 + 15*b
*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b*x*sin(c + d*x)**2*cos(c + d*x)
**4/16 + 5*b*x*cos(c + d*x)**6/16 + 5*b*sin(c + d*x)**5*cos(c + d*x)/(16*d)
 + 5*b*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b*sin(c + d*x)*cos(c + d*
x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**5, True))
```

3.406 $\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=92

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] 3/8*a*x+b*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin
(d*x+c)/d-2/3*b*sin(d*x+c)^3/d+1/5*b*sin(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2635, 8, 2633}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Cos[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (b*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*b*Sin[c + d*x]^3)/(3*d) + (b*Sin[c
 + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + b\int \cos^5(c+dx)\,dx$$

$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int \left(1-2\right.\right.}{}$$

$$= \frac{b\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} -$$

$$= \frac{3ax}{8} + \frac{b\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+}{4d}$$

Mathematica [A] time = 0.13, size = 89, normalized size = 0.97

$$\frac{3a(c+dx)}{8d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Cos[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (2*b*Sin[c + d*x]^3)/(3*d) + (
b*Sin[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])
/(32*d)
```

fricas [A] time = 0.76, size = 64, normalized size = 0.70

$$\frac{45\,adx + \left(24\,b\cos\left(dx+c\right)^4 + 30\,a\cos\left(dx+c\right)^3 + 32\,b\cos\left(dx+c\right)^2 + 45\,a\cos\left(dx+c\right) + 64\,b\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(45*a*d*x + (24*b*cos(d*x + c)^4 + 30*a*cos(d*x + c)^3 + 32*b*cos(d*x
 + c)^2 + 45*a*cos(d*x + c) + 64*b)*sin(d*x + c))/d
```

giac [A] time = 0.48, size = 77, normalized size = 0.84

$$\frac{3}{8}\,ax+\frac{b\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{5\,b\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{5\,b\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x + 1/80*b*sin(5*d*x + 5*c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 5/48*b*si
n(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 5/8*b*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 70, normalized size = 0.76

$$\frac{\frac{b\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.47, size = 69, normalized size = 0.75

$$\frac{15\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a+32\left(3\,\sin(dx+c)^{5}-10\,\sin(dx+c)^{3}+15\,\sin(dx+c\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a + 32*(3
*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*b)/d
```

mupad [B] time = 4.26, size = 115, normalized size = 1.25

$$\frac{3\,a\,x}{8}+\frac{\left(2\,b-\frac{5\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{8\,b}{3}-\frac{a}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\frac{116\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{15}+\left(\frac{a}{2}+\frac{8\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\left(\frac{5\,a}{4}+2\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (tan(c/2 + (d*x)/2)*((5*a)/4 + 2*b) + tan(c/2 + (d*x)/2)^3*(a/2
 + (8*b)/3) - tan(c/2 + (d*x)/2)^9*((5*a)/4 - 2*b) - tan(c/2 + (d*x)/2)^7*(
```

```
a/2 - (8*b)/3) + (116*b*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2)^2
+ 1)^5)
```

sympy [A] time = 1.86, size = 168, normalized size = 1.83

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{8b\sin^5(c+dx)}{15d} + \\ x(a+b\cos(c))\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*b*sin(c + d*x)**5/(15*d) + 4*b*sin(c
 + d*x)**3*cos(c + d*x)**2/(3*d) + b*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d,
0)), (x*(a + b*cos(c))*cos(c)**4, True))
```

3.407 $\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] 3/8*b*x+a*sin(d*x+c)/d+3/8*b*cos(d*x+c)*sin(d*x+c)/d+1/4*b*cos(d*x+c)^3*sin
(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2633, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x]),x]
```

```
[Out] (3*b*x)/8 + (a*Sin[c + d*x])/d + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^3(c+dx)\,dx + b\int \cos^4(c+dx)\,dx$$
$$= \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3b)\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1-x\right.\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} -$$
$$= \frac{3bx}{8} + \frac{a\sin(c+dx)}{d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c}{4d}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 0.96

$$-\frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} + \frac{3b(c+dx)}{8d} + \frac{b\sin(2(c+dx))}{4d} + \frac{b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x]),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*
Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 1.03, size = 53, normalized size = 0.70

$$\frac{9\,bdx + \left(6\,b\cos\left(dx+c\right)^3 + 8\,a\cos\left(dx+c\right)^2 + 9\,b\cos\left(dx+c\right) + 16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*b*d*x + (6*b*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*b*cos(d*x + c)
 + 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.49, size = 62, normalized size = 0.82

$$\frac{3}{8}\,bx + \frac{b\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{b\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{3\,a\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/32*b*sin(4*d*x + 4*c)/d + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*b*sin
(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 60, normalized size = 0.79

$$\frac{b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.90, size = 57, normalized size = 0.75

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)a-3\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d
```

mupad [B] time = 0.58, size = 75, normalized size = 0.99

$$\frac{3\,b\,x}{8}+\frac{2\,a\,\sin(c+d\,x)}{3\,d}+\frac{3\,b\,\cos(c+d\,x)\,\sin(c+d\,x)}{8\,d}+\frac{a\cos(c+d\,x)^2\,\sin(c+d\,x)}{3\,d}+\frac{b\cos(c+d\,x)^3\,\sin(c+d}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x)),x)
```

```
[Out] (3*b*x)/8 + (2*a*sin(c + d*x))/(3*d) + (3*b*cos(c + d*x)*sin(c + d*x))/(8*d
) + (a*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (b*cos(c + d*x)^3*sin(c + d*x))
/(4*d)
```

sympy [A] time = 0.92, size = 144, normalized size = 1.89

$$\begin{cases} \frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3bx\sin^4(c+dx)}{8}+\frac{3bx\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3bx\cos^4(c+dx)}{8}+\frac{3b\sin^3(c+dx)\cos(c+dx)}{8d}+ \\ x\left(a+b\cos(c)\right)\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d + 3
*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x*co
s(c + d*x)**4/8 + 3*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b*sin(c + d*x)
*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**3, True))
```

3.408 $\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] 1/2*a*x+b*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d-1/3*b*sin(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 2635, 8, 2633}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*x)/2 + (b*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (b*Sin
[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx &= a\int \cos^2(c+dx)\,dx+b\int \cos^3(c+dx)\,dx\\
&= \frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx-\frac{b\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(\right.}{d}\\
&= \frac{ax}{2}+\frac{b\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{b\sin^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 57, normalized size = 1.06

$$
\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a*Si
n[2*(c + d*x)])/(4*d)
```

fricas [A] time = 1.19, size = 42, normalized size = 0.78

$$
\frac{3\,adx+\left(2\,b\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+4\,b\right)\sin\left(dx+c\right)}{6\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*b*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*b)*sin(d*x + c))/
d
```

giac [A] time = 0.44, size = 47, normalized size = 0.87

$$
\frac{1}{2}\,ax+\frac{b\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{3\,b\sin\left(dx+c\right)}{4\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/12*b*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*b*sin(
d*x + c)/d
```

maple [A] time = 0.04, size = 49, normalized size = 0.91

$$\frac{\frac{b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*b*(2+cos(d*x+c)^2)*sin(d*x+c)+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

maxima [A] time = 0.50, size = 46, normalized size = 0.85

$$\frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a-4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)b}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a - 4*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*b)/d
```

mupad [B] time = 0.58, size = 55, normalized size = 1.02

$$\frac{a\,x}{2}+\frac{2\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x)),x)
```

```
[Out] (a*x)/2 + (2*b*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) +
(b*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

sympy [A] time = 0.46, size = 92, normalized size = 1.70

$$\begin{cases} \frac{ax\sin^2\left(c+dx\right)}{2}+\frac{ax\cos^2\left(c+dx\right)}{2}+\frac{a\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{2b\sin^3\left(c+dx\right)}{3d}+\frac{b\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x\left(a+b\cos\left(c\right)\right)\cos^2\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) + 2*b*sin(c + d*x)**3/(3*d) + b*sin(c + d*x)*cos(c + d*x)
**2/d, Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**2, True))
```

3.409 $\int \cos(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

```
[Out] 1/2*b*x+a*sin(d*x+c)/d+1/2*b*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2734}

$$\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x]),x]
```

```
[Out] (b*x)/2 + (a*Sin[c + d*x])/d + (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx=\frac{bx}{2}+\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 0.92

$$\frac{4a\sin(c+dx)+b(2(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x]),x]
```

```
[Out] (4*a*Sin[c + d*x] + b*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 0.90, size = 29, normalized size = 0.76

$$\frac{bdx + (b\cos(dx+c) + 2a)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(b*d*x + (b*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.43, size = 31, normalized size = 0.82

$$\frac{1}{2}bx + \frac{b\sin(2dx+2c)}{4d} + \frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] 1/2*b*x + 1/4*b*sin(2*d*x + 2*c)/d + a*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{b\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + \sin(dx+c)\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c)),x)
[Out] 1/d*(b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+sin(d*x+c)*a)
```

maxima [A] time = 0.65, size = 34, normalized size = 0.89

$$\frac{(2dx + 2c + \sin(2dx+2c))b + 4a\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*b + 4*a*sin(d*x + c))/d
```

mupad [B] time = 0.52, size = 31, normalized size = 0.82

$$\frac{b\,x}{2} + \frac{b\,\sin(2\,c+2\,d\,x)}{4\,d} + \frac{a\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x)),x)
```

```
[Out] (b*x)/2 + (b*sin(2*c + 2*d*x))/(4*d) + (a*sin(c + d*x))/d
```

sympy **[A]** time = 0.20, size = 66, normalized size = 1.74

$$\begin{cases} \frac{a\sin(c+dx)}{d} + \frac{bx\sin^2(c+dx)}{2} + \frac{bx\cos^2(c+dx)}{2} + \frac{b\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a+b\cos(c))\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)/d + b*x*sin(c + d*x)**2/2 + b*x*cos(c + d*x)**2/2
 + b*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c),
True))
```

3.410 $\int (a+b\cos(c+dx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\sin(c+dx)}{d}$$

[Out] a*x+b*sin(d*x+c)/d

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2637}

$$ax+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Cos[c + d*x],x]

[Out] a*x + (b*Sin[c + d*x])/d

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\,dx &= ax+b\int \cos(c+dx)\,dx\\ &= ax+\frac{b\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.73

$$ax+\frac{b\sin(c)\cos(dx)}{d}+\frac{b\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Cos[c + d*x],x]

[Out] a*x + (b*Cos[d*x]*Sin[c])/d + (b*Cos[c]*Sin[d*x])/d

fricas [A] time = 0.76, size = 17, normalized size = 1.13

$$\frac{adx+b\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c),x, algorithm="fricas")
[Out] (a*d*x + b*sin(d*x + c))/d
```

giac [A] time = 0.33, size = 15, normalized size = 1.00

$$ax + \frac{b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c),x, algorithm="giac")
[Out] a*x + b*sin(d*x + c)/d
```

maple [A] time = 0.02, size = 16, normalized size = 1.07

$$ax + \frac{b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cos(d*x+c),x)
[Out] a*x+b*sin(d*x+c)/d
```

maxima [A] time = 0.67, size = 15, normalized size = 1.00

$$ax + \frac{b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c),x, algorithm="maxima")
[Out] a*x + b*sin(d*x + c)/d
```

mupad [B] time = 0.47, size = 17, normalized size = 1.13

$$\frac{b\,\sin(c + d\,x) + a\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cos(c + d*x),x)
[Out] (b*sin(c + d*x) + a*d*x)/d
```

sympy [A] time = 0.12, size = 17, normalized size = 1.13

$$ax + b\left(\begin{cases} \frac{\sin{(c+dx)}}{d} & \text{for}\ d \neq 0 \\ x\cos{(c)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))
```

3.411 $\int (a+b\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=16

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+bx$$

```
[Out] b*x+a*arctanh(sin(d*x+c))/d
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2735, 3770}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] b*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\sec(c+dx)\,dx &= bx+a\int \sec(c+dx)\,dx\\ &= bx+\frac{a\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] b*x + (a*ArcTanh[Sin[c + d*x]])/d
```

fricas [B] time = 0.89, size = 36, normalized size = 2.25

$$\frac{2\,bdx + a\log\left(\sin\left(dx + c\right) + 1\right) - a\log\left(-\sin\left(dx + c\right) + 1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

giac [B] time = 0.75, size = 43, normalized size = 2.69

$$\frac{(dx + c)b + a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] ((d*x + c)*b + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)))/d
```

maple [A] time = 0.06, size = 30, normalized size = 1.88

$$bx + \frac{a\ln\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d} + \frac{bc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] b*x+1/d*a*ln(sec(d*x+c)+tan(d*x+c))+b*c/d
```

maxima [A] time = 0.32, size = 28, normalized size = 1.75

$$\frac{(dx + c)b + a\log\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*b + a*log(sec(d*x + c) + tan(d*x + c)))/d
```

mupad [B] time = 0.54, size = 57, normalized size = 3.56

$$\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/cos(c + d*x),x)
```

```
[Out] (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atan(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [B] time = 4.98, size = 49, normalized size = 3.06

$$a\left(\begin{cases} \frac{x\tan(c)\sec(c)}{\tan(c)+\sec(c)}+\frac{x\sec^2(c)}{\tan(c)+\sec(c)} & \text{for } d=0 \\ \frac{\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{otherwise} \end{cases}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*Piecewise((x*tan(c)*sec(c)/(tan(c) + sec(c)) + x*sec(c)**2/(tan(c) + sec(
c)), Eq(d, 0)), (log(tan(c + d*x) + sec(c + d*x))/d, True)) + b*x
```

3.412 $\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=24

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3767, 8, 3770}

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx = a\int \sec^2(c+dx)\,dx + b\int \sec(c+dx)\,dx$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} + \frac{a\tan(c+dx)}{d}$$

Mathematica [**A**] time = 0.01, size = 24, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

fricas [**B**] time = 1.01, size = 60, normalized size = 2.50

$$\frac{b\cos(dx+c)\log(\sin(dx+c)+1) - b\cos(dx+c)\log(-\sin(dx+c)+1) + 2a\sin(dx+c)}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*cos(d*x + c)*log(sin(d*x + c) + 1) - b*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

giac [**B**] time = 0.53, size = 63, normalized size = 2.62

$$\frac{b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.07, size = 32, normalized size = 1.33

$$\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^2,x)
[Out] 1/d*b*ln(sec(d*x+c)+tan(d*x+c))+a*tan(d*x+c)/d
```

maxima [A] time = 0.70, size = 38, normalized size = 1.58

$$\frac{b\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*a*tan(d*x + c))/
d
```

mupad [B] time = 0.51, size = 47, normalized size = 1.96

$$\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/cos(c + d*x)^2,x)
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
(d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\cos\left(c+dx\right)\right)\sec^2\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**2,x)
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.413 $\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=47

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2748, 3768, 3770, 3767, 8}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx = a\int \sec^3(c+dx)\,dx + b\int \sec^2(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{b\,\mathrm{Subst}(\int 1\,dx, x, -$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^3,x]
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

fricas [A] time = 0.94, size = 74, normalized size = 1.57

$$\frac{a\cos(dx+c)^2\log(\sin(dx+c)+1) - a\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2(2b\cos(dx+c)+a)\sin(dx+}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*b*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.61, size = 105, normalized size = 2.23

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2b\,t\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/
2*d*x + 1/2*c) + 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/
d
```

maple [A] time = 0.08, size = 51, normalized size = 1.09

$$\frac{a\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{b\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+b*tan(d*x+c
)/d
```

maxima [A] time = 0.34, size = 58, normalized size = 1.23

$$-\frac{a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,b\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*b*tan(d*x + c))/d
```

mupad [B] time = 1.13, size = 81, normalized size = 1.72

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\left(a-2\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(a+2\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^3*(a - 2*b) + tan(c/2
 + (d*x)/2)*(a + 2*b))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 +
1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))\sec^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.414 $\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=63

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d+1/
3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3767, 3768, 3770}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx = a\int \sec^4(c+dx)\,dx + b\int \sec^3(c+dx)\,dx$$
$$= \frac{b\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}b\int \sec(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1+x^2\right)\right.}{}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{b\sec(c+dx)\tan(c+dx)}{2d} + \frac{a\,\mathrm{t}}{}$$

Mathematica [A] time = 0.17, size = 60, normalized size = 0.95

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.79, size = 88, normalized size = 1.40

$$\frac{3\,b\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right) - 3\,b\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(4\,a\cos\left(dx+c\right)^2 + 3\,b\cos\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*b*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 2*a)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.52, size = 122, normalized size = 1.94

$$\frac{3\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 - 4*
a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.08, size = 72, normalized size = 1.14

$$\frac{2a\tan\left(dx+c\right)}{3d}+\frac{a\tan\left(dx+c\right)\left(\sec^{2}\left(dx+c\right)\right)}{3d}+\frac{b\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/2*b*sec(d*x+c)*tan(d*x
+c)/d+1/2/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.53, size = 70, normalized size = 1.11

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,b\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.34, size = 111, normalized size = 1.76

$$\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\left(2\,a-b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-\frac{4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+\left(2\,a+b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a - b) + tan(c/2
 + (d*x)/2)*(2*a + b) - (4*a*tan(c/2 + (d*x)/2)^3)/3)/(d*(3*tan(c/2 + (d*x)
/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))\sec^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**4, x)
```

3.415 $\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1/
4*a*sec(d*x+c)^3*tan(d*x+c)/d+1/3*b*tan(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3768, 3770, 3767}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*
x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx = a\int \sec^5(c+dx)\,dx + b\int \sec^4(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{b\,\mathrm{Subst}\left(\int \left(1+\right.\right.}{}$$
$$= \frac{b\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} +$$

Mathematica [A] time = 0.23, size = 76, normalized size = 0.89

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\sec(c+dx)\right)}{8d} + \frac{b\left(\frac{1}{3}\tan^3(c+dx) + \tan(c+dx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (b*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 1.04, size = 99, normalized size = 1.16

$$\frac{9\,a\cos(dx+c)^4\log(\sin(dx+c)+1) - 9\,a\cos(dx+c)^4\log(-\sin(dx+c)+1) + 2\left(16\,b\cos(dx+c)^3 + 9\,a\cos\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*b*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*b*cos(d*x
 + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.69, size = 164, normalized size = 1.93

$$\frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x + 1/2*c)^7 +
 9*a*tan(1/2*d*x + 1/2*c)^5 + 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*tan(1/2*d*x
 + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c) + 24*
b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.09, size = 92, normalized size = 1.08

$$\frac{a\left(\sec^{3}\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3a\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{2b\tan\left(dx+c\right)}{3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+2/3*b*tan(d*x+c)/d+1/3/d*b*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.63, size = 95, normalized size = 1.12

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)b-3\,a\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.08, size = 150, normalized size = 1.76

$$\frac{\left(\frac{5\,a}{4}-2\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\left(\frac{3\,a}{4}+\frac{10\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\left(\frac{3\,a}{4}-\frac{10\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\left(\frac{5\,a}{4}+2\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}+\frac{3\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((5*a)/4 + 2*b) + tan(c/2 + (d*x)/2)^7*((5*a)/4 - 2*b)
+ tan(c/2 + (d*x)/2)^3*((3*a)/4 - (10*b)/3) + tan(c/2 + (d*x)/2)^5*((3*a)/4
 + (10*b)/3))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c
/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2
)))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))\sec^5(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**5, x)
```

3.416 $\int (a + b\cos(c + dx))\sec^6(c + dx)\,dx$

Optimal. Leaf size=101

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c+$$

```
[Out] 3/8*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+1/
4*b*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5/d
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2748, 3767, 3768, 3770}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*b*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*a*Tan[c +
d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx = a\int \sec^6(c+dx)\,dx + b\int \sec^5(c+dx)\,dx$$
$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3b)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+\right.\right.}{}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d}$$
$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} +$$

Mathematica [A] time = 0.33, size = 88, normalized size = 0.87

$$\frac{a\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\left(\tanh^{-1}(\sin(c+dx))+\tan(c\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*b*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Ta
n[c + d*x]^5/5))/d
```

fricas [A] time = 1.00, size = 110, normalized size = 1.09

$$\frac{45\,b\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\,b\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(64\,a\cos\left(dx+c\right)^4+45\,b\right.}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(45*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*b*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 2*(64*a*cos(d*x + c)^4 + 45*b*cos(d*x + c)^3 + 32*a*co
s(d*x + c)^2 + 30*b*cos(d*x + c) + 24*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [A] time = 0.53, size = 178, normalized size = 1.76

$$\frac{45\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-160\,a\tan\left(\frac{1}{2}\right.\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(45*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x + 1/2*c)
^9 - 160*a*tan(1/2*d*x + 1/2*c)^7 + 30*b*tan(1/2*d*x + 1/2*c)^7 + 464*a*tan
(1/2*d*x + 1/2*c)^5 - 160*a*tan(1/2*d*x + 1/2*c)^3 - 30*b*tan(1/2*d*x + 1/2
*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.09, size = 112, normalized size = 1.11

$$\frac{8a\tan(dx+c)}{15d}+\frac{a\tan(dx+c)\left(\sec^{4}(dx+c)\right)}{5d}+\frac{4a\tan(dx+c)\left(\sec^{2}(dx+c)\right)}{15d}+\frac{b\left(\sec^{3}(dx+c)\right)\tan(dx+c)}{4d}+\frac{3b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 8/15*a*tan(d*x+c)/d+1/5/d*a*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a*tan(d*x+c)*sec
(d*x+c)^2+1/4*b*sec(d*x+c)^3*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+3/8
/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.69, size = 107, normalized size = 1.06

$$\frac{16\left(3\tan(dx+c)^{5}+10\tan(dx+c)^{3}+15\tan(dx+c)\right)a-15\,b\left(\frac{2\left(3\sin(dx+c)^{3}-5\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\sin(dx+c)^{2}+1}-3\log(\sin(dx+c)+1\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 15*b
*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
+ 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.12, size = 180, normalized size = 1.78

$$\frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\left(2\,a-\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{b}{2}-\frac{8\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{116\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\left(-\frac{8\,a}{3}-\frac{b}{2}\right)\,\mathrm{ta}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] (3*b*atanh(tan(c/2 + (d*x)/2)))/(4*d) - (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4)
 - tan(c/2 + (d*x)/2)^3*((8*a)/3 + b/2) + tan(c/2 + (d*x)/2)^9*(2*a - (5*b)
/4) - tan(c/2 + (d*x)/2)^7*((8*a)/3 - b/2) + (116*a*tan(c/2 + (d*x)/2)^5)/1
5)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x
)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**6, x)
```

3.417 $\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=150

$$\frac{(6a^2+5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+5b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(6a^2+5b^2\right)+\frac{2ab\sin^5(c+dx)}{5d}-$$

```
[Out] 1/16*(6*a^2+5*b^2)*x+2*a*b*sin(d*x+c)/d+1/16*(6*a^2+5*b^2)*cos(d*x+c)*sin(d
*x+c)/d+1/24*(6*a^2+5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*b^2*cos(d*x+c)^5*s
in(d*x+c)/d-4/3*a*b*sin(d*x+c)^3/d+2/5*a*b*sin(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2633, 3014, 2635, 8}

$$\frac{(6a^2+5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+5b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(6a^2+5b^2\right)+\frac{2ab\sin^5(c+dx)}{5d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + 5*b^2)*x)/16 + (2*a*b*Sin[c + d*x])/d + ((6*a^2 + 5*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(16*d) + ((6*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(
24*d) + (b^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (4*a*b*Sin[c + d*x]^3)/(3
*d) + (2*a*b*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^5(c+dx)\,dx + \int \cos^4(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$

$$= \frac{b^2\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}\left(6a^2+5b^2\right)\int \cos^4(c+dx)\,dx - \frac{(2ab)}{\quad}$$

$$= \frac{2ab\sin(c+dx)}{d} + \frac{\left(6a^2+5b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{b^2\cos^5(c+}{\quad}$$

$$= \frac{2ab\sin(c+dx)}{d} + \frac{\left(6a^2+5b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{\left(6a^2+5b^2\right)\text{c}}{\quad}$$

$$= \frac{1}{16}\left(6a^2+5b^2\right)x + \frac{2ab\sin(c+dx)}{d} + \frac{\left(6a^2+5b^2\right)\cos(c+dx)\sin(c+}{16d}$$

Mathematica [A] time = 0.31, size = 123, normalized size = 0.82

$$\frac{5\left(\left(48a^2+45b^2\right)\sin(2(c+dx))+\left(6a^2+9b^2\right)\sin(4(c+dx))+72a^2c+72a^2dx+b^2\sin(6(c+dx))+60b^2c+6}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (1920*a*b*Sin[c + d*x] - 1280*a*b*Sin[c + d*x]^3 + 384*a*b*Sin[c + d*x]^5 +
 5*(72*a^2*c + 60*b^2*c + 72*a^2*d*x + 60*b^2*d*x + (48*a^2 + 45*b^2)*Sin[2
*(c + d*x)] + (6*a^2 + 9*b^2)*Sin[4*(c + d*x)] + b^2*Sin[6*(c + d*x)]))/(96
0*d)
```

fricas [A] time = 0.98, size = 110, normalized size = 0.73

$$\frac{15\left(6a^2+5b^2\right)dx+\left(40b^2\cos(dx+c)^5+96ab\cos(dx+c)^4+128ab\cos(dx+c)^2+10\left(6a^2+5b^2\right)\cos(dx+}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(6*a^2 + 5*b^2)*d*x + (40*b^2*cos(d*x + c)^5 + 96*a*b*cos(d*x + c
)^4 + 128*a*b*cos(d*x + c)^2 + 10*(6*a^2 + 5*b^2)*cos(d*x + c)^3 + 256*a*b
+ 15*(6*a^2 + 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.55, size = 127, normalized size = 0.85

$$\frac{1}{16}\left(6\,a^2+5\,b^2\right)x+\frac{b^2\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{ab\sin\left(5\,dx+5\,c\right)}{40\,d}+\frac{5\,ab\sin\left(3\,dx+3\,c\right)}{24\,d}+\frac{5\,ab\sin\left(dx+c\right)}{4\,d}+\frac{\left(2\,a^2+3\,b^2\right)\sin}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(6*a^2 + 5*b^2)*x + 1/192*b^2*sin(6*d*x + 6*c)/d + 1/40*a*b*sin(5*d*x
+ 5*c)/d + 5/24*a*b*sin(3*d*x + 3*c)/d + 5/4*a*b*sin(d*x + c)/d + 1/64*(2*a
^2 + 3*b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^2 + 15*b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.05, size = 120, normalized size = 0.80

$$\frac{b^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2ab\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(d}{2}\right.}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+2/5*a*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*(
1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.53, size = 120, normalized size = 0.80

$$\frac{30\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^2+128\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+\right.\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/960*(30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 + 128
*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a*b - 5*(4*sin(2*
```

```
d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^
2)/d
```

mupad [B] time = 0.67, size = 143, normalized size = 0.95

$$\frac{3\,a^2\,x}{8}+\frac{5\,b^2\,x}{16}+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{15\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{b^2\,\sin\left(6\right.}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/8 + (5*b^2*x)/16 + (a^2*sin(2*c + 2*d*x))/(4*d) + (a^2*sin(4*c +
4*d*x))/(32*d) + (15*b^2*sin(2*c + 2*d*x))/(64*d) + (3*b^2*sin(4*c + 4*d*x)
)/(64*d) + (b^2*sin(6*c + 6*d*x))/(192*d) + (5*a*b*sin(c + d*x))/(4*d) + (5
*a*b*sin(3*c + 3*d*x))/(24*d) + (a*b*sin(5*c + 5*d*x))/(40*d)
```

sympy [A] time = 4.02, size = 343, normalized size = 2.29

$$\begin{cases} \frac{3a^2x\sin^4(c+dx)}{8}+\frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^2x\cos^4(c+dx)}{8}+\frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{16ab\sin^5(}{15d} \\ x\left(a+b\cos\left(c\right)\right)^2\cos^4\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 16*a*b*sin(c + d*x)**5/
(15*d) + 8*a*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 2*a*b*sin(c + d*x)*c
os(c + d*x)**4/d + 5*b**2*x*sin(c + d*x)**6/16 + 15*b**2*x*sin(c + d*x)**4*
cos(c + d*x)**2/16 + 15*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**2*
x*cos(c + d*x)**6/16 + 5*b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*b**2*
sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b**2*sin(c + d*x)*cos(c + d*x)**
5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**4, True))
```

3.418 $\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=111

$$-\frac{(a^2+2b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{3abx}{4}$$

```
[Out] 3/4*a*b*x+(a^2+b^2)*sin(d*x+c)/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/2*a*b*co
s(d*x+c)^3*sin(d*x+c)/d-1/3*(a^2+2*b^2)*sin(d*x+c)^3/d+1/5*b^2*sin(d*x+c)^5
/d
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2635, 8, 3013, 373}

$$-\frac{(a^2+2b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{3abx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/4 + ((a^2 + b^2)*Sin[c + d*x])/d + (3*a*b*Cos[c + d*x]*Sin[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - ((a^2 + 2*b^2)*Sin[c
+ d*x]^3)/(3*d) + (b^2*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^4(c+dx)\,dx + \int \cos^3(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$

$$= \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}(3ab)\int \cos^2(c+dx)\,dx - \frac{\text{Subst}\left(\int \left(1\right.\right.}{}$$

$$= \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{4}(3ab)\int$$

$$= \frac{3abx}{4} + \frac{\left(a^2+b^2\right)\sin(c+dx)}{d} + \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(}{}$$

Mathematica [A] time = 0.14, size = 85, normalized size = 0.77

$$\frac{-80\left(a^2+2b^2\right)\sin^3(c+dx)+240\left(a^2+b^2\right)\sin(c+dx)+15ab(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (240*(a^2 + b^2)*Sin[c + d*x] - 80*(a^2 + 2*b^2)*Sin[c + d*x]^3 + 48*b^2*Si
n[c + d*x]^5 + 15*a*b*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/(240*d)
```

fricas [A] time = 0.96, size = 86, normalized size = 0.77

$$\frac{45\,abdx+\left(12\,b^2\cos\left(dx+c\right)^4+30\,ab\cos\left(dx+c\right)^3+45\,ab\cos\left(dx+c\right)+4\left(5\,a^2+4\,b^2\right)\cos\left(dx+c\right)^2+40\,a^2+}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(45*a*b*d*x + (12*b^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b*
cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 40*a^2 + 32*b^2)*sin(d*x
+ c))/d
```

giac [A] time = 0.55, size = 102, normalized size = 0.92

$$\frac{3}{4}\,abx+\frac{b^2\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{ab\sin\left(4\,dx+4\,c\right)}{16\,d}+\frac{ab\sin\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(4\,a^2+5\,b^2\right)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{\left(6\,a^2+5\,b^2\right)\sin}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/4*a*b*x + 1/80*b^2*sin(5*d*x + 5*c)/d + 1/16*a*b*sin(4*d*x + 4*c)/d + 1/2
*a*b*sin(2*d*x + 2*c)/d + 1/48*(4*a^2 + 5*b^2)*sin(3*d*x + 3*c)/d + 1/8*(6*
a^2 + 5*b^2)*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 95, normalized size = 0.86

$$\frac{\frac{b^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2ab\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*(2+cos(d*x+c)^2
)*sin(d*x+c))
```

maxima [A] time = 0.34, size = 94, normalized size = 0.85

$$-\frac{80\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)ab-16\left(3\,\sin\left(dx+\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(80*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 15*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 16*(3*sin(d*x + c)^5 - 10*sin(d*x
+ c)^3 + 15*sin(d*x + c))*b^2)/d
```

mupad [B] time = 0.61, size = 117, normalized size = 1.05

$$\frac{3\,a^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{5\,b^2\,\sin\left(c+d\,x\right)}{8\,d}+\frac{3\,a\,b\,x}{4}+\frac{a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{5\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{b^2\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{a\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*sin(c + d*x))/(4*d) + (5*b^2*sin(c + d*x))/(8*d) + (3*a*b*x)/4 + (a^
2*sin(3*c + 3*d*x))/(12*d) + (5*b^2*sin(3*c + 3*d*x))/(48*d) + (b^2*sin(5*c
 + 5*d*x))/(80*d) + (a*b*sin(2*c + 2*d*x))/(2*d) + (a*b*sin(4*c + 4*d*x))/(
16*d)
```

sympy [A] time = 2.02, size = 221, normalized size = 1.99

$$\begin{cases} \frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3abx\sin^4(c+dx)}{4}+\frac{3abx\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{3abx\cos^4(c+dx)}{4}+\frac{3ab\sin^3(c+dx)\cos(c+}{4d} \\ x\left(a+b\cos(c)\right)^2\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2
/d + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2
+ 3*a*b*x*cos(c + d*x)**4/4 + 3*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 5*
a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*b**2*sin(c + d*x)**5/(15*d) + 4*
b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + b**2*sin(c + d*x)*cos(c + d*x)
**4/d, Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**3, True))
```

3.419 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{(4a^2+3b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2+3b^2\right)-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

```
[Out] 1/8*(4*a^2+3*b^2)*x+2*a*b*sin(d*x+c)/d+1/8*(4*a^2+3*b^2)*cos(d*x+c)*sin(d*x
+c)/d+1/4*b^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2633, 3014, 2635, 8}

$$\frac{(4a^2+3b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2+3b^2\right)-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a^2 + 3*b^2)*x)/8 + (2*a*b*Sin[c + d*x])/d + ((4*a^2 + 3*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*b*
Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
```

```
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^3(c+dx)\,dx + \int \cos^2(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$
$$= \frac{b^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\left(4a^2+3b^2\right)\int \cos^2(c+dx)\,dx - \frac{(2ab)}{}$$
$$= \frac{2ab\sin(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{b^2\cos^3(c+d}{4}$$
$$= \frac{1}{8}\left(4a^2+3b^2\right)x + \frac{2ab\sin(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+d}{8d}$$

Mathematica [A] time = 0.16, size = 86, normalized size = 0.85

$$\frac{24\left(a^2+b^2\right)\sin(2(c+dx)) + 48a^2c + 48a^2dx - 64ab\sin^3(c+dx) + 192ab\sin(c+dx) + 3b^2\sin(4(c+dx)) + 36}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (48*a^2*c + 36*b^2*c + 48*a^2*d*x + 36*b^2*d*x + 192*a*b*Sin[c + d*x] - 64*
a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*b^2*Sin[4*(c + d*x
)])/(96*d)
```

fricas [A] time = 0.82, size = 77, normalized size = 0.76

$$\frac{3\left(4a^2+3b^2\right)dx + \left(6b^2\cos(dx+c)^3 + 16ab\cos(dx+c)^2 + 32ab + 3\left(4a^2+3b^2\right)\cos(dx+c)\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(4*a^2 + 3*b^2)*d*x + (6*b^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2
 + 32*a*b + 3*(4*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.46, size = 82, normalized size = 0.81

$$\frac{1}{8}\left(4\,a^{2}+3\,b^{2}\right)x+\frac{b^{2}\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{ab\sin\left(3\,dx+3\,c\right)}{6\,d}+\frac{3\,ab\sin\left(dx+c\right)}{2\,d}+\frac{\left(a^{2}+b^{2}\right)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2 + 3*b^2)*x + 1/32*b^2*sin(4*d*x + 4*c)/d + 1/6*a*b*sin(3*d*x + 3
*c)/d + 3/2*a*b*sin(d*x + c)/d + 1/4*(a^2 + b^2)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.05, size = 89, normalized size = 0.88

$$\frac{b^{2}\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+a^{2}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

maxima [A] time = 0.83, size = 82, normalized size = 0.81

$$\frac{24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}-64\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)ab+3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 64*(sin(d*x + c)^3 - 3*sin(
d*x + c))*a*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b
^2)/d
```

mupad [B] time = 0.60, size = 93, normalized size = 0.92

$$\frac{a^{2}\,x}{2}+\frac{3\,b^{2}\,x}{8}+\frac{a^{2}\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{b^{2}\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{b^{2}\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{3\,a\,b\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(3\,c+3\,d\,x\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (a^2*x)/2 + (3*b^2*x)/8 + (a^2*sin(2*c + 2*d*x))/(4*d) + (b^2*sin(2*c + 2*d
*x))/(4*d) + (b^2*sin(4*c + 4*d*x))/(32*d) + (3*a*b*sin(c + d*x))/(2*d) + (
a*b*sin(3*c + 3*d*x))/(6*d)
```

sympy [A] time = 1.06, size = 211, normalized size = 2.09

$$\begin{cases} \frac{a^2x\sin^2(c+dx)}{2} + \frac{a^2x\cos^2(c+dx)}{2} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{4ab\sin^3(c+dx)}{3d} + \frac{2ab\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3b^2x\sin^4(c+dx)}{8} + \frac{3b^2x\sin \\ x(a+b\cos(c))^2\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) + 4*a*b*sin(c + d*x)**3/(3*d) + 2*a*b*sin(c + d*
x)*cos(c + d*x)**2/d + 3*b**2*x*sin(c + d*x)**4/8 + 3*b**2*x*sin(c + d*x)**
2*cos(c + d*x)**2/4 + 3*b**2*x*cos(c + d*x)**4/8 + 3*b**2*sin(c + d*x)**3*c
os(c + d*x)/(8*d) + 5*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (
x*(a + b*cos(c))**2*cos(c)**2, True))
```

3.420 $\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=71

$$\frac{2(a^2+b^2)\sin(c+dx)}{3d}+\frac{\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+abx$$

```
[Out] a*b*x+2/3*(a^2+b^2)*sin(d*x+c)/d+1/3*a*b*cos(d*x+c)*sin(d*x+c)/d+1/3*(a+b*c
os(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$\frac{2(a^2+b^2)\sin(c+dx)}{3d}+\frac{\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] a*b*x + (2*(a^2 + b^2)*Sin[c + d*x])/(3*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]
)/(3*d) + ((a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx = \frac{(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}\int (2b+2a\cos(c+dx))(a+b\cos($$

$$= abx + \frac{2\left(a^2+b^2\right)\sin(c+dx)}{3d} + \frac{ab\cos(c+dx)\sin(c+dx)}{3d} + \frac{(a+b\cos}{}$$

Mathematica [A] time = 0.15, size = 59, normalized size = 0.83

$$\frac{3\left(4a^2+3b^2\right)\sin(c+dx)+b(12a(c+dx)+6a\sin(2(c+dx))+b\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 + 3*b^2)*Sin[c + d*x] + b*(12*a*(c + d*x) + 6*a*Sin[2*(c + d*x)]
+ b*Sin[3*(c + d*x)]))/(12*d)
```

fricas [A] time = 0.72, size = 52, normalized size = 0.73

$$\frac{3\,abdx+\left(b^2\cos\left(dx+c\right)^2+3\,ab\cos\left(dx+c\right)+3\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*d*x + (b^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 3*a^2 + 2*b^2)*
sin(d*x + c))/d
```

giac [A] time = 0.68, size = 60, normalized size = 0.85

$$abx+\frac{b^2\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{ab\sin\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(4\,a^2+3\,b^2\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a*b*x + 1/12*b^2*sin(3*d*x + 3*c)/d + 1/2*a*b*sin(2*d*x + 2*c)/d + 1/4*(4*a
^2 + 3*b^2)*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 63, normalized size = 0.89

$$\frac{\frac{b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+a^2*sin(d*x+c))
```

maxima [A] time = 0.69, size = 60, normalized size = 0.85

$$\frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))ab-2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)b^{2}+6\,a^{2}\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b - 2*(sin(d*x + c)^3 - 3*sin(d*x
 + c))*b^2 + 6*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.55, size = 72, normalized size = 1.01

$$\frac{a^2\,\sin\left(c+d\,x\right)}{d}+\frac{2\,b^2\,\sin\left(c+d\,x\right)}{3\,d}+a\,b\,x+\frac{b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x))/d + (2*b^2*sin(c + d*x))/(3*d) + a*b*x + (b^2*cos(c + d*
x)^2*sin(c + d*x))/(3*d) + (a*b*cos(c + d*x)*sin(c + d*x))/d
```

sympy [A] time = 0.48, size = 107, normalized size = 1.51

$$\begin{cases} \frac{a^{2}\sin\left(c+dx\right)}{d}+abx\sin^{2}\left(c+dx\right)+abx\cos^{2}\left(c+dx\right)+\frac{ab\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{2b^{2}\sin^{3}\left(c+dx\right)}{3d}+\frac{b^{2}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d} & \text{fo} \\ x\left(a+b\cos\left(c\right)\right)^{2}\cos\left(c\right) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)/d + a*b*x*sin(c + d*x)**2 + a*b*x*cos(c + d*x)
**2 + a*b*sin(c + d*x)*cos(c + d*x)/d + 2*b**2*sin(c + d*x)**3/(3*d) + b**2
*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c), Tr
ue))
```

3.421 $\int (a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] 1/2*(2*a^2+b^2)*x+2*a*b*sin(d*x+c)/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*
x])/(2*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 0.92

$$\frac{2\left(2a^2+b^2\right)(c+dx)+8ab\sin(c+dx)+b^2\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2,x]
```

[Out] (2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*x)])/(4*
d)

fricas [A] time = 0.91, size = 40, normalized size = 0.80

$$\frac{\left(2\,a^2+b^2\right)dx+\left(b^2\cos\left(dx+c\right)+4\,ab\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/2*((2*a^2 + b^2)*d*x + (b^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d

giac [A] time = 0.48, size = 43, normalized size = 0.86

$$\frac{1}{2}\left(2\,a^2+b^2\right)x+\frac{b^2\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{2\,ab\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] 1/2*(2*a^2 + b^2)*x + 1/4*b^2*sin(2*d*x + 2*c)/d + 2*a*b*sin(d*x + c)/d

maple [A] time = 0.04, size = 51, normalized size = 1.02

$$\frac{b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sin\left(dx+c\right)+a^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^2,x)

[Out] 1/d*(b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*sin(d*x+c)+a^2*(d*
x+c))

maxima [A] time = 0.77, size = 44, normalized size = 0.88

$$a^2x+\frac{\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)b^2}{4\,d}+\frac{2\,ab\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] a^2*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*b^2/d + 2*a*b*sin(d*x + c)/d

mupad [B] time = 0.53, size = 42, normalized size = 0.84

$$a^2\,x+\frac{b^2\,x}{2}+\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (b^2*x)/2 + (b^2*sin(2*c + 2*d*x))/(4*d) + (2*a*b*sin(c + d*x))/d
```

sympy [A] time = 0.25, size = 78, normalized size = 1.56

$$\begin{cases} a^2x+\frac{2ab\sin\left(c+dx\right)}{d}+\frac{b^2x\sin^2\left(c+dx\right)}{2}+\frac{b^2x\cos^2\left(c+dx\right)}{2}+\frac{b^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x\left(a+b\cos\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sin(c + d*x)/d + b**2*x*sin(c + d*x)**2/2 + b**2*
x*cos(c + d*x)**2/2 + b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(
a + b*cos(c))**2, True))
```

3.422 $\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx$

Optimal. Leaf size=33

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2abx+\frac{b^2\sin(c+dx)}{d}$$

```
[Out] 2*a*b*x+a^2*arctanh(sin(d*x+c))/d+b^2*sin(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2746, 2735, 3770}

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2abx+\frac{b^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x],x]
```

```
[Out] 2*a*b*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (b^2*Sin[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx = \frac{b^2\sin(c+dx)}{d} + \int \left(a^2+2ab\cos(c+dx)\right)\sec(c+dx)\,dx$$
$$= 2abx + \frac{b^2\sin(c+dx)}{d} + a^2\int \sec(c+dx)\,dx$$
$$= 2abx + \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{b^2\sin(c+dx)}{d}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.39

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + 2abx + \frac{b^2\sin(c)\cos(dx)}{d} + \frac{b^2\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x],x]
```

```
[Out] 2*a*b*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (b^2*Cos[d*x]*Sin[c])/d + (b^2*Co
s[c]*Sin[d*x])/d
```

fricas [A] time = 1.14, size = 52, normalized size = 1.58

$$\frac{4\,abdx + a^2\log\left(\sin\left(dx+c\right)+1\right) - a^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\,b^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) + 2
*b^2*sin(d*x + c))/d
```

giac [B] time = 0.54, size = 78, normalized size = 2.36

$$\frac{2\,(dx+c)ab + a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a*b + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

maple [A] time = 0.08, size = 49, normalized size = 1.48

$$2abx + \frac{b^2 \sin(dx + c)}{d} + \frac{a^2 \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{2abc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c),x)
```

```
[Out] 2*a*b*x+b^2*sin(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b*c
```

maxima [A] time = 0.60, size = 42, normalized size = 1.27

$$\frac{2(dx + c)ab + a^2 \log(\sec(dx + c) + \tan(dx + c)) + b^2 \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(d*x + c)*a*b + a^2*log(sec(d*x + c) + tan(d*x + c)) + b^2*sin(d*x + c))
/d
```

mupad [B] time = 0.55, size = 73, normalized size = 2.21

$$\frac{b^2 \sin(c + d\,x)}{d} + \frac{2\,a^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a\,b \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (b^2*sin(c + d*x))/d + (2*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))^2 \sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x), x)
```

3.423 $\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx$

Optimal. Leaf size=33

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+b^2x$$

```
[Out] b^2*x+2*a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2789, 3770, 3012, 8}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx = (2ab)\int \sec(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + b^2\int 1\,dx$$
$$= b^2x + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d}$$

Mathematica [A] time = 0.08, size = 32, normalized size = 0.97

$$\frac{a^2\tan(c+dx) + 2ab\tanh^{-1}(\sin(c+dx)) + b^2dx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*d*x + 2*a*b*ArcTanh[Sin[c + d*x]] + a^2*Tan[c + d*x])/d
```

fricas [B] time = 0.91, size = 74, normalized size = 2.24

$$\frac{b^2dx\cos(dx+c) + ab\cos(dx+c)\log(\sin(dx+c)+1) - ab\cos(dx+c)\log(-\sin(dx+c)+1) + a^2\sin(dx+c}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*
x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.65, size = 77, normalized size = 2.33

$$\frac{(dx+c)b^2 + 2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*b^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

maple [A] time = 0.08, size = 49, normalized size = 1.48

$$b^2x + \frac{a^2\tan(dx+c)}{d} + \frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{c\,b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x)
```

```
[Out] b^2*x+a^2*tan(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*c*b^2
```

maxima [A] time = 0.42, size = 48, normalized size = 1.45

$$\frac{(dx+c)b^2 + ab\Big(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\Big) + a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)*b^2 + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + a^2*
tan(d*x + c))/d
```

mupad [B] time = 0.57, size = 181, normalized size = 5.48

$$\frac{2\,b^2\,\mathrm{atan}\left(\frac{64\,b^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,a^2\,b^4+64\,b^6}+\frac{256\,a^2\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,a^2\,b^4+64\,b^6}\right)}{d} - \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{128\,a\,b^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512\,a^3\,b^3+128\,a\,b^5}+\frac{512\,a^3\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512\,a^3\,b^3+128\,a\,b^5}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] (2*b^2*atan((64*b^6*tan(c/2 + (d*x)/2))/(64*b^6 + 256*a^2*b^4) + (256*a^2*b
^4*tan(c/2 + (d*x)/2))/(64*b^6 + 256*a^2*b^4)))/d - (2*a^2*tan(c/2 + (d*x)/
2))/(d*(tan(c/2 + (d*x)/2)^2 - 1)) + (4*a*b*atanh((128*a*b^5*tan(c/2 + (d*x
)/2))/(128*a*b^5 + 512*a^3*b^3) + (512*a^3*b^3*tan(c/2 + (d*x)/2))/(128*a*b
^5 + 512*a^3*b^3)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**2, x)
```

3.424 $\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+2b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}$$

```
[Out] 1/2*(a^2+2*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)
*tan(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3767, 8, 3012, 3770}

$$\frac{(a^2+2b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (a^2
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx = (2ab)\int \sec^2(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(a^2+2b^2\right)\int \sec(c+dx)\,dx - \frac{(2ab)\,\mathrm{Su}}{}$$
$$= \frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan}{2d}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.14

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (b^2*ArcTanh[Sin[c + d*x]])/d + (2*a*b*
Tan[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.89, size = 93, normalized size = 1.58

$$\frac{\left(a^2+2\,b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left(a^2+2\,b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(4\,ab\cos\left(d\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 + 2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^2 + 2*b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + a^2)*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.62, size = 127, normalized size = 2.15

$$\frac{\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (a^2 + 2*b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^3 + a^2*tan(1/2*d*x + 1/2*c) + 4*a*b*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.10, size = 78, normalized size = 1.32

$$\frac{a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{2ab\tan\left(dx+c\right)}{d}+\frac{b^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^2*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b*t
an(d*x+c)/d+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.57, size = 87, normalized size = 1.47

$$-\frac{a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,b^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 8*a*b*tan(d*x + c))/d
```

mupad [B] time = 1.17, size = 99, normalized size = 1.68

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+2\,b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(4\,a\,b-a^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+4\,b\,a\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(a^2 + 2*b^2))/d - (tan(c/2 + (d*x)/2)^3*(4*a*b
- a^2) - tan(c/2 + (d*x)/2)*(4*a*b + a^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan
(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))^2 \sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**3,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**3, x)
```

3.425 $\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{(2a^2+3b^2)\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}$$

```
[Out] a*b*arctanh(sin(d*x+c))/d+1/3*(2*a^2+3*b^2)*tan(d*x+c)/d+a*b*sec(d*x+c)*tan
(d*x+c)/d+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2789, 3768, 3770, 3012, 3767, 8}

$$\frac{(2a^2+3b^2)\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + ((2*a^2 + 3*b^2)*Tan[c + d*x])/(3*d) + (a*b
*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx = (2ab)\int \sec^3(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d} + (ab)\int \sec($$
$$= \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\sec^2(c+dx)}{3d}$$
$$= \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan}{d}$$

Mathematica [A] time = 0.22, size = 71, normalized size = 0.89

$$\frac{a^2\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*Ta
n[c + d*x])/d + (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 1.10, size = 100, normalized size = 1.25

$$\frac{3\,ab\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,ab\cos\left(dx+c\right)+\left(2\,a^2\right.\right.}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (2*a^2 + 3*b^2)*cos(d*x + c)^2
 + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.56, size = 178, normalized size = 2.22

$$\frac{3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2*c
)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a^2*tan(1/2*d*x + 1/2*c)^3 - 6*b^2*t
an(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/
2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.11, size = 89, normalized size = 1.11

$$\frac{2a^2\tan(dx+c)}{3d}+\frac{a^2\left(\sec^2(dx+c)\right)\tan(dx+c)}{3d}+\frac{ab\sec(dx+c)\tan(dx+c)}{d}+\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a^2*tan(d*x+c)/d+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d+a*b*sec(d*x+c)*tan(d
*x+c)/d+1/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*tan(d*x+c)
```

maxima [A] time = 0.33, size = 84, normalized size = 1.05

$$\frac{2\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^2-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6\,b^2\tan}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a*b*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*b^2*tan
(d*x + c))/d
```

mupad [B] time = 2.59, size = 141, normalized size = 1.76

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\left(2\,a^2-2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{4\,a^2}{3}-4\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2+2\,a\,b+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] (2*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*a*b
+ 2*b^2) - tan(c/2 + (d*x)/2)^3*((4*a^2)/3 + 4*b^2) + tan(c/2 + (d*x)/2)*(2
*a*b + 2*a^2 + 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))^2 \sec^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**4,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**4, x)
```

3.426 $\int (a+b\cos(c+dx))^2 \sec^5(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{(3a^2+4b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2+4b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{2ab\tan^3(c+$$

```
[Out] 1/8*(3*a^2+4*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/8*(3*a^2+4*b^2
)*sec(d*x+c)*tan(d*x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*b*tan(d*x
+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3767, 3012, 3768, 3770}

$$\frac{(3a^2+4b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2+4b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{2ab\tan^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*Tan[c + d*x])/d + ((
3*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (2*a*b*Tan[c + d*x]^3)/(3*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx = (2ab)\int \sec^4(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$

$$= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(3a^2+4b^2\right)\int \sec^3(c+dx)\,dx - \frac{(2ab)}{}$$

$$= \frac{2ab\tan(c+dx)}{d} + \frac{\left(3a^2+4b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2\sec^3(c+d}{4}$$

$$= \frac{\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2ab\tan(c+dx)}{d} + \frac{\left(3a^2+4b^2\right)\sec(}{}$$

Mathematica [A] time = 0.28, size = 82, normalized size = 0.75

$$\frac{3\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(3a^2+4b^2\right)\sec(c+dx)+6a^2\sec^3(c+dx)+16ab\left(\tan^2(c+\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(3*a^2 + 4*b^2)*
Sec[c + d*x] + 6*a^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d)
```

fricas [A] time = 0.67, size = 133, normalized size = 1.21

$$\frac{3\left(3\,a^2+4\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,a^2+4\,b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(32\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(3*a^2 + 4*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*a^2 + 4
*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 + 16
*a*b*cos(d*x + c) + 3*(3*a^2 + 4*b^2)*cos(d*x + c)^2 + 6*a^2)*sin(d*x + c))
/(d*cos(d*x + c)^4)
```

giac [B] time = 0.63, size = 258, normalized size = 2.35

$$\frac{3\left(3\,a^2+4\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,a^2+4\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\,\mathrm{t}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^2 + 4*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7 -
 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*a^2*tan(
1/2*d*x + 1/2*c)^5 + 80*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*b^2*tan(1/2*d*x + 1
/2*c)^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 - 12
*b^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c) + 48*a*b*tan(1/2*
d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)
/d
```

maple [A] time = 0.10, size = 142, normalized size = 1.29

$$\frac{a^2\left(\sec^3(dx+c)\right)\tan(dx+c)}{4d}+\frac{3a^2\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{4ab\tan(dx+}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^2*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+4/3*a*b*tan(d*x+c)/d+2/3/d*a*b*tan(d*x+c)*sec(d*
x+c)^2+1/2/d*b^2*tan(d*x+c)*sec(d*x+c)+1/2/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.67, size = 144, normalized size = 1.31

$$\frac{32\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)ab-3\,a^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*a^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*b^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.10, size = 184, normalized size = 1.67

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{4}+b^2\right)}{d}+\frac{\left(\frac{5\,a^2}{4}-4\,a\,b+b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,a^2}{4}+\frac{20\,a\,b}{3}-b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,a^2}{4}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((3*a^2)/4 + b^2))/d + (tan(c/2 + (d*x)/2)^5*((2
0*a*b)/3 + (3*a^2)/4 - b^2) + tan(c/2 + (d*x)/2)*(4*a*b + (5*a^2)/4 + b^2)
+ tan(c/2 + (d*x)/2)^7*((5*a^2)/4 - 4*a*b + b^2) - tan(c/2 + (d*x)/2)^3*((2
0*a*b)/3 - (3*a^2)/4 + b^2))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)
/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.427 $\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{(4a^2+5b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2+5b^2)\tan(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan}{}$$

```
[Out] 3/4*a*b*arctanh(sin(d*x+c))/d+1/5*(4*a^2+5*b^2)*tan(d*x+c)/d+3/4*a*b*sec(d*
x+c)*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a^2*sec(d*x+c)^4*ta
n(d*x+c)/d+1/15*(4*a^2+5*b^2)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3768, 3770, 3012, 3767}

$$\frac{(4a^2+5b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2+5b^2)\tan(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^6,x]
```

```
[Out] (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) + ((4*a^2 + 5*b^2)*Tan[c + d*x])/(5*d)
+ (3*a*b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a*b*Sec[c + d*x]^3*Tan[c + d*x
])/(2*d) + (a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*a^2 + 5*b^2)*Tan[c
 + d*x]^3)/(15*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx = (2ab)\int \sec^5(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$$

$$= \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^2\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{2}(3ab)\int$$

$$= \frac{3ab\sec(c+dx)\tan(c+dx)}{4d} + \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^2\sec^4(c}{}$$

$$= \frac{3ab\tanh^{-1}(\sin(c+dx))}{4d} + \frac{\left(4a^2+5b^2\right)\tan(c+dx)}{5d} + \frac{3ab\sec(c+dx)\,t}{4d}$$

Mathematica [A] time = 0.56, size = 118, normalized size = 0.87

$$\frac{a^2\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{ab\tan(c+dx)\sec^3(c+dx)}{2d}+\frac{3ab\left(\tanh^{-1}(\sin(c+dx))+\text{ta}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^6,x]
```

```
[Out] (a*b*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) + (3*a*b*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x]))/(4*d) + (b^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/
d + (a^2*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

fricas [A] time = 1.43, size = 136, normalized size = 1.01

$$\frac{45\,ab\cos(dx+c)^5\log(\sin(dx+c)+1) - 45\,ab\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\left(45\,ab\cos(dx+c)^3 + \right.}{120\,d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="fricas")

[Out] 1/120*(45*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(45*a*b*cos(d*x + c)^3 + 8*(4*a^2 + 5*b^2)*cos(d*x + c)^4 + 30*a*b*cos(d*x + c) + 4*(4*a^2 + 5*b^2)*cos(d*x + c)^2 + 12*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)

giac [B] time = 0.61, size = 272, normalized size = 2.01

$$45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,b^2\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="giac")

[Out] 1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x + 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 - 80*a^2*tan(1/2*d*x + 1/2*c)^7 + 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 160*b^2*tan(1/2*d*x + 1/2*c)^7 + 232*a^2*tan(1/2*d*x + 1/2*c)^5 + 200*b^2*tan(1/2*d*x + 1/2*c)^5 - 80*a^2*tan(1/2*d*x + 1/2*c)^3 - 30*a*b*tan(1/2*d*x + 1/2*c)^3 - 160*b^2*tan(1/2*d*x + 1/2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d

maple [A] time = 0.10, size = 157, normalized size = 1.16

$$\frac{8a^2\tan(dx+c)}{15d}+\frac{a^2\left(\sec^4(dx+c)\right)\tan(dx+c)}{5d}+\frac{4a^2\left(\sec^2(dx+c)\right)\tan(dx+c)}{15d}+\frac{ab\left(\sec^3(dx+c)\right)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x)

[Out] 8/15*a^2*tan(d*x+c)/d+1/5*a^2*sec(d*x+c)^4*tan(d*x+c)/d+4/15*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3*tan(d*x+c)/d+3/4*a*b*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*b^2*tan(d*x+c)+1/3/d*b^2*tan(d*x+c)*sec(d*x+c)^2

maxima [A] time = 0.55, size = 132, normalized size = 0.98

$$\frac{8\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^2+40\left(\tan(dx+c)^3+3\tan(dx+c)\right)b^2-15\,ab\left(\frac{2\left(3\sin(\right.}{\sin(dx+}\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40*
(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [**B**] time = 3.23, size = 221, normalized size = 1.64

$$\frac{3\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\left(2\,a^2-\frac{5\,a\,b}{2}+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{8\,a^2}{3}+a\,b-\frac{16\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{116\,a^2}{15}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\mathrm{tan}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^6,x)
```

```
[Out] (3*a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d) - (tan(c/2 + (d*x)/2)^5*((116*a^2)/
15 + (20*b^2)/3) + tan(c/2 + (d*x)/2)^9*(2*a^2 - (5*a*b)/2 + 2*b^2) - tan(c
/2 + (d*x)/2)^3*(a*b + (8*a^2)/3 + (16*b^2)/3) - tan(c/2 + (d*x)/2)^7*((8*a
^2)/3 - a*b + (16*b^2)/3) + tan(c/2 + (d*x)/2)*((5*a*b)/2 + 2*a^2 + 2*b^2))
/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/
2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.428 $\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=170

$$-\frac{a\left(a^2+6b^2\right)\sin^3(c+dx)}{3d}+\frac{a\left(a^2+3b^2\right)\sin(c+dx)}{d}+\frac{b\left(18a^2+5b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\left(18a^2+5b^2\right)\sin}{1}$$

```
[Out] 9/8*a^2*b*x+5/16*b^3*x+a*(a^2+3*b^2)*sin(d*x+c)/d+1/16*b*(18*a^2+5*b^2)*cos
(d*x+c)*sin(d*x+c)/d+1/24*b*(18*a^2+5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*b^
3*cos(d*x+c)^5*sin(d*x+c)/d-1/3*a*(a^2+6*b^2)*sin(d*x+c)^3/d+3/5*a*b^2*sin(
d*x+c)^5/d
```

Rubi [A] time = 0.21, antiderivative size = 193, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3023, 2748, 2633, 2635, 8}

$$-\frac{a\left(5a^2+12b^2\right)\sin^3(c+dx)}{15d}+\frac{a\left(5a^2+12b^2\right)\sin(c+dx)}{5d}+\frac{b\left(18a^2+5b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\left(18a^2+5b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (b*(18*a^2 + 5*b^2)*x)/16 + (a*(5*a^2 + 12*b^2)*Sin[c + d*x])/(5*d) + (b*(1
8*a^2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (b*(18*a^2 + 5*b^2)*Cos[
c + d*x]^3*Sin[c + d*x])/(24*d) + (13*a*b^2*Cos[c + d*x]^4*Sin[c + d*x])/(3
0*d) + (b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])*Sin[c + d*x])/(6*d) - (a*(5
*a^2 + 12*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^3(c+dx)\left(2a\left(\right.\right.$$
$$= \frac{13ab^2\cos^4(c+dx)\sin(c+dx)}{30d} + \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))\sin(}{6d}$$
$$= \frac{13ab^2\cos^4(c+dx)\sin(c+dx)}{30d} + \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))\sin(}{6d}$$
$$= \frac{b\left(18a^2+5b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{13ab^2\cos^4(c+dx)\sin(c+dx}{30d}$$
$$= \frac{a\left(5a^2+12b^2\right)\sin(c+dx)}{5d} + \frac{b\left(18a^2+5b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{b}{}$$
$$= \frac{1}{16}b\left(18a^2+5b^2\right)x + \frac{a\left(5a^2+12b^2\right)\sin(c+dx)}{5d} + \frac{b\left(18a^2+5b^2\right)\cos(c}{16d}$$

Mathematica [A] time = 0.33, size = 159, normalized size = 0.94

$$\frac{80a^3\sin(3(c+dx)) + 45\left(16a^2b+5b^3\right)\sin(2(c+dx)) + 360a\left(2a^2+5b^2\right)\sin(c+dx) + 90a^2b\sin(4(c+dx)) + 10}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (1080*a^2*b*c + 300*b^3*c + 1080*a^2*b*d*x + 300*b^3*d*x + 360*a*(2*a^2 + 5
*b^2)*Sin[c + d*x] + 45*(16*a^2*b + 5*b^3)*Sin[2*(c + d*x)] + 80*a^3*Sin[3*
(c + d*x)] + 300*a*b^2*Sin[3*(c + d*x)] + 90*a^2*b*Sin[4*(c + d*x)] + 45*b^
3*Sin[4*(c + d*x)] + 36*a*b^2*Sin[5*(c + d*x)] + 5*b^3*Sin[6*(c + d*x)])/(9
60*d)
```

fricas [A] time = 1.03, size = 132, normalized size = 0.78

$$\frac{15\left(18\,a^2b+5\,b^3\right)dx + \left(40\,b^3\cos\left(dx+c\right)^5 + 144\,ab^2\cos\left(dx+c\right)^4 + 10\left(18\,a^2b+5\,b^3\right)\cos\left(dx+c\right)^3 + 160\,a^3 + 38}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(18*a^2*b + 5*b^3)*d*x + (40*b^3*cos(d*x + c)^5 + 144*a*b^2*cos(d
*x + c)^4 + 10*(18*a^2*b + 5*b^3)*cos(d*x + c)^3 + 160*a^3 + 384*a*b^2 + 16
*(5*a^3 + 12*a*b^2)*cos(d*x + c)^2 + 15*(18*a^2*b + 5*b^3)*cos(d*x + c))*si
n(d*x + c))/d
```

giac [A] time = 0.59, size = 150, normalized size = 0.88

$$\frac{b^3\sin(6\,dx+6\,c)}{192\,d}+\frac{3\,ab^2\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{16}\left(18\,a^2b+5\,b^3\right)x+\frac{3\left(2\,a^2b+b^3\right)\sin(4\,dx+4\,c)}{64\,d}+\frac{\left(4\,a^3+15\,ab^2\right)\mathrm{si}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*b^3*sin(6*d*x + 6*c)/d + 3/80*a*b^2*sin(5*d*x + 5*c)/d + 1/16*(18*a^2
*b + 5*b^3)*x + 3/64*(2*a^2*b + b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*a^3 + 15*
a*b^2)*sin(3*d*x + 3*c)/d + 3/64*(16*a^2*b + 5*b^3)*sin(2*d*x + 2*c)/d + 3/
8*(2*a^3 + 5*a*b^2)*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 145, normalized size = 0.85

$$\frac{b^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3b^2a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3a^2b\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*b^2*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a
^2*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^3*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.61, size = 145, normalized size = 0.85

$$-\frac{320\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^3-90\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^2b-192\left(3\,\mathrm{sin}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 90*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 192*(3*sin(d*x + c)^5 - 10*sin(
d*x + c)^3 + 15*sin(d*x + c))*a*b^2 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60
*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^3)/d
```

mupad [B] time = 2.10, size = 380, normalized size = 2.24

$$\frac{\left(2\,a^3-\frac{15\,a^2\,b}{4}+6\,a\,b^2-\frac{11\,b^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{22\,a^3}{3}-\frac{21\,a^2\,b}{4}+14\,a\,b^2+\frac{5\,b^3}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(12\,a^3-\frac{3\,a^2\,b}{2}+\frac{15}{}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^11*(6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - (11*b^3)/8) + tan(
c/2 + (d*x)/2)^3*(14*a*b^2 + (21*a^2*b)/4 + (22*a^3)/3 - (5*b^3)/24) + tan(
c/2 + (d*x)/2)^9*(14*a*b^2 - (21*a^2*b)/4 + (22*a^3)/3 + (5*b^3)/24) + tan(
c/2 + (d*x)/2)^5*((156*a*b^2)/5 + (3*a^2*b)/2 + 12*a^3 + (15*b^3)/4) + tan(
c/2 + (d*x)/2)^7*((156*a*b^2)/5 - (3*a^2*b)/2 + 12*a^3 - (15*b^3)/4) + tan(
c/2 + (d*x)/2)*(6*a*b^2 + (15*a^2*b)/4 + 2*a^3 + (11*b^3)/8))/(d*(6*tan(c/2
 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(
c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) +
(b*atan((b*tan(c/2 + (d*x)/2)*(18*a^2 + 5*b^2))/(8*((9*a^2*b)/4 + (5*b^3)/8
)))*(18*a^2 + 5*b^2))/(8*d) - (b*(18*a^2 + 5*b^2)*(atan(tan(c/2 + (d*x)/2))
 - (d*x)/2))/(8*d)
```

sympy [A] time = 3.93, size = 393, normalized size = 2.31

$$\begin{cases} \frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^2(c+dx)}{d}+\frac{9a^2bx\sin^4(c+dx)}{8}+\frac{9a^2bx\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{9a^2bx\cos^4(c+dx)}{8}+\frac{9a^2b\sin^3(c+dx)\cos(}{8d} \\ x\,(a+b\cos(c))^3\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d + 9*a**2*b*x*sin(c + d*x)**4/8 + 9*a**2*b*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 9*a**2*b*x*cos(c + d*x)**4/8 + 9*a**2*b*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 15*a**2*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*a*b**2*sin(c + d
*x)**5/(5*d) + 4*a*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*a*b**2*sin(c
+ d*x)*cos(c + d*x)**4/d + 5*b**3*x*sin(c + d*x)**6/16 + 15*b**3*x*sin(c +
d*x)**4*cos(c + d*x)**2/16 + 15*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
 5*b**3*x*cos(c + d*x)**6/16 + 5*b**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) +
 5*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b**3*sin(c + d*x)*cos(c
+ d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**3*cos(c)**3, True))
```

3.429 $\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=180

$$-\frac{(3a^2-16b^2)\sin(c+dx)(a+b\cos(c+dx))^2}{60bd}-\frac{a(6a^2-71b^2)\sin(c+dx)\cos(c+dx)}{120d}+\frac{1}{8}ax(4a^2+9b^2)-\frac{(3a^4-$$

```
[Out] 1/8*a*(4*a^2+9*b^2)*x-1/30*(3*a^4-52*a^2*b^2-16*b^4)*sin(d*x+c)/b/d-1/120*a
*(6*a^2-71*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/60*(3*a^2-16*b^2)*(a+b*cos(d*x+c)
)^2*sin(d*x+c)/b/d-1/20*a*(a+b*cos(d*x+c))^3*sin(d*x+c)/b/d+1/5*(a+b*cos(d*
x+c))^4*sin(d*x+c)/b/d
```

Rubi [A] time = 0.22, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2791, 2753, 2734}

$$-\frac{(-52a^2b^2+3a^4-16b^4)\sin(c+dx)}{30bd}-\frac{(3a^2-16b^2)\sin(c+dx)(a+b\cos(c+dx))^2}{60bd}-\frac{a(6a^2-71b^2)\sin(c+dx)}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (a*(4*a^2 + 9*b^2)*x)/8 - ((3*a^4 - 52*a^2*b^2 - 16*b^4)*Sin[c + d*x])/(30*
b*d) - (a*(6*a^2 - 71*b^2)*Cos[c + d*x]*Sin[c + d*x])/(120*d) - ((3*a^2 - 1
6*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*b*d) - (a*(a + b*Cos[c + d*
x])^3*Sin[c + d*x])/(20*b*d) + ((a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d
)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5bd} + \frac{\int(4b-a\cos(c+dx))(a+b\cos(c+a}{5b}$$

$$= -\frac{a(a+b\cos(c+dx))^3\sin(c+dx)}{20bd} + \frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5bd} +$$

$$= -\frac{\left(3a^2-16b^2\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60bd} - \frac{a(a+b\cos(c+dx))^3\,\mathrm{s}}{20bd}$$

$$= \frac{1}{8}a\left(4a^2+9b^2\right)x - \frac{\left(3a^4-52a^2b^2-16b^4\right)\sin(c+dx)}{30bd} - \frac{a\left(6a^2-71b^2\right)\mathrm{c}}{}$$

Mathematica [A] time = 0.33, size = 130, normalized size = 0.72

$$\frac{120\left(a^3+3ab^2\right)\sin(2(c+dx))+240a^3c+240a^3dx+60b\left(18a^2+5b^2\right)\sin(c+dx)+120a^2b\sin(3(c+dx))+45a}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (240*a^3*c + 540*a*b^2*c + 240*a^3*d*x + 540*a*b^2*d*x + 60*b*(18*a^2 + 5*b
^2)*Sin[c + d*x] + 120*(a^3 + 3*a*b^2)*Sin[2*(c + d*x)] + 120*a^2*b*Sin[3*(
c + d*x)] + 50*b^3*Sin[3*(c + d*x)] + 45*a*b^2*Sin[4*(c + d*x)] + 6*b^3*Sin
[5*(c + d*x)])/(480*d)
```

fricas [A] time = 1.45, size = 110, normalized size = 0.61

$$\frac{15\left(4a^3+9ab^2\right)dx+\left(24b^3\cos(dx+c)^4+90ab^2\cos(dx+c)^3+240a^2b+64b^3+8\left(15a^2b+4b^3\right)\cos(dx+c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(4*a^3 + 9*a*b^2)*d*x + (24*b^3*cos(d*x + c)^4 + 90*a*b^2*cos(d*x
 + c)^3 + 240*a^2*b + 64*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2 + 15*(4*
a^3 + 9*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.57, size = 124, normalized size = 0.69

$$\frac{b^3 \sin(5\,dx+5\,c)}{80\,d}+\frac{3\,ab^2 \sin(4\,dx+4\,c)}{32\,d}+\frac{1}{8}\left(4\,a^3+9\,ab^2\right)x+\frac{\left(12\,a^2b+5\,b^3\right)\sin(3\,dx+3\,c)}{48\,d}+\frac{\left(a^3+3\,ab^2\right)\sin(2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/80*b^3*sin(5*d*x + 5*c)/d + 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*a^3 +
9*a*b^2)*x + 1/48*(12*a^2*b + 5*b^3)*sin(3*d*x + 3*c)/d + 1/4*(a^3 + 3*a*b^
2)*sin(2*d*x + 2*c)/d + 1/8*(18*a^2*b + 5*b^3)*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 123, normalized size = 0.68

$$\frac{\frac{b^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3b^2a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^2b\left(2+\cos^2(dx+c)\right)\sin(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*b^2*a*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^2*b*(2+cos(d*x+c)^2
)*sin(d*x+c)+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.67, size = 119, normalized size = 0.66

$$\frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3-480\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^2b+45\,(12\,dx+12\,c+\sin(4\,dx+4\,c}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 480*(sin(d*x + c)^3 - 3*s
in(d*x + c))*a^2*b + 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2
*c))*a*b^2 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*b^
3)/d
```

mupad [B] time = 2.05, size = 319, normalized size = 1.77

$$\frac{\left(-a^3+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-2\,a^3+16\,a^2\,b-\frac{3\,a\,b^2}{2}+\frac{8\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(20\,a^2\,b+\frac{116\,b^3}{15}\right)\,\mathrm{ta}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((3*a*b^2)/2 + 16*a^2*b + 2*a^3 + (8*b^3)/3) - tan(c/
2 + (d*x)/2)^7*((3*a*b^2)/2 - 16*a^2*b + 2*a^3 - (8*b^3)/3) + tan(c/2 + (d*
x)/2)*((15*a*b^2)/4 + 6*a^2*b + a^3 + 2*b^3) + tan(c/2 + (d*x)/2)^5*(20*a^2
*b + (116*b^3)/15) - tan(c/2 + (d*x)/2)^9*((15*a*b^2)/4 - 6*a^2*b + a^3 - 2
*b^3))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
(d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (a*atan
((a*tan(c/2 + (d*x)/2)*(4*a^2 + 9*b^2))/(4*((9*a*b^2)/4 + a^3)))*(4*a^2 + 9
*b^2))/(4*d) - (a*(4*a^2 + 9*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*
d)
```

sympy [A] time = 2.18, size = 284, normalized size = 1.58

$$\begin{cases} \frac{a^3 x \sin^2(c+dx)}{2} + \frac{a^3 x \cos^2(c+dx)}{2} + \frac{a^3 \sin(c+dx)\cos(c+dx)}{2d} + \frac{2a^2 b \sin^3(c+dx)}{d} + \frac{3a^2 b \sin(c+dx)\cos^2(c+dx)}{d} + \frac{9ab^2 x \sin^4(c+dx)}{8} + \frac{9ab^2 x \mathrm{s}}{} \\ x\left(a+b\cos(c)\right)^3 \cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**2/2 + a**3*x*cos(c + d*x)**2/2 + a**3*sin(c
 + d*x)*cos(c + d*x)/(2*d) + 2*a**2*b*sin(c + d*x)**3/d + 3*a**2*b*sin(c +
d*x)*cos(c + d*x)**2/d + 9*a*b**2*x*sin(c + d*x)**4/8 + 9*a*b**2*x*sin(c +
d*x)**2*cos(c + d*x)**2/4 + 9*a*b**2*x*cos(c + d*x)**4/8 + 9*a*b**2*sin(c +
 d*x)**3*cos(c + d*x)/(8*d) + 15*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
+ 8*b**3*sin(c + d*x)**5/(15*d) + 4*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3
*d) + b**3*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**3*
cos(c)**2, True))
```

3.430 $\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=121

$$\frac{a\left(a^2+4b^2\right)\sin(c+dx)}{2d}+\frac{b\left(2a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}bx\left(4a^2+b^2\right)+\frac{\sin(c+dx)(a+b\cos(c+dx))}{4d}$$

```
[Out] 3/8*b*(4*a^2+b^2)*x+1/2*a*(a^2+4*b^2)*sin(d*x+c)/d+1/8*b*(2*a^2+3*b^2)*cos(
d*x+c)*sin(d*x+c)/d+1/4*a*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/4*(a+b*cos(d*x+
c))^3*sin(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$\frac{a\left(a^2+4b^2\right)\sin(c+dx)}{2d}+\frac{b\left(2a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}bx\left(4a^2+b^2\right)+\frac{\sin(c+dx)(a+b\cos(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*x)/8 + (a*(a^2 + 4*b^2)*Sin[c + d*x])/(2*d) + (b*(2*a^2
+ 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(a + b*Cos[c + d*x])^2*Sin[c
 + d*x])/(4*d) + ((a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\int (3b+3a\cos(c+dx))(a+b\cos(c$$
$$= \frac{a(a+b\cos(c+dx))^2\sin(c+dx)}{4d} + \frac{(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{12}$$
$$= \frac{3}{8}b\left(4a^2+b^2\right)x + \frac{a\left(a^2+4b^2\right)\sin(c+dx)}{2d} + \frac{b\left(2a^2+3b^2\right)\cos(c+dx)\,\mathrm{si}}{8d}$$

Mathematica [A] time = 0.27, size = 100, normalized size = 0.83

$$\frac{8a\left(4a^2+9b^2\right)\sin(c+dx) + b\left(8\left(3a^2+b^2\right)\sin(2(c+dx)) + 48a^2c + 48a^2dx + 8ab\sin(3(c+dx)) + b^2\sin(4(c+\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (8*a*(4*a^2 + 9*b^2)*Sin[c + d*x] + b*(48*a^2*c + 12*b^2*c + 48*a^2*d*x + 1
2*b^2*d*x + 8*(3*a^2 + b^2)*Sin[2*(c + d*x)] + 8*a*b*Sin[3*(c + d*x)] + b^2
*Sin[4*(c + d*x)]))/(32*d)
```

fricas [A] time = 1.38, size = 84, normalized size = 0.69

$$\frac{3\left(4a^2b+b^3\right)dx + \left(2b^3\cos(dx+c)^3 + 8ab^2\cos(dx+c)^2 + 8a^3 + 16ab^2 + 3\left(4a^2b+b^3\right)\cos(dx+c)\right)\sin(dx+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(4*a^2*b + b^3)*d*x + (2*b^3*cos(d*x + c)^3 + 8*a*b^2*cos(d*x + c)^2
 + 8*a^3 + 16*a*b^2 + 3*(4*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.51, size = 96, normalized size = 0.79

$$\frac{b^3\sin(4dx+4c)}{32d} + \frac{ab^2\sin(3dx+3c)}{4d} + \frac{3}{8}\left(4a^2b+b^3\right)x + \frac{\left(3a^2b+b^3\right)\sin(2dx+2c)}{4d} + \frac{\left(4a^3+9ab^2\right)\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/32*b^3*sin(4*d*x + 4*c)/d + 1/4*a*b^2*sin(3*d*x + 3*c)/d + 3/8*(4*a^2*b +
 b^3)*x + 1/4*(3*a^2*b + b^3)*sin(2*d*x + 2*c)/d + 1/4*(4*a^3 + 9*a*b^2)*si
n(d*x + c)/d
```

maple [A] time = 0.04, size = 102, normalized size = 0.84

$$\frac{b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2a\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+b^2*a
*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+a^3*sin(d*x+c))
```

maxima [A] time = 0.36, size = 95, normalized size = 0.79

$$\frac{24\,(2\,dx+2\,c+\sin\,(2\,dx+2\,c))a^2b-32\left(\sin\,(dx+c)^3-3\,\sin\,(dx+c)\right)ab^2+(12\,dx+12\,c+\sin\,(4\,dx+4\,c)+}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/32*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b - 32*(sin(d*x + c)^3 - 3*si
n(d*x + c))*a*b^2 + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*b^3 + 32*a^3*sin(d*x + c))/d
```

mupad [B] time = 1.95, size = 279, normalized size = 2.31

$$\frac{\left(2\,a^3-3\,a^2\,b+6\,a\,b^2-\frac{5\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(6\,a^3-3\,a^2\,b+10\,a\,b^2+\frac{3\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(6\,a^3+3\,a^2\,b+10\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*(6*a*b^2 - 3*a^2*b + 2*a^3 - (5*b^3)/4) + tan(c/2 + (
d*x)/2)^3*(10*a*b^2 + 3*a^2*b + 6*a^3 - (3*b^3)/4) + tan(c/2 + (d*x)/2)^5*(
10*a*b^2 - 3*a^2*b + 6*a^3 + (3*b^3)/4) + tan(c/2 + (d*x)/2)*(6*a*b^2 + 3*a
^2*b + 2*a^3 + (5*b^3)/4))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2
)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atan((3*b*
tan(c/2 + (d*x)/2)*(4*a^2 + b^2))/(4*(3*a^2*b + (3*b^3)/4)))*(4*a^2 + b^2))
/(4*d) - (3*b*(4*a^2 + b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d)
```

sympy [A] time = 1.08, size = 233, normalized size = 1.93

$$\begin{cases} \frac{a^3\sin(c+dx)}{d} + \frac{3a^2bx\sin^2(c+dx)}{2} + \frac{3a^2bx\cos^2(c+dx)}{2} + \frac{3a^2b\sin(c+dx)\cos(c+dx)}{2d} + \frac{2ab^2\sin^3(c+dx)}{d} + \frac{3ab^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3b^3x \\ x(a+b\cos(c))^3\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)/d + 3*a**2*b*x*sin(c + d*x)**2/2 + 3*a**2*b*x*
cos(c + d*x)**2/2 + 3*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*a*b**2*sin
(c + d*x)**3/d + 3*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*b**3*x*sin(c +
 d*x)**4/8 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b**3*x*cos(c +
d*x)**4/8 + 3*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b**3*sin(c + d*x)
*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**3*cos(c), True))
```

3.431 $\int (a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=76

$$a^3x+\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3\sin^3(c+dx)}{3d}$$

```
[Out] a^3*x+3/2*a*b^2*x+b*(3*a^2+b^2)*sin(d*x+c)/d+3/2*a*b^2*cos(d*x+c)*sin(d*x+c
)/d-1/3*b^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$\frac{2b\left(4a^2+b^2\right)\sin(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2+3b^2\right)+\frac{5ab^2\sin(c+dx)\cos(c+dx)}{6d}+\frac{b\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 + (2*b*(4*a^2 + b^2)*Sin[c + d*x])/(3*d) + (5*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])
/(3*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\,dx = \frac{b(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}\int (a+b\cos(c+dx))\left(3a^2+2b^2+5ab\cos(c+\right.$$
$$= \frac{1}{2}a\left(2a^2+3b^2\right)x + \frac{2b\left(4a^2+b^2\right)\sin(c+dx)}{3d} + \frac{5ab^2\cos(c+dx)\sin(c+dx)}{6d} + \frac{b(a+}{}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 1.05

$$\frac{12a^3c+12a^3dx+9b\left(4a^2+b^2\right)\sin(c+dx)+9ab^2\sin(2(c+dx))+18ab^2c+18ab^2dx+b^3\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sin[
c + d*x] + 9*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 0.69, size = 66, normalized size = 0.87

$$\frac{3\left(2a^3+3ab^2\right)dx+\left(2b^3\cos(dx+c)^2+9ab^2\cos(dx+c)+18a^2b+4b^3\right)\sin(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*d*x + (2*b^3*cos(d*x + c)^2 + 9*a*b^2*cos(d*x + c)
 + 18*a^2*b + 4*b^3)*sin(d*x + c))/d
```

giac [A] time = 0.40, size = 72, normalized size = 0.95

$$\frac{b^3\sin(3dx+3c)}{12d} + \frac{3ab^2\sin(2dx+2c)}{4d} + \frac{1}{2}\left(2a^3+3ab^2\right)x + \frac{3\left(4a^2b+b^3\right)\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*sin(3*d*x + 3*c)/d + 3/4*a*b^2*sin(2*d*x + 2*c)/d + 1/2*(2*a^3 + 3
*a*b^2)*x + 3/4*(4*a^2*b + b^3)*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 76, normalized size = 1.00

$$\frac{\frac{b^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + 3b^2a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) + 3a^2b\sin(dx+c) + a^3(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*b^2*a*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+3*a^2*b*sin(d*x+c)+a^3*(d*x+c))
```

maxima [A] time = 0.61, size = 72, normalized size = 0.95

$$a^3x + \frac{3(2dx + 2c + \sin(2dx + 2c))ab^2}{4d} - \frac{\left(\sin(dx + c)^3 - 3\sin(dx + c)\right)b^3}{3d} + \frac{3a^2b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b^2/d - 1/3*(sin(d*x + c)^3
- 3*sin(d*x + c))*b^3/d + 3*a^2*b*sin(d*x + c)/d
```

mupad [B] time = 0.60, size = 77, normalized size = 1.01

$$a^3\,x + \frac{3\,b^3\,\sin(c + d\,x)}{4\,d} + \frac{b^3\,\sin(3\,c + 3\,d\,x)}{12\,d} + \frac{3\,a\,b^2\,x}{2} + \frac{3\,a\,b^2\,\sin(2\,c + 2\,d\,x)}{4\,d} + \frac{3\,a^2\,b\,\sin(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3,x)
```

```
[Out] a^3*x + (3*b^3*sin(c + d*x))/(4*d) + (b^3*sin(3*c + 3*d*x))/(12*d) + (3*a*b
^2*x)/2 + (3*a*b^2*sin(2*c + 2*d*x))/(4*d) + (3*a^2*b*sin(c + d*x))/d
```

sympy [A] time = 0.52, size = 128, normalized size = 1.68

$$\begin{cases} a^3x + \frac{3a^2b\sin(c+dx)}{d} + \frac{3ab^2x\sin^2(c+dx)}{2} + \frac{3ab^2x\cos^2(c+dx)}{2} + \frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2b^3\sin^3(c+dx)}{3d} + \frac{b^3\sin(c+dx)\cos^2(c+dx)}{d} \\ x\left(a + b\cos(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sin(c + d*x)/d + 3*a*b**2*x*sin(c + d*x)**2/2
+ 3*a*b**2*x*cos(c + d*x)**2/2 + 3*a*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) +
 2*b**3*sin(c + d*x)**3/(3*d) + b**3*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d,
0)), (x*(a + b*cos(c))**3, True))
```

3.432 $\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{1}{2}bx\left(6a^2+b^2\right)+\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

```
[Out] 1/2*b*(6*a^2+b^2)*x+a^3*arctanh(sin(d*x+c))/d+5/2*a*b^2*sin(d*x+c)/d+1/2*b^
2*(a+b*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2793, 3023, 2735, 3770}

$$\frac{1}{2}bx\left(6a^2+b^2\right)+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x],x]
```

```
[Out] (b*(6*a^2 + b^2)*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (5*a*b^2*Sin[c + d*
x])/(2*d) + (b^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx = \frac{b^2(a+b\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int\left(2a^3+b\left(6a^2+b^2\right)\cos(c+dx\right)$$
$$=\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2(a+b\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int\left(2a^3+b\left(6\right.\right.$$
$$=\frac{1}{2}b\left(6a^2+b^2\right)x+\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2(a+b\cos(c+dx))\sin(c+dx)}{2d}$$
$$=\frac{1}{2}b\left(6a^2+b^2\right)x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2(a+b}{}$$

Mathematica [A] time = 0.14, size = 105, normalized size = 1.44

$$\frac{-4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+2b\left(6a^2+b^2\right)(c+}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x],x]
```

```
[Out] (2*b*(6*a^2 + b^2)*(c + d*x) - 4*a^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 4*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a*b^2*Sin[c + d*x]
 + b^3*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.85, size = 72, normalized size = 0.99

$$\frac{a^3\log\left(\sin\left(dx+c\right)+1\right)-a^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,a^2b+b^3\right)dx+\left(b^3\cos\left(dx+c\right)+6\,ab^2\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="fricas")
```

[Out] 1/2*(a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (6*a^2*b + b^
3)*d*x + (b^3*cos(d*x + c) + 6*a*b^2)*sin(d*x + c))/d

giac [B] time = 0.51, size = 137, normalized size = 1.88

$$\frac{2\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(6\,a^{2}b+b^{3}\right)(dx+c)+\frac{2\left(6\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b^{3}\,t}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="giac")

[Out] 1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + (6*a^2*b + b^3)*(d*x + c) + 2*(6*a*b^2*tan(1/2*d*x + 1/2*c)
^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d

maple [A] time = 0.08, size = 90, normalized size = 1.23

$$\frac{a^{3}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+3a^{2}bx+\frac{3a^{2}bc}{d}+\frac{3a\,b^{2}\sin\left(dx+c\right)}{d}+\frac{b^{3}\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{b^{3}x}{2}+\frac{c\,b^{3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^3*sec(d*x+c),x)

[Out] 1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b*x+3/d*a^2*b*c+3*a*b^2*sin(d*x+c)/
d+1/2/d*b^3*cos(d*x+c)*sin(d*x+c)+1/2*b^3*x+1/2/d*c*b^3

maxima [A] time = 0.34, size = 69, normalized size = 0.95

$$\frac{12\,(dx+c)a^{2}b+(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))b^{3}+4\,a^{3}\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+12\,ab^{2}\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="maxima")

[Out] 1/4*(12*(d*x + c)*a^2*b + (2*d*x + 2*c + sin(2*d*x + 2*c))*b^3 + 4*a^3*log(
sec(d*x + c) + tan(d*x + c)) + 12*a*b^2*sin(d*x + c))/d

mupad [B] time = 0.72, size = 123, normalized size = 1.68

$$\frac{2\,a^{3}\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^{3}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^{3}\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b^{2}\,\sin\left(c+d\,x\right)}{d}+\frac{6\,a^{2}\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (2*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(2*c + 2*d*x))/(4*d) + (3*a*b^2
*sin(c + d*x))/d + (6*a^2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))^3 \sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x), x)
```

3.433 $\int (a+b\cos(c+dx))^3 \sec^2(c+dx)\,dx$

Optimal. Leaf size=68

$$-\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}+3ab^2x$$

```
[Out] 3*a*b^2*x+3*a^2*b*arctanh(sin(d*x+c))/d-b*(a^2-b^2)*sin(d*x+c)/d+a^2*(a+b*c
os(d*x+c))*tan(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2792, 3023, 2735, 3770}

$$-\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}+3ab^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b*(a^2 - b^2)*Sin[c + d*x]
)/d + (a^2*(a + b*Cos[c + d*x])*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))\tan(c+dx)}{d} + \int \Big(3a^2b + 3ab^2\cos(c+dx) - b\Big(a^2$$

$$= -\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d} + \frac{a^2(a+b\cos(c+dx))\tan(c+dx)}{d} + \int \Big(3a^2$$

$$= 3ab^2x - \frac{b\left(a^2-b^2\right)\sin(c+dx)}{d} + \frac{a^2(a+b\cos(c+dx))\tan(c+dx)}{d} + \Big($$

$$= 3ab^2x + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\left(a^2-b^2\right)\sin(c+dx)}{d} + \frac{a^2(a+b$$

Mathematica [A] time = 0.35, size = 88, normalized size = 1.29

$$\frac{a^3\tan(c+dx) + 3ab\left(-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + a\log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right) + b}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

```
[Out] (3*a*b*(b*c + b*d*x - a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + a*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b^3*Sin[c + d*x] + a^3*Tan[c + d*x])/
d
```

fricas [A] time = 1.08, size = 94, normalized size = 1.38

$$\frac{6\,ab^2dx\cos(dx+c) + 3\,a^2b\cos(dx+c)\log(\sin(dx+c)+1) - 3\,a^2b\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,\big(b}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*d*x*cos(d*x + c) + 3*a^2*b*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a^2*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(b^3*cos(d*x + c) + a^3)*
sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.58, size = 129, normalized size = 1.90

$$\frac{3\,(dx+c)ab^2+3\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx\right.\right.}{\tan\left(\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a*b^2 + 3*a^2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*b*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*tan
(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

maple [A] time = 0.08, size = 68, normalized size = 1.00

$$3a\,b^2x+\frac{a^3\tan\left(dx+c\right)}{d}+\frac{b^3\sin\left(dx+c\right)}{d}+\frac{3a^2b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3a\,b^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x)
```

```
[Out] 3*a*b^2*x+a^3*tan(d*x+c)/d+1/d*b^3*sin(d*x+c)+3/d*a^2*b*ln(sec(d*x+c)+tan(d
*x+c))+3/d*a*b^2*c
```

maxima [A] time = 0.89, size = 66, normalized size = 0.97

$$\frac{6\,(dx+c)ab^2+3\,a^2b\Big(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\Big)+2\,b^3\sin\left(dx+c\right)+2\,a^3\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a*b^2 + 3*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 2*b^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

mupad [B] time = 0.62, size = 97, normalized size = 1.43

$$\frac{b^3\sin(c+d\,x)}{d}+\frac{a^3\sin(c+d\,x)}{d\cos(c+d\,x)}+\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (b^3*sin(c + d*x))/d + (a^3*sin(c + d*x))/(d*cos(c + d*x)) + (6*a*b^2*atan(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**2, x)
```

3.434 $\int (a + b\cos(c + dx))^3 \sec^3(c + dx)\, dx$

Optimal. Leaf size=79

$$\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}+b^3x$$

```
[Out] b^3*x+1/2*a*(a^2+6*b^2)*arctanh(sin(d*x+c))/d+5/2*a^2*b*tan(d*x+c)/d+1/2*a^
2*(a+b*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2792, 3021, 2735, 3770}

$$\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

```
[Out] b^3*x + (a*(a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^2*b*Tan[c + d*
x])/(2*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(5a^2b+a\left(a^2+6b\right.\right.$$

$$=\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2(a+b\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}$$

$$=b^3x+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2(a+b\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}$$

$$=b^3x+\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2(a+b}{}$$

Mathematica [A] time = 0.18, size = 55, normalized size = 0.70

$$\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))+a^2\tan(c+dx)(a\sec(c+dx)+6b)+2b^3dx}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^3*d*x + a*(a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]] + a^2*(6*b + a*Sec[c + d
*x])*Tan[c + d*x])/(2*d)
```

fricas [A] time = 1.12, size = 112, normalized size = 1.42

$$\frac{4\,b^3dx\cos\left(dx+c\right)^2+\left(a^3+6\,ab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(a^3+6\,ab^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*d*x*cos(d*x + c)^2 + (a^3 + 6*a*b^2)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (a^3 + 6*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^
2*b*cos(d*x + c) + a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.70, size = 143, normalized size = 1.81

$$\frac{2\,(dx+c)b^3 + \left(a^3+6\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(a^3+6\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*b^3 + (a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 (a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x +
1/2*c)^3 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + 6*a^
2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.10, size = 95, normalized size = 1.20

$$\frac{a^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{3a^2b\tan(dx+c)}{d}+\frac{3b^2a\ln(\sec(dx+c)+\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b
*tan(d*x+c)/d+3/d*b^2*a*ln(sec(d*x+c)+tan(d*x+c))+b^3*x+1/d*c*b^3
```

maxima [A] time = 0.63, size = 101, normalized size = 1.28

$$\frac{4\,(dx+c)b^3 - a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 6\,ab^2\left(\log(\sin(dx+c)+1) - \log\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*b^3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 12*a^2*b*tan(d*x + c))/d
```

mupad [B] time = 0.67, size = 136, normalized size = 1.72

$$\frac{a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a^2\,b\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^3*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2)
 + (6*a*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a^2*b*sin(
c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**3,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**3, x)
```

3.435 $\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$

Optimal. Leaf size=109

$$\frac{a(2a^2+9b^2)\tan(c+dx)}{3d}+\frac{b(3a^2+2b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{6d}+\frac{a^2\tan(c+dx)\sec^2(}{}$$

```
[Out] 1/2*b*(3*a^2+2*b^2)*arctanh(sin(d*x+c))/d+1/3*a*(2*a^2+9*b^2)*tan(d*x+c)/d+
7/6*a^2*b*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*(a+b*cos(d*x+c))*sec(d*x+c)^2*tan
(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3021, 2748, 3767, 8, 3770}

$$\frac{a(2a^2+9b^2)\tan(c+dx)}{3d}+\frac{b(3a^2+2b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{6d}+\frac{a^2\tan(c+dx)\sec^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

```
[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*a^2 + 9*b^2)*Tan[c
+ d*x])/(3*d) + (7*a^2*b*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a^2*(a + b*Cos
[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
```

```
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \left(7a^2b+a\left(2a^2+\right.\right.\\ &= \frac{7a^2b\sec(c+dx)\tan(c+dx)}{6d}+\frac{a^2(a+b\cos(c+dx))\sec^2(c+dx)\tan(\\ &= \frac{7a^2b\sec(c+dx)\tan(c+dx)}{6d}+\frac{a^2(a+b\cos(c+dx))\sec^2(c+dx)\tan(\\ &= \frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{7a^2b\sec(c+dx)\tan(c+dx)}{6d}+\frac{a^2}{}\\ &= \frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\left(2a^2+9b^2\right)\tan(c+dx)}{3d}+\frac{7a^2b}{}\end{aligned}$$

Mathematica [A] time = 0.26, size = 70, normalized size = 0.64

$$\frac{\left(9a^2b+6b^3\right)\tanh^{-1}(\sin(c+dx))+a\tan(c+dx)\left(2a^2\tan^2(c+dx)+6a^2+9ab\sec(c+dx)+18b^2\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

```
[Out] ((9*a^2*b + 6*b^3)*ArcTanh[Sin[c + d*x]] + a*Tan[c + d*x]*(6*a^2 + 18*b^2 +
 9*a*b*Sec[c + d*x] + 2*a^2*Tan[c + d*x]^2))/(6*d)
```

fricas [A] time = 1.08, size = 126, normalized size = 1.16

$$\frac{3\left(3a^2b+2b^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(3a^2b+2b^3\right)\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(9a\right.}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(3*a^2*b + 2*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*a^2*b
 + 2*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a^2*b*cos(d*x + c) +
 2*a^3 + 2*(2*a^3 + 9*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

giac [B] time = 0.55, size = 205, normalized size = 1.88

$$\frac{3\left(3a^2b+2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(3a^2b+2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9a^2\right.}{}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^2*b +
2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a^3*tan(1/2*d*x + 1/2*c)^5
 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*a^3
*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2*d
*x + 1/2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.10, size = 118, normalized size = 1.08

$$\frac{2a^3\tan(dx+c)}{3d}+\frac{a^3\tan(dx+c)\left(\sec^2(dx+c)\right)}{3d}+\frac{3a^2b\sec(dx+c)\tan(dx+c)}{2d}+\frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a^3*tan(d*x+c)/d+1/3/d*a^3*tan(d*x+c)*sec(d*x+c)^2+3/2*a^2*b*sec(d*x+c)
*tan(d*x+c)/d+3/2/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*b^2*a*tan(d*x+c)+1/
d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.83, size = 113, normalized size = 1.04

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3-9a^2b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6b^3(}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^2*b*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*b^3*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*a*b^2*tan(d*x + c))/d
```

mupad [B] time = 2.63, size = 157, normalized size = 1.44

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(3a^2b+2b^3\right)}{d}-\frac{\left(2a^3-3a^2b+6ab^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{4a^3}{3}-12ab^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(3*a^2*b + 2*b^3))/d - (tan(c/2 + (d*x)/2)^5*(6*
a*b^2 - 3*a^2*b + 2*a^3) - tan(c/2 + (d*x)/2)^3*(12*a*b^2 + (4*a^3)/3) + ta
n(c/2 + (d*x)/2)*(6*a*b^2 + 3*a^2*b + 2*a^3))/(d*(3*tan(c/2 + (d*x)/2)^2 -
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.436 $\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$

Optimal. Leaf size=133

$$\frac{b(2a^2+b^2)\tan(c+dx)}{d}+\frac{3a(a^2+4b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a(a^2+4b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{3a^2b\tan(c+$$

```
[Out] 3/8*a*(a^2+4*b^2)*arctanh(sin(d*x+c))/d+b*(2*a^2+b^2)*tan(d*x+c)/d+3/8*a*(a
^2+4*b^2)*sec(d*x+c)*tan(d*x+c)/d+3/4*a^2*b*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a
^2*(a+b*cos(d*x+c))*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{b(2a^2+b^2)\tan(c+dx)}{d}+\frac{3a(a^2+4b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a(a^2+4b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{3a^2b\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

```
[Out] (3*a*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (b*(2*a^2 + b^2)*Tan[c +
d*x])/d + (3*a*(a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^2*b*Se
c[c + d*x]^2*Tan[c + d*x])/(4*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int \left(9a^2b+3a\left(a^2+4\right.\right.$$
$$= \frac{3a^2b\sec^2(c+dx)\tan(c+dx)}{4d} + \frac{a^2(a+b\cos(c+dx))\sec^3(c+dx)\tan(c}{4d}$$
$$= \frac{3a^2b\sec^2(c+dx)\tan(c+dx)}{4d} + \frac{a^2(a+b\cos(c+dx))\sec^3(c+dx)\tan(c}{4d}$$
$$= \frac{3a\left(a^2+4b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{3a^2b\sec^2(c+dx)\tan(c+dx)}{4d} +$$
$$= \frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\left(2a^2+b^2\right)\tan(c+dx)}{d} + \frac{3a\left(a^2+\right.}{}$$

Mathematica [A] time = 0.42, size = 90, normalized size = 0.68

$$\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(2a^3\sec^3(c+dx)+8b\left(a^2\tan^2(c+dx)+3a^2+b^2\right)+3a\left(a^2+4b\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

```
[Out] (3*a*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(a^2 + 4*b^2)*
Sec[c + d*x] + 2*a^3*Sec[c + d*x]^3 + 8*b*(3*a^2 + b^2 + a^2*Tan[c + d*x]^2
)))/(8*d)
```

fricas [A] time = 1.04, size = 140, normalized size = 1.05

$$\frac{3\left(a^3+4\,ab^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^3+4\,ab^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,a^2b\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a^3 + 4*a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 + 4*a
*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2*b*cos(d*x + c) + 8*(
2*a^2*b + b^3)*cos(d*x + c)^3 + 2*a^3 + 3*(a^3 + 4*a*b^2)*cos(d*x + c)^2)*s
in(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.69, size = 330, normalized size = 2.48

$$3\left(a^3+4\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(a^3+4\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,a^2b\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 + 4*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(a^3 + 4*a*b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 2
4*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*b^3*ta
n(1/2*d*x + 1/2*c)^7 + 3*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*b^3*tan(1/2*d*x + 1/2*c)^
5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b
^2*tan(1/2*d*x + 1/2*c)^3 - 24*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*tan(1/2*d
*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c)
 + 8*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.11, size = 160, normalized size = 1.20

$$\frac{a^3\left(\sec^3(dx+c)\right)\tan(dx+c)}{4d}+\frac{3a^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{2a^2b\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^3*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^3
*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*b*tan(d*x+c)/d+a^2*b*sec(d*x+c)^2*tan(d*x+
c)/d+3/2/d*b^2*a*tan(d*x+c)*sec(d*x+c)+3/2/d*b^2*a*ln(sec(d*x+c)+tan(d*x+c)
)+1/d*b^3*tan(d*x+c)
```

maxima [A] time = 0.80, size = 158, normalized size = 1.19

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2b-a^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b - a^3*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a*b^2*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*b^3*tan(d*x
 + c))/d
```

mupad [B] time = 4.22, size = 224, normalized size = 1.68

$$\frac{\left(\frac{5\,a^3}{4}-6\,a^2\,b+3\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,a^3}{4}+10\,a^2\,b-3\,a\,b^2+6\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,a^3}{4}-10\,a^2\,b-3\,a\,\ldots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*(3*a*b^2 - 6*a^2*b + (5*a^3)/4 - 2*b^3) - tan(c/2 + (
d*x)/2)^3*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3) + tan(c/2 + (d*x)/2)^5*(
10*a^2*b - 3*a*b^2 + (3*a^3)/4 + 6*b^3) + tan(c/2 + (d*x)/2)*(3*a*b^2 + 6*a
^2*b + (5*a^3)/4 + 2*b^3))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atanh(tan(
c/2 + (d*x)/2))*(a^2 + 4*b^2))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.437 $\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{a\left(4a^2+15b^2\right)\tan^3(c+dx)}{15d}+\frac{a\left(4a^2+15b^2\right)\tan(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(9a^2+4b^2\right)\tan(}{8}$$

```
[Out] 1/8*b*(9*a^2+4*b^2)*arctanh(sin(d*x+c))/d+1/5*a*(4*a^2+15*b^2)*tan(d*x+c)/d
+1/8*b*(9*a^2+4*b^2)*sec(d*x+c)*tan(d*x+c)/d+11/20*a^2*b*sec(d*x+c)^3*tan(d
*x+c)/d+1/5*a^2*(a+b*cos(d*x+c))*sec(d*x+c)^4*tan(d*x+c)/d+1/15*a*(4*a^2+15
*b^2)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3021, 2748, 3767, 3768, 3770}

$$\frac{a\left(4a^2+15b^2\right)\tan^3(c+dx)}{15d}+\frac{a\left(4a^2+15b^2\right)\tan(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(9a^2+4b^2\right)\tan(}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

```
[Out] (b*(9*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(4*a^2 + 15*b^2)*Tan[c
 + d*x])/(5*d) + (b*(9*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (11*
a^2*b*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c
 + d*x]^4*Tan[c + d*x])/(5*d) + (a*(4*a^2 + 15*b^2)*Tan[c + d*x]^3)/(15*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
```

```
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \sec^6(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{5}\int \left(11a^2b + a\left(4a^2\right.\right.$$
$$= \frac{11a^2b\sec^3(c+dx)\tan(c+dx)}{20d} + \frac{a^2(a+b\cos(c+dx))\sec^4(c+dx)\tan}{5d}$$
$$= \frac{11a^2b\sec^3(c+dx)\tan(c+dx)}{20d} + \frac{a^2(a+b\cos(c+dx))\sec^4(c+dx)\tan}{5d}$$
$$= \frac{b\left(9a^2+4b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{11a^2b\sec^3(c+dx)\tan(c+dx)}{20d}$$
$$= \frac{b\left(9a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\left(4a^2+15b^2\right)\tan(c+dx)}{5d} + \frac{b\left(9\right.}{}$$

Mathematica [A] time = 0.90, size = 120, normalized size = 0.71

$$\frac{15b\left(9a^2+4b^2\right)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(8a\left(5\left(2a^2+3b^2\right)\tan^2(c+dx)+15\left(a^2+3b^2\right)+3a^2\tan^4(c\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

```
[Out] (15*b*(9*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*b*(9*a^2 + 4
*b^2)*Sec[c + d*x] + 90*a^2*b*Sec[c + d*x]^3 + 8*a*(15*(a^2 + 3*b^2) + 5*(2
*a^2 + 3*b^2)*Tan[c + d*x]^2 + 3*a^2*Tan[c + d*x]^4)))/(120*d)
```

fricas [A] time = 0.89, size = 170, normalized size = 1.01

$$\frac{15\left(9a^2b+4b^3\right)\cos(dx+c)^5\log(\sin(dx+c)+1) - 15\left(9a^2b+4b^3\right)\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(9*a^2*b + 4*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(9*a^
2*b + 4*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(4*a^3 + 15*a*b^
2)*cos(d*x + c)^4 + 90*a^2*b*cos(d*x + c) + 15*(9*a^2*b + 4*b^3)*cos(d*x +
c)^3 + 24*a^3 + 8*(4*a^3 + 15*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

giac [B] time = 0.66, size = 367, normalized size = 2.17

$$15\left(9\,a^{2}b+4\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(9\,a^{2}b+4\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(120\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(15*(9*a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(9*a^2*
b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*a^3*tan(1/2*d*x + 1/
2*c)^9 - 225*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^
9 - 60*b^3*tan(1/2*d*x + 1/2*c)^9 - 160*a^3*tan(1/2*d*x + 1/2*c)^7 + 90*a^2
*b*tan(1/2*d*x + 1/2*c)^7 - 960*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*b^3*tan(
1/2*d*x + 1/2*c)^7 + 464*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*a*b^2*tan(1/2*d*
x + 1/2*c)^5 - 160*a^3*tan(1/2*d*x + 1/2*c)^3 - 90*a^2*b*tan(1/2*d*x + 1/2*
c)^3 - 960*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*b^3*tan(1/2*d*x + 1/2*c)^3 +
120*a^3*tan(1/2*d*x + 1/2*c) + 225*a^2*b*tan(1/2*d*x + 1/2*c) + 360*a*b^2*t
an(1/2*d*x + 1/2*c) + 60*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

maple [A] time = 0.12, size = 206, normalized size = 1.22

$$\frac{8a^{3}\tan\left(dx+c\right)}{15d}+\frac{a^{3}\tan\left(dx+c\right)\left(\sec^{4}\left(dx+c\right)\right)}{5d}+\frac{4a^{3}\tan\left(dx+c\right)\left(\sec^{2}\left(dx+c\right)\right)}{15d}+\frac{3a^{2}b\left(\sec^{3}\left(dx+c\right)\right)\tan\left(dx+\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x)
```

```
[Out] 8/15*a^3*tan(d*x+c)/d+1/5/d*a^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a^3*tan(d*x+
c)*sec(d*x+c)^2+3/4*a^2*b*sec(d*x+c)^3*tan(d*x+c)/d+9/8*a^2*b*sec(d*x+c)*ta
n(d*x+c)/d+9/8/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*b^2*a*tan(d*x+c)+1/d*b
^2*a*tan(d*x+c)*sec(d*x+c)^2+1/2/d*b^3*tan(d*x+c)*sec(d*x+c)+1/2/d*b^3*ln(s
ec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.65, size = 181, normalized size = 1.07

$$16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{3}+240\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)ab^{2}-45\,a^{2}b\left(\frac{2(}{\mathrm{si}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2 - 45*a^2*b*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.22, size = 260, normalized size = 1.54

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{9\,a^2\,b}{4}+b^3\right)}{d}-\frac{\left(2\,a^3-\frac{15\,a^2\,b}{4}+6\,a\,b^2-b^3\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{8\,a^3}{3}+\frac{3\,a^2\,b}{2}-16\,a\,b^2+2\,b^3\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((9*a^2*b)/4 + b^3))/d - (tan(c/2 + (d*x)/2)^9*(
6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - b^3) - tan(c/2 + (d*x)/2)^3*(16*a*b^2 + (3
*a^2*b)/2 + (8*a^3)/3 + 2*b^3) - tan(c/2 + (d*x)/2)^7*(16*a*b^2 - (3*a^2*b)
/2 + (8*a^3)/3 - 2*b^3) + tan(c/2 + (d*x)/2)*(6*a*b^2 + (15*a^2*b)/4 + 2*a^
3 + b^3) + tan(c/2 + (d*x)/2)^5*(20*a*b^2 + (116*a^3)/15))/(d*(5*tan(c/2 +
(d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2
+ (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.438 $\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=247

$$\frac{b^2\left(37a^2+6b^2\right)\sin(c+dx)\cos^4(c+dx)}{35d}+\frac{ab\left(6a^2+5b^2\right)\sin(c+dx)\cos^3(c+dx)}{6d}+\frac{ab\left(6a^2+5b^2\right)\sin(c+dx)\cos}{4d}$$

```
[Out] 1/4*a*b*(6*a^2+5*b^2)*x+1/35*(35*a^4+168*a^2*b^2+24*b^4)*sin(d*x+c)/d+1/4*a
*b*(6*a^2+5*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/6*a*b*(6*a^2+5*b^2)*cos(d*x+c)^3
*sin(d*x+c)/d+1/35*b^2*(37*a^2+6*b^2)*cos(d*x+c)^4*sin(d*x+c)/d+8/21*a*b^3*
cos(d*x+c)^5*sin(d*x+c)/d+1/7*b^2*cos(d*x+c)^4*(a+b*cos(d*x+c))^2*sin(d*x+c
)/d-1/105*(35*a^4+168*a^2*b^2+24*b^4)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.40, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2793, 3033, 3023, 2748, 2633, 2635, 8}

$$-\frac{\left(168a^2b^2+35a^4+24b^4\right)\sin^3(c+dx)}{105d}+\frac{\left(168a^2b^2+35a^4+24b^4\right)\sin(c+dx)}{35d}+\frac{b^2\left(37a^2+6b^2\right)\sin(c+dx)\cos^4(}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (a*b*(6*a^2 + 5*b^2)*x)/4 + ((35*a^4 + 168*a^2*b^2 + 24*b^4)*Sin[c + d*x])/
(35*d) + (a*b*(6*a^2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a*b*(6*a^
2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (b^2*(37*a^2 + 6*b^2)*Cos[c
 + d*x]^4*Sin[c + d*x])/(35*d) + (8*a*b^3*Cos[c + d*x]^5*Sin[c + d*x])/(21*
d) + (b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) - ((35*
a^4 + 168*a^2*b^2 + 24*b^4)*Sin[c + d*x]^3)/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx = \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{1}{7}\int \cos^3(c+dx)(a+$$

$$= \frac{8ab^3\cos^5(c+dx)\sin(c+dx)}{21d} + \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))^2\sin($$

$$= \frac{b^2\left(37a^2+6b^2\right)\cos^4(c+dx)\sin(c+dx)}{35d} + \frac{8ab^3\cos^5(c+dx)\sin(c+dx}{21d}$$

$$= \frac{b^2\left(37a^2+6b^2\right)\cos^4(c+dx)\sin(c+dx)}{35d} + \frac{8ab^3\cos^5(c+dx)\sin(c+dx}{21d}$$

$$= \frac{ab\left(6a^2+5b^2\right)\cos^3(c+dx)\sin(c+dx)}{6d} + \frac{b^2\left(37a^2+6b^2\right)\cos^4(c+dx)s}{35d}$$

$$= \frac{\left(35a^4+168a^2b^2+24b^4\right)\sin(c+dx)}{35d} + \frac{ab\left(6a^2+5b^2\right)\cos(c+dx)\sin(c}{4d}$$

$$= \frac{1}{4}ab\left(6a^2+5b^2\right)x + \frac{\left(35a^4+168a^2b^2+24b^4\right)\sin(c+dx)}{35d} + \frac{ab\left(6a^2+5\right.}{}$$

Mathematica [A] time = 0.41, size = 181, normalized size = 0.73

$$\frac{1680ab\left(6a^2+5b^2\right)(c+dx)+21b^2\left(24a^2+7b^2\right)\sin(5(c+dx))+420ab\left(16a^2+15b^2\right)\sin(2(c+dx))+420ab\left(2a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (1680*a*b*(6*a^2 + 5*b^2)*(c + d*x) + 105*(48*a^4 + 240*a^2*b^2 + 35*b^4)*S
in[c + d*x] + 420*a*b*(16*a^2 + 15*b^2)*Sin[2*(c + d*x)] + 35*(16*a^4 + 120
*a^2*b^2 + 21*b^4)*Sin[3*(c + d*x)] + 420*a*b*(2*a^2 + 3*b^2)*Sin[4*(c + d*
x)] + 21*b^2*(24*a^2 + 7*b^2)*Sin[5*(c + d*x)] + 140*a*b^3*Sin[6*(c + d*x)]
 + 15*b^4*Sin[7*(c + d*x)])/(6720*d)
```

fricas [A] time = 0.89, size = 171, normalized size = 0.69

$$\frac{105\left(6\,a^3b+5\,ab^3\right)dx+\left(60\,b^4\cos\left(dx+c\right)^6+280\,ab^3\cos\left(dx+c\right)^5+72\left(7\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+280\,a^4+13\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/420*(105*(6*a^3*b + 5*a*b^3)*d*x + (60*b^4*cos(d*x + c)^6 + 280*a*b^3*cos
(d*x + c)^5 + 72*(7*a^2*b^2 + b^4)*cos(d*x + c)^4 + 280*a^4 + 1344*a^2*b^2
```

```
+ 192*b^4 + 70*(6*a^3*b + 5*a*b^3)*cos(d*x + c)^3 + 4*(35*a^4 + 168*a^2*b^2
 + 24*b^4)*cos(d*x + c)^2 + 105*(6*a^3*b + 5*a*b^3)*cos(d*x + c))*sin(d*x +
 c))/d
```

giac [A] time = 0.65, size = 197, normalized size = 0.80

$$\frac{b^4 \sin(7\,dx+7\,c)}{448\,d}+\frac{ab^3 \sin(6\,dx+6\,c)}{48\,d}+\frac{1}{4}\left(6\,a^3b+5\,ab^3\right)x+\frac{\left(24\,a^2b^2+7\,b^4\right)\sin(5\,dx+5\,c)}{320\,d}+\frac{\left(2\,a^3b+3\,ab^3\right)\sin}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/448*b^4*sin(7*d*x + 7*c)/d + 1/48*a*b^3*sin(6*d*x + 6*c)/d + 1/4*(6*a^3*b
 + 5*a*b^3)*x + 1/320*(24*a^2*b^2 + 7*b^4)*sin(5*d*x + 5*c)/d + 1/16*(2*a^3
*b + 3*a*b^3)*sin(4*d*x + 4*c)/d + 1/192*(16*a^4 + 120*a^2*b^2 + 21*b^4)*si
n(3*d*x + 3*c)/d + 1/16*(16*a^3*b + 15*a*b^3)*sin(2*d*x + 2*c)/d + 1/64*(48
*a^4 + 240*a^2*b^2 + 35*b^4)*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 190, normalized size = 0.77

$$\frac{\frac{b^4\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+4a\,b^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{6a^2b^2}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(1/7*b^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+
c)+4*a*b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+6/5*a^2*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
+4*a^3*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a
^4*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.66, size = 192, normalized size = 0.78

$$-\frac{560\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^4-210\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3b-672\left(3\,\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1680*(560*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 210*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3*b - 672*(3*sin(d*x + c)^5 - 10*si
```

```
n(d*x + c)^3 + 15*sin(d*x + c))*a^2*b^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x
 - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a*b^3 + 48*(5*sin(d*x +
 c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*b^4)/d
```

mupad [B] time = 2.07, size = 476, normalized size = 1.93

$$\frac{\left(2\,a^4-5\,a^3\,b+12\,a^2\,b^2-\frac{11\,a\,b^3}{2}+2\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(\frac{28\,a^4}{3}-12\,a^3\,b+40\,a^2\,b^2-\frac{14\,a\,b^3}{3}+4\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*(24*a^4 + (424*b^4)/35 + (624*a^2*b^2)/5) + tan(c/2 +
 (d*x)/2)^13*(2*a^4 - 5*a^3*b - (11*a*b^3)/2 + 2*b^4 + 12*a^2*b^2) + tan(c/
2 + (d*x)/2)^3*((14*a*b^3)/3 + 12*a^3*b + (28*a^4)/3 + 4*b^4 + 40*a^2*b^2)
+ tan(c/2 + (d*x)/2)^11*((28*a^4)/3 - 12*a^3*b - (14*a*b^3)/3 + 4*b^4 + 40*
a^2*b^2) + tan(c/2 + (d*x)/2)^5*((85*a*b^3)/6 + 9*a^3*b + (58*a^4)/3 + (86*
b^4)/5 + (452*a^2*b^2)/5) + tan(c/2 + (d*x)/2)^9*((58*a^4)/3 - 9*a^3*b - (8
5*a*b^3)/6 + (86*b^4)/5 + (452*a^2*b^2)/5) + tan(c/2 + (d*x)/2)*((11*a*b^3)
/2 + 5*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21
*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 +
 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14
 + 1)) + (a*b*atan((a*b*tan(c/2 + (d*x)/2)*(6*a^2 + 5*b^2))/(2*((5*a*b^3)/2
 + 3*a^3*b)))*(6*a^2 + 5*b^2))/(2*d) - (a*b*(6*a^2 + 5*b^2)*(atan(tan(c/2 +
 (d*x)/2)) - (d*x)/2))/(2*d)
```

sympy [A] time = 6.45, size = 495, normalized size = 2.00

$$\begin{cases} \frac{2a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3a^3bx\sin^4(c+dx)}{2}+3a^3bx\sin^2(c+dx)\cos^2(c+dx)+\frac{3a^3bx\cos^4(c+dx)}{2}+\frac{3a^3b\sin^3}{} \\ x\,(a+b\cos(c))^4\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((2*a**4*sin(c + d*x)**3/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**2
/d + 3*a**3*b*x*sin(c + d*x)**4/2 + 3*a**3*b*x*sin(c + d*x)**2*cos(c + d*x)
**2 + 3*a**3*b*x*cos(c + d*x)**4/2 + 3*a**3*b*sin(c + d*x)**3*cos(c + d*x)/
(2*d) + 5*a**3*b*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 16*a**2*b**2*sin(c +
d*x)**5/(5*d) + 8*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 6*a**2*b**2
```

```
*sin(c + d*x)*cos(c + d*x)**4/d + 5*a*b**3*x*sin(c + d*x)**6/4 + 15*a*b**3*
x*sin(c + d*x)**4*cos(c + d*x)**2/4 + 15*a*b**3*x*sin(c + d*x)**2*cos(c + d
*x)**4/4 + 5*a*b**3*x*cos(c + d*x)**6/4 + 5*a*b**3*sin(c + d*x)**5*cos(c +
d*x)/(4*d) + 10*a*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 11*a*b**3*si
n(c + d*x)*cos(c + d*x)**5/(4*d) + 16*b**4*sin(c + d*x)**7/(35*d) + 8*b**4*
sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*b**4*sin(c + d*x)**3*cos(c + d*x)
**4/d + b**4*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(a + b*cos(c))**
4*cos(c)**3, True))
```

3.439 $\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=235

$$-\frac{\left(4a^2-25b^2\right)\sin(c+dx)(a+b\cos(c+dx))^3}{120bd}-\frac{a\left(4a^2-53b^2\right)\sin(c+dx)(a+b\cos(c+dx))^2}{120bd}-\frac{a\left(4a^4-121a^2b^2\right.}{6}$$

```
[Out] 1/16*(8*a^4+36*a^2*b^2+5*b^4)*x-1/60*a*(4*a^4-121*a^2*b^2-128*b^4)*sin(d*x+
c)/b/d-1/240*(8*a^4-178*a^2*b^2-75*b^4)*cos(d*x+c)*sin(d*x+c)/d-1/120*a*(4*
a^2-53*b^2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/b/d-1/120*(4*a^2-25*b^2)*(a+b*cos
(d*x+c))^3*sin(d*x+c)/b/d-1/30*a*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d+1/6*(a+b
*cos(d*x+c))^5*sin(d*x+c)/b/d
```

Rubi [A] time = 0.32, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2791, 2753, 2734}

$$-\frac{a\left(-121a^2b^2+4a^4-128b^4\right)\sin(c+dx)}{60bd}-\frac{\left(4a^2-25b^2\right)\sin(c+dx)(a+b\cos(c+dx))^3}{120bd}-\frac{a\left(4a^2-53b^2\right)\sin(c+d}{120b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 36*a^2*b^2 + 5*b^4)*x)/16 - (a*(4*a^4 - 121*a^2*b^2 - 128*b^4)*Si
n[c + d*x])/(60*b*d) - ((8*a^4 - 178*a^2*b^2 - 75*b^4)*Cos[c + d*x]*Sin[c +
 d*x])/(240*d) - (a*(4*a^2 - 53*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
120*b*d) - ((4*a^2 - 25*b^2)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(120*b*d)
 - (a*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d) + ((a + b*Cos[c + d*x])
^5*Sin[c + d*x])/(6*b*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx = \frac{(a+b\cos(c+dx))^5\sin(c+dx)}{6bd} + \frac{\int (5b-a\cos(c+dx))(a+b\cos(c+$$

$$= -\frac{a(a+b\cos(c+dx))^4\sin(c+dx)}{30bd} + \frac{(a+b\cos(c+dx))^5\sin(c+dx)}{6bd}$$

$$= -\frac{\left(4a^2-25b^2\right)(a+b\cos(c+dx))^3\sin(c+dx)}{120bd} - \frac{a(a+b\cos(c+dx))^4}{30bd}$$

$$= -\frac{a\left(4a^2-53b^2\right)(a+b\cos(c+dx))^2\sin(c+dx)}{120bd} - \frac{\left(4a^2-25b^2\right)(a+b}{}$$

$$= \frac{1}{16}\left(8a^4+36a^2b^2+5b^4\right)x - \frac{a\left(4a^4-121a^2b^2-128b^4\right)\sin(c+dx)}{60bd}$$

Mathematica [A] time = 0.44, size = 156, normalized size = 0.66

$$\frac{45b^2\left(4a^2+b^2\right)\sin(4(c+dx))+480ab\left(6a^2+5b^2\right)\sin(c+dx)+80ab\left(4a^2+5b^2\right)\sin(3(c+dx))+60\left(8a^4+3\right.}{960}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (60*(8*a^4 + 36*a^2*b^2 + 5*b^4)*(c + d*x) + 480*a*b*(6*a^2 + 5*b^2)*Sin[c
+ d*x] + 15*(16*a^4 + 96*a^2*b^2 + 15*b^4)*Sin[2*(c + d*x)] + 80*a*b*(4*a^2
 + 5*b^2)*Sin[3*(c + d*x)] + 45*b^2*(4*a^2 + b^2)*Sin[4*(c + d*x)] + 48*a*b
^3*Sin[5*(c + d*x)] + 5*b^4*Sin[6*(c + d*x)])/(960*d)
```

fricas [A] time = 0.81, size = 150, normalized size = 0.64

$$\frac{15\left(8a^4+36a^2b^2+5b^4\right)dx+\left(40b^4\cos(dx+c)^5+192ab^3\cos(dx+c)^4+640a^3b+512ab^3+10\left(36a^2b^2+5\right.\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*d*x + (40*b^4*cos(d*x + c)^5 + 192*a
*b^3*cos(d*x + c)^4 + 640*a^3*b + 512*a*b^3 + 10*(36*a^2*b^2 + 5*b^4)*cos(d
*x + c)^3 + 64*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^2 + 15*(8*a^4 + 36*a^2*b^2
+ 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.60, size = 168, normalized size = 0.71

$$\frac{b^4\sin(6\,dx+6\,c)}{192\,d}+\frac{ab^3\sin(5\,dx+5\,c)}{20\,d}+\frac{1}{16}\left(8\,a^4+36\,a^2b^2+5\,b^4\right)x+\frac{3\left(4\,a^2b^2+b^4\right)\sin(4\,dx+4\,c)}{64\,d}+\frac{\left(4\,a^3b+5\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/192*b^4*sin(6*d*x + 6*c)/d + 1/20*a*b^3*sin(5*d*x + 5*c)/d + 1/16*(8*a^4
+ 36*a^2*b^2 + 5*b^4)*x + 3/64*(4*a^2*b^2 + b^4)*sin(4*d*x + 4*c)/d + 1/12*
(4*a^3*b + 5*a*b^3)*sin(3*d*x + 3*c)/d + 1/64*(16*a^4 + 96*a^2*b^2 + 15*b^4
)*sin(2*d*x + 2*c)/d + 1/2*(6*a^3*b + 5*a*b^3)*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 174, normalized size = 0.74

$$\frac{b^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a\,b^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+6a^2b^2\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a
^2*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a^3
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

maxima [A] time = 0.58, size = 170, normalized size = 0.72

$$\frac{240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4-1280\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^3b+180\,(12\,dx+12\,c+\sin(4\,dx+4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*
sin(d*x + c))*a^3*b + 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x +
 2*c))*a^2*b^2 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c
))*a*b^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 4
8*sin(2*d*x + 2*c))*b^4)/d
```

mupad [B] time = 0.84, size = 214, normalized size = 0.91

$$\frac{a^4\,x}{2}+\frac{5\,b^4\,x}{16}+\frac{9\,a^2\,b^2\,x}{4}+\frac{a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{15\,b^4\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,b^4\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{b^4\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^4,x)
```

```
[Out] (a^4*x)/2 + (5*b^4*x)/16 + (9*a^2*b^2*x)/4 + (a^4*sin(2*c + 2*d*x))/(4*d) +
 (15*b^4*sin(2*c + 2*d*x))/(64*d) + (3*b^4*sin(4*c + 4*d*x))/(64*d) + (b^4*
sin(6*c + 6*d*x))/(192*d) + (5*a*b^3*sin(3*c + 3*d*x))/(12*d) + (a^3*b*sin(
3*c + 3*d*x))/(3*d) + (a*b^3*sin(5*c + 5*d*x))/(20*d) + (3*a^2*b^2*sin(2*c
+ 2*d*x))/(2*d) + (3*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (5*a*b^3*sin(c + d*
x))/(2*d) + (3*a^3*b*sin(c + d*x))/d
```

sympy [A] time = 4.02, size = 459, normalized size = 1.95

$$\begin{cases} \frac{a^4 x \sin^2\left(c+dx\right)}{2} + \frac{a^4 x \cos^2\left(c+dx\right)}{2} + \frac{a^4 \sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} + \frac{8a^3 b \sin^3\left(c+dx\right)}{3d} + \frac{4a^3 b \sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d} + \frac{9a^2 b^2 x \sin^4\left(c+dx\right)}{4} + \\ x\left(a+b\cos\left(c\right)\right)^4\cos^2\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x*sin(c + d*x)**2/2 + a**4*x*cos(c + d*x)**2/2 + a**4*sin(c
 + d*x)*cos(c + d*x)/(2*d) + 8*a**3*b*sin(c + d*x)**3/(3*d) + 4*a**3*b*sin(
c + d*x)*cos(c + d*x)**2/d + 9*a**2*b**2*x*sin(c + d*x)**4/4 + 9*a**2*b**2*
x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*a**2*b**2*x*cos(c + d*x)**4/4 + 9*a
**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*a**2*b**2*sin(c + d*x)*cos
(c + d*x)**3/(4*d) + 32*a*b**3*sin(c + d*x)**5/(15*d) + 16*a*b**3*sin(c + d
*x)**3*cos(c + d*x)**2/(3*d) + 4*a*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*
b**4*x*sin(c + d*x)**6/16 + 15*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 +
15*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**4*x*cos(c + d*x)**6/16
+ 5*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*b**4*sin(c + d*x)**3*cos(c
 + d*x)**3/(6*d) + 11*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)),
(x*(a + b*cos(c))**4*cos(c)**2, True))
```

3.440 $\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=170

$$\frac{(3a^2+4b^2)\sin(c+dx)(a+b\cos(c+dx))^2}{15d}+\frac{ab(6a^2+29b^2)\sin(c+dx)\cos(c+dx)}{30d}+\frac{1}{2}abx(4a^2+3b^2)+\frac{2(3a^4}{}$$

```
[Out] 1/2*a*b*(4*a^2+3*b^2)*x+2/15*(3*a^4+28*a^2*b^2+4*b^4)*sin(d*x+c)/d+1/30*a*b
*(6*a^2+29*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/15*(3*a^2+4*b^2)*(a+b*cos(d*x+c))
^2*sin(d*x+c)/d+1/5*a*(a+b*cos(d*x+c))^3*sin(d*x+c)/d+1/5*(a+b*cos(d*x+c))^
4*sin(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$\frac{2(28a^2b^2+3a^4+4b^4)\sin(c+dx)}{15d}+\frac{(3a^2+4b^2)\sin(c+dx)(a+b\cos(c+dx))^2}{15d}+\frac{ab(6a^2+29b^2)\sin(c+dx)\cos}{30d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (a*b*(4*a^2 + 3*b^2)*x)/2 + (2*(3*a^4 + 28*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(
15*d) + (a*b*(6*a^2 + 29*b^2)*Cos[c + d*x]*Sin[c + d*x])/(30*d) + ((3*a^2 +
 4*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(15*d) + (a*(a + b*Cos[c + d*x
])^3*Sin[c + d*x])/(5*d) + ((a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx = \frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}\int (4b+4a\cos(c+dx))(a+b\cos($$
$$= \frac{a(a+b\cos(c+dx))^3\sin(c+dx)}{5d} + \frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5d} +$$
$$= \frac{(3a^2+4b^2)(a+b\cos(c+dx))^2\sin(c+dx)}{15d} + \frac{a(a+b\cos(c+dx))^3\sin}{5d}$$
$$= \frac{1}{2}ab(4a^2+3b^2)x + \frac{2(3a^4+28a^2b^2+4b^4)\sin(c+dx)}{15d} + \frac{ab(6a^2+29}{}$$

Mathematica [A] time = 0.49, size = 133, normalized size = 0.78

$$\frac{30(8a^4+36a^2b^2+5b^4)\sin(c+dx)+b(480a^3c+480a^3dx+5(24a^2b+5b^3)\sin(3(c+dx))+240a(a^2+b^2)\mathrm{s}}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (30*(8*a^4 + 36*a^2*b^2 + 5*b^4)*Sin[c + d*x] + b*(480*a^3*c + 360*a*b^2*c
+ 480*a^3*d*x + 360*a*b^2*d*x + 240*a*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(24*
a^2*b + 5*b^3)*Sin[3*(c + d*x)] + 30*a*b^2*Sin[4*(c + d*x)] + 3*b^3*Sin[5*(
c + d*x)]))/(240*d)
```

fricas [A] time = 1.04, size = 121, normalized size = 0.71

$$\frac{15(4a^3b+3ab^3)dx+(6b^4\cos(dx+c)^4+30ab^3\cos(dx+c)^3+30a^4+120a^2b^2+16b^4+4(15a^2b^2+2b^4)\mathrm{c}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(4*a^3*b + 3*a*b^3)*d*x + (6*b^4*cos(d*x + c)^4 + 30*a*b^3*cos(d*x
 + c)^3 + 30*a^4 + 120*a^2*b^2 + 16*b^4 + 4*(15*a^2*b^2 + 2*b^4)*cos(d*x +
c)^2 + 15*(4*a^3*b + 3*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.64, size = 134, normalized size = 0.79

$$\frac{b^4\sin(5dx+5c)}{80d}+\frac{ab^3\sin(4dx+4c)}{8d}+\frac{1}{2}(4a^3b+3ab^3)x+\frac{(24a^2b^2+5b^4)\sin(3dx+3c)}{48d}+\frac{(a^3b+ab^3)\sin(2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/80*b^4*sin(5*d*x + 5*c)/d + 1/8*a*b^3*sin(4*d*x + 4*c)/d + 1/2*(4*a^3*b +
 3*a*b^3)*x + 1/48*(24*a^2*b^2 + 5*b^4)*sin(3*d*x + 3*c)/d + (a^3*b + a*b^3
)*sin(2*d*x + 2*c)/d + 1/8*(8*a^4 + 36*a^2*b^2 + 5*b^4)*sin(d*x + c)/d
```

maple [A] time = 0.04, size = 138, normalized size = 0.81

$$\frac{\frac{b^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4a\,b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^2b^2\left(2+\cos^2\left(dx+c\right)\right)\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a*b^3*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*b^2*(2+cos(d*x+
c)^2)*sin(d*x+c)+4*a^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*sin(
d*x+c))
```

maxima [A] time = 0.67, size = 133, normalized size = 0.78

$$\frac{120\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^3b-240\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2b^2+15\,(12\,dx+12\,c+\sin\left(4\,dx+4\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3*b - 240*(sin(d*x + c)^3 - 3
*sin(d*x + c))*a^2*b^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a*b^3 + 8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*b^4 + 120*a^4*sin(d*x + c))/d
```

mupad [B] time = 2.04, size = 363, normalized size = 2.14

$$\frac{\left(2\,a^4-4\,a^3\,b+12\,a^2\,b^2-5\,a\,b^3+2\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(8\,a^4-8\,a^3\,b+32\,a^2\,b^2-2\,a\,b^3+\frac{8\,b^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(12*a^4 + (116*b^4)/15 + 40*a^2*b^2) + tan(c/2 + (d*x
)/2)^9*(2*a^4 - 4*a^3*b - 5*a*b^3 + 2*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2
)^3*(2*a*b^3 + 8*a^3*b + 8*a^4 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/
2)^7*(8*a^4 - 8*a^3*b - 2*a*b^3 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)
/2)*(5*a*b^3 + 4*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)
/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (a*b*atan((a*b*tan(c/2 + (d*x)/2)*(
4*a^2 + 3*b^2))/(3*a*b^3 + 4*a^3*b))*(4*a^2 + 3*b^2))/d - (a*b*(4*a^2 + 3*b
^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/d
```

sympy [A] time = 2.25, size = 301, normalized size = 1.77

$$\begin{cases} \frac{a^4 \sin(c+dx)}{d} + 2a^3bx\sin^2(c+dx) + 2a^3bx\cos^2(c+dx) + \frac{2a^3b\sin(c+dx)\cos(c+dx)}{d} + \frac{4a^2b^2\sin^3(c+dx)}{d} + \frac{6a^2b^2\sin(c+dx)\,\mathrm{co}}{d} \\ x(a+b\cos(c))^4\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)/d + 2*a**3*b*x*sin(c + d*x)**2 + 2*a**3*b*x*co
s(c + d*x)**2 + 2*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*a**2*b**2*sin(c +
d*x)**3/d + 6*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*a*b**3*x*sin(c +
 d*x)**4/2 + 3*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*a*b**3*x*cos(c
+ d*x)**4/2 + 3*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*a*b**3*sin(c
+ d*x)*cos(c + d*x)**3/(2*d) + 8*b**4*sin(c + d*x)**5/(15*d) + 4*b**4*sin(c
 + d*x)**3*cos(c + d*x)**2/(3*d) + b**4*sin(c + d*x)*cos(c + d*x)**4/d, Ne(
d, 0)), (x*(a + b*cos(c))**4*cos(c), True))
```

3.441 $\int (a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{ab\left(19a^2+16b^2\right)\sin(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(8a^4+24a^2b^2+3b^4\right)+\frac{b\sin(c+dx)(a}{}$$

```
[Out] 1/8*(8*a^4+24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2+16*b^2)*sin(d*x+c)/d+1/24*b^
2*(26*a^2+9*b^2)*cos(d*x+c)*sin(d*x+c)/d+7/12*a*b*(a+b*cos(d*x+c))^2*sin(d*
x+c)/d+1/4*b*(a+b*cos(d*x+c))^3*sin(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2656, 2753, 2734}

$$\frac{ab\left(19a^2+16b^2\right)\sin(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)+\frac{b\sin(c+dx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 + 16*b^2)*Sin[c + d*x])/(
6*d) + (b^2*(26*a^2 + 9*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (7*a*b*(a
+ b*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + (b*(a + b*Cos[c + d*x])^3*Sin[c
+ d*x])/(4*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\,dx = \frac{b(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\int (a+b\cos(c+dx))^2\left(4a^2+3b^2+7ab\cos(c\right.$$
$$= \frac{7ab(a+b\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{b(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{12}\int (a+$$
$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x + \frac{ab\left(19a^2+16b^2\right)\sin(c+dx)}{6d} + \frac{b^2\left(26a^2+9b^2\right)\cos(c+}{24d}$$

Mathematica [A] time = 0.21, size = 104, normalized size = 0.76

$$\frac{24b^2\left(6a^2+b^2\right)\sin(2(c+dx))+96ab\left(4a^2+3b^2\right)\sin(c+dx)+12\left(8a^4+24a^2b^2+3b^4\right)(c+dx)+32ab^3\sin(3(}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sin[c +
 d*x] + 24*b^2*(6*a^2 + b^2)*Sin[2*(c + d*x)] + 32*a*b^3*Sin[3*(c + d*x)] +
 3*b^4*Sin[4*(c + d*x)])/(96*d)
```

fricas [A] time = 0.77, size = 96, normalized size = 0.70

$$\frac{3\left(8a^4+24a^2b^2+3b^4\right)dx+\left(6b^4\cos(dx+c)^3+32ab^3\cos(dx+c)^2+96a^3b+64ab^3+9\left(8a^2b^2+b^4\right)\cos(d\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*d*x + (6*b^4*cos(d*x + c)^3 + 32*a*b^3
*cos(d*x + c)^2 + 96*a^3*b + 64*a*b^3 + 9*(8*a^2*b^2 + b^4)*cos(d*x + c))*s
in(d*x + c))/d
```

giac [A] time = 0.54, size = 107, normalized size = 0.78

$$\frac{b^4\sin(4dx+4c)}{32d}+\frac{ab^3\sin(3dx+3c)}{3d}+\frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x+\frac{\left(6a^2b^2+b^4\right)\sin(2dx+2c)}{4d}+\frac{\left(4a^3b+3ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/32*b^4*sin(4*d*x + 4*c)/d + 1/3*a*b^3*sin(3*d*x + 3*c)/d + 1/8*(8*a^4 + 24*a^2*b^2 + 3*b^4)*x + 1/4*(6*a^2*b^2 + b^4)*sin(2*d*x + 2*c)/d + (4*a^3*b + 3*a*b^3)*sin(d*x + c)/d

maple [A] time = 0.04, size = 116, normalized size = 0.85

$$\frac{b^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a\,b^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^4,x)

[Out] 1/d*(b^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^2*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^3*b*sin(d*x+c)+a^4*(d*x+c))

maxima [A] time = 0.70, size = 111, normalized size = 0.81

$$a^4x+\frac{3\,(2\,dx+2\,c+\sin\,(2\,dx+2\,c))a^2b^2}{2\,d}-\frac{4\left(\sin\,(dx+c)^3-3\,\sin\,(dx+c)\right)ab^3}{3\,d}+\frac{(12\,dx+12\,c+\sin\,(4\,dx+4\,c)+}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="maxima")

[Out] a^4*x + 3/2*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b^2/d - 4/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^3/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b^4/d + 4*a^3*b*sin(d*x + c)/d

mupad [B] time = 0.66, size = 123, normalized size = 0.90

$$a^4\,x+\frac{3\,b^4\,x}{8}+3\,a^2\,b^2\,x+\frac{b^4\,\sin\,(2\,c+2\,d\,x)}{4\,d}+\frac{b^4\,\sin\,(4\,c+4\,d\,x)}{32\,d}+\frac{a\,b^3\,\sin\,(3\,c+3\,d\,x)}{3\,d}+\frac{3\,a^2\,b^2\,\sin\,(2\,c+2\,d\,x)}{2\,d}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x))^4,x)

[Out] a^4*x + (3*b^4*x)/8 + 3*a^2*b^2*x + (b^4*sin(2*c + 2*d*x))/(4*d) + (b^4*sin(4*c + 4*d*x))/(32*d) + (a*b^3*sin(3*c + 3*d*x))/(3*d) + (3*a^2*b^2*sin(2*c + 2*d*x))/(2*d) + (3*a*b^3*sin(c + d*x))/d + (4*a^3*b*sin(c + d*x))/d

sympy [A] time = 1.11, size = 240, normalized size = 1.75

$$\begin{cases} a^4x + \frac{4a^3b\sin(c+dx)}{d} + 3a^2b^2x\sin^2(c+dx) + 3a^2b^2x\cos^2(c+dx) + \frac{3a^2b^2\sin(c+dx)\cos(c+dx)}{d} + \frac{8ab^3\sin^3(c+dx)}{3d} + \frac{4ab^3 \text{s} \\ x(a+b\cos(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sin(c + d*x)/d + 3*a**2*b**2*x*sin(c + d*x)**2
 + 3*a**2*b**2*x*cos(c + d*x)**2 + 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)/d
+ 8*a*b**3*sin(c + d*x)**3/(3*d) + 4*a*b**3*sin(c + d*x)*cos(c + d*x)**2/d
+ 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
 3*b**4*x*cos(c + d*x)**4/8 + 3*b**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5
*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**4,
True))
```

3.442 $\int (a+b\cos(c+dx))^4 \sec(c+dx)\,dx$

Optimal. Leaf size=107

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+2abx\left(2a^2+b^2\right)+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{3d}+\frac{b^2\sin(c+a}{}$$

```
[Out] 2*a*b*(2*a^2+b^2)*x+a^4*arctanh(sin(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*sin(d*
x+c)/d+4/3*a*b^3*cos(d*x+c)*sin(d*x+c)/d+1/3*b^2*(a+b*cos(d*x+c))^2*sin(d*x
+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2793, 3033, 3023, 2735, 3770}

$$\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+2abx\left(2a^2+b^2\right)+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{3d}+\frac{b^2\sin(c+a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x],x]
```

```
[Out] 2*a*b*(2*a^2 + b^2)*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Sin[c + d*x])/(3*d) + (4*a*b^3*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (b^2*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx = \frac{b^2(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+b\cos(c+dx))\left(3a^3+b\left(9\right.\right.$$

$$= \frac{4ab^3\cos(c+dx)\sin(c+dx)}{3d}+\frac{b^2(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{6}\int$$

$$= \frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+\frac{4ab^3\cos(c+dx)\sin(c+dx)}{3d}+\frac{b^2(a+b\,\mathrm{c}}{}$$

$$= 2ab\left(2a^2+b^2\right)x+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+\frac{4ab^3\cos(c+dx)\sin(c}{3d}$$

$$= 2ab\left(2a^2+b^2\right)x+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.16, size = 128, normalized size = 1.20

$$\frac{-12a^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+24ab\left(2a^2+b^2\right)(}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x],x]
```

```
[Out] (24*a*b*(2*a^2 + b^2)*(c + d*x) - 12*a^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*a^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*b^2*(8*a^2 + b
^2)*Sin[c + d*x] + 12*a*b^3*Sin[2*(c + d*x)] + b^4*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 1.15, size = 98, normalized size = 0.92

$$\frac{3\,a^4\log\left(\sin\left(dx+c\right)+1\right)-3\,a^4\log\left(-\sin\left(dx+c\right)+1\right)+12\left(2\,a^3b+ab^3\right)dx+2\left(b^4\cos\left(dx+c\right)^2+6\,ab^3\cos\left(dx\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1) + 12*(2*a^3
*b + a*b^3)*d*x + 2*(b^4*cos(d*x + c)^2 + 6*a*b^3*cos(d*x + c) + 18*a^2*b^2
 + 2*b^4)*sin(d*x + c))/d
```

giac [B] time = 0.59, size = 212, normalized size = 1.98

$$3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+6\left(2\,a^3b+ab^3\right)(dx+c)+\frac{2\left(18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 6*(2*a^3*b + a*b^3)*(d*x + c) + 2*(18*a^2*b^2*tan(1/2*d*x +
 1/2*c)^5 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*x + 1/2*c)^5 +
 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^4*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*
b^2*tan(1/2*d*x + 1/2*c) + 6*a*b^3*tan(1/2*d*x + 1/2*c) + 3*b^4*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.08, size = 131, normalized size = 1.22

$$\frac{a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+4a^3bx+\frac{4a^3bc}{d}+\frac{6a^2b^2\sin\left(dx+c\right)}{d}+\frac{2a\,b^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{d}+2a\,b^3x+\frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c),x)
```

```
[Out] 1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b*x+4/d*a^3*b*c+6/d*a^2*b^2*sin(d*x
+c)+2*a*b^3*cos(d*x+c)*sin(d*x+c)/d+2*a*b^3*x+2/d*a*b^3*c+1/3/d*sin(d*x+c)*
cos(d*x+c)^2*b^4+2/3/d*b^4*sin(d*x+c)
```

maxima [A] time = 0.68, size = 95, normalized size = 0.89

$$\frac{12\,(dx+c)a^3b+3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))ab^3-\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)b^4+3\,a^4\log\left(\sec\left(dx+c\right)+\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*(12*(d*x + c)*a^3*b + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b^3 - (sin(d
*x + c)^3 - 3*sin(d*x + c))*b^4 + 3*a^4*log(sec(d*x + c) + tan(d*x + c)) +
18*a^2*b^2*sin(d*x + c))/d
```

mupad [B] time = 0.82, size = 158, normalized size = 1.48

$$\frac{3\,b^4\,\sin\left(c+d\,x\right)}{4\,d}+\frac{2\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^4\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{4\,a\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^4/cos(c + d*x),x)
```

```
[Out] (3*b^4*sin(c + d*x))/(4*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/d + (b^4*sin(3*c + 3*d*x))/(12*d) + (a*b^3*sin(2*c + 2*d*x))/d + (6*
a^2*b^2*sin(c + d*x))/d + (4*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (8*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos\left(c+dx\right))^4\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**4*sec(c + d*x), x)
```

3.443 $\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}b^2x\left(12a^2+b^2\right)+$$

```
[Out] 1/2*b^2*(12*a^2+b^2)*x+4*a^3*b*arctanh(sin(d*x+c))/d-2*a*b*(a^2-2*b^2)*sin(
d*x+c)/d-1/2*b^2*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/d+a^2*(a+b*cos(d*x+c))^2
*tan(d*x+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3033, 3023, 2735, 3770}

$$-\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}b^2x\left(12a^2+b^2\right)+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*(12*a^2 + b^2)*x)/2 + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*(a^2
- 2*b^2)*Sin[c + d*x])/d - (b^2*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(2
*d) + (a^2*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))^2\tan(c+dx)}{d}+\int (a+b\cos(c+dx))\left(4a^2b+3ab\right.$$

$$= -\frac{b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^2(a+b\cos(c+dx))^2\tan(c+$$

$$= -\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2d}+$$

$$= \frac{1}{2}b^2\left(12a^2+b^2\right)x-\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{b^2\left(2a^2-b^2\right)\cos(c+$$

$$= \frac{1}{2}b^2\left(12a^2+b^2\right)x+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\left(a^2-2b^2\right)\sin(c+$$

Mathematica [A] time = 0.66, size = 119, normalized size = 1.04

$$\frac{4a^4\tan(c+dx)+2b\left(-8a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(b*(12*a^2 + b^2)*(c + d*x) - 8*a^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 8*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a*b^3*Sin[c +
 d*x] + b^4*Sin[2*(c + d*x)] + 4*a^4*Tan[c + d*x])/(4*d)
```

fricas [A] time = 1.22, size = 116, normalized size = 1.02

$$\frac{4\,a^3b\cos(dx+c)\log(\sin(dx+c)+1)-4\,a^3b\cos(dx+c)\log(-\sin(dx+c)+1)+\left(12\,a^2b^2+b^4\right)dx\cos(dx+c}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^3*b*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a^3*b*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (12*a^2*b^2 + b^4)*d*x*cos(d*x + c) + (b^4*cos(d*x + c
)^2 + 8*a*b^3*cos(d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.55, size = 170, normalized size = 1.49

$$\frac{8\,a^3b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,a^3b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\left(12\,a^2b^2+b^4\right)(dx+c)+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(8*a^3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^3*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
 + (12*a^2*b^2 + b^4)*(d*x + c) + 2*(8*a*b^3*tan(1/2*d*x + 1/2*c)^3 - b^4*t
an(1/2*d*x + 1/2*c)^3 + 8*a*b^3*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.09, size = 109, normalized size = 0.96

$$\frac{a^4\tan(dx+c)}{d}+\frac{4a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+6a^2b^2x+\frac{6a^2b^2c}{d}+\frac{4a\,b^3\sin(dx+c)}{d}+\frac{b^4\cos(dx+c)\sin(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x)
```

```
[Out] a^4*tan(d*x+c)/d+4/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*b^2*x+6/d*a^2*b^
2*c+4/d*a*b^3*sin(d*x+c)+1/2/d*b^4*cos(d*x+c)*sin(d*x+c)+1/2*b^4*x+1/2/d*b^
4*c
```

maxima [A] time = 0.63, size = 90, normalized size = 0.79

$$\frac{24\,(dx+c)a^2b^2 + (2\,dx + 2\,c + \sin(2\,dx + 2\,c))b^4 + 8\,a^3b\Big(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\Big) + 16\,ab}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(24*(d*x + c)*a^2*b^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*b^4 + 8*a^3*b*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a*b^3*sin(d*x + c) + 4
*a^4*tan(d*x + c))/d
```

mupad [B] time = 0.71, size = 150, normalized size = 1.32

$$\frac{b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{12\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{b^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^2,x)
```

```
[Out] (b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^4*sin(c + d*x))/(d
*cos(c + d*x)) + (12*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
 + (4*a*b^3*sin(c + d*x))/d + (b^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (8*a^
3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))^4 \sec^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**4*sec(c + d*x)**2, x)
```

3.444 $\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$

Optimal. Leaf size=108

$$\frac{3a^3b\tan(c+dx)}{d}-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{2d}+\frac{a^2\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

```
[Out] 4*a*b^3*x+1/2*a^2*(a^2+12*b^2)*arctanh(sin(d*x+c))/d-1/2*b^2*(a^2-2*b^2)*si
n(d*x+c)/d+3*a^3*b*tan(d*x+c)/d+1/2*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)*tan(d
*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3031, 3023, 2735, 3770}

$$-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{2d}+\frac{a^2\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3b\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

```
[Out] 4*a*b^3*x + (a^2*(a^2 + 12*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*(a^2 -
2*b^2)*Sin[c + d*x])/(2*d) + (3*a^3*b*Tan[c + d*x])/d + (a^2*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a+b\cos(c+dx\\ &= \frac{3a^3b\tan(c+dx)}{d}+\frac{a^2(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}\\ &= -\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{2d}+\frac{3a^3b\tan(c+dx)}{d}+\frac{a^2(a+b\cos(c+dx))^2}{}\\ &= 4ab^3x-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{2d}+\frac{3a^3b\tan(c+dx)}{d}+\frac{a^2(a+b\cos(c}{}\\ &= 4ab^3x+\frac{a^2\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx}{2d}\end{aligned}$$

Mathematica [A] time = 2.44, size = 174, normalized size = 1.61

$$\frac{16a^3b\tan(c+dx)+a\left(\frac{a^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{a^3}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}-2a\left(a^2+12b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

```
[Out] (a*(16*b^3*c + 16*b^3*d*x - 2*a*(a^2 + 12*b^2)*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 2*a*(a^2 + 12*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ a^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^3/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2) + 4*b^4*Sin[c + d*x] + 16*a^3*b*Tan[c + d*x])/(4*d)
```

fricas [A] time = 0.75, size = 130, normalized size = 1.20

$$\frac{16\,ab^3dx\cos\left(dx+c\right)^2+\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a*b^3*d*x*cos(d*x + c)^2 + (a^4 + 12*a^2*b^2)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - (a^4 + 12*a^2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*(2*b^4*cos(d*x + c)^2 + 8*a^3*b*cos(d*x + c) + a^4)*sin(d*x + c))/(d*co
s(d*x + c)^2)
```

giac [A] time = 0.86, size = 177, normalized size = 1.64

$$\frac{8\,(dx+c)ab^3+\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\left(a^4+12\,a^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(a^4+12\,a^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a*b^3 + 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2
 + 1) + (a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (a^4 + 12*a
^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^4*tan(1/2*d*x + 1/2*c)^3
- 8*a^3*b*tan(1/2*d*x + 1/2*c)^3 + a^4*tan(1/2*d*x + 1/2*c) + 8*a^3*b*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.10, size = 114, normalized size = 1.06

$$\frac{a^4 \sec(dx+c)\tan(dx+c)}{2d}+\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{4a^3 b\tan(dx+c)}{d}+\frac{6a^2b^2 \ln(\sec(dx+c)+\tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^4*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b
*tan(d*x+c)/d+6/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4*a*b^3*x+4/d*a*b^3*c+1
/d*b^4*sin(d*x+c)
```

maxima [A] time = 0.86, size = 115, normalized size = 1.06

$$\frac{16\,(dx+c)ab^3-a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12\,a^2b^2\left(\log(\sin(dx+c)+1)\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a*b^3 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + 4*b^4*sin(d*x + c) + 16*a^3*b*tan(d*x + c))/d
```

mupad [B] time = 0.72, size = 152, normalized size = 1.41

$$\frac{b^4\,\sin(c+d\,x)}{d}+\frac{a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\sin(c+d\,x)}{2\,d\cos(c+d\,x)^2}+\frac{12\,a^2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^3,x)
```

```
[Out] (b^4*sin(c + d*x))/d + (a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
 + (a^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (12*a^2*b^2*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d + (4*a^3*b*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.445 $\int (a+b\cos(c+dx))^4 \sec^4(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{4a^3b\tan(c+dx)\sec(c+dx)}{3d}+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{2ab\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{}$$

```
[Out] b^4*x+2*a*b*(a^2+2*b^2)*arctanh(sin(d*x+c))/d+1/3*a^2*(2*a^2+17*b^2)*tan(d*
x+c)/d+4/3*a^3*b*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*(a+b*cos(d*x+c))^2*sec(d*x
+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3031, 3021, 2735, 3770}

$$\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{2ab\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^3b\tan(c+dx)\sec(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

```
[Out] b^4*x + (2*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^
2)*Tan[c + d*x])/(3*d) + (4*a^3*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (a^2*(
a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\cos(c+dx)\\ &= \frac{4a^3b\sec(c+dx)\tan(c+dx)}{3d}+\frac{a^2(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c\\ &= \frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{4a^3b\sec(c+dx)\tan(c+dx)}{3d}+\frac{a^2(a+b\,\mathrm{c}\\ &= b^4x+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{4a^3b\sec(c+dx)\tan(c+dx)}{3d}+\frac{a^2(a\\ &= b^4x+\frac{2ab\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.39, size = 77, normalized size = 0.67

$$\frac{a^4\tan^3(c+dx)+6ab\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))+3a^2\tan(c+dx)\left(a^2+2ab\sec(c+dx)+6b^2\right)+3b^4dx}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

```
[Out] (3*b^4*d*x + 6*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]] + 3*a^2*(a^2 + 6*b^2
 + 2*a*b*Sec[c + d*x])*Tan[c + d*x] + a^4*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 1.23, size = 138, normalized size = 1.20

$$\frac{3\,b^4dx\cos\left(dx+c\right)^3+3\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)^3\log\left(-\mathrm{s}\right.}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*d*x*cos(d*x + c)^3 + 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3*log(sin(
d*x + c) + 1) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 (6*a^3*b*cos(d*x + c) + a^4 + 2*(a^4 + 9*a^2*b^2)*cos(d*x + c)^2)*sin(d*x
+ c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.62, size = 221, normalized size = 1.92

$$3\left(dx+c\right)b^4+6\left(a^3b+2\,ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\left(a^3b+2\,ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^4\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*b^4 + 6*(a^3*b + 2*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 6*(a^3*b + 2*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*a^4*tan(
1/2*d*x + 1/2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*
x + 1/2*c)^5 - 2*a^4*tan(1/2*d*x + 1/2*c)^3 - 36*a^2*b^2*tan(1/2*d*x + 1/2*
c)^3 + 3*a^4*tan(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b
^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.10, size = 135, normalized size = 1.17

$$\frac{2a^4\tan\left(dx+c\right)}{3d}+\frac{a^4\tan\left(dx+c\right)\left(\sec^2\left(dx+c\right)\right)}{3d}+\frac{2a^3b\sec\left(dx+c\right)\tan\left(dx+c\right)}{d}+\frac{2a^3b\ln\left(\sec\left(dx+c\right)+\tan\left(dx\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a^4*tan(d*x+c)/d+1/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2+2*a^3*b*sec(d*x+c)*t
an(d*x+c)/d+2/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*tan(d*x+c)+4/d*
a*b^3*ln(sec(d*x+c)+tan(d*x+c))+b^4*x+1/d*b^4*c
```

maxima [A] time = 0.63, size = 125, normalized size = 1.09

$$\frac{\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{4}+3\,(dx+c)b^{4}-3\,a^{3}b\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-\right.\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 3*(d*x + c)*b^4 - 3*a^3*b*(2*s
in(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) + 6*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*a^2*b
^2*tan(d*x + c))/d
```

mupad [B] time = 0.78, size = 185, normalized size = 1.61

$$\frac{2\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}+\frac{8\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^4,x)
```

```
[Out] (2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a^4*sin(c + d*x)
)/(3*d*cos(c + d*x)) + (a^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (8*a*b^3*a
tanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*a^3*b*atanh(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a^3*b*sin(c + d*x))/(d*cos(c + d*x)^2)
+ (6*a^2*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.446 $\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$

Optimal. Leaf size=154

$$\frac{5a^3b\tan(c+dx)\sec^2(c+dx)}{6d}+\frac{4ab\left(2a^2+3b^2\right)\tan(c+dx)}{3d}+\frac{a^2\left(3a^2+22b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^2\tan(c$$

```
[Out] 1/8*(3*a^4+24*a^2*b^2+8*b^4)*arctanh(sin(d*x+c))/d+4/3*a*b*(2*a^2+3*b^2)*ta
n(d*x+c)/d+1/8*a^2*(3*a^2+22*b^2)*sec(d*x+c)*tan(d*x+c)/d+5/6*a^3*b*sec(d*x
+c)^2*tan(d*x+c)/d+1/4*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.34, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{4ab\left(2a^2+3b^2\right)\tan(c+dx)}{3d}+\frac{\left(24a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\left(3a^2+22b^2\right)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a*b*(2*a^2
+ 3*b^2)*Tan[c + d*x])/(3*d) + (a^2*(3*a^2 + 22*b^2)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (5*a^3*b*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a^2*(a + b*Cos[
c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (a+b\cos(c+d$$
$$= \frac{5a^3b\sec^2(c+dx)\tan(c+dx)}{6d} + \frac{a^2(a+b\cos(c+dx))^2\sec^3(c+dx)\tan$$
$$= \frac{a^2\left(3a^2+22b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{5a^3b\sec^2(c+dx)\tan(c+dx}{6d}$$
$$= \frac{a^2\left(3a^2+22b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{5a^3b\sec^2(c+dx)\tan(c+dx}{6d}$$
$$= \frac{\left(3a^4+24a^2b^2+8b^4\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2\left(3a^2+22b^2\right)\sec(c+d}{8d}$$
$$= \frac{\left(3a^4+24a^2b^2+8b^4\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4ab\left(2a^2+3b^2\right)\tan(c+d}{3d}$$

Mathematica [A] time = 0.49, size = 101, normalized size = 0.66

$$\frac{3\left(3a^4+24a^2b^2+8b^4\right)\tanh^{-1}(\sin(c+dx))+a\tan(c+dx)\left(6a^3\sec^3(c+dx)+32b\left(3\left(a^2+b^2\right)+a^2\tan^2(c+d\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]] + a*Tan[c + d*x]*(9*a
*(a^2 + 8*b^2)*Sec[c + d*x] + 6*a^3*Sec[c + d*x]^3 + 32*b*(3*(a^2 + b^2) +
a^2*Tan[c + d*x]^2)))/(24*d)
```

fricas [A] time = 0.91, size = 163, normalized size = 1.06

$$\frac{3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(
32*a^3*b*cos(d*x + c) + 6*a^4 + 32*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3 + 9*(
a^4 + 8*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.70, size = 360, normalized size = 2.34

$$3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,a^4+24\,a^2b^2+8\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(3*a^4 + 24*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^
4*tan(1/2*d*x + 1/2*c)^7 - 96*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*tan
(1/2*d*x + 1/2*c)^7 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*a^4*tan(1/2*d*x +
 1/2*c)^5 + 160*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 72*a^2*b^2*tan(1/2*d*x + 1/2
*c)^5 + 288*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^4*tan(1/2*d*x + 1/2*c)^3 - 1
60*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*a
*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c) + 96*a^3*b*tan(1/
2*d*x + 1/2*c) + 72*a^2*b^2*tan(1/2*d*x + 1/2*c) + 96*a*b^3*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.11, size = 188, normalized size = 1.22

$$\frac{a^4\left(\sec^3\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3a^4\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{8a^3b\tan\left(dx+\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^4*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^4
*ln(sec(d*x+c)+tan(d*x+c))+8/3*a^3*b*tan(d*x+c)/d+4/3*a^3*b*sec(d*x+c)^2*ta
n(d*x+c)/d+3/d*a^2*b^2*tan(d*x+c)*sec(d*x+c)+3/d*a^2*b^2*ln(sec(d*x+c)+tan(
d*x+c))+4/d*a*b^3*tan(d*x+c)+1/d*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.98, size = 187, normalized size = 1.21

$$64\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3b-3\,a^4\left(\frac{2\left(3\,\sin\left(dx+c\right)^3-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b - 3*a^4*(2*(3*sin(d*x + c)
^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*b^4*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*a*b^3*tan(d*x + c))/d
```

mupad [B] time = 4.29, size = 245, normalized size = 1.59

$$\frac{\left(\frac{5\,a^4}{4}-8\,a^3\,b+6\,a^2\,b^2-8\,a\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,a^4}{4}+\frac{40\,a^3\,b}{3}-6\,a^2\,b^2+24\,a\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,a^4}{4}-\frac{40\,a^3\,b}{3}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3*b + (5*a^4)/4 + 6*a^2*b^2) - tan(c/2 +
 (d*x)/2)^7*(8*a*b^3 + 8*a^3*b - (5*a^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)/2
)^3*(24*a*b^3 + (40*a^3*b)/3 - (3*a^4)/4 + 6*a^2*b^2) + tan(c/2 + (d*x)/2)^
5*(24*a*b^3 + (40*a^3*b)/3 + (3*a^4)/4 - 6*a^2*b^2))/(d*(6*tan(c/2 + (d*x)/
2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)
^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*((3*a^4)/4 + 2*b^4 + 6*a^2*b^2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.447 $\int (a + b\cos(c + dx))^4 \sec^6(c + dx)\,dx$

Optimal. Leaf size=188

$$\frac{3a^3b\tan(c+dx)\sec^3(c+dx)}{5d} + \frac{ab\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\left(4a^2+27b^2\right)\tan(c+dx)\sec^2(c+dx)}{15d} + \frac{ab}{}$$

```
[Out] 1/2*a*b*(3*a^2+4*b^2)*arctanh(sin(d*x+c))/d+1/15*(8*a^4+60*a^2*b^2+15*b^4)*
tan(d*x+c)/d+1/2*a*b*(3*a^2+4*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*a^2*(4*a^2+
27*b^2)*sec(d*x+c)^2*tan(d*x+c)/d+3/5*a^3*b*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a
^2*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A] time = 0.36, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2792, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\left(60a^2b^2+8a^4+15b^4\right)\tan(c+dx)}{15d} + \frac{ab\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\left(4a^2+27b^2\right)\tan(c+dx)\sec^2(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

```
[Out] (a*b*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + ((8*a^4 + 60*a^2*b^2 +
15*b^4)*Tan[c + d*x])/(15*d) + (a*b*(3*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*
x])/(2*d) + (a^2*(4*a^2 + 27*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (3*
a^3*b*Sec[c + d*x]^3*Tan[c + d*x])/(5*d) + (a^2*(a + b*Cos[c + d*x])^2*Sec[
c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
```

```
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int (a+b\cos(c+dx)$$

$$=\frac{3a^3b\sec^3(c+dx)\tan(c+dx)}{5d}+\frac{a^2(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(}{5d}$$

$$=\frac{a^2\left(4a^2+27b^2\right)\sec^2(c+dx)\tan(c+dx)}{15d}+\frac{3a^3b\sec^3(c+dx)\tan(c+dx}{5d}$$

$$=\frac{a^2\left(4a^2+27b^2\right)\sec^2(c+dx)\tan(c+dx)}{15d}+\frac{3a^3b\sec^3(c+dx)\tan(c+dx}{5d}$$

$$=\frac{ab\left(3a^2+4b^2\right)\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^2\left(4a^2+27b^2\right)\sec^2(c+dx)\,\mathrm{ta}}{15d}$$

$$=\frac{ab\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\left(8a^4+60a^2b^2+15b^4\right)\tan(c+dx}{15d}$$

Mathematica [**A**] time = 0.75, size = 125, normalized size = 0.66

$$\frac{15ab\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(6a^4\tan^4(c+dx)+30a^3b\sec^3(c+dx)+20a^2\left(a^2+3b^2\right)\tan\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

```
[Out] (15*a*b*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(30*(a^4 + 6*a
^2*b^2 + b^4) + 15*a*b*(3*a^2 + 4*b^2)*Sec[c + d*x] + 30*a^3*b*Sec[c + d*x]
^3 + 20*a^2*(a^2 + 3*b^2)*Tan[c + d*x]^2 + 6*a^4*Tan[c + d*x]^4))/(30*d)
```

fricas [**A**] time = 1.10, size = 182, normalized size = 0.97

$$\frac{15\left(3\,a^3b+4\,ab^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,a^3b+4\,ab^3\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/60*(15*(3*a^3*b + 4*a*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*a
^3*b + 4*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*a^3*b*cos(d*x
 + c) + 2*(8*a^4 + 60*a^2*b^2 + 15*b^4)*cos(d*x + c)^4 + 6*a^4 + 15*(3*a^3*
b + 4*a*b^3)*cos(d*x + c)^3 + 4*(2*a^4 + 15*a^2*b^2)*cos(d*x + c)^2)*sin(d*
x + c))/(d*cos(d*x + c)^5)
```

giac [B] time = 0.72, size = 461, normalized size = 2.45

$$\frac{15\left(3\,a^{3}b+4\,ab^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(3\,a^{3}b+4\,ab^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(30\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*a^3
*b + 4*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x +
1/2*c)^9 - 75*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 60*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 - 40
*a^4*tan(1/2*d*x + 1/2*c)^7 + 30*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*a^2*b^2
*tan(1/2*d*x + 1/2*c)^7 + 120*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 120*b^4*tan(1/
2*d*x + 1/2*c)^7 + 116*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 180*b^4*tan(1/2*d*x + 1/2*c)^5 - 40*a^4*tan(1/2*d*x + 1/2*c)^
3 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 -
120*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 120*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*
tan(1/2*d*x + 1/2*c) + 75*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 60*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.12, size = 225, normalized size = 1.20

$$\frac{8a^{4}\tan\left(dx+c\right)}{15d}+\frac{a^{4}\tan\left(dx+c\right)\left(\sec^{4}\left(dx+c\right)\right)}{5d}+\frac{4a^{4}\tan\left(dx+c\right)\left(\sec^{2}\left(dx+c\right)\right)}{15d}+\frac{a^{3}b\left(\sec^{3}\left(dx+c\right)\right)\tan\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x)
```

```
[Out] 8/15*a^4*tan(d*x+c)/d+1/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a^4*tan(d*x+
c)*sec(d*x+c)^2+a^3*b*sec(d*x+c)^3*tan(d*x+c)/d+3/2*a^3*b*sec(d*x+c)*tan(d*
x+c)/d+3/2/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^2*b^2*tan(d*x+c)+2/d*a^2
*b^2*tan(d*x+c)*sec(d*x+c)^2+2/d*a*b^3*tan(d*x+c)*sec(d*x+c)+2/d*a*b^3*ln(s
ec(d*x+c)+tan(d*x+c))+1/d*b^4*tan(d*x+c)
```

maxima [A] time = 1.20, size = 195, normalized size = 1.04

$$\frac{4\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{4}+120\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{2}b^{2}-15\,a^{3}b\left(\frac{2\,(3}{\sin}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 - 15*a^3*b*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*b^4*tan(d*x +
 c))/d
```

mupad [B] time = 4.46, size = 304, normalized size = 1.62

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(3\,a^3\,b+4\,a\,b^3\right)}{d}-\frac{\left(2\,a^4-5\,a^3\,b+12\,a^2\,b^2-4\,a\,b^3+2\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{8\,a^4}{3}+2\,a^3\,b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^6,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(4*a*b^3 + 3*a^3*b))/d - (tan(c/2 + (d*x)/2)^5*(
(116*a^4)/15 + 12*b^4 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 5*a^3*b
 - 4*a*b^3 + 2*b^4 + 12*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(8*a*b^3 + 2*a^3*b
+ (8*a^4)/3 + 8*b^4 + 32*a^2*b^2) - tan(c/2 + (d*x)/2)^7*((8*a^4)/3 - 2*a^3
*b - 8*a*b^3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(4*a*b^3 + 5*a^3*b
+ 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d
*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*
x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.448 $\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$

Optimal. Leaf size=222

$$\frac{7a^3b\tan(c+dx)\sec^4(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\tan^3(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\tan(c+dx)}{5d}+\frac{a^2\left(5a^2+32b^2\right)\tan}{2}$$

```
[Out] 1/16*(5*a^4+36*a^2*b^2+8*b^4)*arctanh(sin(d*x+c))/d+4/5*a*b*(4*a^2+5*b^2)*t
an(d*x+c)/d+1/16*(5*a^4+36*a^2*b^2+8*b^4)*sec(d*x+c)*tan(d*x+c)/d+1/24*a^2*
(5*a^2+32*b^2)*sec(d*x+c)^3*tan(d*x+c)/d+7/15*a^3*b*sec(d*x+c)^4*tan(d*x+c)
/d+1/6*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)^5*tan(d*x+c)/d+4/15*a*b*(4*a^2+5*b
^2)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.38, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3031, 3021, 2748, 3767, 3768, 3770}

$$\frac{4ab\left(4a^2+5b^2\right)\tan^3(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\tan(c+dx)}{5d}+\frac{\left(36a^2b^2+5a^4+8b^4\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^2\left(5a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

```
[Out] ((5*a^4 + 36*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a*b*(4*a^2
 + 5*b^2)*Tan[c + d*x])/(5*d) + ((5*a^4 + 36*a^2*b^2 + 8*b^4)*Sec[c + d*x]*
Tan[c + d*x])/(16*d) + (a^2*(5*a^2 + 32*b^2)*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) + (7*a^3*b*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (a^2*(a + b*Cos[c +
d*x])^2*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a*b*(4*a^2 + 5*b^2)*Tan[c +
 d*x]^3)/(15*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))^2\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{6}\int (a+b\cos(c+d$$

$$= \frac{7a^3b\sec^4(c+dx)\tan(c+dx)}{15d}+\frac{a^2(a+b\cos(c+dx))^2\sec^5(c+dx)\tan}{6d}$$

$$= \frac{a^2\left(5a^2+32b^2\right)\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{7a^3b\sec^4(c+dx)\tan(c+d}{15d}$$

$$= \frac{a^2\left(5a^2+32b^2\right)\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{7a^3b\sec^4(c+dx)\tan(c+d}{15d}$$

$$= \frac{\left(5a^4+36a^2b^2+8b^4\right)\sec(c+dx)\tan(c+dx)}{16d}+\frac{a^2\left(5a^2+32b^2\right)\sec^3(c}{24d}$$

$$= \frac{\left(5a^4+36a^2b^2+8b^4\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4ab\left(4a^2+5b^2\right)\tan(c+d}{5d}$$

Mathematica [A] time = 1.01, size = 154, normalized size = 0.69

$$\frac{15\left(5a^4+36a^2b^2+8b^4\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(40a^4\sec^5(c+dx)+64ab\left(5\left(2a^2+b^2\right)\tan^2(c+dx\right.\right.}{240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

```
[Out] (15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(
5*a^4 + 36*a^2*b^2 + 8*b^4)*Sec[c + d*x] + 10*a^2*(5*a^2 + 36*b^2)*Sec[c +
d*x]^3 + 40*a^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(2*a^2 + b^2)*T
an[c + d*x]^2 + 3*a^2*Tan[c + d*x]^4)))/(240*d)
```

fricas [A] time = 1.10, size = 217, normalized size = 0.98

$$\frac{15\left(5\,a^4+36\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(5\,a^4+36\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^6\log\left(-\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1)
 - 15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) +
```

```
2*(128*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^5 + 192*a^3*b*cos(d*x + c) + 15*(5*
a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 40*a^4 + 64*(4*a^3*b + 5*a*b^3)*
cos(d*x + c)^3 + 10*(5*a^4 + 36*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*c
os(d*x + c)^6)
```

giac [B] time = 0.78, size = 592, normalized size = 2.67

$$\frac{15\left(5\,a^4+36\,a^2b^2+8\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(5\,a^4+36\,a^2b^2+8\,b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(16
5*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2
*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*b^4*
tan(1/2*d*x + 1/2*c)^11 + 25*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*a^3*b*tan(1/
2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*a*b^3*tan(1/2
*d*x + 1/2*c)^9 - 360*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d*x + 1/
2*c)^7 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*
c)^7 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1/2*c)^7 +
 450*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*a
^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*b^4
*tan(1/2*d*x + 1/2*c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*a^3*b*tan(1/
2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*a*b^3*tan(1/2
*d*x + 1/2*c)^3 - 360*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*a^4*tan(1/2*d*x + 1/
2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*d*x + 1/2*c) +
960*a*b^3*tan(1/2*d*x + 1/2*c) + 120*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 0.12, size = 302, normalized size = 1.36

$$\frac{a^4\left(\sec^5\left(dx+c\right)\right)\tan\left(dx+c\right)}{6d}+\frac{5a^4\left(\sec^3\left(dx+c\right)\right)\tan\left(dx+c\right)}{24d}+\frac{5a^4\sec\left(dx+c\right)\tan\left(dx+c\right)}{16d}+\frac{5a^4\ln\left(\sec\left(dx+c\right.\right.}{16\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x)
```

```
[Out] 1/6*a^4*sec(d*x+c)^5*tan(d*x+c)/d+5/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+5/16*a
^4*sec(d*x+c)*tan(d*x+c)/d+5/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+32/15*a^3*b
*tan(d*x+c)/d+4/5*a^3*b*sec(d*x+c)^4*tan(d*x+c)/d+16/15*a^3*b*sec(d*x+c)^2*
```

```
tan(d*x+c)/d+3/2/d*a^2*b^2*tan(d*x+c)*sec(d*x+c)^3+9/4/d*a^2*b^2*tan(d*x+c)
*sec(d*x+c)+9/4/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/3/d*a*b^3*tan(d*x+c)+
4/3/d*a*b^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*b^4*tan(d*x+c)*sec(d*x+c)+1/2/d*b
^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.76, size = 275, normalized size = 1.24

$$\frac{128\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{3}b+640\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)ab^{3}-5\,a^{4}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3*b +
 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 5*a^4*(2*(15*sin(d*x + c)^5
- 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 180*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2
*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) -
 120*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.31, size = 370, normalized size = 1.67

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{5\,a^4}{8}+\frac{9\,a^2\,b^2}{2}+b^4\right)}{d}+\frac{\left(\frac{11\,a^4}{8}-8\,a^3\,b+\frac{15\,a^2\,b^2}{2}-8\,a\,b^3+b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{5\,a^4}{24}+\frac{56\,a^3\,b}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^7,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((5*a^4)/8 + b^4 + (9*a^2*b^2)/2))/d + (tan(c/2
+ (d*x)/2)^9*((88*a*b^3)/3 + (56*a^3*b)/3 + (5*a^4)/24 - 3*b^4 - (21*a^2*b^
2)/2) - tan(c/2 + (d*x)/2)^3*((88*a*b^3)/3 + (56*a^3*b)/3 - (5*a^4)/24 + 3*
b^4 + (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*(48*a*b^3 + (208*a^3*b)/5 + (1
5*a^4)/4 + 2*b^4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)^7*((15*a^4)/4 - (208*a^3
*b)/5 - 48*a*b^3 + 2*b^4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3
*b + (11*a^4)/8 + b^4 + (15*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^11*((11*a^4)/8
 - 8*a^3*b - 8*a*b^3 + b^4 + (15*a^2*b^2)/2))/(d*(15*tan(c/2 + (d*x)/2)^4 -
 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8
 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.449 $\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{2a^5\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\,\sqrt{a+b}}-\frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d}+\frac{\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8b^3d}+\frac{x\left(8a^4+4a^2b^2\right.}{8b^5}$$

```
[Out] 1/8*(8*a^4+4*a^2*b^2+3*b^4)*x/b^5-1/3*a*(3*a^2+2*b^2)*sin(d*x+c)/b^4/d+1/8*
(4*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/b^3/d-1/3*a*cos(d*x+c)^2*sin(d*x+c)/b^2
/d+1/4*cos(d*x+c)^3*sin(d*x+c)/b/d-2*a^5*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2
*c)/(a+b)^(1/2))/b^5/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3049, 3023, 2735, 2659, 205}

$$-\frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d}-\frac{2a^5\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\,\sqrt{a+b}}+\frac{\left(4a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8b^3d}+\frac{x\left(4a^2b^2+8a^4\right.}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((8*a^4 + 4*a^2*b^2 + 3*b^4)*x)/(8*b^5) - (2*a^5*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^5*Sqrt[a + b]*d) - (a*(3*a^2 + 2*b
^2)*Sin[c + d*x])/(3*b^4*d) + ((4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(
8*b^3*d) - (a*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(4*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\int \frac{\cos^2(c+dx)\left(3a+3b\cos(c+dx)-4a\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{4b}$$

$$= -\frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\int \frac{\cos(c+dx)\left(-8a^2+ab\cos(c+dx)+3\right.}{a+b\cos(c+}}{12b^2}$$

$$= \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(}{4bd}$$

$$= -\frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos^2(c+dx)\sin}{3b^2d}$$

$$= \frac{\left(8a^4+4a^2b^2+3b^4\right)x}{8b^5} - \frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+}{8b^3d}$$

$$= \frac{\left(8a^4+4a^2b^2+3b^4\right)x}{8b^5} - \frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+}{8b^3d}$$

$$= \frac{\left(8a^4+4a^2b^2+3b^4\right)x}{8b^5} - \frac{2a^5\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^5\sqrt{a+b}\,d} - \frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d} + \Big($$

Mathematica [A] time = 0.64, size = 153, normalized size = 0.79

$$\frac{-24ab\left(4a^2+3b^2\right)\sin(c+dx)+24b^2\left(a^2+b^2\right)\sin(2(c+dx))+\frac{192a^5\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+12\left(8a^4+4a^2b^2+3\right.}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a^4 + 4*a^2*b^2 + 3*b^4)*(c + d*x) + (192*a^5*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 24*a*b*(4*a^2 + 3*b^2)*S
in[c + d*x] + 24*b^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a*b^3*Sin[3*(c + d*x)
] + 3*b^4*Sin[4*(c + d*x)])/(96*b^5*d)
```

fricas [A] time = 1.72, size = 479, normalized size = 2.48

$$\left[-\frac{12\sqrt{-a^2+b^2}\,a^5\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)-3\left(8a^6-4a^4b^2-a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/24*(12*sqrt(-a^2 + b^2)*a^5*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos
(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 +
2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 3*(8*a^6 - 4*a^4*
b^2 - a^2*b^4 - 3*b^6)*d*x + (24*a^5*b - 8*a^3*b^3 - 16*a*b^5 - 6*(a^2*b^4
- b^6)*cos(d*x + c)^3 + 8*(a^3*b^3 - a*b^5)*cos(d*x + c)^2 - 3*(4*a^4*b^2 -
 a^2*b^4 - 3*b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d), -1/24*(2
4*sqrt(a^2 - b^2)*a^5*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
 + c))) - 3*(8*a^6 - 4*a^4*b^2 - a^2*b^4 - 3*b^6)*d*x + (24*a^5*b - 8*a^3*b
^3 - 16*a*b^5 - 6*(a^2*b^4 - b^6)*cos(d*x + c)^3 + 8*(a^3*b^3 - a*b^5)*cos(
d*x + c)^2 - 3*(4*a^4*b^2 - a^2*b^4 - 3*b^6)*cos(d*x + c))*sin(d*x + c))/((
a^2*b^5 - b^7)*d)]
```

giac [B] time = 0.62, size = 393, normalized size = 2.04

$$\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a^5}{\sqrt{a^2-b^2}\,b^5}+\frac{3\left(8a^4+4a^2b^2+3b^4\right)(dx+c)}{b^5}-\frac{2\left(24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12a^2b\tan\left(\frac{1}{2}d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^5/(sqrt(a^2
 - b^2)*b^5) + 3*(8*a^4 + 4*a^2*b^2 + 3*b^4)*(d*x + c)/b^5 - 2*(24*a^3*tan(
1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2*d*x
 + 1/2*c)^7 + 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 72*a^3*tan(1/2*d*x + 1/2*c)^5
 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*b^
3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*b*tan(1/2
*d*x + 1/2*c)^3 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*b^3*tan(1/2*d*x + 1/2
*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 24*a*
b^2*tan(1/2*d*x + 1/2*c) - 15*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2
*c)^2 + 1)^4*b^4))/d
```

maple [B] time = 0.06, size = 672, normalized size = 3.48

$$-\frac{2a^5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^5\sqrt{(a-b)\,(a+b)}}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{d\,b^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*a^5/b^5/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^3-1/d/
b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2-2/d/b^2/(1+tan(1/2*
d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a-5/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*t
an(1/2*d*x+1/2*c)^7-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5
*a^3-10/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a+3/4/d/b/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-1/d/b^3/(1+tan(1/2*d*x+1/2*c
)^2)^4*tan(1/2*d*x+1/2*c)^5*a^2+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)^3*a^2-3/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-6/
d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a^3-10/3/d/b^2/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)*a^2+5/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)-2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^3-2/d/b^2/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a+2/d/b^5*arctan(tan(1/2*d*x+1/2
*c))*a^4+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+3/4/d/b*arctan(tan(1/2*d*x+
1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.72, size = 474, normalized size = 2.46

$$\frac{\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d}+\frac{\sin\left(4\,c+4\,d\,x\right)}{32\,b\,d}+\frac{3\,\mathrm{atan}\left(\frac{40\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b^6+15\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^8+9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^{10}}{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(40\,a^4\,b^5+15\,a^2\,b^7+9\,b^9\right)}\right)}{4\,b\,d}-\frac{a\,\sin\left(3\,c+3\,d\,x\right)}{12\,b^2\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x)),x)
```

```
[Out] sin(2*c + 2*d*x)/(4*b*d) + sin(4*c + 4*d*x)/(32*b*d) + (3*atan((9*b^10*sin(
c/2 + (d*x)/2) + 15*a^2*b^8*sin(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*x)
```

```
/2))/(b*cos(c/2 + (d*x)/2)*(9*b^9 + 15*a^2*b^7 + 40*a^4*b^5))))/(4*b*d) - (
a*sin(3*c + 3*d*x))/(12*b^2*d) - (a^3*sin(c + d*x))/(b^4*d) + (a^2*sin(2*c
+ 2*d*x))/(4*b^3*d) + (a^2*atan((9*b^10*sin(c/2 + (d*x)/2) + 15*a^2*b^8*sin
(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*x)/2))/(b*cos(c/2 + (d*x)/2)*(9*b
^9 + 15*a^2*b^7 + 40*a^4*b^5))))/(b^3*d) + (2*a^4*atan((9*b^10*sin(c/2 + (d
*x)/2) + 15*a^2*b^8*sin(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*x)/2))/(b*
cos(c/2 + (d*x)/2)*(9*b^9 + 15*a^2*b^7 + 40*a^4*b^5))))/(b^5*d) - (3*a*sin(
c + d*x))/(4*b^2*d) - (a^5*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/
2))*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^5*d*(b^2 - a^2)^(1/2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.450 $\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\,\sqrt{a+b}}-\frac{ax\left(2a^2+b^2\right)}{2b^4}+\frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

```
[Out] -1/2*a*(2*a^2+b^2)*x/b^4+1/3*(3*a^2+2*b^2)*sin(d*x+c)/b^3/d-1/2*a*cos(d*x+c
)*sin(d*x+c)/b^2/d+1/3*cos(d*x+c)^2*sin(d*x+c)/b/d+2*a^4*arctan((a-b)^(1/2)
*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d}+\frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\,\sqrt{a+b}}-\frac{ax\left(2a^2+b^2\right)}{2b^4}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{\sin(c+dx)\cos^2(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3*a^2 + 2*b^2)*Sin[c + d
*x])/(3*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]^2*
Sin[c + d*x])/(3*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)\left(2a+2b\cos(c+dx)-3a\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{3b}$$

$$= -\frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{-3a^2+ab\cos(c+dx)+2\left(3a^2+2b^2\right)c}{a+b\cos(c+dx)}}{6b^2}$$

$$= \frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3bd} + \int$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)}{3b}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)}{3b}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d} + \frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d} - a\cos(c+$$

Mathematica [A] time = 0.33, size = 122, normalized size = 0.82

$$\frac{-6a\left(2a^2+b^2\right)(c+dx) + 3b\left(4a^2+3b^2\right)\sin(c+dx) - \frac{24a^4\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 3ab^2\sin(2(c+dx)) + b^3\sin(3}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-6*a*(2*a^2 + b^2)*(c + d*x) - (24*a^4*ArcTanh[((a - b)*Tan[(c + d*x)/2])/
Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sin[c + d*x] - 3*
a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*b^4*d)
```

fricas [A] time = 1.17, size = 400, normalized size = 2.70

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^4\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2+2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right) + 3\left(2a^5-a^3b^2-ab^4\right.}{6\left(a^2b^4-b^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(2*a^5 - a^3*b^2
- a*b^4)*d*x - (6*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^2*b^3 - b^5)*cos(d*x + c
)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d),
1/6*(6*sqrt(a^2 - b^2)*a^4*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*si
n(d*x + c))) - 3*(2*a^5 - a^3*b^2 - a*b^4)*d*x + (6*a^4*b - 2*a^2*b^3 - 4*b
^5 + 2*(a^2*b^3 - b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^4 - b^6)*d)]
```

giac [A] time = 0.48, size = 249, normalized size = 1.68

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a^4}{\sqrt{a^2-b^2}\,b^4}+\frac{3\left(2a^3+ab^2\right)(dx+c)}{b^4}-\frac{2\left(6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+\cdots\right.}{\cdots}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^4/(sqrt(a^2
 - b^2)*b^4) + 3*(2*a^3 + a*b^2)*(d*x + c)/b^4 - 2*(6*a^2*tan(1/2*d*x + 1/2
*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*a^
2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x
 + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

maple [B] time = 0.06, size = 367, normalized size = 2.48

$$\frac{2a^4\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^4\sqrt{(a-b)\,(a+b)}}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{d\,b^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4\,\cdots}{d\,b^3\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*a^4/b^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*a^2+1/d/b
^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*a+2/d/b/(1+tan(1/2*d*x+1
```

```
/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)^3*a^2+4/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+2/
d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*a^2+2/d/b/(1+tan(1/2*d*
x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2
*d*x+1/2*c)*a-2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3-1/d/b^2*arctan(tan(1/2
*d*x+1/2*c))*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.18, size = 203, normalized size = 1.37

$$\frac{3\sin(c+dx)}{4bd}+\frac{\sin(3c+3dx)}{12bd}-\frac{a\sin(2c+2dx)}{4b^2d}+\frac{a^2\sin(c+dx)}{b^3d}-\frac{2a^3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{b^4d}-\frac{a\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x)),x)
```

```
[Out] (3*sin(c + d*x))/(4*b*d) + sin(3*c + 3*d*x)/(12*b*d) - (a*sin(2*c + 2*d*x))
/(4*b^2*d) + (a^2*sin(c + d*x))/(b^3*d) - (2*a^3*atan(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)))/(b^4*d) - (a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/(b^2*d) + (a^4*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))*1i)/(c
os(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^4*d*(b^2 - a^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.451 $\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\,\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)}{2b^3}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] 1/2*(2*a^2+b^2)*x/b^3-a*sin(d*x+c)/b^2/d+1/2*cos(d*x+c)*sin(d*x+c)/b/d-2*a^
3*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^3/d/(a-b)^(1/2)/(a+b
)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2793, 3023, 2735, 2659, 205}

$$-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\,\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)}{2b^3}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*Sin[c + d*x])/(b^2*d) + (C
os[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{\cos(c+dx)\sin(c+dx)}{2bd}+\frac{\int \frac{a+b\cos(c+dx)-2a\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}\\
&= -\frac{a\sin(c+dx)}{b^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}+\frac{\int \frac{ab+\left(2a^2+b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{2b^2}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{a\sin(c+dx)}{b^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}-\frac{a^3\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^3}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{a\sin(c+dx)}{b^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}-\frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,d\right.}{b^3d}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 97, normalized size = 0.88

$$\frac{2\left(2a^2+b^2\right)(c+dx)+\frac{8a^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-4ab\sin(c+dx)+b^2\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(2*a^2 + b^2)*(c + d*x) + (8*a^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*x)]
)/(4*b^3*d)
```

fricas [A] time = 0.97, size = 334, normalized size = 3.04

$$\left[-\frac{\sqrt{-a^2+b^2}\,a^3\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(2\,a^4-a^2b^2-b^4\right)dx+}{2\left(a^2b^3-b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*a^4 - a^2*b^2 - b^
4)*d*x + (2*a^3*b - 2*a*b^3 - (a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^2*b^3 - b^5)*d), -1/2*(2*sqrt(a^2 - b^2)*a^3*arctan(-(a*cos(d*x + c) + b
)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^4 - a^2*b^2 - b^4)*d*x + (2*a^3*b
- 2*a*b^3 - (a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d)
]
```

giac [A] time = 0.53, size = 177, normalized size = 1.61

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}\,b^3}+\frac{\left(2\,a^2+b^2\right)(dx+c)}{b^3}-\frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^3/(sqrt(a^2 -
 b^2)*b^3) + (2*a^2 + b^2)*(d*x + c)/b^3 - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 +
b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

maple [B] time = 0.06, size = 222, normalized size = 2.02

$$-\frac{2a^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^3\sqrt{(a-b)\,(a+b)}}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*a^3/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*a-1/d/b/
(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/b^2/(1+tan(1/2*d*x+1/2*
c)^2)^2*tan(1/2*d*x+1/2*c)*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1
/2*c)+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+1/d/b*arctan(tan(1/2*d*x+1/2*c
))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.07, size = 168, normalized size = 1.53

$$\frac{\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d}+\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d}-\frac{a\,\sin\left(c+d\,x\right)}{b^2\,d}-\frac{a^3\,\mathrm{atan}\left(\frac{\left(a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{b^3\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x)),x)
```

```
[Out] atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(b*d) + sin(2*c + 2*d*x)/(4*b*d
) + (2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) - (a*sin(c
+ d*x))/(b^2*d) - (a^3*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))*
1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^3*d*(b^2 - a^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.452 $\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sin(c+dx)}{bd}$$

```
[Out] -a*x/b^2+sin(d*x+c)/b/d+2*a^2*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(
1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2746, 12, 2735, 2659, 205}

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
Sqrt[a - b]*b^2*Sqrt[a + b]*d) + Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{\sin(c+dx)}{bd} - \frac{\int \frac{a\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b} \\
&= \frac{\sin(c+dx)}{bd} - \frac{a\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b} \\
&= -\frac{ax}{b^2} + \frac{\sin(c+dx)}{bd} + \frac{a^2\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2} \\
&= -\frac{ax}{b^2} + \frac{\sin(c+dx)}{bd} + \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2 d} \\
&= -\frac{ax}{b^2} + \frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d} + \frac{\sin(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 72, normalized size = 0.95

$$\frac{-\frac{2a^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - a(c+dx) + b\sin(c+dx)}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)) - (2*a^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + b*Sin[c + d*x])/(b^2*d)
```

fricas [A] time = 1.10, size = 269, normalized size = 3.54

$$\left[-\frac{\sqrt{-a^2+b^2}\,a^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+2\left(a^3-ab^2\right)dx-2\left(a^2\right.}{2\left(a^2b^2-b^4\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(a^3 - a*b^2)*d*x -
 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), (sqrt(a^2 - b^2)*a^2*ar
ctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*b^2)*
d*x + (a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

giac [A] time = 0.55, size = 126, normalized size = 1.66

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}\,b^2}+\frac{(dx+c)a}{b^2}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^2/(sqrt(a^2 - b^
2)*b^2) + (d*x + c)*a/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 + 1)*b))/d
```

maple [A] time = 0.05, size = 102, normalized size = 1.34

$$\frac{2a^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^2\sqrt{(a-b)\,(a+b)}}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*a^2/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))+2/d/b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d/b^2*arctan(
tan(1/2*d*x+1/2*c))*a
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 0.91, size = 190, normalized size = 2.50

$$\frac{\sin\left(c+d\,x\right)}{b\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d}-\frac{a^2\,\mathrm{atan}\left(\frac{1{}\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b-2{}\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2+1{}\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(b^2-a^2\right)}^{3/2}+a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}-a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)\,2{}\mathrm{i}}{b^2\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x)),x)
```

```
[Out] sin(c + d*x)/(b*d) - (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^2
*d) - (a^2*atan((b^3*sin(c/2 + (d*x)/2)*1i - a*b^2*sin(c/2 + (d*x)/2)*2i +
a^2*b*sin(c/2 + (d*x)/2)*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + a^2*co
s(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2)))*2i)/(b^2*d*(b^2 - a^2)^(1/2))
```

sympy [**A**] time = 122.50, size = 1744, normalized size = 22.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (d*x*tan(c/2 + d*
x/2)**3/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + d*x*tan(c/2 + d*
x/2)/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 3*tan(c/2 + d*x/2)*
*2/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 1/(b*d*tan(c/2 + d*x/
```

```
2)**3 + b*d*tan(c/2 + d*x/2)), Eq(a, -b)), ((x*sin(c + d*x)**2/2 + x*cos(c
+ d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))/a, Eq(b, 0)), (x*cos(c)**2/(
a + b*cos(c)), Eq(d, 0)), (-d*x*tan(c/2 + d*x/2)**2/(b*d*tan(c/2 + d*x/2)**
2 + b*d) - d*x/(b*d*tan(c/2 + d*x/2)**2 + b*d) + tan(c/2 + d*x/2)**3/(b*d*t
an(c/2 + d*x/2)**2 + b*d) + 3*tan(c/2 + d*x/2)/(b*d*tan(c/2 + d*x/2)**2 + b
*d), Eq(a, b)), (-a**2*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2
/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt
(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*
x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - a**2*d*x*sqrt(-a/(a - b) -
 b/(a - b))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*
b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*t
an(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + a**2*log(-sqrt(
-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a*b**2*d*s
qrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b)
- b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b*
*3*d*sqrt(-a/(a - b) - b/(a - b))) + a**2*log(-sqrt(-a/(a - b) - b/(a - b))
 + tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2
)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(
a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - a**2*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a*
b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/
(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)
**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - a**2*log(sqrt(-a/(a - b) - b/(
a - b)) + tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2
+ d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b
) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) +
 a*b*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a
/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a
 - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*s
qrt(-a/(a - b) - b/(a - b))) + a*b*d*x*sqrt(-a/(a - b) - b/(a - b))/(a*b**2
*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a -
 b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2
- b**3*d*sqrt(-a/(a - b) - b/(a - b))) + 2*a*b*sqrt(-a/(a - b) - b/(a - b))
*tan(c/2 + d*x/2)/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**
2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a -
 b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - 2*b**2*sq
rt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)/(a*b**2*d*sqrt(-a/(a - b) - b/(
a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*
d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b)
 - b/(a - b))), True))
```

3.453 $\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] x/b-2*a*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b/d/(a-b)^(1/2)/
(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2735, 2659, 205}

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x]),x]
```

```
[Out] x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]
*b*Sqrt[a + b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}$$
$$= \frac{x}{b} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$
$$= \frac{x}{b} - \frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.09, size = 58, normalized size = 0.98

$$\frac{\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + c + dx}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (c + d*x + (2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[
-a^2 + b^2])/(b*d)
```

fricas [A] time = 1.23, size = 223, normalized size = 3.78

$$\left[\frac{2\left(a^2-b^2\right)dx - \sqrt{-a^2+b^2}\,a\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(a^2b-b^3\right)d}, \frac{\left(a^2-b^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*a*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/((a^2
*b - b^3)*d), ((a^2 - b^2)*d*x - sqrt(a^2 - b^2)*a*arctan(-(a*cos(d*x + c)
+ b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2*b - b^3)*d)]
```

giac [B] time = 0.55, size = 240, normalized size = 4.07

$$-\frac{\frac{\left(\sqrt{a^2-b^2}\,(2\,a-b)|a-b|+\sqrt{a^2-b^2}\,|a-b||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{\frac{2\,a+\sqrt{-4\,(a+b)(a-b)+4\,a^2}}{a-b}}}\right)\right)}{\left(a^2-2\,ab+b^2\right)b^2+\left(a^3-2\,a^2b+ab^2\right)|b|}+\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{\frac{2\,a-\sqrt{-4\,(a+b)(a-b)+4\,a^2}}{a-b}}}\right)\right)(2\,a- \cdots}{b^2-a|b|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((sqrt(a^2 - b^2)*(2*a - b)*abs(a - b) + sqrt(a^2 - b^2)*abs(a - b)*abs(b)
)*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*
c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/((a^2 - 2*a*b +
 b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (pi*floor(1/2*(d*x + c)/pi +
1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a - sqrt(-4*(a + b)*
(a - b) + 4*a^2))/(a - b))))*(2*a - b - abs(b))/(b^2 - a*abs(b)))/d
```

maple [A] time = 0.05, size = 67, normalized size = 1.14

$$-\frac{2a\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{db\sqrt{(a-b)\,(a+b)}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*a/b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))+2/d/b*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.78, size = 99, normalized size = 1.68

$$\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{2\,a\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{b\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*cos(c + d*x)),x)
```

```
[Out] (2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*a*atanh((a*sin(c
/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
)))/(b*d*(b^2 - a^2)^(1/2))
```

sympy [A] time = 24.70, size = 320, normalized size = 5.42

$$\begin{cases} \tilde{\infty}x \\ \frac{x}{b}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} \\ \frac{x}{b}+\frac{1}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} \\ \frac{\sin(c+dx)}{ad} \\ \frac{x\cos(c)}{a+b\cos(c)} \\ \frac{adx\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{a\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{bdx\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b - tan(c/2 + d*x/2)/
(b*d), Eq(a, b)), (x/b + 1/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), (sin(c + d*x
)/(a*d), Eq(b, 0)), (x*cos(c)/(a + b*cos(c)), Eq(d, 0)), (a*d*x*sqrt(-a/(a
- b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a -
 b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))
/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b)))
 + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a
 - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - b*d*x*sqrt(-a/(
a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a
 - b) - b/(a - b))), True))
```

3.454 $\int \frac{1}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] 2*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^(1
/2)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2659, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
+ b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\cos(c+dx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{d\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2]
*d)
```

fricas [A] time = 1.13, size = 175, normalized size = 3.57

$$\left[-\frac{\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(a^2-b^2\right)d}, \frac{\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)}{\sqrt{a^2-b^2}\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)
^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))/((a^2 - b^2)*d), arctan(-(a*
cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))/(sqrt(a^2 - b^2)*d)]
```

giac [A] time = 0.46, size = 78, normalized size = 1.59

$$-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\text{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

maple [A] time = 0.04, size = 44, normalized size = 0.90

$$\frac{2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.52, size = 43, normalized size = 0.88

$$\frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a-b)}{\sqrt{a^2-b^2}}\right)}{d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)),x)
```

```
[Out] (2*atan((tan(c/2 + (d*x)/2)*(a - b))/(a^2 - b^2)^(1/2)))/(d*(a^2 - b^2)^(1/
2))
```

sympy [A] time = 4.02, size = 172, normalized size = 3.51

$$\begin{cases} \frac{\tilde{\infty}x}{\cos(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{1}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } a = -b \\ \frac{x}{a+b\cos(c)} & \text{for } d = 0 \\ \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for } a = b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(b*d*tan(c/2 +
 d*x/2)), Eq(a, -b)), (x/(a + b*cos(c)), Eq(d, 0)), (tan(c/2 + d*x/2)/(b*d)
, Eq(a, b)), (log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sq
rt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a - b) - b/(a - b))) - log(sqrt(-
a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b)
) - b*d*sqrt(-a/(a - b) - b/(a - b))), True))
```

3.455 $\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] arctanh(sin(d*x+c))/a/d-2*b*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/
2))/a/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2747, 3770, 2659, 205}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sq
rt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\int \sec(c+dx)\,dx}{a} - \frac{b\int \frac{1}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$
$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{\tanh^{-1}(\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.08, size = 102, normalized size = 1.50

$$\frac{\frac{2b\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2
] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]])/(a*d)
```

fricas [A] time = 1.08, size = 278, normalized size = 4.09

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right) - \left(a^2-b^2\right)\log\left(\sin\left(dx+\right.\right.}{2\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (a^2 - b^2)*log(sin(d*x
 + c) + 1) + (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d), -1/2*(2
*sqrt(a^2 - b^2)*b*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x +
c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log(-sin(d*x + c) +
1))/((a^3 - a*b^2)*d)]
```

giac [B] time = 0.90, size = 119, normalized size = 1.75

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a
) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x + 1/2*c) - 1
))/a)/d
```

maple [A] time = 0.07, size = 88, normalized size = 1.29

$$-\frac{2b\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{da\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/a*b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.84, size = 99, normalized size = 1.46

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{a\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))),x)
```

```
[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*b*atanh((a*sin(
c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
))))/(a*d*(b^2 - a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

3.456 $\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}$$

```
[Out] -b*arctanh(sin(d*x+c))/a^2/d+2*b^2*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a
+b)^(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)+tan(d*x+c)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 12, 2747, 3770, 2659, 205}

$$\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]
*Sqrt[a + b]*d) - (b*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{\tan(c+dx)}{ad} - \frac{\int \frac{b\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a} \\
&= \frac{\tan(c+dx)}{ad} - \frac{b\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a} \\
&= \frac{\tan(c+dx)}{ad} - \frac{b\int \sec(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2} \\
&= -\frac{b\tanh^{-1}(\sin(c+dx))}{a^2 d} + \frac{\tan(c+dx)}{ad} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+
\end{aligned}
$$

$$
= \frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d} - \frac{b\tanh^{-1}(\sin(c+dx))}{a^2 d} + \frac{\tan(c+dx)}{ad}
$$

Mathematica [A] time = 0.38, size = 115, normalized size = 1.35

$$-\frac{\frac{2b^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+a\tan(c+dx)+b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\ldots\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-2*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
b^2] + b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) + a*Tan[c + d*x])/(a^2*d)
```

fricas [B] time = 1.30, size = 382, normalized size = 4.49

$$\left[-\frac{\sqrt{-a^2+b^2}\,b^2\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+\left(a^2b-b^3\right)\ldots}{2\left(a^4-a^2b^2\right)d\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (a^2*b -
 b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (a^2*b - b^3)*cos(d*x + c)*log(-
sin(d*x + c) + 1) - 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*
x + c)), 1/2*(2*sqrt(a^2 - b^2)*b^2*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c)))*cos(d*x + c) - (a^2*b - b^3)*cos(d*x + c)*log(sin(d*x
 + c) + 1) + (a^2*b - b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3 - a
*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c))]
```

giac [B] time = 0.64, size = 153, normalized size = 1.80

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}\,a^2}+\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{2\tan\left(\frac{1}{2}\ldots\right.}{\left(\tan\left(\frac{1}{2}\,dx\ldots\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^2/(sqrt(a^2 - b^
2)*a^2) + b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a
))/d
```

maple [A] time = 0.08, size = 134, normalized size = 1.58

$$\frac{2b^2 \arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^2\sqrt{(a-b)\,(a+b)}}-\frac{1}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}-\frac{1}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\ln\left(\tan\left(\frac{d}{2}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*b^2/a^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))-1/a/d/(tan(1/2*d*x+1/2*c)-1)+1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)-1)-1
/a/d/(tan(1/2*d*x+1/2*c)+1)-1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.05, size = 324, normalized size = 3.81

$$\frac{a^3\,\sin\left(c+d\,x\right)-a\,b^2\,\sin\left(c+d\,x\right)}{a^2\,d\,\cos\left(c+d\,x\right)\,\left(a^2-b^2\right)}-\frac{2\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+2\,b^2\,\mathrm{atanh}\left(\frac{a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)
```

```
[Out] (a^3*sin(c + d*x) - a*b^2*sin(c + d*x))/(a^2*d*cos(c + d*x)*(a^2 - b^2)) -
(2*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 2*b^3*atanh(sin(c/2
```

```
 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 2*b^2*atanh((a^5*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + 2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c/2 +
 (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
) - a^3*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(b^2 - a^2)^(1/2)
)/(a^2*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

3.457 $\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=119

$$-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tan(c+dx)}{a^2d}+\frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] 1/2*(a^2+2*b^2)*arctanh(sin(d*x+c))/a^3/d-2*b^3*arctan((a-b)^(1/2)*tan(1/2*
d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+b)^(1/2)-b*tan(d*x+c)/a^2/d+1/
2*sec(d*x+c)*tan(d*x+c)/a/d
```

Rubi [A] time = 0.32, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\,\sqrt{a+b}}+\frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{b\tan(c+dx)}{a^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*b^3*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b
]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b*Tan
[c + d*x])/(a^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-2b+a\cos(c+dx)+b\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2a}$$

$$= -\frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(a^2+2b^2+ab\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{2a^2}$$

$$= -\frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} - \frac{b^3\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^3} + \frac{\left(a^2+2b^2\right)\int \sec(c}{2a^3}$$

$$= \frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(2b^3\right)\mathrm{Su}}{}$$

$$= -\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{b\tan(c+dx)}{a^2d} + \mathrm{se}$$

Mathematica [A] time = 1.06, size = 238, normalized size = 2.00

$$\frac{\frac{8b^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((8*b^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b
^2] - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
 + 4*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a*b*
Tan[c + d*x])/(4*a^3*d)
```

fricas [A] time = 1.68, size = 459, normalized size = 3.86

$$\left[-\frac{2\sqrt{-a^2+b^2}\,b^3\cos(dx+c)^2\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right) - \left(a^4+a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a^2 + b^2)*b^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (a^4
 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^4 + a^2*b^2 -
 2*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(a^4 - a^2*b^2 - 2*(a^3*b
 - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2), -
1/4*(4*sqrt(a^2 - b^2)*b^3*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*si
n(d*x + c)))*cos(d*x + c)^2 - (a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) + (a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(-sin(d*x + c)
+ 1) - 2*(a^4 - a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a
^5 - a^3*b^2)*d*cos(d*x + c)^2)]
```

giac [A] time = 0.74, size = 211, normalized size = 1.77

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b^3}{\sqrt{a^2-b^2}\,a^3}+\frac{\left(a^2+2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{\left(a^2+2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^3/(sqrt(a^2 -
 b^2)*a^3) + (a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (a^2 +
2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(a*tan(1/2*d*x + 1/2*c)^3
 + 2*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

maple [B] time = 0.11, size = 262, normalized size = 2.20

$$-\frac{2b^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^3\sqrt{(a-b)\,(a+b)}}+\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b}{d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)+
```

```
1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*b-1/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*l
n(tan(1/2*d*x+1/2*c)-1)*b^2-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2+1/2/a/d/(tan(1
/2*d*x+1/2*c)+1)+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*b+1/2/a/d*ln(tan(1/2*d*x+1/
2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*b^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 1.76, size = 1087, normalized size = 9.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))),x)
```

```
[Out] (a*(sin(c + d*x)/2 + atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/2 + (atan
h(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2))/(d*(a^2 - b^
2)*(cos(2*c + 2*d*x)/2 + 1/2)) + ((b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/2 - (b^2*sin(c + d*x))/2 + (b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2))*cos(2*c + 2*d*x))/2)/(a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2
)) - (b*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (b
^3*atan(((a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/
2)*(b^2 - a^2)^(3/2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b
^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2
 - a^2)^(1/2) - 3*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*
sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2) - a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (d*
x)/2)*(a*b^2 - a^3)*(a^7 - 3*a^3*b^4 + 2*a^5*b^2)))*1i)/(a^3*d*(b^2 - a^2)^
(1/2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/(a^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (b^3*sin(2*c +
 2*d*x))/(2*a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (b^3*atan(((a^9
*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2
)^(3/2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
 - 3*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) -
```

```
a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2
- a^3)*(a^7 - 3*a^3*b^4 + 2*a^5*b^2)))*cos(2*c + 2*d*x)*1i)/(a^3*d*(b^2 - a
^2)^(1/2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (b^4*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*cos(2*c + 2*d*x))/(a^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

3.458 $\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{\left(2a^2+3b^2\right)\tan(c+}{3a^3d}$$

```
[Out] -1/2*b*(a^2+2*b^2)*arctanh(sin(d*x+c))/a^4/d+2*b^4*arctan((a-b)^(1/2)*tan(1
/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(2*a^2+3*b^2)*
tan(d*x+c)/a^3/d-1/2*b*sec(d*x+c)*tan(d*x+c)/a^2/d+1/3*sec(d*x+c)^2*tan(d*x
+c)/a/d
```

Rubi [A] time = 0.52, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\,\sqrt{a+b}}+\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d}-\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{b\tan(c+dx)\sec(c+}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b^4*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]
*Sqrt[a + b]*d) - (b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + ((2*a
^2 + 3*b^2)*Tan[c + d*x])/(3*a^3*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*
d) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-3b+2a\cos(c+dx)+2b\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx}{3a}$$

$$= -\frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(2\left(2a^2+3b^2\right)+ab\cos(c+dx)-3b^2\cos\right.}{a+b\cos(c+dx}}{6a^2}$$

$$= \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \int$$

$$= \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{b^4}{}$$

$$= -\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+}{2a^2d}$$

$$= \frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}\,d} - \frac{b\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{\left(2a^2+3b^2\right)\tan(c}{3a^3d}$$

Mathematica [A] time = 2.46, size = 258, normalized size = 1.64

$$\frac{\frac{1}{2}\sec^3(c+dx)\left(4a\sin(c+dx)\left(\left(2a^2+3b^2\right)\cos(2(c+dx))+4a^2-3ab\cos(c+dx)+3b^2\right)+9b\left(a^2+2b^2\right)\cos(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-24*b^4*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
 b^2] + (Sec[c + d*x]^3*(9*b*(a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*b*(a
^2 + 2*b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*a*(4*a^2 + 3*b^2 - 3*a*b*Cos[c
+ d*x] + (2*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*a^4*d)
```

fricas [A] time = 1.35, size = 535, normalized size = 3.41

$$\left[-\frac{6\sqrt{-a^2+b^2}\,b^4\cos(dx+c)^3\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2+2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)+3\left(a^4b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*sqrt(-a^2 + b^2)*b^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
 + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(
a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(a^4*b +
a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^5 - 2*a^3*b
^2 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^3)*cos
(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), 1/12*(12*sqrt(
a^2 - b^2)*b^4*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))
*cos(d*x + c)^3 - 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(sin(d*x +
c) + 1) + 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
 + 2*(2*a^5 - 2*a^3*b^2 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 - 3*
(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x +
c)^3)]
```

giac [B] time = 0.67, size = 286, normalized size = 1.82

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b^4}{\sqrt{a^2-b^2}\,a^4}+\frac{3\left(a^2b+2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{3\left(a^2b+2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\right.\right.\right.}{a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^4/(sqrt(a^2
 - b^2)*a^4) + 3*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3
*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(6*a^2*tan(1/2*
d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^
5 - 4*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*ta
n(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

maple [B] time = 0.11, size = 400, normalized size = 2.55

$$\frac{2b^4\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^4\sqrt{(a-b)\,(a+b)}}-\frac{1}{3da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{b}{2d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{}{ad\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*b^4/a^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))-1/3/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2
-1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2*b-1/a/d/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^
2/(tan(1/2*d*x+1/2*c)-1)*b-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*b^2+1/2/d*b/a^2*l
n(tan(1/2*d*x+1/2*c)-1)+1/d*b^3/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a/(tan(1
/2*d*x+1/2*c)+1)^3+1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tan(1/2*d*x+
1/2*c)+1)^2*b-1/a/d/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)
*b-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b^2-1/2/d*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)-
1/d*b^3/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 2.68, size = 991, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

```
[Out] (a^5*(sin(c + d*x)/2 + sin(3*c + 3*d*x)/6) - a^4*((b*sin(2*c + 2*d*x))/4 +
(b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4 + (3*b*
cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4) - a^2*((3*b^3
*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - (b^3*sin(2*
c + 2*d*x))/4 + (b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c +
 3*d*x))/4) - a^3*((b^2*sin(c + d*x))/4 - (b^2*sin(3*c + 3*d*x))/12) - a*((
b^4*sin(c + d*x))/4 + (b^4*sin(3*c + 3*d*x))/4) + (3*b^5*cos(c + d*x)*atanh
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (b^5*atanh(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (3*b^4*atanh((a^9*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*b^9*
sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*sin
```

```
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2
)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^8*b*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a^7 - 3*a^3*
b^4 + 2*a^5*b^2)))*cos(c + d*x)*(b^2 - a^2)^(1/2))/2 + (b^4*atanh((a^9*sin(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/
2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*
a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^8*b
*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a
^7 - 3*a^3*b^4 + 2*a^5*b^2)))*cos(3*c + 3*d*x)*(b^2 - a^2)^(1/2))/2)/(a^4*d
*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*cos(c + d*x)), x)
```

3.459 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=266

$$-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(4a^2-b^2\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)}-\frac{ax\left(4a^2+b^2\right)}{b^5}-\frac{a\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{b^3d\left(a^2-b^2\right)}$$

```
[Out] -a*(4*a^2+b^2)*x/b^5+2*a^4*(4*a^2-5*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2
*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^5/(a+b)^(3/2)/d+1/3*(12*a^4-7*a^2*b^2-2*b^4)
*sin(d*x+c)/b^4/(a^2-b^2)/d-a*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2-b^
2)/d+1/3*(4*a^2-b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)/d-a^2*cos(d*x+c)
^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.72, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(-7a^2b^2+12a^4-2b^4\right)\sin(c+dx)}{3b^4d\left(a^2-b^2\right)}+\frac{2a^4\left(4a^2-5b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((a*(4*a^2 + b^2)*x)/b^5) + (2*a^4*(4*a^2 - 5*b^2)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^5*(a + b)^(3/2)*d) + ((12*a^4
 - 7*a^2*b^2 - 2*b^4)*Sin[c + d*x])/(3*b^4*(a^2 - b^2)*d) - (a*(2*a^2 - b^2
)*Cos[c + d*x]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((4*a^2 - b^2)*Cos[c + d
*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(3a^2-ab\cos(c+dx)-\left(4a^2-b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(4a^2-b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)}{}}{}$$

$$= -\frac{a\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(4a^2-b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2}{b\left(a^2\right.}$$

$$= \frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{3b^4\left(a^2-b^2\right)d} - \frac{a\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(4a^2-b\right.}{}$$

$$= -\frac{a\left(4a^2+b^2\right)x}{b^5} + \frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{3b^4\left(a^2-b^2\right)d} - \frac{a\left(2a^2-b^2\right)\cos(c+dx)\sin(}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(4a^2+b^2\right)x}{b^5} + \frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{3b^4\left(a^2-b^2\right)d} - \frac{a\left(2a^2-b^2\right)\cos(c+dx)\sin(}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(4a^2+b^2\right)x}{b^5} + \frac{2a^4\left(4a^2-5b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^5(a+b)^{3/2}d} + \frac{\left(12a^4-7a^2b^2-2b^4\right)}{3b^4\left(a^2-b^2\right.}$$

Mathematica [C] time = 0.90, size = 176, normalized size = 0.66

$$\frac{\frac{12a^5b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + 9b\left(4a^2+b^2\right)\sin(c+dx) + \frac{24a^4\left(4a^2-5b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - 6ab^2\sin(2(c+dx)) - 12a(}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-12*a*(2*a - I*b)*(2*a + I*b)*(c + d*x) + (24*a^4*(4*a^2 - 5*b^2)*ArcTanh[
((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 9*b*(4*a
^2 + b^2)*Sin[c + d*x] + (12*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Co
s[c + d*x])) - 6*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*b^5*d)
```

fricas [A] time = 1.30, size = 747, normalized size = 2.81

$$\left[-\frac{6\left(4\,a^7b-7\,a^5b^3+2\,a^3b^5+ab^7\right)dx\cos\left(dx+c\right)+6\left(4\,a^8-7\,a^6b^2+2\,a^4b^4+a^2b^6\right)dx+3\left(4\,a^7-5\,a^5b^2+\left(4\,a^6\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(4*a^7*b - 7*a^5*b^3 + 2*a^3*b^5 + a*b^7)*d*x*cos(d*x + c) + 6*(4*
a^8 - 7*a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*d*x + 3*(4*a^7 - 5*a^5*b^2 + (4*a^6*
b - 5*a^4*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
 + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(
12*a^7*b - 19*a^5*b^3 + 5*a^3*b^5 + 2*a*b^7 + (a^4*b^4 - 2*a^2*b^6 + b^8)*c
os(d*x + c)^3 - 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + 2*(3*a^6*b
^2 - 5*a^4*b^4 + a^2*b^6 + b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a
^2*b^8 + b^10)*d*cos(d*x + c) + (a^5*b^5 - 2*a^3*b^7 + a*b^9)*d), -1/3*(3*(
4*a^7*b - 7*a^5*b^3 + 2*a^3*b^5 + a*b^7)*d*x*cos(d*x + c) + 3*(4*a^8 - 7*a^
6*b^2 + 2*a^4*b^4 + a^2*b^6)*d*x - 3*(4*a^7 - 5*a^5*b^2 + (4*a^6*b - 5*a^4*
b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 -
 b^2)*sin(d*x + c))) - (12*a^7*b - 19*a^5*b^3 + 5*a^3*b^5 + 2*a*b^7 + (a^4*
b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^3 - 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos
(d*x + c)^2 + 2*(3*a^6*b^2 - 5*a^4*b^4 + a^2*b^6 + b^8)*cos(d*x + c))*sin(d
*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c) + (a^5*b^5 - 2*a^3*b^
7 + a*b^9)*d)]
```

giac [A] time = 0.93, size = 333, normalized size = 1.25

$$\frac{\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^2b^4-b^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{6\left(4\,a^6-5\,a^4b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^5-b^7\right)\sqrt{a^2-b^2}}-\frac{3\left(4\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^5*tan(1/2*d*x + 1/2*c)/((a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 6*(4*a^6 - 5*a^4*b^2)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^5 - b^7)*sqrt(a^2 - b^2)) -
3*(4*a^3 + a*b^2)*(d*x + c)/b^5 + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a*b*t
```

```
an(1/2*d*x + 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*tan(1/2*d*x +
 1/2*c)^3 + 2*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c) - 3*a
*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 + 1)^3*b^4))/d
```

maple [A] time = 0.07, size = 504, normalized size = 1.89

$$\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^4\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+\frac{8a^6\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^5\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}-\frac{10a^4\,\mathrm{arc}}{d\,b^3\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*a^5/b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)+8/d*a^6/b^5/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-10/d*a^4/b^3/(a-b)/(a+b)/((a-b)*(a+
b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+6/d/b^4/(1+t
an(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*a^2+2/d/b^3/(1+tan(1/2*d*x+1/2*
c)^2)^3*tan(1/2*d*x+1/2*c)^5*a+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d
*x+1/2*c)^5+12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*a^2+4/
3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+6/d/b^4/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*a^2-2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)*a+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-8
/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^3-2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.21, size = 3852, normalized size = 14.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^2,x)

[Out] - ((2*tan(c/2 + (d*x)/2)^3*(8*a*b^4 - 6*a^4*b - 36*a^5 - b^5 + 7*a^2*b^3 +
19*a^3*b^2))/(3*b^4*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)^7*(4*a^5 - 2*a
^4*b + b^5 + a^2*b^3 - 3*a^3*b^2))/(b^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*
x)/2)^5*(8*a*b^4 + 6*a^4*b - 36*a^5 + b^5 - 7*a^2*b^3 + 19*a^3*b^2))/(3*b^4
*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(b^5 - 4*a^5 - 2*a^4*b + a^2*b^3
+ 3*a^3*b^2))/(b^4*(a + b)*(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^8*(a -
b) + tan(c/2 + (d*x)/2)^2*(4*a + 2*b) + tan(c/2 + (d*x)/2)^6*(4*a - 2*b) +
6*a*tan(c/2 + (d*x)/2)^4)) - (2*a*atan(((a*(4*a^2 + b^2)*((32*tan(c/2 + (d*
x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7
+ 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2))/(a*b^10 +
b^11 - a^2*b^9 - a^3*b^8) + (a*(4*a^2 + b^2)*((32*(a*b^17 + a^3*b^15 - 5*a^
4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b^11 - 4*a^8*b^10))/(a*b^14 + b^15
 - a^2*b^13 - a^3*b^12) - (a*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a*b^15 - 2
*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10)*32i)/(b^5*(a
*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*1i)/b^5))/b^5 + (a*(4*a^2 + b^2)*((32*t
an(c/2 + (d*x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 -
 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2)
)/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (a*(4*a^2 + b^2)*((32*(a*b^17 + a^3
*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b^11 - 4*a^8*b^10))/(a
*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (a*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(
2*a*b^15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10)*
32i)/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*1i)/b^5))/b^5)/((64*(64*a^1
4 - 32*a^13*b + 5*a^6*b^8 - 5*a^7*b^7 + 31*a^8*b^6 - 6*a^9*b^5 + 12*a^10*b^
4 + 48*a^11*b^3 - 112*a^12*b^2))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (a
*(4*a^2 + b^2)*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*
a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a
^9*b^3 - 48*a^10*b^2))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (a*(4*a^2 + b^
2)*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b
^11 - 4*a^8*b^10))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (a*tan(c/2 + (d*
x)/2)*(4*a^2 + b^2)*(2*a*b^15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^
5*b^11 - 2*a^6*b^10)*32i)/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*1i)/b^
5)*1i)/b^5 + (a*(4*a^2 + b^2)*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^11*b
+ a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2
*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) -
 (a*(4*a^2 + b^2)*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6
*b^12 + 2*a^7*b^11 - 4*a^8*b^10))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (
a*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a*b^15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*
a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10)*32i)/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^
3*b^8)))*1i)/b^5)*1i)/b^5))*(4*a^2 + b^2))/(b^5*d) - (a^4*atan(((a^4*(4*a^2
 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 3
2*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^
7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2))/(a*b^10 + b^11 - a^2*b^9 - a
^3*b^8) + (a^4*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^
```

```
12 + 2*a^7*b^11 - 4*a^8*b^10))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (32*
a^4*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^
15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10))/((a*b
^10 + b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*
(4*a^2 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7
 - a^6*b^5))*1i)/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5) + (a^4*(4*a^2 - 5
*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^
11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^
5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2))/(a*b^10 + b^11 - a^2*b^9 - a^3*b
^8) - (a^4*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 +
 2*a^7*b^11 - 4*a^8*b^10))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (32*a^4*
tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^15 -
 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10))/((a*b^10
+ b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(4*a
^2 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a
^6*b^5))*1i)/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))/((64*(64*a^14 - 32*a
^13*b + 5*a^6*b^8 - 5*a^7*b^7 + 31*a^8*b^6 - 6*a^9*b^5 + 12*a^10*b^4 + 48*a
^11*b^3 - 112*a^12*b^2))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (a^4*(4*a^
2 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*a^12 -
32*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a
^7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2))/(a*b^10 + b^11 - a^2*b^9 -
a^3*b^8) + (a^4*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b
^12 + 2*a^7*b^11 - 4*a^8*b^10))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (32
*a^4*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b
^15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10))/((a*
b^10 + b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))
*(4*a^2 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^11 - 3*a^2*b^9 + 3*a^4*b^
7 - a^6*b^5)))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5) + (a^4*(4*a^2 - 5*b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^11
*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5
+ 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8
) - (a^4*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2
*a^7*b^11 - 4*a^8*b^10))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (32*a^4*ta
n(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^15 - 2
*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10))/((a*b^10 +
b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(4*a^2
 - 5*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6
*b^5)))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(4*a^2 - 5*b^2)*(-(a + b
)^3*(a - b)^3)^(1/2)*2i)/(d*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.460 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=166

$$\frac{x(6a^2+b^2)}{2b^4}-\frac{a^4\sin(c+dx)}{b^3d(a^2-b^2)(a+b\cos(c+dx))}-\frac{2a^3(3a^2-4b^2)\tanh^{-1}\left(\frac{(a-b)\sin(c+dx)}{\sqrt{b^2-a^2}(\cos(c+dx)+1)}\right)}{b^4d(b^2-a^2)^{3/2}}-\frac{2a\sin(c+dx)}{b^3d}+\frac{\text{si}}{}$$

```
[Out] 1/2*(6*a^2+b^2)*x/b^4-2*a^3*(3*a^2-4*b^2)*arctanh((a-b)*sin(d*x+c)/(1+cos(d
*x+c))/(-a^2+b^2)^(1/2))/b^4/(-a^2+b^2)^(3/2)/d-2*a*sin(d*x+c)/b^3/d+1/2*co
s(d*x+c)*sin(d*x+c)/b^2/d-a^4*sin(d*x+c)/b^3/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.43, antiderivative size = 213, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3049, 3023, 2735, 2659, 205}

$$-\frac{a(3a^2-2b^2)\sin(c+dx)}{b^3d(a^2-b^2)}-\frac{2a^3(3a^2-4b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{(3a^2-b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*x)/(2*b^4) - (2*a^3*(3*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) - (a*(3*a^2
 - 2*b^2)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*a^2 - b^2)*Cos[c + d*x]*S
in[c + d*x])/(2*b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^2*Sin[c + d*x])/(b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2a^2-ab\cos(c+dx)-\left(3a^2-b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \int \frac{-a\left(3a^2-b^2\right)}{}$$

$$= -\frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx}{b\left(a^2-b^2\right)d(a+}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{}{b(}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{}{b(}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} - \frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx}{b^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.78, size = 144, normalized size = 0.87

$$\frac{-\frac{4a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + 2\left(6a^2+b^2\right)(c+dx) - \frac{8a^3\left(3a^2-4b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - 8ab\sin(c+dx) + b^2\sin(2(c+}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(6*a^2 + b^2)*(c + d*x) - (8*a^3*(3*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c
 + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - 8*a*b*Sin[c + d*x] - (4
*a^4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*Sin[2*(c
+ d*x)])/(4*b^4*d)
```

fricas [A] time = 1.05, size = 651, normalized size = 3.92

$$\left[\frac{\left(6a^6b-11a^4b^3+4a^2b^5+b^7\right)dx\cos(dx+c) + \left(6a^7-11a^5b^2+4a^3b^4+ab^6\right)dx - \left(3a^6-4a^4b^2+\left(3a^5b-4\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d*x*cos(d*x + c) + (6*a^7 -
11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d*x - (3*a^6 - 4*a^4*b^2 + (3*a^5*b - 4*a^3
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*a^6*b - 1
0*a^4*b^3 + 4*a^2*b^5 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 + 3*(a^5
*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7
 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d), 1/2*((6*a^6*b -
11*a^4*b^3 + 4*a^2*b^5 + b^7)*d*x*cos(d*x + c) + (6*a^7 - 11*a^5*b^2 + 4*a^
3*b^4 + a*b^6)*d*x - 2*(3*a^6 - 4*a^4*b^2 + (3*a^5*b - 4*a^3*b^3)*cos(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
+ c))) - (6*a^6*b - 10*a^4*b^3 + 4*a^2*b^5 - (a^4*b^3 - 2*a^2*b^5 + b^7)*co
s(d*x + c)^2 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/
((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)
*d)]
```

giac [A] time = 0.57, size = 262, normalized size = 1.58

$$-\frac{\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{4\left(3\,a^5-4\,a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{(6}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) -
 (6*a^2 + b^2)*(d*x + c)/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1/2*d*
x + 1/2*c)^3 + 4*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/((tan(1/2
*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

maple [B] time = 0.06, size = 358, normalized size = 2.16

$$-\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^3\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{6a^5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^4\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}+\frac{8a^3\,\mathrm{arc}}{d\,b^2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)-6/d*a^5/b^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1
/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+8/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+
b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-4/d/b^3/(1+t
an(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*a-1/d/b^2/(1+tan(1/2*d*x+1/2*c)
^2)^2*tan(1/2*d*x+1/2*c)^3-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1
/2*c)*a+1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+6/d/b^4*arcta
n(tan(1/2*d*x+1/2*c))*a^2+1/d/b^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.04, size = 3751, normalized size = 22.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (atan(((((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^
2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 1
20*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + ((a^2*6i + b^2*1i)*((8*(2*
b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 1
2*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (4*tan(c/2 + (d*x)/2)*(a
^2*6i + b^2*1i)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*
b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))))/(2*b^4))*(a^2*6
i + b^2*1i)*1i)/(2*b^4) + (((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a
*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^
4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - ((a^2*6
i + b^2*1i)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b
^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (4*ta
n(c/2 + (d*x)/2)*(a^2*6i + b^2*1i)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 1
```

6*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)
)))/(2*b^4))*(a^2*6i + b^2*1i)*1i)/(2*b^4))/((16*(108*a^11 - 54*a^10*b + 4*
a^3*b^8 - 4*a^4*b^7 + 41*a^5*b^6 - 9*a^6*b^5 + 63*a^7*b^4 + 81*a^8*b^3 - 21
6*a^9*b^2))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (((8*tan(c/2 + (d*x)/2)*
(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6
 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2
*b^7 - a^3*b^6) + ((a^2*6i + b^2*1i)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12
 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^
2*b^10 - a^3*b^9) - (4*tan(c/2 + (d*x)/2)*(a^2*6i + b^2*1i)*(8*a*b^13 - 8*a
^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 +
 b^9 - a^2*b^7 - a^3*b^6))))/(2*b^4))*(a^2*6i + b^2*1i))/(2*b^4) + (((8*tan
(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*
b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a
*b^8 + b^9 - a^2*b^7 - a^3*b^6) - ((a^2*6i + b^2*1i)*((8*(2*b^15 + 6*a^2*b^
13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*
b^11 + b^12 - a^2*b^10 - a^3*b^9) + (4*tan(c/2 + (d*x)/2)*(a^2*6i + b^2*1i)
*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8
))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))))/(2*b^4))*(a^2*6i + b^2*1i))/(2
b^4)))(a^2*6i + b^2*1i)*1i)/(b^4*d) - ((tan(c/2 + (d*x)/2)*(3*a^3*b - 3*a
*b^3 + 6*a^4 + b^4 - 5*a^2*b^2))/((a*b^3 - b^4)*(a + b)) + (tan(c/2 + (d*x)
/2)^5*(3*a*b^3 - 3*a^3*b + 6*a^4 + b^4 - 5*a^2*b^2))/((a*b^3 - b^4)*(a + b)
) - (2*tan(c/2 + (d*x)/2)^3*(b^4 - 6*a^4 + 3*a^2*b^2))/(b*(a*b^2 - b^3)*(a
+ b)))/(d*(a + b + tan(c/2 + (d*x)/2)^2*(3*a + b) + tan(c/2 + (d*x)/2)^6*(a
 - b) + tan(c/2 + (d*x)/2)^4*(3*a - b))) + (a^3*atan(((a^3*(3*a^2 - 4*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2
*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*
b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a^3*
((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*
b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^3*tan(c/2 +
(d*x)/2)*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^1
2 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2
*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)
/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^3*(3*a^2 - 4*b^2)*(-(a + b)^
3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b
^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*
a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^3*((8*(2*b^1
5 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a
^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (8*a^3*tan(c/2 + (d*x)/2)*(
3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3
*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3
b^6)(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*(-(a + b)^
3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)/(b^10 - 3
*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))/((16*(108*a^11 - 54*a^10*b + 4*a^3*b^8 - 4
*a^4*b^7 + 41*a^5*b^6 - 9*a^6*b^5 + 63*a^7*b^4 + 81*a^8*b^3 - 216*a^9*b^2))

```
/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a
 - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10
+ 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*
b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a^3*((8*(2*b^15 +
6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b
^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^3*tan(c/2 + (d*x)/2)*(3*a^
2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^1
1 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6
)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a
 - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^
8 + 3*a^4*b^6 - a^6*b^4) + (a^3*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2
)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8
- 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8
*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^3*((8*(2*b^15 + 6*a^2*b^13 -
16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11
+ b^12 - a^2*b^10 - a^3*b^9) + (8*a^3*tan(c/2 + (d*x)/2)*(3*a^2 - 4*b^2)*(-
(a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^
10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a
^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2
))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6
 - a^6*b^4)))*(3*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(d*(b^10 - 3
*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.461 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$\frac{(2a^2-b^2)\sin(c+dx)}{b^2d(a^2-b^2)}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{2ax}{b^3}$$

```
[Out] -2*a*x/b^3+2*a^2*(2*a^2-3*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^
(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+(2*a^2-b^2)*sin(d*x+c)/b^2/(a^2-b^2)/d
-a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3023, 2735, 2659, 205}

$$\frac{(2a^2-b^2)\sin(c+dx)}{b^2d(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*a*x)/b^3 + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + ((2*a^2 - b^2)*Sin[c +
 d*x])/(b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{a^2-ab\cos(c+dx)-\left(2a^2-b^2\right)\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{a^2b+2a\left(a^2-b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{2ax}{b^3} + \frac{\left(2a^2-b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(a^2\left(2a^2-3b^2\right)\right)}{b^3\left(a^2\right.}$$

$$= -\frac{2ax}{b^3} + \frac{\left(2a^2-b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(2a^2\left(2a^2-3b^2\right)\right.}{}$$

$$= -\frac{2ax}{b^3} + \frac{2a^2\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{\left(2a^2-b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\cos}{b\left(a^2-\right.}$$

Mathematica [A] time = 0.73, size = 113, normalized size = 0.73

$$\frac{\sin(c+dx)\left(\frac{a^3b}{(a-b)(a+b)(a+b\cos(c+dx))}+b\right)+\frac{2a^2\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-2a(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*a*(c + d*x) + (2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])
/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (b + (a^3*b)/((a - b)*(a + b)*(a +
 b*Cos[c + d*x])))*Sin[c + d*x])/(b^3*d)
```

fricas [A] time = 1.74, size = 554, normalized size = 3.57

$$\left[-\frac{4\left(a^5b-2a^3b^3+ab^5\right)dx\cos(dx+c)+4\left(a^6-2a^4b^2+a^2b^4\right)dx+\left(2a^5-3a^3b^2+\left(2a^4b-3a^2b^3\right)\cos(dx+c\right.}{2\left(\left(a^4b^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^6 - 2*a^4*b^2
+ a^2*b^4)*d*x + (2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*s
qrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*
sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^5*b - 3*a^3*b^3 + a*b^5 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b
^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d), -(2*(a^5*b - 2
*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d*x - (2
*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^5*b - 3*a^3
*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^
4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

giac [B] time = 1.12, size = 847, normalized size = 5.46

$$\frac{\left(4\,a^{6}b^{2}-2\,a^{5}b^{3}-9\,a^{4}b^{4}+4\,a^{3}b^{5}+5\,a^{2}b^{6}-2\,ab^{7}+2\,a^{3}\left|-a^{2}b^{3}+b^{5}\right|-a^{2}b\left|-a^{2}b^{3}+b^{5}\right|-2\,ab^{2}\left|-a^{2}b^{3}+b^{5}\right|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\cdots}{\sqrt{\frac{2\,a^{3}b^{2}-2\,ab^{4}+\sqrt{-4\left(a^{3}b^{2}+a^{2}b^{3}-\cdots\right.}}{a^{3}b\cdots}}}\right.\right.}{a^{3}b^{2}\left|-a^{2}b^{3}+b^{5}\right|-ab^{4}\left|-a^{2}b^{3}+b^{5}\right|+\left(a^{2}b^{3}-b^{5}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4 + 4*a^3*b^5 + 5*a^2*b^6 - 2*a*b^7 + 2*a
^3*abs(-a^2*b^3 + b^5) - a^2*b*abs(-a^2*b^3 + b^5) - 2*a*b^2*abs(-a^2*b^3 +
 b^5))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x +
 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 + sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^
5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a
^2*b^3 - a*b^4 + b^5))))/(a^3*b^2*abs(-a^2*b^3 + b^5) - a*b^4*abs(-a^2*b^3
+ b^5) + (a^2*b^3 - b^5)^2) - ((2*a^3 - a^2*b - 2*a*b^2)*sqrt(a^2 - b^2)*ab
s(-a^2*b^3 + b^5)*abs(-a + b) - (4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4 + 4*a^3*
b^5 + 5*a^2*b^6 - 2*a*b^7)*sqrt(a^2 - b^2)*abs(-a + b))*(pi*floor(1/2*(d*x
+ c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 -
2*a*b^4 - sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*
b^4 + b^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/(
(a^2*b^3 - b^5)^2*(a^2 - 2*a*b + b^2) - (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 -
a*b^6)*abs(-a^2*b^3 + b^5)) + 2*(2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1
/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x + 1/2*c)
^3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*tan(1/
2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*t
```

```
an(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)
))/d
```

maple [A] time = 0.06, size = 238, normalized size = 1.54

$$\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^2\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+\frac{4a^4\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^3\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}-\frac{6a^2\arcta}{db\,(a-b)\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)+4/d*a^4/b^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-6/d*a^2/b/(a-b)/(a+b)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/b^2*tan(1/2
*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.12, size = 3180, normalized size = 20.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^2,x)
```

```
[Out] - ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - 2*a^3 - b^3))/(b^2*(a + b)*(a -
 b)) + (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - 2*a^3 + b^3))/(b^2*(a + b)*(a
 - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) + 2*a*tan(c/2 + (d*x)/2)^2
)) - (4*a*atan(((2*a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 -
 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 -
 a^3*b^4) + (a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^
```

7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a*tan(c/2 + (d*x)/2)*(
2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)*64i)
/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*2i)/b^3))/b^3 + (2*a*((32*tan(c/2
 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b
^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a*((32*(2*a*b^11 - 3
*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^
2*b^7 - a^3*b^6) + (a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9
 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)*64i)/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^
3*b^4)))*2i)/b^3))/b^3)/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4 + 6*a^5*b^3 - 20
*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a*((32*tan(c/2 + (d*x)/2)*(
8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b
^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a*((32*(2*a*b^11 - 3*a^2*b^10 - 3
*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b
^6) - (a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8
+ 2*a^5*b^7 - 2*a^6*b^6)*64i)/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*2i)/
b^3)*2i)/b^3 + (a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*
a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^
3*b^4) - (a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 -
 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a*tan(c/2 + (d*x)/2)*(2*a
*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)*64i)/(b
^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*2i)/b^3)*2i)/b^3)))/(b^3*d) - (a^2*a
tan(((a^2*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5
+ 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) +
 (a^2*(2*a^2 - 3*b^2)*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 +
 a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a^2*tan(c/2
+ (d*x)/2)*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b
^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b
^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3*(a - b)
^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*(-(a +
b)^3*(a - b)^3)^(1/2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a^2*((
32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4
+ 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a^2*(2*a^2
 - 3*b^2)*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2
*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a^2*tan(c/2 + (d*x)/2)*(
2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*
b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4
)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2))/(
b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)
^3)^(1/2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(8*a^8 - 4*a^7*
b + 12*a^4*b^4 + 6*a^5*b^3 - 20*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)
 - (a^2*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 +
5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (
a^2*(2*a^2 - 3*b^2)*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a
^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a^2*tan(c/2 +
(d*x)/2)*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^1

```
0 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5
 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3*(a - b)^3
)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*(-(a + b)
^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a^2*((32*ta
n(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*
a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a^2*(2*a^2 - 3*
b^2)*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*
b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a^2*tan(c/2 + (d*x)/2)*(2*a^2
 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 +
 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^
9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 -
 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(
1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(2*a^2 - 3*b^2)*(-(a + b)^3
*(a - b)^3)^(1/2)*2i)/(d*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.462 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=108

$$-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x}{b^2}$$

```
[Out] x/b^2-2*a*(a^2-2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a
-b)^(3/2)/b^2/(a+b)^(3/2)/d-a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2790, 2735, 2659, 205}

$$-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] x/b^2 - (2*a*(a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - (a^2*Sin[c + d*x])/(b*(a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{ab+\left(a^2-b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}-\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\left(a\left(a^2-2b^2\right)\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}\\
&= \frac{x}{b^2}-\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\left(2a\left(a^2-2b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\right.}{b^2\left(a^2-b^2\right)d}\\
&= \frac{x}{b^2}-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d}-\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.41, size = 103, normalized size = 0.95

$$\frac{-\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{a^2b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}+c+dx}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (c + d*x - (2*a*(a^2 - 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2
+ b^2]])/(-a^2 + b^2)^(3/2) - (a^2*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*
Cos[c + d*x])))/(b^2*d)
```

fricas [B] time = 1.41, size = 470, normalized size = 4.35

$$\left[\frac{2\left(a^4b-2\,a^2b^3+b^5\right)dx\cos\left(dx+c\right)+2\left(a^5-2\,a^3b^2+ab^4\right)dx-\left(a^4-2\,a^2b^2+\left(a^3b-2\,ab^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2}}{2\left(\left(a^4b^3-2\,a^2b^5+b^7\right)d\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*d*x*cos(d*x + c) + 2*(a^5 - 2*a^3*b^2 + a
*b^4)*d*x - (a^4 - 2*a^2*b^2 + (a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(-a^2 +
b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 +
 b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2
+ 2*a*b*cos(d*x + c) + a^2)) - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^4*b^3
- 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((a^4
*b - 2*a^2*b^3 + b^5)*d*x*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*x - (a
^4 - 2*a^2*b^2 + (a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^4*b - a^2*b^3)*sin(
d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^
4 + a*b^6)*d)]
```

giac [A] time = 0.58, size = 175, normalized size = 1.62

$$-\frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{2\left(a^3-2\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}-\frac{dx+c}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 + a + b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) - (d*x + c)
/b^2)/d
```

maple [B] time = 0.06, size = 200, normalized size = 1.85

$$-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{2a^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^2\left(a-b\right)\left(a+b\right)\sqrt{\left(a-b\right)\left(a+b\right)}}+\frac{4a\,\mathrm{arcta}}{d\left(a-b\right)\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)-2/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+4/d*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2
)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/b^2*arctan(tan(1
/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.21, size = 2872, normalized size = 26.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*atan((((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b
^6 - a^2*b^4 - a^3*b^3) - (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*
b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 -
 a^3*b^2)))*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b
^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))
/b^2 - ((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6
 - a^2*b^4 - a^3*b^3) + (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^
7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a
^3*b^2)))*1i)/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6
 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))/b
^2)/((64*(2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^6 - a^
2*b^4 - a^3*b^3) + (((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))
/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b
^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5
- a^2*b^3 - a^3*b^2)))*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b -
2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3
```

```
- a^3*b^2))*1i)/b^2 + (((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^
4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^
2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b
^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b
 - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b
^3 - a^3*b^2))*1i)/b^2)))/(b^2*d) + (a*atan(((a*(a^2 - 2*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 +
3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (a*
(a^2 - 2*b^2)*((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5
+ b^6 - a^2*b^4 - a^3*b^3) - (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*(-(a +
b)^3*(a - b)^3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*
b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a
^4*b^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*
b^4 - a^6*b^2))*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (a*(a^2 - 2*b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b -
 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3
 - a^3*b^2) - (a*(a^2 - 2*b^2)*((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 +
a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*a*tan(c/2 + (d*x)/2)*(a^2
 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4
*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8
- 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3
*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2
))/((64*(2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^6 - a^2
*b^4 - a^3*b^3) + (a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/
2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a
^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (a*(a^2 - 2*b^2)*((32*(2*a*b^8
 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3)
- (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*
b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 +
 b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(-(a +
b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a
^2*b^6 + 3*a^4*b^4 - a^6*b^2) - (a*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/
2)*((32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4
*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (a*(a^2 - 2*b^2)
*((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b
^4 - a^3*b^3) + (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*(-(a + b)^3*(a - b)^
3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b
^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*
b^2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2
)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2 - 2*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*2i)/(d*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)) - (2*a^2*tan(
c/2 + (d*x)/2))/(d*(a + b)*(a*b - b^2)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b
)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.463 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{a\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -2*b*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(
3/2)/d+a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2754, 12, 2659, 205}

$$\frac{a\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
 + b)^(3/2)*d) + (a*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{b}{a+b\cos(c+dx)}\,dx}{-a^2+b^2} \\
&= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{b\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d} \\
&= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 83, normalized size = 0.98

$$\frac{\frac{a\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} - \frac{2b\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
3/2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/d
```

fricas [A] time = 1.00, size = 321, normalized size = 3.78

$$\left[\frac{\left(b^2\cos(dx+c)+ab\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+2\,(a\right.$$
$$\overline{2\left(\left(a^4b-2\,a^2b^3+b^5\right)d\cos(dx+c)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (
2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d
*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2
*(a^3 - a*b^2)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a
^5 - 2*a^3*b^2 + a*b^4)*d), -((b^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*arct
an(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*b^2)*si
n(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 +
a*b^4)*d)]
```

giac [A] time = 0.63, size = 135, normalized size = 1.59

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)b}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(a^2 - b^2)^(3/2)
 + a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*
c)^2 + a + b)*(a^2 - b^2)))/d
```

maple [A] time = 0.05, size = 116, normalized size = 1.36

$$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{2b\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)-2/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)
*(a-b)/((a-b)*(a+b))^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 0.74, size = 99, normalized size = 1.16

$$\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(a-b\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}-\frac{2\,b\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)}{2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*a*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b + tan(c/2 + (d*x)/2)^2*(
a - b))) - (2*b*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a -
 b)^(1/2))))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.464 $\int \frac{1}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] 2*a*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3
/2)/d-b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 12, 2659, 205}

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
+ b)^(3/2)*d) - (b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^2}\,dx &= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\int \frac{a}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{a\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\
&= \frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 84, normalized size = 0.98

$$\frac{\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-2),x]
```

```
[Out] ((2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3
/2) - (b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/d
```

fricas [A] time = 1.35, size = 320, normalized size = 3.72

$$\left[\frac{\left(ab\cos(dx+c)+a^2\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)-2\left(a^2\right.}{2\left(\left(a^4b-2a^2b^3+b^5\right)d\cos(dx+c)+\left(a^5-2a^3b^2+ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cos(d*x + c) + a^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (
2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d
*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2
*(a^2*b - b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a
^5 - 2*a^3*b^2 + a*b^4)*d), ((a*b*cos(d*x + c) + a^2)*sqrt(a^2 - b^2)*arcta
n(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^2*b - b^3)*sin
(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a
*b^4)*d)]
```

giac [A] time = 0.50, size = 135, normalized size = 1.57

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a/(a^2 - b^2)^(3/2
) + b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 + a + b)*(a^2 - b^2)))/d
```

maple [A] time = 0.04, size = 116, normalized size = 1.35

$$-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+\frac{2a\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\left(a-b\right)\left(a+b\right)\sqrt{\left(a-b\right)\left(a+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)+2/d*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.71, size = 99, normalized size = 1.15

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)}{2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(a-b\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*a*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a - b)^(1/2)))
)/(d*(a + b)^(3/2)*(a - b)^(3/2)) - (2*b*tan(c/2 + (d*x)/2))/(d*(a + b)*(a
- b)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(-2), x)
```

3.465 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=118

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

```
[Out] -2*b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-
b)^(3/2)/(a+b)^(3/2)/d+arctanh(sin(d*x+c))/a^2/d+b^2*sin(d*x+c)/a/(a^2-b^2)
/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2802, 3001, 3770, 2659, 205}

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*(a - b)^(3/2)*(a + b)^(3/2)*d) + ArcTanh[Sin[c + d*x]]/(a^2*d) + (b^2*Sin
[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2-b^2-ab\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \sec(c+dx)\,dx}{a^2} - \frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2b\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \ldots\right.}{a^2\left(\ldots\right.}$$

$$= -\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b^2\sin(c+\ldots}{a\left(a^2-b^2\right)d(a+b\ldots}$$

Mathematica [A] time = 0.37, size = 146, normalized size = 1.24

$$\frac{\frac{2b(b^2-2a^2)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{(b^2-a^2)^{3/2}}+\frac{ab^2\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*b*(-2*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*Sin[c + d*x])/((a - b)*(a + b)*(a +
b*Cos[c + d*x])))/(a^2*d)
```

fricas [B] time = 1.86, size = 592, normalized size = 5.02

$$\left[-\frac{\left(2a^3b-ab^3+\left(2a^2b^2-b^4\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}\left(a\cos(dx+c)+b\right)\sin}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*
log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)
*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + a^2)) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b
^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b
 - 2*a^2*b^3 + b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^3*b^2 - a*b
^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*
a^5*b^2 + a^3*b^4)*d), -1/2*(2*(2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cos(d*x
 + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*
x + c))) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c
))*log(sin(d*x + c) + 1) + (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 +
b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^3*b^2 - a*b^4)*sin(d*x + c
))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b
^4)*d)]
```

giac [A] time = 0.70, size = 198, normalized size = 1.68

$$\frac{\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{2\left(2\,a^2b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{a^2-b^2}}+\frac{\log\left(\left|\tan\left(\cdots\right.\right.\right.}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*b^2*tan(1/2*d*x + 1/2*c)/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*ta
n(1/2*d*x + 1/2*c)^2 + a + b)) - 2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + log(abs(tan(
1/2*d*x + 1/2*c) + 1))/a^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2)/d
```

maple [B] time = 0.09, size = 221, normalized size = 1.87

$$\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{4b\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}+\frac{2b^3\arctan\left(\cdots\right.}{d\,a^2\,(a-b)\,(a+b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)-4/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/
2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)
*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/a^2*ln(tan(1/2*d*
x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 5.98, size = 2886, normalized size = 24.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] - (atan((((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a
^6 - a^3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a
^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 -
a^3*b^2)))/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 -
5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/
a^2 - ((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6
- a^3*b^3 - a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*
b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3
*b^2)))/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a
^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2
)/((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^
3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5
+ 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2
)))/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b
^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))/a^2 - (64*(
2*a^4*b - a*b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b + a^6 - a^3*b^3 - a^
4*b^2) + (((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^
6 - a^3*b^3 - a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^
5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a
^3*b^2)))/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5
*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))/a^2))
*2i)/(a^2*d) - (b*atan(((b*((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5
 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3
*b^2) + (b*(2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^
2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b
^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*
b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6
 + 3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/(
a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + (b*((32*tan(c/2 + (d*x)/2)*(a^6 -
2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^
5 - a^2*b^3 - a^3*b^2) - (b*(2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3
*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*b*tan(c/2 + (d
*x)/2)*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*
a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3
*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2
))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a -
```

```
b)^3)^(1/2)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))/((64*(2*a^4*b - a*
b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (b*
((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^
3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (b*(2*a^2 - b^2)*((
32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3
- a^4*b^2) + (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^
(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)
)/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2
)))*(-(a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*
(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*
a^6*b^2) + (b*((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*
a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (b*(2
*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b +
a^6 - a^3*b^3 - a^4*b^2) - (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*(-(a + b)
^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^
3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b
^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4
- 3*a^6*b^2))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 +
3*a^4*b^4 - 3*a^6*b^2)))*(2*a^2 - b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(d*
(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)) - (2*b^2*tan(c/2 + (d*x)/2))/(d*(a
 + b)*(a*b - a^2)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**2, x)
```

3.466 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(a^2-2b^2)\tan(c+dx)}{a^2d(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{2b^2(3a^2-2b^2)\tan^{-1}\left(\sqrt{a-b}\right.}{a^3d(a-b)^{3/2}(a+b}$$

```
[Out] 2*b^2*(3*a^2-2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/
(a-b)^(3/2)/(a+b)^(3/2)/d-2*b*arctanh(sin(d*x+c))/a^3/d+(a^2-2*b^2)*tan(d*x
+c)/a^2/(a^2-b^2)/d+b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.41, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^2(3a^2-2b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{(a^2-2b^2)\tan(c+dx)}{a^2d(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}-\frac{2b\tanh^{-1}(\mathrm{s}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b^2*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - (2*b*ArcTanh[Sin[c + d*x]])/(a^3*d) +
 ((a^2 - 2*b^2)*Tan[c + d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2-2b^2-ab\cos(c+dx)+b^2\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-2b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-2b\left(a^2-b^2\right)+ab^2\cos(c+dx\right.}{a+b\cos(c+dx)}}{a^2\left(a^2-b^2\right.}$$

$$= \frac{\left(a^2-2b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(2b)\int \sec(c+dx)\,dx}{a^3} +$$

$$= -\frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(a^2-2b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx}$$

$$= \frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(a^2-2b^2\right)\tan}{a^2\left(a^2-\right.}$$

Mathematica [A] time = 0.97, size = 163, normalized size = 1.05

$$\frac{-\frac{2b^2\left(2b^2-3a^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - \frac{ab^3\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + a\tan(c+dx) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^
2]])/(-a^2 + b^2)^(3/2) + 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*
b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a*b^3*Sin[c + d*x])/((a - b)*
(a + b)*(a + b*Cos[c + d*x])) + a*Tan[c + d*x])/(a^3*d)
```

fricas [B] time = 2.11, size = 750, normalized size = 4.84

$$\left[-\frac{\left(\left(3a^2b^3-2b^5\right)\cos(dx+c)^2+\left(3a^3b^2-2ab^4\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2+2\sqrt{}}{b^2\cos(dx+c)^2+2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(((3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x
+ c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)
^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*((a^4*b^2 - 2*a^2*b^4 +
b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x
 + c) + 1) - 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^3
*b^3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^6 - 2*a^4*b^2 + a
^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b
- 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos
(d*x + c)), (((3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^3*b^2 - 2*a*b^4)*co
s(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*s
in(d*x + c))) - ((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^
3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((a^4*b^2 - 2*a^2*b^4
+ b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(-sin(
d*x + c) + 1) + (a^6 - 2*a^4*b^2 + a^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*
cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2
 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]
```

giac [B] time = 0.70, size = 332, normalized size = 2.14

$$-\frac{2\left(\frac{\left(3\,a^{2}b^{2}-2\,b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{5}-a^{3}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((3*a^2*b^2 - 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2))
)/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*t
an(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*x +
1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*ta
n(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4
 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^
2*b^2)) + b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3)/d
```

maple [A] time = 0.09, size = 271, normalized size = 1.75

$$-\frac{2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+\frac{6b^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{da\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}-\frac{4b^4\,\mathrm{arc}}{d\,a^3\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)+6/d*b^2/a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-4/d*b^4/a^3/(a-b)/(a+b)/((a-b)*(a+b)
)^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*b/a^3*ln(t
an(1/2*d*x+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/
2*c)+1)-2/d*b/a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 5.92, size = 3176, normalized size = 20.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (b*atan(((b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*
b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2)
 - (2*b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^
10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (64*b*tan(c/2 + (d*x)/2)*(2*a^
11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a
^6*b + a^7 - a^4*b^3 - a^5*b^2))))/a^3)*2i)/a^3 + (b*((32*tan(c/2 + (d*x)/2
)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^
```

6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (2*b*((32*(2*a^11*b - 2*a^6*b^6
 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 -
a^7*b^2) + (64*b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a
^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2))))
/a^3)*2i)/a^3)/((64*(8*b^8 - 4*a*b^7 - 20*a^2*b^6 + 6*a^3*b^5 + 12*a^4*b^4)
)/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (2*b*((32*tan(c/2 + (d*x)/2)*(8*b^8 -
 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a
^6*b + a^7 - a^4*b^3 - a^5*b^2) - (2*b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5
 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) -
 (64*b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4
*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2))))/a^3))/a^3
 + (2*b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5
+ 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (
2*b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b
^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (64*b*tan(c/2 + (d*x)/2)*(2*a^11*b
 - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b
 + a^7 - a^4*b^3 - a^5*b^2))))/a^3))/a^3))*4i)/(a^3*d) - ((2*tan(c/2 + (d*x
)/2)^3*(a*b^2 + a^2*b - a^3 - 2*b^3))/(a^2*(a + b)*(a - b)) + (2*tan(c/2 +
(d*x)/2)*(a*b^2 - a^2*b - a^3 + 2*b^3))/(a^2*(a + b)*(a - b)))/(d*(a + b -
tan(c/2 + (d*x)/2)^4*(a - b) - 2*b*tan(c/2 + (d*x)/2)^2)) + (b^2*atan(((b^2
*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4
*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b^2*(3*
a^2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 -
 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b^2*tan(c/2 + (d*x)/2
)*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^
7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*
b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)
)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a -
 b)^3)^(1/2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b^2*((32*tan(c/
2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5
*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b^2*(3*a^2 - 2*b^2)
((32(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2)
)/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2 - 2
b^2)(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^
8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 -
a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(a^9 - a^3
*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)
*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))/((64*(8*b^8 - 4*a*b^7 - 20*a^
2*b^6 + 6*a^3*b^5 + 12*a^4*b^4))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^2*(
(32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b
^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b^2*(3*a^
2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3
*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b^2*tan(c/2 + (d*x)/2)*
(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*
b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^

```
2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2))/
(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b
)^3)^(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) - (b^2*((32*tan(c/2 + (
d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3
+ 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b^2*(3*a^2 - 2*b^2)*((32
*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^
8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2 - 2*b^2)
*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4
 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b
^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(a^9 - a^3*b^6
+ 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^
9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^
3)^(1/2)*2i)/(d*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**2, x)
```

3.467 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$\frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2a^2d(a^2-b^2)}+\frac{b^2\tan(c+dx)\sec(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))}+\frac{(a^2+6b^2)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{2b^3(4a^2-\ldots}{a^4\ldots}$$

```
[Out] -2*b^3*(4*a^2-3*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4
/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*(a^2+6*b^2)*arctanh(sin(d*x+c))/a^4/d-b*(2*a
^2-3*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d+1/2*(a^2-3*b^2)*sec(d*x+c)*tan(d*x+c)/
a^2/(a^2-b^2)/d+b^2*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.68, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^3(4a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b(2a^2-3b^2)\tan(c+dx)}{a^3d(a^2-b^2)}+\frac{(a^2+6b^2)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(a^2-3b^2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^3*(4*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]]
)/(2*a^4*d) - (b*(2*a^2 - 3*b^2)*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2
- 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*
x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2-3b^2-ab\cos(c+dx)+2b^2\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-2b\left(2a^2-3b\right.\right.}{}}{}$$

$$= -\frac{b\left(2a^2-3b^2\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\sec(c+dx)}{a\left(a^2-b^2\right)d(a+}$$

$$= -\frac{b\left(2a^2-3b^2\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\sec(c+dx)}{a\left(a^2-b^2\right)d(a+}$$

$$= \frac{\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{b\left(2a^2-3b^2\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\sec(c+dx}{2a^2\left(a^2-b^2\right.}$$

$$= -\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{b\left(2a\right.}{}$$

Mathematica [A] time = 5.52, size = 285, normalized size = 1.31

$$\frac{\frac{8b^3\left(3b^2-4a^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2
]])/(-a^2 + b^2)^(3/2) - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1
2*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + 12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^
2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + (4*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])
) - 8*a*b*Tan[c + d*x])/(4*a^4*d)
```

fricas [B] time = 3.27, size = 899, normalized size = 4.14

$$\left[-\frac{2\left(\left(4\,a^{2}b^{4}-3\,b^{6}\right)\cos\left(dx+c\right)^{3}+\left(4\,a^{3}b^{3}-3\,ab^{5}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{-a^{2}+b^{2}}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^{2}-b^{2}\right)\cos(dx+c)^{2}-}{b^{2}\cos(dx+c)}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*((4*a^2*b^4 - 3*b^6)*cos(d*x + c)^3 + (4*a^3*b^3 - 3*a*b^5)*cos(d*
x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^6*b + 4*a^4*b^3 -
11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^
6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b + 4*a^4*b^3 - 11*a^2*b^5
 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^7 - 2*a^5*b^2 + a^3*b^4 - 2*(2*a^5*b
^2 - 5*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2 - 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*
cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3
 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2), -1/4*(4*((4*a^2*b^4 - 3*b
^6)*cos(d*x + c)^3 + (4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*
arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((a^6*b + 4*
a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^
4 + 6*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b + 4*a^4*b^3 -
11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^
6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^7 - 2*a^5*b^2 + a^3*b^4 -
2*(2*a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2 - 3*(a^6*b - 2*a^4*b^3 +
 a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(
d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2)]
```

giac [A] time = 0.96, size = 293, normalized size = 1.35

$$\frac{\frac{4\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{5}-a^{3}b^{2}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b\right)}+\frac{4\left(4\,a^{2}b^{3}-3\,b^{5}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}-a^{4}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2*
```

```
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) +
(a^2 + 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (a^2 + 6*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/
2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) - 4*b*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

maple [A] time = 0.13, size = 401, normalized size = 1.85

$$\frac{2b^4 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{8b^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^2\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}+\frac{6b^5 \operatorname{arcta}}{d\,a^4\,(a-b)\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)-8/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+6/d*b^5/a^4/(a-b)/(a+b)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+1/2/d/a^2/(ta
n(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)+2/d/a^3/(tan(1/2*d*x
+1/2*c)-1)*b-1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)-3/d/a^4*ln(tan(1/2*d*x+1/2*
c)-1)*b^2-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tan(1/2*d*x+1/2*c)+
1)+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b+1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)+3/d/
a^4*ln(tan(1/2*d*x+1/2*c)+1)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.93, size = 3699, normalized size = 17.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)

[Out] - ((tan(c/2 + (d*x)/2)*(3*a*b^3 - 3*a^3*b + a^4 + 6*b^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)) + (tan(c/2 + (d*x)/2)^5*(3*a^3*b - 3*a*b^3 + a^4 + 6*b^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(a^4 - 6*b^4 + 3*a^2*b^2))/(a*(a^2*b - a^3)*(a + b)))/(d*(a + b - tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) + tan(c/2 + (d*x)/2)^6*(a - b))) - (atan((((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*1i)/(2*a^4) + ((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*1i)/(2*a^4))/((16*(108*b^11 - 54*a*b^10 - 216*a^2*b^9 + 81*a^3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^5 - 4*a^7*b^4 + 4*a^8*b^3))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - ((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4)))/(2*a^4) + ((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4)))/(2*a^4)))*(a^2 + 6*b^2)*1i)/(a^4*d) - (b^3*atan(((b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (8*b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*

```
b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^
8*b^2)))*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*
a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^3*(
4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 -
2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*
a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a
^7*b^2) - (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11
*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8
*b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a^1
3*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^
8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(
4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 -
3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))/((16*(108*b^11 -
54*a*b^10 - 216*a^2*b^9 + 81*a^3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^5
- 4*a^7*b^4 + 4*a^8*b^3))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (b^3*(4*a^
2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^
9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*
b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b
^2) + (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4
 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (8*b^3
*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b
- 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b
+ a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^
2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^
8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) - (b^3*(4*a^2 - 3*b^2)*(-
(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^
9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*
b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^3*((
8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^
3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8*b^3*tan(c/2 + (d
*x)/2)*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 +
 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b
^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)*(-
(a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))/(a^1
0 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^
3)^(1/2)*2i)/(d*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**2, x)
```

3.468 $\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=270

$$\frac{\left(a^2-4b^2\right)\tan(c+dx)\sec^2(c+dx)}{3a^2d\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{b\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{2b^4\left(5a^2\right.}{}$$

```
[Out] 2*b^4*(5*a^2-4*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/
(a-b)^(3/2)/(a+b)^(3/2)/d-b*(a^2+4*b^2)*arctanh(sin(d*x+c))/a^5/d+1/3*(2*a^
4+7*a^2*b^2-12*b^4)*tan(d*x+c)/a^4/(a^2-b^2)/d-b*(a^2-2*b^2)*sec(d*x+c)*tan
(d*x+c)/a^3/(a^2-b^2)/d+1/3*(a^2-4*b^2)*sec(d*x+c)^2*tan(d*x+c)/a^2/(a^2-b^
2)/d+b^2*sec(d*x+c)^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.97, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^4\left(5a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(7a^2b^2+2a^4-12b^4\right)\tan(c+dx)}{3a^4d\left(a^2-b^2\right)}-\frac{b\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b^4*(5*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]
])/(a^5*d) + ((2*a^4 + 7*a^2*b^2 - 12*b^4)*Tan[c + d*x])/(3*a^4*(a^2 - b^2)
*d) - (b*(a^2 - 2*b^2)*Sec[c + d*x]*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a
^2 - 4*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sec^2(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2-4b^2-ab\cos(c+dx)+3b^2\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-4b^2\right)\sec^2(c+dx)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^2(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-6b\left(a^2-2b\right.\right.}{}}{}$$

$$= -\frac{b\left(a^2-2b^2\right)\sec(c+dx)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-4b^2\right)\sec^2(c+dx)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\,\mathrm{s}}{a\left(a^2\right.}$$

$$= \frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\sec(c+dx)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-4b^2\right.}{}$$

$$= \frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\sec(c+dx)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-4b^2\right.}{}$$

$$= -\frac{b\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right.}{}$$

$$= \frac{2b^4\left(5a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{\left(2a^4\right.}{}$$

Mathematica [A] time = 6.15, size = 499, normalized size = 1.85

$$-\frac{b^5\sin(c+dx)}{a^4d(a-b)(a+b)(a+b\cos(c+dx))} + \frac{a-6b}{12a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{6b-a}{12a^3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^4*(5*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/(a^5*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((a^2*b + 4*b^3)*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]])/(a^5*d) + ((-(a^2*b) - 4*b^3)*Log[Cos[(c + d*x)/2
```

```
] + Sin[(c + d*x)/2]])/(a^5*d) + (a - 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^2) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])^3) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3) + (-a + 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2
*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2])/(3*a^4*d*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])) + (2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2])/(3
*a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^5*Sin[c + d*x])/(a^4*(a
- b)*(a + b)*d*(a + b*Cos[c + d*x]))
```

fricas [A] time = 2.77, size = 1001, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(3*((5*a^2*b^5 - 4*b^7)*cos(d*x + c)^4 + (5*a^3*b^4 - 4*a*b^6)*cos(d*
x + c)^3)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*((a^6*b^2 + 2*a^4*b^
4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*
a*b^7)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((a^6*b^2 + 2*a^4*b^4 - 7*
a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)
*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(a^8 - 2*a^6*b^2 + a^4*b^4 + (2
*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 + 2*(a^8 + a^6*b
^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^2 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5
)*cos(d*x + c))*sin(d*x + c))/((a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)
^4 + (a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^3), 1/6*(6*((5*a^2*b^5 - 4
*b^7)*cos(d*x + c)^4 + (5*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3)*sqrt(a^2 - b^2
)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*((a^6*b^
2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*
a^3*b^5 + 4*a*b^7)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) + 3*((a^6*b^2 + 2*
a^4*b^4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^
5 + 4*a*b^7)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) + 2*(a^8 - 2*a^6*b^2 +
a^4*b^4 + (2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 + 2*
(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^2 - 2*(a^7*b - 2*a^5*b
^3 + a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9*b - 2*a^7*b^3 + a^5*b^5)*d*
cos(d*x + c)^4 + (a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^3)]
```

giac [A] time = 1.39, size = 368, normalized size = 1.36

$$-\frac{\frac{6\,b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{6}-a^{4}b^{2}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b\right)}+\frac{6\left(5\,a^{2}b^{4}-4\,b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{7}-a^{5}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - a^5*b^2)*sqrt(a^2 - b^2)) +
 3*(a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(a^2*b + 4*b^
3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5
 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a^2*tan(
1/2*d*x + 1/2*c)^3 - 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/
2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 - 1)^3*a^4))/d
```

maple [B] time = 0.12, size = 535, normalized size = 1.98

$$-\frac{2b^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^4\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+\frac{10b^4\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^3\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}-\frac{8b^6\,\mathrm{arc}}{d\,a^5\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*b^5/a^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)+10/d*b^4/a^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-8/d*b^6/a^5/(a-b)/(a+b)/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/3/d/a^2/(
tan(1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/d/a^3/(tan(1/2
*d*x+1/2*c)-1)^2*b-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*
c)-1)*b-3/d/a^4/(tan(1/2*d*x+1/2*c)-1)*b^2+1/d*b/a^3*ln(tan(1/2*d*x+1/2*c)-
1)+4/d*b^3/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3+
1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+1/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2*b-1/d/a
^2/(tan(1/2*d*x+1/2*c)+1)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b-3/d/a^4/(tan(1/2
*d*x+1/2*c)+1)*b^2-1/d*b/a^3*ln(tan(1/2*d*x+1/2*c)+1)-4/d*b^3/a^5*ln(tan(1/
2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.09, size = 3843, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*cos(c + d*x))^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^7*(a^5 - 2*a*b^4 + 4*b^5 - 3*a^2*b^3 + a^3*b^2))/(a^
4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*a*b^4 - 8*a^4*b + a^5 + 36*
b^5 - 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)
/2)^5*(6*a*b^4 + 8*a^4*b + a^5 - 36*b^5 + 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(
a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(a^5 - 2*a*b^4 - 4*b^5 + 3*a^2*b^3
+ a^3*b^2))/(a^4*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^8*(a - b)
 - tan(c/2 + (d*x)/2)^2*(2*a + 4*b) + tan(c/2 + (d*x)/2)^6*(2*a - 4*b) + 6*
b*tan(c/2 + (d*x)/2)^4)) + (b*atan(((b*(a^2 + 4*b^2)*((32*tan(c/2 + (d*x)/2
)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 +
 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11
 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^10*b^8 + 2*a^11
*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a
^12*b^3 - a^13*b^2) + (32*b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a^15*b - 2*
a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/(a^5*(a^10*b
 + a^11 - a^8*b^3 - a^9*b^2))))/a^5)*1i)/a^5 + (b*(a^2 + 4*b^2)*((32*tan(c/
2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 -
2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^
10*b + a^11 - a^8*b^3 - a^9*b^2) - (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^10*b
^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b
 + a^15 - a^12*b^3 - a^13*b^2) - (32*b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*
a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/(
a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2))))/a^5)*1i)/a^5)/((64*(64*b^14 - 32
*a*b^13 - 112*a^2*b^12 + 48*a^3*b^11 + 12*a^4*b^10 - 6*a^5*b^9 + 31*a^6*b^8
 - 5*a^7*b^7 + 5*a^8*b^6))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*(a^2
+ 4*b^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^
3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*
b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^2)*((3
2*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4
+ a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (32*b*tan(c/2 + (d*x)/
2)*(a^2 + 4*b^2)*(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*
b^3 - 2*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2))))/a^5))/a^5 -
(b*(a^2 + 4*b^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10
 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4
```

- 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b*(a^2 + 4*
b^2)*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a
^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (32*b*tan(c/2
+ (d*x)/2)*(a^2 + 4*b^2)*(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 -
4*a^13*b^3 - 2*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2))))/a^5)
)/a^5))*(a^2 + 4*b^2)*2i)/(a^5*d) + (b^4*atan(((b^4*(5*a^2 - 4*b^2)*(-(a +
b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2
*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8
*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b^4*((
32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4
+ a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (32*b^4*tan(c/2 + (d*
x)/2)*(5*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*a^10*b^6 +
2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a^11 - a^8*
b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*(
-(a + b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))*1i)/
(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2) + (b^4*(5*a^2 - 4*b^2)*(-(a + b)^3
(a - b)^3)^(1/2)((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^1
0 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4
- 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b^4*((32*(
a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a
^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (32*b^4*tan(c/2 + (d*x)/2
)*(5*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*a^10*b^6 + 2*a
^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a^11 - a^8*b^3
- a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*(-(a
+ b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))*1i)/(a^1
1 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))/((64*(64*b^14 - 32*a*b^13 - 112*a^2*b
^12 + 48*a^3*b^11 + 12*a^4*b^10 - 6*a^5*b^9 + 31*a^6*b^8 - 5*a^7*b^7 + 5*a^
8*b^6))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b^4*(5*a^2 - 4*b^2)*(-(a +
b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^
2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^
8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b^4*(
(32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^
4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (32*b^4*tan(c/2 + (d
x)/2)(5*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*a^10*b^6
+ 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a^11 - a^8
*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))/(a
^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2) - (b^4*(5*a^2 - 4*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10
+ 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 -
2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b^4*((32*(a^
17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^1
5*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (32*b^4*tan(c/2 + (d*x)/2)*
(5*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*a^10*b^6 + 2*a^1
1*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 -

```
a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*(-(a +
b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))/(a^11 - a
^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*(-(a + b)^3*(a - b)^3)^(1
/2)*2i)/(d*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*cos(c + d*x))**2, x)
```

3.469 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=300

$$-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{a^2\left(4a^2-7b^2\right)\sin(c+dx)\cos^2(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{x\left(12a^2+b^2\right)}{2b^5}-\frac{3a\left(4a^4-7a^2b^2+2b\ldots\right.}{2b^4d\left(a^2-\ldots\right.}$$

```
[Out] 1/2*(12*a^2+b^2)*x/b^5-a^3*(12*a^4-29*a^2*b^2+20*b^4)*arctan((a-b)^(1/2)*ta
n(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^5/(a+b)^(5/2)/d-3/2*a*(4*a^4-7*
a^2*b^2+2*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d+1/2*(6*a^4-10*a^2*b^2+b^4)*cos(
d*x+c)*sin(d*x+c)/b^3/(a^2-b^2)^2/d-1/2*a^2*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-
b^2)/d/(a+b*cos(d*x+c))^2-1/2*a^2*(4*a^2-7*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2
/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.78, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3047, 3049, 3023, 2735, 2659, 205}

$$-\frac{3a\left(-7a^2b^2+4a^4+2b^4\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}-\frac{a^3\left(-29a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\left(4a^2-7b^2\right)\sin(\ldots}{2b^2d\left(a^2-b^2\right)^2(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((12*a^2 + b^2)*x)/(2*b^5) - (a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(Sq
rt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*
d) - (3*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^2*d)
 + ((6*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2
)^2*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])^2) - (a^2*(4*a^2 - 7*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(2*b^2*(a^2
- b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(3a^2-2ab\cos(c+dx)-2\left(2a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(4a^2-7b^2\right)\cos^2(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \int$$

$$= \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} -$$

$$= -\frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(c+}{2b^3\left(a^2-b^2\right)^2}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(c+}{2b^3\left(a^2-b^2\right)^2}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{a^3\left(12a^4-29a^2b^2+20b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^5(a+b)^{5/2}d} - \frac{3a\left(4a^4-7a\right.}{2b^4}$$

Mathematica [A] time = 2.02, size = 199, normalized size = 0.66

$$\frac{\frac{2a^5b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + 2\left(12a^2+b^2\right)(c+dx) + \frac{2a^4b\left(10b^2-7a^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{4a^3\left(12a^4-29a^2b^2+20b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{4b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(12*a^2 + b^2)*(c + d*x) + (4*a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTanh
[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 12*a*b*
Sin[c + d*x] + (2*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])
```

```
^2) + (2*a^4*b*(-7*a^2 + 10*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*
Cos[c + d*x])) + b^2*Sin[2*(c + d*x)])/(4*b^5*d)
```

fricas [A] time = 1.21, size = 1161, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 - b^10)*d*x*cos(d
*x + c)^2 + 4*(12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d*x*
cos(d*x + c) + 2*(12*a^10 - 35*a^8*b^2 + 33*a^6*b^4 - 9*a^4*b^6 - a^2*b^8)*
d*x - (12*a^9 - 29*a^7*b^2 + 20*a^5*b^4 + (12*a^7*b^2 - 29*a^5*b^4 + 20*a^3
*b^6)*cos(d*x + c)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)*cos(d*x + c))
*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 -
2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(12*a^9*b - 33*a^7*b^3 + 27*a
^5*b^5 - 6*a^3*b^7 - (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(d*x + c)^
3 + 4*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*cos(d*x + c)^2 + (18*a^8*b^
2 - 50*a^6*b^4 + 43*a^4*b^6 - 11*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6
*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5
*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*a^6*b^7 + 3*a^4*b
^9 - a^2*b^11)*d), 1/2*((12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 -
 b^10)*d*x*cos(d*x + c)^2 + 2*(12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b
^7 - a*b^9)*d*x*cos(d*x + c) + (12*a^10 - 35*a^8*b^2 + 33*a^6*b^4 - 9*a^4*b
^6 - a^2*b^8)*d*x - (12*a^9 - 29*a^7*b^2 + 20*a^5*b^4 + (12*a^7*b^2 - 29*a^
5*b^4 + 20*a^3*b^6)*cos(d*x + c)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)
*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2
)*sin(d*x + c))) - (12*a^9*b - 33*a^7*b^3 + 27*a^5*b^5 - 6*a^3*b^7 - (a^6*b
^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(d*x + c)^3 + 4*(a^7*b^3 - 3*a^5*b^5
+ 3*a^3*b^7 - a*b^9)*cos(d*x + c)^2 + (18*a^8*b^2 - 50*a^6*b^4 + 43*a^4*b^6
 - 11*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^
11 - b^13)*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)
*d*cos(d*x + c) + (a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]
```

giac [B] time = 1.89, size = 1735, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(((12*a^6 - 6*a^5*b - 23*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 - a*b^5 + b
^6)*sqrt(a^2 - b^2)*abs(a^4*b^5 - 2*a^2*b^7 + b^9)*abs(-a + b) + (24*a^11*b
```

```
^4 - 12*a^10*b^5 - 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68*a^6*b^9 - 11
1*a^5*b^10 + 42*a^4*b^11 + 28*a^3*b^12 - 8*a^2*b^13 + a*b^14 - b^15)*sqrt(a
^2 - b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*tan(1/2
*d*x + 1/2*c)/sqrt((4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 + sqrt(-16*(a^5*b^4 + a
^4*b^5 - 2*a^3*b^6 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^
6 + 2*a^2*b^7 + a*b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2))/(a^5*b^
4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9)))))/((a^4*b^5 - 2*a^2*b^7
 + b^9)^2*(a^2 - 2*a*b + b^2) + (a^7*b^4 - 2*a^6*b^5 - a^5*b^6 + 4*a^4*b^7
- a^3*b^8 - 2*a^2*b^9 + a*b^10)*abs(a^4*b^5 - 2*a^2*b^7 + b^9)) - (24*a^11*
b^4 - 12*a^10*b^5 - 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68*a^6*b^9 - 1
11*a^5*b^10 + 42*a^4*b^11 + 28*a^3*b^12 - 8*a^2*b^13 + a*b^14 - b^15 - 12*a
^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*a^5*b*abs(a^4*b^5 - 2*a^2*b^7 + b^9)
+ 23*a^4*b^2*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*a^3*b^3*abs(a^4*b^5 - 2*a^
2*b^7 + b^9) - 10*a^2*b^4*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + a*b^5*abs(a^4*b^
5 - 2*a^2*b^7 + b^9) - b^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9))*(pi*floor(1/2*(d
*x + c)/pi + 1/2) + arctan(2*tan(1/2*d*x + 1/2*c)/sqrt((4*a^5*b^4 - 8*a^3*b
^6 + 4*a*b^8 - sqrt(-16*(a^5*b^4 + a^4*b^5 - 2*a^3*b^6 - 2*a^2*b^7 + a*b^8
+ b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9) + 16*(a^5*
b^4 - 2*a^3*b^6 + a*b^8)^2))/(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a
*b^8 - b^9)))))/(a^5*b^4*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 2*a^3*b^6*abs(a^4*
b^5 - 2*a^2*b^7 + b^9) + a*b^8*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - (a^4*b^5 -
2*a^2*b^7 + b^9)^2) + 2*(12*a^7*tan(1/2*d*x + 1/2*c)^7 - 18*a^6*b*tan(1/2*d
*x + 1/2*c)^7 - 17*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 33*a^4*b^3*tan(1/2*d*x
+ 1/2*c)^7 - 2*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 13*a^2*b^5*tan(1/2*d*x + 1/
2*c)^7 + 4*a*b^6*tan(1/2*d*x + 1/2*c)^7 + b^7*tan(1/2*d*x + 1/2*c)^7 + 36*a
^7*tan(1/2*d*x + 1/2*c)^5 - 18*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 67*a^5*b^2*ta
n(1/2*d*x + 1/2*c)^5 + 29*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 26*a^3*b^4*tan(1
/2*d*x + 1/2*c)^5 - 5*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 4*a*b^6*tan(1/2*d*x
+ 1/2*c)^5 - 3*b^7*tan(1/2*d*x + 1/2*c)^5 + 36*a^7*tan(1/2*d*x + 1/2*c)^3 +
 18*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 67*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 29*a
^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 26*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*b
^5*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 3*b^7*tan(1/2*
d*x + 1/2*c)^3 + 12*a^7*tan(1/2*d*x + 1/2*c) + 18*a^6*b*tan(1/2*d*x + 1/2*c
) - 17*a^5*b^2*tan(1/2*d*x + 1/2*c) - 33*a^4*b^3*tan(1/2*d*x + 1/2*c) - 2*a
^3*b^4*tan(1/2*d*x + 1/2*c) + 13*a^2*b^5*tan(1/2*d*x + 1/2*c) + 4*a*b^6*tan
(1/2*d*x + 1/2*c) - b^7*tan(1/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*
(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/
2*c)^2 + a + b)^2))/d
```

maple [B] time = 0.07, size = 802, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -6/d*a^6/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+10/d*a
^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*a^6/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*a^5/b^3/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*t
an(1/2*d*x+1/2*c)+10/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-12/d*a^7/b^5/(a^4-2*a^2*
b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))+29/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*
d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-20/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a-b)
*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-6/d/b^4/
(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*a-1/d/b^3/(1+tan(1/2*d*x+1/
2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d
*x+1/2*c)*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+12/d/b^5*
arctan(tan(1/2*d*x+1/2*c))*a^2+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 8.67, size = 5962, normalized size = 19.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^3,x)
```

```
[Out] (atan(((((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b - 2*a*b^13 + b^14 + 2
1*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7
*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 1104*a^11*b^3 - 1104*a^
12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10
 - a^6*b^9 - a^7*b^8) + (((4*(4*b^21 + 28*a^2*b^19 - 80*a^3*b^18 - 120*a^4*
b^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 - 100*a^8*b^13 + 212*a^9*
b^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^
16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*tan(c/2 + (d*x)/2)
```

*(a^2*12i + b^2*1i)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48 *a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b ^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^ 10 - a^6*b^9 - a^7*b^8)))*(a^2*12i + b^2*1i))/(2*b^5))*(a^2*12i + b^2*1i)*1 i)/(2*b^5) + (((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b - 2*a*b^13 + b^ 14 + 21*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 + 18*a^6*b^8 + 8 72*a^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 1104*a^11*b^3 - 1 104*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^ 5*b^10 - a^6*b^9 - a^7*b^8) - (((4*(4*b^21 + 28*a^2*b^19 - 80*a^3*b^18 - 12 0*a^4*b^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 - 100*a^8*b^13 + 21 2*a^9*b^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^19 - 3*a^2*b^17 - 3* a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) + (4*tan(c/2 + (d *x)/2)*(a^2*12i + b^2*1i)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^1 6 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8* a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3* a^5*b^10 - a^6*b^9 - a^7*b^8)))*(a^2*12i + b^2*1i))/(2*b^5))*(a^2*12i + b^2 *1i)*1i)/(2*b^5))/((8*(1728*a^15 - 864*a^14*b + 20*a^3*b^12 - 20*a^4*b^11 + 411*a^5*b^10 - 11*a^6*b^9 + 1314*a^7*b^8 + 2326*a^8*b^7 - 7829*a^9*b^6 - 4 770*a^10*b^5 + 11700*a^11*b^4 + 3456*a^12*b^3 - 7344*a^13*b^2))/(a*b^18 + b ^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^ 12) - (((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b - 2*a*b^13 + b^14 + 21 *a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7* b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 1104*a^11*b^3 - 1104*a^1 2*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) + (((4*(4*b^21 + 28*a^2*b^19 - 80*a^3*b^18 - 120*a^4*b ^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 - 100*a^8*b^13 + 212*a^9*b ^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^1 6 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*tan(c/2 + (d*x)/2)* (a^2*12i + b^2*1i)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48* a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^ 10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^1 0 - a^6*b^9 - a^7*b^8)))*(a^2*12i + b^2*1i))/(2*b^5))*(a^2*12i + b^2*1i))/(2*b^5) + (((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b - 2*a*b^13 + b^14 + 21*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 + 18*a^6*b^8 + 872*a ^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 1104*a^11*b^3 - 1104* a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^ 10 - a^6*b^9 - a^7*b^8) - (((4*(4*b^21 + 28*a^2*b^19 - 80*a^3*b^18 - 120*a^ 4*b^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 - 100*a^8*b^13 + 212*a^ 9*b^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3* b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) + (4*tan(c/2 + (d*x)/ 2)*(a^2*12i + b^2*1i)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10 *b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5* b^10 - a^6*b^9 - a^7*b^8)))*(a^2*12i + b^2*1i))/(2*b^5))*(a^2*12i + b^2*1i))/(2*b^5)))*(a^2*12i + b^2*1i)*1i)/(b^5*d) - ((tan(c/2 + (d*x)/2)^3*(18*a^6

*b - 4*a*b^6 + 36*a^7 + 3*b^7 + 5*a^2*b^5 + 26*a^3*b^4 - 29*a^4*b^3 - 67*a^5*b^2))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) - (tan(c/2 + (d*x)/2)^5*(4*a*b^6 + 18*a^6*b - 36*a^7 + 3*b^7 + 5*a^2*b^5 - 26*a^3*b^4 - 29*a^4*b^3 + 67*a^5*b^2))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) - (tan(c/2 + (d*x)/2)^7*(5*a*b^5 + 6*a^5*b - 12*a^6 + b^6 - 8*a^2*b^4 - 10*a^3*b^3 + 23*a^4*b^2))/((a*b^4 - b^5)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(5*a*b^5 + 6*a^5*b + 12*a^6 - b^6 + 8*a^2*b^4 - 10*a^3*b^3 - 23*a^4*b^2))/((a + b)*(b^6 - 2*a*b^5 + a^2*b^4)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^4*(6*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^2*(4*a*b + 4*a^2) - tan(c/2 + (d*x)/2)^6*(4*a*b - 4*a^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (a^3*atan(((a^3*((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b - 2*a*b^13 + b^14 + 21*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 1104*a^11*b^3 - 1104*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) + (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(4*b^21 + 28*a^2*b^19 - 80*a^3*b^18 - 120*a^4*b^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 - 100*a^8*b^13 + 212*a^9*b^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*a^3*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/((b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(12*a^4 + 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*1i)/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)) + (a^3*((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b - 2*a*b^13 + b^14 + 21*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 1104*a^11*b^3 - 1104*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) - (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(4*b^21 + 28*a^2*b^19 - 80*a^3*b^18 - 120*a^4*b^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 - 100*a^8*b^13 + 212*a^9*b^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) + (4*a^3*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/((b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(12*a^4 + 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*1i)/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))/((8*(1728*a^15 - 864*a^14*b + 20*a^3*b^12 - 20*a^4*b^11 + 411*a^5*b^10 - 11*a^6*b^9 + 1314*a^7*b^8 + 2326*a^8*b^7 - 7829*a^9*b^6 - 4770*a^10*b^5 + 11700*a^11*b^4 + 3456*a^

```
12*b^3 - 7344*a^13*b^2))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b
^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (a^3*((8*tan(c/2 + (d*x)/2)*(288*
a^14 - 288*a^13*b - 2*a*b^13 + b^14 + 21*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^
10 - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 +
1538*a^10*b^4 + 1104*a^11*b^3 - 1104*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13
 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) + (a^3*(-(a +
b)^5*(a - b)^5)^(1/2)*((4*(4*b^21 + 28*a^2*b^19 - 80*a^3*b^18 - 120*a^4*b^1
7 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a^7*b^14 - 100*a^8*b^13 + 212*a^9*b^1
2 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16
+ 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*a^3*tan(c/2 + (d*x)/2
)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)*(8*a*b^19 - 8
*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*
b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/((b^15 - 5*a^2*b^13 + 10*a^
4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b^14 + b^15 - 3*a^2*b^13 - 3
*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(12*a^4 + 20*b^4
 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^
7 - a^10*b^5)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2)
)/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5))
 + (a^3*((8*tan(c/2 + (d*x)/2)*(288*a^14 - 288*a^13*b - 2*a*b^13 + b^14 + 2
1*a^2*b^12 - 40*a^3*b^11 + 74*a^4*b^10 - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7
*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^10*b^4 + 1104*a^11*b^3 - 1104*a^
12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10
 - a^6*b^9 - a^7*b^8) - (a^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(4*b^21 + 28*
a^2*b^19 - 80*a^3*b^18 - 120*a^4*b^17 + 276*a^5*b^16 + 164*a^6*b^15 - 360*a
^7*b^14 - 100*a^8*b^13 + 212*a^9*b^12 + 24*a^10*b^11 - 48*a^11*b^10))/(a*b^
18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 -
a^7*b^12) + (4*a^3*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4
+ 20*b^4 - 29*a^2*b^2)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 +
 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^1
0*b^10))/((b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*
b^5)*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a
^6*b^9 - a^7*b^8)))*(12*a^4 + 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 +
 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(-(a + b)^5*(a - b)^5)^
(1/2)*(12*a^4 + 20*b^4 - 29*a^2*b^2))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 -
 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5))))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*a^4
 + 20*b^4 - 29*a^2*b^2)*1i)/(d*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^
9 + 5*a^8*b^7 - a^10*b^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.470 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-$$

```
[Out] -3*a*x/b^4+3*a^2*(2*a^4-5*a^2*b^2+4*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2
*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/d+1/2*(3*a^2-2*b^2)*sin(d*x+c)
/b^3/(a^2-b^2)/d-1/2*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x
+c))^2+3/2*a^3*(a^2-2*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.49, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3031, 3023, 2735, 2659, 205}

$$\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{3a^2\left(-5a^2b^2+2a^4+4b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-3*a*x)/b^4 + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) + ((3*a^2 -
2*b^2)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^2*Sin[c + d*
x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*a^3*(a^2 - 2*b^2)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(2a^2-2ab\cos(c+dx)-\left(3a^2-2b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int \frac{3a^2b(}{}}{}$$

$$= \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)s}{2b^3\left(a^2-b^2\right)^2 d(a+}$$

$$= -\frac{3ax}{b^4} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a^3\left(a^2\right.}{2b^3\left(a^2-b\right.}$$

$$= -\frac{3ax}{b^4} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a^3\left(a^2\right.}{2b^3\left(a^2-b\right.}$$

$$= -\frac{3ax}{b^4} + \frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d} + \frac{\left(3a^2-2b^2\right)\sin(c+dx}{2b^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 1.48, size = 177, normalized size = 0.80

$$\frac{-\frac{a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} - \frac{6a^2\left(2a^4-5a^2b^2+4b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{a^3b\left(5a^2-8b^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - 6a(c+dx) + 2b\sin(c}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-6*a*(c + d*x) - (6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*Sin[c + d*x] - (a^
4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a^3*b*(5*a^2
- 8*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b^4*d
)
```

fricas [B] time = 1.32, size = 1029, normalized size = 4.66

$$\left[-\frac{12\left(a^7b^2-3a^5b^4+3a^3b^6-ab^8\right)dx\cos\left(dx+c\right)^2+24\left(a^8b-3a^6b^3+3a^4b^5-a^2b^7\right)dx\cos\left(dx+c\right)+12\left(a^9-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c)^2 + 24
*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c) + 12*(a^9 - 3*a
^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*x + 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (2*a
^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(d*x + c)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4
*a^3*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*a^8
*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b
^7 - b^9)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*
cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*co
s(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)
+ (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(6*(a^7*b^2 - 3*a^5
*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c)^2 + 12*(a^8*b - 3*a^6*b^3 + 3*a^
4*b^5 - a^2*b^7)*d*x*cos(d*x + c) + 6*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^
6)*d*x - 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*
b^6)*cos(d*x + c)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c))*sqr
t(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) -
 (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 +
3*a^2*b^7 - b^9)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*
a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^1
2)*d*cos(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*
x + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d)]
```

giac [A] time = 1.14, size = 354, normalized size = 1.60

$$-\frac{\frac{3\left(2a^6-5a^4b^2+4a^2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4-2a^2b^6+b^8\right)\sqrt{a^2-b^2}}-\frac{4a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7a^4b^2\mathrm{t}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*a^6*tan(1/
2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*tan(1/2*d*x + 1/2*c) +
 5*a^5*b*tan(1/2*d*x + 1/2*c) - 7*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*a^3*b^3*
tan(1/2*d*x + 1/2*c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + 3*(d*x + c)*a/b^4 - 2*tan(1/2*d*
x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

maple [B] time = 0.07, size = 679, normalized size = 3.07

$$\frac{4a^5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^3\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^2(a-b)\left(a^2+2ab+b^2\right)}-\frac{a^4\left(\mathrm{ta}\right.}{d\,b^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{d}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-8/d*a^3
/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d*a^4/b^2/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1
/2*d*x+1/2*c)-8/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+6/d*a^6/b^4/(a^4-2*a^2*b^2+b^4)
/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1
5/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))+12/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/b^3*tan(1/2*d*
x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/b^4*a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.23, size = 5350, normalized size = 24.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(2*a*b^4 - 3*a^4*b + 6*a^5 - 2*b^5 + 4*a^2*b^3 - 12*
a^3*b^2))/((a*b^3 - b^4)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*a*b^4 + 3*a^4*
b + 6*a^5 + 2*b^5 - 4*a^2*b^3 - 12*a^3*b^2))/((a + b)*(b^5 - 2*a*b^4 + a^2*
b^3)) + (2*tan(c/2 + (d*x)/2)^3*(6*a^6 - 2*b^6 + 6*a^2*b^4 - 13*a^4*b^2))/(
b*(a*b^2 - b^3)*(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a*b
 + 3*a^2 - b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - ta
n(c/2 + (d*x)/2)^4*(2*a*b - 3*a^2 + b^2))) - (6*a*atan(((3*a*((8*tan(c/2 +
(d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 + 288
*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288*a^10
*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a
^6*b^7 - a^7*b^6) + (a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 + 26*a^4*b
^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*
a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^
11 - a^6*b^10 - a^7*b^9) - (a*tan(c/2 + (d*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 3
2*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8
*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*24i)/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a
^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*3i)/b^4))/b^4 + (3*a
*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 3
6*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9
*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9
+ 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*
b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 +
 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*
b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*tan(c/2 + (d*x)/2)*(8*a*b^17 -
 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^
7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*24i)/(b^4*(a*b^12 + b^13 - 3
*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*3i)/b
^4))/b^4)/((48*(36*a^12 - 18*a^11*b + 72*a^4*b^8 + 72*a^5*b^7 - 234*a^6*b^6
 - 126*a^7*b^5 + 288*a^8*b^4 + 81*a^9*b^3 - 162*a^10*b^2))/(a*b^15 + b^16 -
 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) -
(a*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 +
 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a
^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^
9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^
3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10
```

```
 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^
4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (a*tan(c/2 + (d*x)/2)*(8*a*b^17
 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*
a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*24i)/(b^4*(a*b^12 + b^13 -
 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*3i)
/b^4)*3i)/b^4 + (a*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^1
0 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441
*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3
*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (a*((24*(4*a*b^17 - 8*
a^2*b^16 - 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^
11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3
*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*tan(c/2 + (d
*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 4
8*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*24i)/(b^4*
(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7
- a^7*b^6)))*3i)/b^4)*3i)/b^4)))/(b^4*d) - (a^2*atan(((a^2*(-(a + b)^5*(a -
 b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72
*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b
^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10
+ 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (3*a^2*((24*(4*a*b^17 - 8*a^
2*b^16 - 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11
 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a
^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (12*a^2*tan(c/2 +
 (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*(8*a*b^1
7 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32
*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10
*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11
- 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(-(a + b)^5*(a
- b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b
^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4*b^4 - 5*a^2*b^2)*3i)/
(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) +
 (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^1
1*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 4
32*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*
a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (3*a^2
*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32
*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b
^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^
9) + (12*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4
 - 5*a^2*b^2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b
^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((
b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^1
2 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*
b^6)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 -
 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4
```

```
*b^4 - 5*a^2*b^2)*3i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*
a^8*b^6 - a^10*b^4)))/((48*(36*a^12 - 18*a^11*b + 72*a^4*b^8 + 72*a^5*b^7 -
 234*a^6*b^6 - 126*a^7*b^5 + 288*a^8*b^4 + 81*a^9*b^3 - 162*a^10*b^2))/(a*b
^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 -
 a^7*b^9) - (3*a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*
a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 28
8*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^1
2 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*
b^6) + (3*a^2*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 + 26*a^4*b^14 + 14*
a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8)
)/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*
b^10 - a^7*b^9) - (12*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(
2*a^4 + 4*b^4 - 5*a^2*b^2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^
14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*
a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^1
0*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a
^6*b^7 - a^7*b^6)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2
))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)
))*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^
6*b^8 + 5*a^8*b^6 - a^10*b^4)) + (3*a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*ta
n(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b
^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 -
288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5
*b^8 - a^6*b^7 - a^7*b^6) - (3*a^2*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^1
5 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*
a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^1
2 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (12*a^2*tan(c/2 + (d*x)/2)*(-(a + b)
^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*(8*a*b^17 - 8*a^2*b^16 - 32
*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*
b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*
b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^
4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^
4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 +
5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12
+ 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))))*(-(a + b)^5*(a - b)^5
)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*3i)/(d*(b^14 - 5*a^2*b^12 + 10*a^4*b^10
 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.471 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=179

$$-\frac{a^2(2a^2-5b^2)\sin(c+dx)}{2b^2d(a^2-b^2)^2(a+b\cos(c+dx))}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{a(2a^4-5a^2b^2+6b^4)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c\right.}{\sqrt{a+b}}\right.}{b^3d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] x/b^3-a*(2*a^4-5*a^2*b^2+6*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-
b^2)/d/(a+b*cos(d*x+c))^2-1/2*a^2*(2*a^2-5*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/
d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3021, 2735, 2659, 205}

$$-\frac{a(-5a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(2a^2-5b^2)\sin(c+dx)}{2b^2d(a^2-b^2)^2(a+b\cos(c+dx))}-\frac{a^2\sin(c+dx)\cos(c+}{2bd(a^2-b^2)(a+b\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] x/b^3 - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*Cos[c + d*x]*Sin
[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(2*a^2 - 5*b^2
)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{a^2-2ab\cos(c+dx)-2\left(a^2-b^2\right)\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{ab\left(a^2-4\right.}{}}{}$$

$$= \frac{x}{b^3} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\left(a\left(2\right.\right.}{}$$

$$= \frac{x}{b^3} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\left(a\left(2\right.\right.}{}$$

$$= \frac{x}{b^3} - \frac{a\left(2a^4-5a^2b^2+6b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+d}$$

Mathematica [A] time = 1.12, size = 149, normalized size = 0.83

$$\frac{\frac{2a\left(2a^4-5a^2b^2+6b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} - \frac{a^2b\sin(c+dx)\left(2a^3+3b\left(a^2-2b^2\right)\cos(c+dx)-5ab^2\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + 2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(c + d*x) + (2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - (a^2*b*(2*a^3 - 5*a*b^2 + 3
*b*(a^2 - 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Co
s[c + d*x])^2))/(2*b^3*d)
```

fricas [B] time = 1.09, size = 913, normalized size = 5.10

$$\left[\frac{4\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)dx\cos(dx+c)^2 + 8\left(a^7b-3a^5b^3+3a^3b^5-ab^7\right)dx\cos(dx+c) + 4\left(a^8-3a^6b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 + 8*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^8 - 3*a^6*b^2 +
 3*a^4*b^4 - a^2*b^6)*d*x - (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4 + (2*a^5*b^2 - 5
*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*co
s(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b
^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^7*b - 7*a^5*b
^3 + 5*a^3*b^5 + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sin(d*x
+ c))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b
^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5
+ 3*a^4*b^7 - a^2*b^9)*d), 1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d
*x*cos(d*x + c)^2 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x +
 c) + 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x - (2*a^7 - 5*a^5*b^2 +
6*a^3*b^4 + (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(2*a^6*b -
 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x +
c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5
+ 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5
 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6
+ 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a
^2*b^9)*d)]
```

giac [A] time = 1.04, size = 319, normalized size = 1.78

$$\frac{\frac{\left(2\,a^{5}-5\,a^{3}b^{2}+6\,ab^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}b^{3}-2\,a^{2}b^{5}+b^{7}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,a^{3}b^{2}\tan(}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
- b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 - b^2)) - (2*a^5*tan(1/2*d*x
 + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x + 1/2*
c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c) + 3*a^
4*b*tan(1/2*d*x + 1/2*c) - 5*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*a^2*b^3*tan(1
/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (d*x + c)/b^3)/d
```

maple [B] time = 0.06, size = 639, normalized size = 3.57

$$-\frac{2a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^2(a-b)\left(a^2+2ab+b^2\right)}+\frac{a^3\left(\mathrm{ta}\right.}{db\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+6/d*a^2/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*a^3/b/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*
x+1/2*c)+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-b)
*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+5/d*a^3/
b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/(
(a-b)*(a+b))^(1/2))-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(
tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/b^3*arctan(tan(1/2*d*x+1/
2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.80, size = 5102, normalized size = 28.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^3,x)
```

[Out] (2*atan((((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11
+ 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 +
b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)
- (tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 +
48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b
^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6
- a^6*b^5 - a^7*b^4)))*1i)/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b -
8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*
a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8
+ 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))/b^3 - ((((8*(12*a*b^14 - 4*b^
15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a
^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 +
3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (tan(c/2 + (d*x)/2)*(8*a*b^15
- 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*
a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*
a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3
- (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 +
32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^
2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^
5 - a^7*b^4))/b^3)/((((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 -
6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6)
)/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^
7 - a^7*b^6) - (tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 3
2*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^
7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7
+ 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10
- 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a
^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9
- 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3 + (((((8*
(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 +
4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^
11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (tan(c/2 + (
d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 -
48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a
*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^
7*b^4)))*1i)/b^3 - (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^
10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^
7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3
*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3 + (16*(12*a*b^8 - 2*a^8*b + 4*a^9 +
24*a^2*b^7 - 34*a^3*b^6 - 26*a^4*b^5 + 36*a^5*b^4 + 13*a^6*b^3 - 18*a^7*b^2
))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b
^7 - a^7*b^6))))/(b^3*d) + ((tan(c/2 + (d*x)/2)^3*(a^3*b - 2*a^4 + 6*a^2*b^
2))/((a*b^2 - b^3)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(a^3*b + 2*a^4 - 6*a^2*
b^2))/((a + b)*(b^4 - 2*a*b^3 + a^2*b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2
*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) +

(a*atan(((a*((8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + (a*((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)) + (a*((8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) - (a*((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))/((16*(12*a*b^8 - 2*a^8*b + 4*a^9 + 24*a^2*b^7 - 34*a^3*b^6 - 26*a^4*b^5 + 36*a^5*b^4 + 13*a^6*b^3 - 18*a^7*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (a*((8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + (a*((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(-(a

```
 + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11
 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)) - (a*((8*tan(c/2 + (d*x
)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^
4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11
- 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) - (a*(
(8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^1
0 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2
*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (4*a*tan(
c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*(8*
a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10
 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 +
 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9
 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*(-(a + b)^5*(a
- b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b
^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a
^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 +
5*a^8*b^5 - a^10*b^3))))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^
2*b^2)*1i)/(d*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^
10*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.472 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$\frac{(a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{a(a^2-4b^2)\sin(c+dx)}{2bd(a^2-b^2)^2(a+b\cos(c+dx))}$$

```
[Out] (a^2+2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/
(a+b)^(5/2)/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*a*(a^
2-4*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2790, 2754, 12, 2659, 205}

$$\frac{(a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{a(a^2-4b^2)\sin(c+dx)}{2bd(a^2-b^2)^2(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(5/2)*(a + b)^(5/2)*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2) + (a*(a^2 - 4*b^2)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*C
os[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{2ab+\left(a^2-2b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{b(a^2+2}{a+b\cos(}}{2b\left(a^2-\right.}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(a^2+2b^2\right.}{2}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(a^2+2b^2\right.}{}$$

$$= \frac{\left(a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2\right.}{2b\left(a^2-\right.}$$

Mathematica [A] time = 0.55, size = 115, normalized size = 0.77

$$\frac{\frac{a\sin(c+dx)\left(\left(a^2-4b^2\right)\cos(c+dx)-3ab\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} - \frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-
a^2 + b^2)^(5/2) + (a*(-3*a*b + (a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
(a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2))/(2*d)
```

fricas [A] time = 0.82, size = 587, normalized size = 3.94

$$\left[-\frac{\left(a^4+2a^2b^2+\left(a^2b^2+2b^4\right)\cos(dx+c)^2+2\left(a^3b+2ab^3\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos}{b^2}\right.}{4\left(\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)d\cos(dx+c)^2+2\left(a^7b-3a^5b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((a^4 + 2*a^2*b^2 + (a^2*b^2 + 2*b^4)*cos(d*x + c)^2 + 2*(a^3*b + 2*a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(3*a^4*b -
 3*a^2*b^3 - (a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3
+ 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6)*d), 1/2*((a^4 + 2*a^2*b^2 + (a^2*b^2 + 2*b^4)*cos(d*x + c)^2 + 2*(a^3*b
+ 2*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt
(a^2 - b^2)*sin(d*x + c))) - (3*a^4*b - 3*a^2*b^3 - (a^5 - 5*a^3*b^2 + 4*a*
b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d
*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c)
+ (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

giac [A] time = 0.74, size = 250, normalized size = 1.68

$$-\frac{\frac{\left(\pi\left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)\left(a^2+2\,b^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(a^2 + 2*b^2)/((a^4
- 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x
+ 1/2*c) + 3*a^2*b*tan(1/2*d*x + 1/2*c) + 4*a*b^2*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
+ a + b)^2))/d
```

maple [B] time = 0.05, size = 400, normalized size = 2.68

$$-\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^2\left(a-b\right)\left(a^2+2ab+b^2\right)}-\frac{4a\left(\tan^3\right.}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b+1/d*a^2/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1
/2*d*x+1/2*c)-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(
a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b+1/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/(a^
4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 2.99, size = 203, normalized size = 1.36

$$\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\,a\,b+b^2\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{5/2}}\right)\,\left(a^2+2\,b^2\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(a^2+4\,b\,a\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a\,b-a^2\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^3,x)
```

```
[Out] (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)
*(a - b)^(5/2)))*(a^2 + 2*b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((tan(c/2
 + (d*x)/2)^3*(4*a*b + a^2))/((a + b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(4*a
*b - a^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*
(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.473 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{(a^2+2b^2)\sin(c+dx)}{2d(a^2-b^2)^2(a+b\cos(c+dx))}+\frac{a\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{3ab\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] -3*a*b*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)
^(5/2)/d+1/2*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*(a^2+2*b^2)*si
n(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2754, 12, 2659, 205}

$$\frac{(a^2+2b^2)\sin(c+dx)}{2d(a^2-b^2)^2(a+b\cos(c+dx))}+\frac{a\sin(c+dx)}{2d(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{3ab\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-3*a*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*
(a + b)^(5/2)*d) + (a*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2
) + ((a^2 + 2*b^2)*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{2b-a\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+2b^2\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int -\frac{3ab}{a+b\cos(c+}}{2\left(a^2-b^2\right.}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+2b^2\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{(3ab)\int \frac{}{a+b\,\mathrm{co}}}{2\left(a^2-\right.}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+2b^2\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{(3ab)\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{3ab\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+2b^2\right)}{2\left(a^2-b^2\right)^2 d(a}$$

Mathematica [A] time = 0.38, size = 115, normalized size = 0.86

$$\frac{\frac{\sin(c+dx)\left(b\left(a^2+2b^2\right)\cos(c+dx)+a\left(2a^2+b^2\right)\right)}{(a+b\cos(c+dx))^2}+\frac{6ab\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}}{2d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((6*a*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b
^2] + ((a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a +
b*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

fricas [B] time = 0.86, size = 555, normalized size = 4.14

$$\left[-\frac{3\left(ab^3\cos(dx+c)^2+2a^2b^2\cos(dx+c)+a^3b\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}\,(a\cos(dx}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right.}{4\left(\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)d\cos(dx+c)^2+2\left(a^7b-3a^5b^3+3a^3b^5\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a*b^3*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a^3*b)*sqrt(-a^2
+ b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2
 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^5 - a^3*b^2 - a*b^4 + (a^4*b + a^2*
b^3 - 2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d
*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), -1/2*(3*(a*b^3*cos(d*
x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a^3*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d
*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^5 - a^3*b^2 - a*b^4 + (
a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4
+ 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*
b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

giac [B] time = 0.63, size = 271, normalized size = 2.02

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)ab}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b^3\tan}{\left(a^4-2a^2b^2+b^4\right)\left(a\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a*b/((a^4 - 2*a^2*b
^2 + b^4)*sqrt(a^2 - b^2)) + (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*
d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^
3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2*tan(1/2
*d*x + 1/2*c) + 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

maple [B] time = 0.05, size = 475, normalized size = 3.54

$$\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^2(a-b)\left(a^2+2ab+b^2\right)}+\frac{a\left(\tan^3\left(\frac{dx}{2}\right.\right.}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b+2/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3*b^2+2/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^
2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b+2/d/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)
*tan(1/2*d*x+1/2*c)*b^2-3/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.15, size = 207, normalized size = 1.54

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^2+a\,b+2\,b^2\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-a\,b+2\,b^2\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}-\frac{3\,a\,b\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(\right.}{2\,\sqrt{a+b}\,\left(a-b\right)\ldots}\right.}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(a*b + 2*a^2 + 2*b^2))/((a + b)^2*(a - b)) + (tan(c/
2 + (d*x)/2)*(2*a^2 - a*b + 2*b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*
b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*
b + b^2) + a^2 + b^2)) - (3*a*b*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 -
 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2))))/(d*(a + b)^(5/2)*(a - b)^(
5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.474 $\int \frac{1}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

```
[Out] (2*a^2+b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/
(a+b)^(5/2)/d-1/2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-3/2*a*b*sin(d
*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 205}

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(5/2)*(a + b)^(5/2)*d) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c +
d*x])^2) - (3*a*b*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3}\,dx = -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{\int \frac{-2a+b\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{3ab\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}+\frac{\int \frac{2a^2+b^2}{a+b\cos(c-}}{2\left(a^2-\right.}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{3ab\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}+\frac{\left(2a^2+b^2\right)}{2\left(\right.}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{3ab\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}+\frac{\left(2a^2+b^2\right)}{}$$

$$= \frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}-\frac{3}{2\left(a^2-b^{2}\right.}$$

Mathematica [A] time = 0.40, size = 113, normalized size = 0.85

$$\frac{\frac{b\sin(c+dx)\left(-4a^2-3ab\cos(c+dx)+b^2\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}-\frac{2\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-3),x]
```

```
[Out] ((-2*(2*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-
a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/((a
 - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2))/(2*d)
```

fricas [B] time = 0.79, size = 585, normalized size = 4.40

$$\left[-\frac{\left(2a^4+a^2b^2+\left(2a^2b^2+b^4\right)\cos(dx+c)^2+2\left(2a^3b+ab^3\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos}{b^2}\right.}{4\left(\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)d\cos(dx+c)^2+2\left(a^7b-3a^5b^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^4 + a^2*b^2 + (2*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^3*b + a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(4*a^4*b -
 5*a^2*b^3 + b^5 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^
2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)
*d), 1/2*((2*a^4 + a^2*b^2 + (2*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^3*b
+ a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a
^2 - b^2)*sin(d*x + c))) - (4*a^4*b - 5*a^2*b^3 + b^5 + 3*(a^3*b^2 - a*b^4)
*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos
(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a
^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

giac [B] time = 0.54, size = 251, normalized size = 1.89

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)\left(2a^2+b^2\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(2*a^2 + b^2)/((a^4
- 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^2*b*tan(1/2*
d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
+ a + b)^2))/d
```

maple [B] time = 0.05, size = 400, normalized size = 3.01

$$-\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(\tan^3\left(\right.\right.}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3*b-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2-4/d/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2
*d*x+1/2*c)*b+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+
b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b^2+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+1/d/(a^
4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 2.88, size = 203, normalized size = 1.53

$$\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)\left(a^2-2ab+b^2\right)}{2\sqrt{a+b}\,(a-b)^{5/2}}\right)\left(2a^2+b^2\right)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(b^2+4ab\right)}{(a+b)^2\,(a-b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(4ab-b^2\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{d\left(2ab+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2-2ab\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x))^3,x)
```

```
[Out] (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)
*(a - b)^(5/2)))*(2*a^2 + b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((tan(c/2
 + (d*x)/2)^3*(4*a*b + b^2))/((a + b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(4*a
*b - b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*
(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.475 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{b\left(6a^4-5a^2b^2+2\right.}{a^3d(a}$$

```
[Out] -b*(6*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2
))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+arctanh(sin(d*x+c))/a^3/d+1/2*b^2*sin(d*x+
c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*sin(d*x+c)/a^2/(a
^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.46, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt
[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d)) + ArcTanh[Sin[c + d*x]]/(a^3
*d) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b^2*
(5*a^2 - 2*b^2)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2-b^2\right)-2ab\cos(c+dx)+b^2\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{\left(2\left(a^2-\right.\right.}{}}{}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \sec(c}{a}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\sin(}{2a^2\left(a^2-b^2\right)^2 d(a+b\,c}$$

$$= -\frac{b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{}{2a\left(a^2-\right.}$$

Mathematica [A] time = 1.14, size = 192, normalized size = 1.05

$$\frac{\frac{2b\left(6a^4-5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{ab^2\sin(c+dx)\left(6a^3+b\left(5a^2-2b^2\right)\cos(c+dx)-3ab^2\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*(6*a^3 - 3*a*b^2 +
b*(5*a^2 - 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*C
os[c + d*x])^2))/(2*a^3*d)
```

fricas [B] time = 2.76, size = 1142, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*c
os(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a^2
 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^
2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6
+ (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5
*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*(a^8 - 3*
a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos
(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(-
sin(d*x + c) + 1) - 2*(6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 + (5*a^5*b^3 - 7*a
^3*b^5 + 2*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5
*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*
b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), -1/2*((6
*a^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(d*x +
c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*ar
ctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^8 - 3*a^6*b
^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x
+ c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*
x + c) + 1) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4
 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b
^6 + (5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^
2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b
^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 -
a^5*b^6)*d)]
```

giac [B] time = 1.27, size = 344, normalized size = 1.89

$$\frac{\left(6\,a^4b-5\,a^2b^3+2\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}+\frac{6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab^4\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
- b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) + (6*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*x +
1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) +
5*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5*tan(1
```

```
/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^
3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)/d
```

maple [B] time = 0.10, size = 660, normalized size = 3.63

$$\frac{6\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{2}}{d\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^{2}(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{b^{3}\left(\tan^{3}\right.}{da\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*b^2+1/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-2/d*b^4/a^2/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b^2-1/d*b^3/a/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c)-2/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)
/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+5/d*b^3/a/(a^
4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(t
an(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-
1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.06, size = 5090, normalized size = 27.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] - (atan((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4
*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b +
a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)
 - (8*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 -
32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14
*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*
a^8*b^3 - 3*a^9*b^2)))/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*
b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b
^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b
^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3 - ((((8*(12*a^14*b - 4*a^1
5 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b
^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*
b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^15*
b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^1
1*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*
b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 + (8*t
an(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^
3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a
^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^
9*b^2))*1i)/a^3)/((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8
*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a
^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a
^11*b^2) - (8*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^
8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3
- 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*
b^4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*
b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 -
52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 +
 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))/a^3 + (((8*(12*a^14*b - 4*
a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^1
1*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a
^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^
15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*
a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a
^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 + (
8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32
*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))
/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3
*a^9*b^2))/a^3 - (16*(12*a^8*b - 2*a*b^8 + 4*b^9 - 18*a^2*b^7 + 13*a^3*b^6
+ 36*a^4*b^5 - 26*a^5*b^4 - 34*a^6*b^3 + 24*a^7*b^2))/(a^12*b + a^13 - a^6*
b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*2i)/(a^3
```

*d) - ((tan(c/2 + (d*x)/2)^3*(a*b^3 - 2*b^4 + 6*a^2*b^2))/((a^2*b - a^3)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(a*b^3 + 2*b^4 - 6*a^2*b^2))/((a + b)*(a^4 - 2*a^3*b + a^2*b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (b*((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)) + (b*((8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (b*((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))/((16*(12*a^8*b - 2*a*b^8 + 4*b^9 - 18*a^2*b^7 + 13*a^3*b^6 + 36*a^4*b^5 - 26*a^5*b^4 - 34*a^6*b^3 + 24*a^7*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (b*((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*

```
b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a
^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(-(a +
b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*
a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1
/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b
^6 + 10*a^9*b^4 - 5*a^11*b^2)) - (b*((8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*
b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 -
52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 +
 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (b*((8*(12*a^14*b - 4*a^1
5 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b
^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*
b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (4*b*tan(c/2 + (d*x)/2)*(-(a +
 b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^15*b - 8*a^6*b^10 +
 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*
b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6
 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5
+ 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4
+ 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9
*b^4 - 5*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^
2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)
)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*1i)/(d*(a^13 -
 a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**3, x)
```

3.476 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$-\frac{3b\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{3b^2\left(4a^4-5a\right.}{}$$

```
[Out] 3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(
1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d-3*b*arctanh(sin(d*x+c))/a^4/d+1/2*(2*a^
4-11*a^2*b^2+6*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*tan(d*x+c)/a/(a^2-
b^2)/d/(a+b*cos(d*x+c))^2+3/2*b^2*(2*a^2-b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/
(a+b*cos(d*x+c))
```

Rubi [A] time = 0.78, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(-11a^2b^2+2a^4+6b^4\right)\tan(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{3b^2\left(2a^2-b^2\right)\tan(}{2a^2d\left(a^2-b^2\right)^2(a+b\,c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) - (3*b*ArcTanh[Sin[c + d*x]
])/(a^4*d) + ((2*a^4 - 11*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)
^2*d) + (b^2*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*
b^2*(2*a^2 - b^2)*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2a^2-3b^2-2ab\cos(c+dx)+2b^2\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{\left(2a^4-11\right.}{}}{}$$

$$= \frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3b^2\left(2\right.}{2a^2\left(a^2-\right.}$$

$$= \frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3b^2\left(2\right.}{2a^2\left(a^2-\right.}$$

$$= -\frac{3b\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\tan(c\ }{2a\left(a^2-b^2\right)d(a+\ }$$

$$= \frac{3b^2\left(4a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d} - \frac{3b\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{\left(2a^4\right.}{}$$

Mathematica [A] time = 4.21, size = 205, normalized size = 0.88

$$-\frac{\frac{6b^2\left(4a^4-5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{ab^3\sin(c+dx)\left(8a^3+b\left(7a^2-4b^2\right)\cos(c+dx)-5ab^2\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} - 2a\tan(c+dx) - 6b\log\left(\cos\left(\frac{1}{2}\right.\right.}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/2*((6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])
/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 6*b*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 6*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^3*(8*a^3 -
 5*a*b^2 + b*(7*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)
^2*(a + b*Cos[c + d*x])^2) - 2*a*Tan[c + d*x])/(a^4*d)
```

fricas [B] time = 2.50, size = 1346, normalized size = 5.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*((4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(d*x + c)^3 + 2*(4*a^5*b^3 - 5
*a^3*b^5 + 2*a*b^7)*cos(d*x + c)^2 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*co
s(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b
^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*((a^6*b^3 - 3*a^4*
b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6
- a*b^8)*cos(d*x + c)^2 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x
 + c))*log(sin(d*x + c) + 1) - 6*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c
os(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2
+ (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c)
 + 1) - 2*(2*a^9 - 6*a^7*b^2 + 6*a^5*b^4 - 2*a^3*b^6 + (2*a^7*b^2 - 13*a^5*
b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4
*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2 - 3*a^8*b^4 + 3*a^
6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5
*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*
x + c)), 1/2*(3*((4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(d*x + c)^3 + 2*(4*a^5*
b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(d*x + c)^2 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*
b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 -
 b^2)*sin(d*x + c))) - 3*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x +
 c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2 + (a^8*b
 - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3
*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a
^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5
 - a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*a^9 - 6*a^7*b^2 + 6*a
^5*b^4 - 2*a^3*b^6 + (2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(d*
x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(d*x + c))*si
n(d*x + c))/((a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3
+ 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3
*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c))]
```

giac [A] time = 1.28, size = 380, normalized size = 1.64

$$-\frac{\frac{3\left(4a^4b^2-5a^2b^4+2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^8-2a^6b^2+a^4b^4\right)\sqrt{a^2-b^2}}+\frac{8a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5ab\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(a^2 - b^2)) + (8*a^3*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^5*tan(1/2*d
*x + 1/2*c)^3 + 4*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*
c) + 7*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*a*b^5*tan(1/2*d*x + 1/2*c) - 4*b^6*
tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + 3*b*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 3*b*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*tan(1/2*d*x + 1
/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

maple [B] time = 0.10, size = 712, normalized size = 3.07

$$-\frac{8b^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^2(a-b)\left(a^2+2ab+b^2\right)}-\frac{b^4\left(\text{ta}\ldots\right.}{d\,a^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\ldots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -8/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2
+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+4/d*b^5/
a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3-8/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d*b^4/a^2/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/
2*d*x+1/2*c)+4/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+12/d/(a^4-2*a^2*b^2+b^4)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*b^2-15/
d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)
*(a-b)/((a-b)*(a+b))^(1/2))+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(
1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/a^3/(tan(1/2*
d*x+1/2*c)-1)+3/d*b/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*c
)+1)-3/d*b/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.45, size = 5347, normalized size = 23.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^3),x)
```

```
[Out] (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 28
8*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*
b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8
*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (3*b*((24*(4*a^17*b - 4*a^8*b
^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26
*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6
+ 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (24*b*tan(c/2 + (d*x
)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^
12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/(a^4*(a^12*
b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*
b^2))))/a^4)*3i)/a^4 + (b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288
*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7
*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a
^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (3*b*((24*(4*a^
17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14
*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b
^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (24*b*
tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^
11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2
))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10
*b^3 - 3*a^11*b^2))))/a^4)*3i)/a^4)/((48*(36*b^12 - 18*a*b^11 - 162*a^2*b^1
0 + 81*a^3*b^9 + 288*a^4*b^8 - 126*a^5*b^7 - 234*a^6*b^6 + 72*a^7*b^5 + 72*
a^8*b^4))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3
*a^13*b^3 - 3*a^14*b^2) - (3*b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11
- 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 28
8*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^
7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (3*b*((24*
(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6
 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 -
a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (
24*b*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 -
```

32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^1
6*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3
*a^10*b^3 - 3*a^11*b^2))))/a^4))/a^4 + (3*b*((8*tan(c/2 + (d*x)/2)*(72*b^12
 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288
*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b +
a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)
 + (3*b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7
- 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^1
5*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*
a^14*b^2) + (24*b*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 3
2*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^1
5*b^3 - 8*a^16*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 +
3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2))))/a^4))/a^4))*6i)/(a^4*d) - ((tan(c/2
 + (d*x)/2)^5*(3*a*b^4 - 2*a^4*b + 2*a^5 - 6*b^5 + 12*a^2*b^3 - 4*a^3*b^2))
/((a^3*b - a^4)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(3*a*b^4 + 2*a^4*b + 2*a^5
 + 6*b^5 - 12*a^2*b^3 - 4*a^3*b^2))/((a + b)*(a^5 - 2*a^4*b + a^3*b^2)) + (
2*tan(c/2 + (d*x)/2)^3*(2*a^6 - 6*b^6 + 13*a^2*b^4 - 6*a^4*b^2))/(a*(a^2*b
- a^3)*(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a*b - a^2 +
3*b^2) - tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (
d*x)/2)^4*(2*a*b + a^2 - 3*b^2))) + (b^2*atan(((b^2*(-(a + b)^5*(a - b)^5)^
(1/2)*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*
b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 -
72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 +
 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (3*b^2*((24*(4*a^17*b - 4*a^8*b^10
+ 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^1
4*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*
a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (12*b^2*tan(c/2 + (d*x)/
2)*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^17*b - 8*a
^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5
 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 -
 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6
+ 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(-(a + b)^5*(a - b)^5)
^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^
8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(4*a^4 + 2*b^4 - 5*a^2*b^2)*3i)/(2*(a^1
4 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) + (b^2*(
-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 2
88*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a
^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 -
 a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (3*b^2*((24*(
4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6
+ 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a
^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (1
2*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^
2*b^2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*
a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 -

a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(4*a^4 + 2*b^4 - 5*a^2*b^2)*3i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))/((48*(36*b^12 - 18*a*b^11 - 162*a^2*b^10 + 81*a^3*b^9 + 288*a^4*b^8 - 126*a^5*b^7 - 234*a^6*b^6 + 72*a^7*b^5 + 72*a^8*b^4))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (3*b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (3*b^2*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (12*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) + (3*b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (3*b^2*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (12*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))))*(-(a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*3i)/(d*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**3, x)
```

3.477 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=305

$$\frac{b^2\left(7a^2-4b^2\right)\tan(c+dx)\sec(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^5d}-\frac{3b\left(\right.}{}$$

```
[Out] -b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*(a^2+12*b^2)*arctanh(sin(d*x+c))/
a^5/d-3/2*b*(2*a^4-7*a^2*b^2+4*b^4)*tan(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(a^4-1
0*a^2*b^2+6*b^4)*sec(d*x+c)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*sec(d*x+c)
*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b^2*(7*a^2-4*b^2)*sec(d*x+
c)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 1.08, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\tan(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(a^2+12b^2\right)\tanh^{-1}}{2a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d)) + ((a^2 + 12*b^2)*ArcTa
nh[Sin[c + d*x]])/(2*a^5*d) - (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Tan[c + d*x]
)/(2*a^4*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^2 + 6*b^4)*Sec[c + d*x]*Tan[c
+ d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2
 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b^2*(7*a^2 - 4*b^2)*Sec[c + d*x]*Tan[c
 + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2-2b^2\right)-2ab\cos(c+dx)+3b^2\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(7a^2-4b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \int \frac{(2(a^4\ldots}{\ldots}$$

$$= \frac{\left(a^4-10a^2b^2+6b^4\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \ldots$$

$$= -\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2\ldots\right.}{\ldots}$$

$$= -\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+\ldots}{2a^5d}$$

Mathematica [A] time = 6.18, size = 427, normalized size = 1.40

$$-\frac{3b\sin\left(\frac{1}{2}(c+dx)\right)}{a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{3b\sin\left(\frac{1}{2}(c+dx)\right)}{a^4d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{b^4\sin(c+dx)}{2a^3d(a-b)(a+b)(a+b\cos\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/(a^5*(a^2 - b^2)^2*Sqrt[-a^2 + b^2]*d) + ((-a^2 - 12*b^2)*Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^5*d) + ((a^2 + 12*b^2)*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]])/(2*a^5*d) + 1/(4*a^3*d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^2) - (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])) - 1/(4*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) -
 (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^
4*Sin[c + d*x])/(2*a^3*(a - b)*(a + b)*d*(a + b*Cos[c + d*x])^2) + (3*(3*a^
2*b^4*Sin[c + d*x] - 2*b^6*Sin[c + d*x]))/(2*a^4*(a - b)^2*(a + b)^2*d*(a +
 b*Cos[c + d*x]))
```

fricas [**B**] time = 6.04, size = 1524, normalized size = 5.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(((20*a^4*b^5 - 29*a^2*b^7 + 12*b^9)*cos(d*x + c)^4 + 2*(20*a^5*b^4 -
 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c)^3 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*
b^7)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^
2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) -
a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^8*b^2 +
 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x + c)^4 + 2*(a^9*b +
 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x + c)^3 + (a^10 + 9
*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*
x + c) + 1) + ((a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*co
s(d*x + c)^4 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*c
os(d*x + c)^3 + (a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*c
os(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a
^4*b^6 - 3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(d*x + c)^3 -
(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c)^2 - 4*(a^9
*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^
2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*
b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^
4 - a^7*b^6)*d*cos(d*x + c)^2), -1/4*(2*((20*a^4*b^5 - 29*a^2*b^7 + 12*b^9)
*cos(d*x + c)^4 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c)^3 + (
20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arcta
n(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((a^8*b^2 + 9*a^6
*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x + c)^4 + 2*(a^9*b + 9*a^7
*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x + c)^3 + (a^10 + 9*a^8*b
^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*x + c)
 + 1) + ((a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x
+ c)^4 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x
 + c)^3 + (a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x
```

```
 + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6
 - 3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(d*x + c)^3 - (11*a^
8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c)^2 - 4*(a^9*b - 3
*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^2 - 3*
a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*b^3 +
3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^
7*b^6)*d*cos(d*x + c)^2)]
```

giac [B] time = 1.52, size = 801, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(20*a^4*b^3 - 29*a^2*b^5 + 12*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^2)) + 2*(a^7*
tan(1/2*d*x + 1/2*c)^7 + 4*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*a^5*b^2*tan(1/
2*d*x + 1/2*c)^7 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*b^4*tan(1/2*d*
x + 1/2*c)^7 - 17*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*a*b^6*tan(1/2*d*x + 1
/2*c)^7 + 12*b^7*tan(1/2*d*x + 1/2*c)^7 + 3*a^7*tan(1/2*d*x + 1/2*c)^5 + 4*
a^6*b*tan(1/2*d*x + 1/2*c)^5 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 26*a^4*b^
3*tan(1/2*d*x + 1/2*c)^5 - 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 67*a^2*b^5*t
an(1/2*d*x + 1/2*c)^5 + 18*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*b^7*tan(1/2*d*
x + 1/2*c)^5 + 3*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*b*tan(1/2*d*x + 1/2*c)^
3 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 -
29*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*
a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*b^7*tan(1/2*d*x + 1/2*c)^3 + a^7*tan(1/2*
d*x + 1/2*c) - 4*a^6*b*tan(1/2*d*x + 1/2*c) - 13*a^5*b^2*tan(1/2*d*x + 1/2*
c) + 2*a^4*b^3*tan(1/2*d*x + 1/2*c) + 33*a^3*b^4*tan(1/2*d*x + 1/2*c) + 17*
a^2*b^5*tan(1/2*d*x + 1/2*c) - 18*a*b^6*tan(1/2*d*x + 1/2*c) - 12*b^7*tan(1
/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b
*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (a^2 + 1
2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - (a^2 + 12*b^2)*log(abs(tan(
1/2*d*x + 1/2*c) - 1))/a^5)/d
```

maple [B] time = 0.14, size = 845, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 10/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*b^
6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*
b+b^2)*tan(1/2*d*x+1/2*c)^3+10/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*b^5/a^3/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*t
an(1/2*d*x+1/2*c)-6/d*b^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-20/d*b^3/a/(a^4-2*a^2*b^2
+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/
2))+29/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x
+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-12/d*b^7/a^5/(a^4-2*a^2*b^2+b^4)/((a-b)*
(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+1/2/d/a^3
/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d/a^4/(tan(1/2
*d*x+1/2*c)-1)*b-1/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)-6/d/a^5*ln(tan(1/2*d*x+
1/2*c)-1)*b^2-1/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3/(tan(1/2*d*x+1/2
*c)+1)+3/d/a^4/(tan(1/2*d*x+1/2*c)+1)*b+1/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)+
6/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.16, size = 5910, normalized size = 19.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(18*a*b^6 - 4*a^6*b + 3*a^7 + 36*b^7 - 67*a^2*b^5 -
29*a^3*b^4 + 26*a^4*b^3 + 5*a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2))
 + (tan(c/2 + (d*x)/2)^5*(18*a*b^6 + 4*a^6*b + 3*a^7 - 36*b^7 + 67*a^2*b^5
- 29*a^3*b^4 - 26*a^4*b^3 + 5*a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2
)) - (tan(c/2 + (d*x)/2)^7*(6*a*b^5 + 5*a^5*b + a^6 - 12*b^6 + 23*a^2*b^4 -
 10*a^3*b^3 - 8*a^4*b^2))/((a^4*b - a^5)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(
6*a*b^5 + 5*a^5*b - a^6 + 12*b^6 - 23*a^2*b^4 - 10*a^3*b^3 + 8*a^4*b^2))/((
```

a + b)*(a^6 - 2*a^5*b + a^4*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) - tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) + tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (atan((((a^2 + 12*b^2)*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2 + 12*b^2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2 + 12*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*1i)/(2*a^5) + ((a^2 + 12*b^2)*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + ((a^2 + 12*b^2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2 + 12*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*1i)/(2*a^5))/((8*(1728*b^15 - 864*a*b^14 - 7344*a^2*b^13 + 3456*a^3*b^12 + 11700*a^4*b^11 - 4770*a^5*b^10 - 7829*a^6*b^9 + 2326*a^7*b^8 + 1314*a^8*b^7 - 11*a^9*b^6 + 411*a^10*b^5 - 20*a^11*b^4 + 20*a^12*b^3))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - ((a^2 + 12*b^2)*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2 + 12*b^2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2 + 12*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5)))/(2*a^5) + ((a^2 + 12*b^2)*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b

^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + ((a^2 + 12*b^2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2 + 12*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5)))/(2*a^5)))*(a^2 + 12*b^2)*1i)/(a^5*d) - (b^3*atan(((b^3*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*b^3*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*1i)/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)) + (b^3*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*b^3*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*1i)/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))/((8*(1728*b^15 - 864*a*b^14 - 7344*a^2*b^13 + 3456*a^3*b^12 + 11700*a^4*b^11 - 4770*a^5*b^10 - 7829*a^6*b^9 + 2326*a^7*b^8 + 1314*a^8*b^7 - 11*a^9*b

```
^6 + 411*a^10*b^5 - 20*a^11*b^4 + 20*a^12*b^3))/(a^18*b + a^19 - a^12*b^7 -
 a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (b^3*((8*t
an(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12
+ 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7
+ 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^1
4*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a
^13*b^2) - (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((4*(4*a^21 - 48*a^10*b^11 + 2
4*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 2
76*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a
^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (
4*b^3*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29
*a^2*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7
- 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^
15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b
+ a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*
b^2)))*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10
*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4
 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*
a^11*b^4 - 5*a^13*b^2)) + (b^3*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 28
8*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*
a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^
4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10
*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (b^3*(-(a + b)^5*(a - b)^5)^
(1/2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b
^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^
3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^1
5*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*b^3*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a
 - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a
^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b
^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6
+ 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5
 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(20*a^4 + 12*b^4 - 29*a^2*b^2))/
(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*
(-(a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5
*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2))))*(-(a + b)^5*(
a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*1i)/(d*(a^15 - a^5*b^10 + 5*
a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**3, x)
```

3.478 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=307

$$-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{a^2\left(4a^2-9b^2\right)\sin(c+dx)\cos^2(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+$$

```
[Out] -4*a*x/b^5+a^2*(8*a^6-28*a^4*b^2+35*a^2*b^4-20*b^6)*arctan((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2)/d+1/6*(12*a^4-23*a^
2*b^2+6*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d-1/3*a^2*cos(d*x+c)^3*sin(d*x+c)/b
/(a^2-b^2)/d/(a+b*cos(d*x+c))^3-1/6*a^2*(4*a^2-9*b^2)*cos(d*x+c)^2*sin(d*x+
c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/2*a^3*(4*a^4-11*a^2*b^2+12*b^4)*s
in(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.90, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3047, 3031, 3023, 2735, 2659, 205}

$$\frac{\left(-23a^2b^2+12a^4+6b^4\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-28a^4b^2+35a^2b^4+8a^6-20b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(4a^2-9b^2\right.}{6b^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (-4*a*x)/b^5 + (a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*ArcTan[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d)
 + ((12*a^4 - 23*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - (
a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3)
 - (a^2*(4*a^2 - 9*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d
*(a + b*Cos[c + d*x])^2) + (a^3*(4*a^4 - 11*a^2*b^2 + 12*b^4)*Sin[c + d*x])
/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(3a^2-3ab\cos(c+dx)-\left(4a^2-3b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\cos^2(c+dx)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int}{}$$

$$= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\cos^2(c+dx)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a^3}{2}$$

$$= \frac{\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(4a^2-\right.}{6b^2\left(a^2\right.}$$

$$= -\frac{4ax}{b^5} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} -$$

$$= -\frac{4ax}{b^5} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} -$$

$$= -\frac{4ax}{b^5} + \frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d} + \frac{\left(12a^4-23a\right.}{6b}$$

Mathematica [A] time = 5.78, size = 240, normalized size = 0.78

$$\frac{\frac{2a^5b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^3} + \frac{5a^4b\left(3b^2-2a^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + \frac{6a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} + \frac{a^3b\left(26a^4-71a^2b^2\right.}{(a-b)^3(a+b)^3(a}}{6b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (-24*a*(c + d*x) + (6*a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*ArcTan
h[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + 6*b*Si
n[c + d*x] + (2*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3
```

```
) + (5*a^4*b*(-2*a^2 + 3*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos
[c + d*x])^2) + (a^3*b*(26*a^4 - 71*a^2*b^2 + 60*b^4)*Sin[c + d*x])/((a - b
)^3*(a + b)^3*(a + b*Cos[c + d*x])))/(6*b^5*d)
```

fricas [B] time = 1.27, size = 1593, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(48*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d
*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d
*x*cos(d*x + c)^2 + 144*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b
^9)*d*x*cos(d*x + c) + 48*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*
b^8)*d*x + 3*(8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6 + (8*a^8*b^3 -
28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c)^3 + 3*(8*a^9*b^2 - 28*a^
7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + 3*(8*a^10*b - 28*a^8*b^3
+ 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*
x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
+ b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 a^2)) - 2*(24*a^11*b - 92*a^9*b^3 + 133*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9 +
 6*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)^3 + (
44*a^9*b^3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x +
 c)^2 + 3*(20*a^10*b^2 - 77*a^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10
)*cos(d*x + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b
^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*
b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*
a^4*b^12 + a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4
*a^5*b^11 + a^3*b^13)*d), -1/6*(24*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3
*b^9 + a*b^11)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 -
4*a^4*b^8 + a^2*b^10)*d*x*cos(d*x + c)^2 + 72*(a^11*b - 4*a^9*b^3 + 6*a^7*b
^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d*x + c) + 24*(a^12 - 4*a^10*b^2 + 6*a^8*
b^4 - 4*a^6*b^6 + a^4*b^8)*d*x - 3*(8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a
^5*b^6 + (8*a^8*b^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c)^3
+ 3*(8*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + 3*(
8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c))*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (24*a^11
*b - 92*a^9*b^3 + 133*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9 + 6*(a^8*b^4 - 4*a^6
*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)^3 + (44*a^9*b^3 - 169*a^
7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c)^2 + 3*(20*a^10*
b^2 - 77*a^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c))*sin
(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d
*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*co
s(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14
```

```
)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^1
3)*d)]
```

giac [A] time = 1.32, size = 563, normalized size = 1.83

$$-\frac{\frac{3\left(8a^{8}-28a^{6}b^{2}+35a^{4}b^{4}-20a^{2}b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}b^{5}-3a^{4}b^{7}+3a^{2}b^{9}-b^{11}\right)\sqrt{a^{2}-b^{2}}}-\frac{18a^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-42a^{8}b\tan\left(\frac{1}{2}dx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*
sqrt(a^2 - b^2)) - (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*x +
1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x + 1/
2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x + 1/2*
c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*a^9*tan(1/2*d*x + 1/2*c)^3 -
152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 236*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 1
20*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) + 42*a^8*b*
tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*b^3*tan(1/
2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*tan(1/2*d*x
+ 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b
^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3)
 + 12*(d*x + c)*a/b^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1
)*b^4))/d
```

maple [B] time = 0.08, size = 1396, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5-18/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+5/d*a^4/b/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b
^3)*tan(1/2*d*x+1/2*c)^5+20/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
```

```
)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+12/d*a^7/
b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-116/3/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3+40/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^
2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+6/d*a^7/b^4/(a*tan(1/2*d*
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
n(1/2*d*x+1/2*c)+2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-18/d*a^5/b^2/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c)-5/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+20/d*a^3
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+8/d*a^8/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((
a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-28/d
*a^6/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+35/d*a^4/b/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
-20/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/b^4*tan(1/2*d*x+1/2*c)/(1+tan(1
/2*d*x+1/2*c)^2)-8/d/b^5*a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.90, size = 7494, normalized size = 24.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^4,x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^3*(12*a^7*b - 72*a^8 - 18*b^8 + 72*a^2*b^6 + 60*a^3*
b^5 - 273*a^4*b^4 - 47*a^5*b^3 + 236*a^6*b^2))/(3*b^4*(a + b)^2*(a - b)^3)
- (tan(c/2 + (d*x)/2)^5*(12*a^7*b + 72*a^8 + 18*b^8 - 72*a^2*b^6 + 60*a^3*b
^5 + 273*a^4*b^4 - 47*a^5*b^3 - 236*a^6*b^2))/(3*b^4*(a + b)^3*(a - b)^2) +
```

(tan(c/2 + (d*x)/2)*(2*a*b^6 - 4*a^6*b - 8*a^7 + 2*b^7 - 6*a^2*b^5 - 26*a^
3*b^4 + 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)*(a - b)^3) + (tan(c/2 + (d*x
)/2)^7*(2*a*b^6 + 4*a^6*b - 8*a^7 - 2*b^7 + 6*a^2*b^5 - 26*a^3*b^4 - 11*a^4
*b^3 + 24*a^5*b^2))/(b^4*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/
2 + (d*x)/2)^4*(6*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4*a^3 -
2*b^3) + tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3 + tan(c
/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (8*a*atan(((4*a*((8*tan
(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^
4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a
^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768
*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5
*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^
11*b^8) + (a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*
a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^
10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22
 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17
 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (a*tan(
c/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5
*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^1
0*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10)*32i)/(b^5
*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*
a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*4
i)/b^5))/b^5 + (4*a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*
b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*
b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^
12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*
b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 +
 5*a^9*b^10 - a^10*b^9 - a^11*b^8) - (a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a
^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*
b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b
^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19
+ 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*
b^13 - a^11*b^12) + (a*tan(c/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b
^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^
16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^
11 - 8*a^14*b^10)*32i)/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a
^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10
 - a^10*b^9 - a^11*b^8)))*4i)/b^5))/b^5)/((32*(128*a^16 - 64*a^15*b + 320*a
^4*b^12 + 480*a^5*b^11 - 1520*a^6*b^10 - 1280*a^7*b^9 + 3088*a^8*b^8 + 1602
*a^9*b^7 - 3472*a^10*b^6 - 1088*a^11*b^5 + 2288*a^12*b^4 + 400*a^13*b^3 - 8
32*a^14*b^2))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a
^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 -
 a^11*b^12) - (a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^1
4 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9
 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*

b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^1
6 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*
a^9*b^10 - a^10*b^9 - a^11*b^8) + (a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*
b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^1
6 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11
 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 1
0*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^1
3 - a^11*b^12) - (a*tan(c/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21
 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16
+ 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11
- 8*a^14*b^10)*32i)/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*
b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 -
a^10*b^9 - a^11*b^8)))*4i)/b^5)*4i)/b^5 + (a*((8*tan(c/2 + (d*x)/2)*(128*a^
16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 -
 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6
- 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b
^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 1
0*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) - (a*((16*(8*a*
b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18
 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13
+ 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 -
5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*
b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) + (a*tan(c/2 + (d*x)/2)*(8*a*b^2
3 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 -
160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 +
 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10)*32i)/(b^5*(a*b^18 + b^19 - 5*a^2
*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12
+ 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*4i)/b^5)*4i)/b^5)))/(b^5
*d) - (a^2*atan(((a^2*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^
2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^
7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*
a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^
3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11
 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) + (a^2*((16*(8*a*b^23 - 20*a^2*b^22 -
36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*
a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^
13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b
^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a
^10*b^13 - a^11*b^12) - (4*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1
/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*(8*a*b^23 - 8*a^2*b^22 - 48*
a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a
^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^
13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 3
5*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2*b^
17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5

*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*(-(a + b)^7*(a - b)^7)^(1/2
)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7*a^2*b^17 + 21*a^
4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))
(-(a + b)^7(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*1i
)/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10
*b^9 + 7*a^12*b^7 - a^14*b^5)) + (a^2*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 12
8*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^
6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*
a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5
*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b
^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) - (a^2*((16*(8*a*b^23
- 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87
*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*
a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3
*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15
+ 5*a^9*b^14 - a^10*b^13 - a^11*b^12) + (4*a^2*tan(c/2 + (d*x)/2)*(-(a + b)
^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*(8*a*b^23 -
8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*
a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*
a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 -
35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 +
b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 -
10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*(-(a + b)^7
(a - b)^7)^(1/2)(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7*
a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b
^7 - a^14*b^5)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4
- 28*a^4*b^2)*1i)/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^
8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))/((32*(128*a^16 - 64*a^15*b
+ 320*a^4*b^12 + 480*a^5*b^11 - 1520*a^6*b^10 - 1280*a^7*b^9 + 3088*a^8*b^8
+ 1602*a^9*b^7 - 3472*a^10*b^6 - 1088*a^11*b^5 + 2288*a^12*b^4 + 400*a^13*
b^3 - 832*a^14*b^2))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19
+ 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10
*b^13 - a^11*b^12) - (a^2*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 6
4*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 192
0*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1
920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 -
5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*
b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) + (a^2*((16*(8*a*b^23 - 20*a^2*b^2
2 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 +
217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 +
4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a
^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14
- a^10*b^13 - a^11*b^12) - (4*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7
)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*(8*a*b^23 - 8*a^2*b^22 -
48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 1

```
60*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 +
8*a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13
 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^
2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12
 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*(-(a + b)^7*(a - b)^7)^
(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7*a^2*b^17 + 2
1*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^
5)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2
))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^1
0*b^9 + 7*a^12*b^7 - a^14*b^5)) + (a^2*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 1
28*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a
^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920
*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 -
5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*
b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) - (a^2*((16*(8*a*b^23
 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 8
7*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52
*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^
3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15
 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) + (4*a^2*tan(c/2 + (d*x)/2)*(-(a + b
)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*(8*a*b^23 -
 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160
*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48
*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15
- 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18
+ b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13
- 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*(-(a + b)^
7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7
*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*
b^7 - a^14*b^5)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4
 - 28*a^4*b^2))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*
b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5))))*(-(a + b)^7*(a - b)^7)^(1/2)
*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*1i)/(d*(b^19 - 7*a^2*b^17 + 21*
a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.479 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=250

$$-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\sin(c+dx)}{6b^3d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}$$

```
[Out] x/b^4-a*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)/d-1/3*a^2*cos(d*x+c)^2*sin(d*
x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a^3*(3*a^2-8*b^2)*sin(d*x+c)/b^3/
(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2-1/6*a^2*(9*a^4-28*a^2*b^2+34*b^4)*sin(d*x+
c)/b^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.57, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3031, 3021, 2735, 2659, 205}

$$-\frac{a\left(-7a^4b^2+8a^2b^4+2a^6-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a^3\left(3a^2-8b\right.}{6b^3d\left(a^2-b^2\right)^2(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] x/b^4 - (a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - (a^2*Cos[
c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a^3*
(3*a^2 - 8*b^2)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2
) - (a^2*(9*a^4 - 28*a^2*b^2 + 34*b^4)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^3*d
*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(2a^2-3ab\cos(c+dx)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}\,d\cdots}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{\int \frac{2a^2b\cdots}{\cdots}}{\cdots}$$

$$= -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{a^2\left(9a\cdots\right.}{6b^3\left(\cdots\right.}$$

$$= \frac{x}{b^4} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{a^2\cdots}{6\cdots}$$

$$= \frac{x}{b^4} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{a^2\cdots}{6\cdots}$$

$$= \frac{x}{b^4} - \frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d} - \frac{a^2\cos^2(c+dx)\,\mathrm{s}\cdots}{3b\left(a^2-b^2\right)d(a+b\cdots}$$

Mathematica [A] time = 2.73, size = 227, normalized size = 0.91

$$\frac{-\frac{2a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^3} + \frac{a^2b\left(-11a^4+32a^2b^2-36b^4\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))} + \frac{a^3b\left(7a^2-12b^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} - \frac{6a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\tanh^{-1}\left(\frac{(a-\cdots}{\cdots}\right.}{\left(b^2-a^2\right)^{7/2}}\cdots}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (6*(c + d*x) - (6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((a - b
)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - (2*a^4*b*Sin[c
+ d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + (a^3*b*(7*a^2 - 12*b^2)*
Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + (a^2*b*(-11*a^
4 + 32*a^2*b^2 - 36*b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Cos[c +
d*x])))/(6*b^4*d)
```

fricas [B] time = 1.29, size = 1445, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*x*cos(d*x
+ c)^3 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*cos(
d*x + c)^2 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*
cos(d*x + c) + 12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x
- 3*(2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6 + (2*a^7*b^3 - 7*a^5*b^5 +
8*a^3*b^7 - 8*a*b^9)*cos(d*x + c)^3 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^4*b^6
- 8*a^2*b^8)*cos(d*x + c)^2 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5*b^5 - 8*a^3*b^
7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*c
os(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
+ 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*a^10*b - 2
3*a^8*b^3 + 43*a^6*b^5 - 26*a^4*b^7 + (11*a^8*b^3 - 43*a^6*b^5 + 68*a^4*b^7
 - 36*a^2*b^9)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3
*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^
2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a
^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 -
4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 -
 4*a^5*b^10 + a^3*b^12)*d), 1/6*(6*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2
*b^9 + b^11)*d*x*cos(d*x + c)^3 + 18*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*d*x*cos(d*x + c)^2 + 18*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c) + 6*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4
*a^5*b^6 + a^3*b^8)*d*x - 3*(2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6 + (
2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9)*cos(d*x + c)^3 + 3*(2*a^8*b^2
- 7*a^6*b^4 + 8*a^4*b^6 - 8*a^2*b^8)*cos(d*x + c)^2 + 3*(2*a^9*b - 7*a^7*b^
3 + 8*a^5*b^5 - 8*a^3*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
 + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^10*b - 23*a^8*b^3 + 43*a^
6*b^5 - 26*a^4*b^7 + (11*a^8*b^3 - 43*a^6*b^5 + 68*a^4*b^7 - 36*a^2*b^9)*co
s(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*cos(d*x + c
))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*
cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)
*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*
b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3
*b^12)*d)]
```

giac [B] time = 2.11, size = 531, normalized size = 2.12

$$\frac{3\left(2a^7-7a^5b^2+8a^3b^4-8ab^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6b^4-3a^4b^6+3a^2b^8-b^{10}\right)\sqrt{a^2-b^2}}-\frac{6a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15a^7b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a
^2 - b^2)) - (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^
5 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 -
6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a
^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 12*a^8*tan(1/2*d*x + 1/2*c)^3 - 56*a^6*b^2*
tan(1/2*d*x + 1/2*c)^3 + 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 72*a^2*b^6*ta
n(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1/2*d*x +
1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x + 1/2*c) -
 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c) + 36*a^2*
b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*(d*x + c)/b^4)
/d
```

maple [B] time = 0.08, size = 1356, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+1/d*a^5/b^2/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-4/d*a^3/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5-12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-4/d*a^6/b^3
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2
+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+44/3/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)
```

```
^3-24/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*
a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-2/d*a^6/b^3/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/
2*d*x+1/2*c)-1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+6/d*a^4/b/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)
*tan(1/2*d*x+1/2*c)+4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-12/d*a^2*b/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)-2/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+7/d*a^5/b^2
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))-8/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(
a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+8/d*a*b^2/
(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)
*(a-b)/((a-b)*(a+b))^(1/2))+2/d/b^4*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.37, size = 7247, normalized size = 28.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (2*atan((((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^1
7 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 +
 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5
*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b
^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (tan(c/2 + (d*x)/2)
*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^
6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^
11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 -
 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7
```

*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4 + (8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))/b^4 - ((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4 - (8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))/b^4)/((((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4 + (8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))*1i)/b^4 + (((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4 - (8*t

an(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48
*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8
*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b
^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 1
0*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))*1i)/b^4 + (16*(1
6*a*b^12 - 2*a^12*b + 4*a^13 + 48*a^2*b^11 - 64*a^3*b^10 - 64*a^4*b^9 + 110
*a^5*b^8 + 66*a^6*b^7 - 110*a^7*b^6 - 34*a^8*b^5 + 70*a^9*b^4 + 11*a^10*b^3
 - 26*a^11*b^2))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 1
0*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^1
0 - a^11*b^9))))/(b^4*d) - ((tan(c/2 + (d*x)/2)^5*(2*a^6 - a^5*b + 12*a^2*b
^4 + 4*a^3*b^3 - 6*a^4*b^2))/((a*b^3 - b^4)*(a + b)^3) + (4*tan(c/2 + (d*x)
/2)^3*(3*a^6 + 18*a^2*b^4 - 11*a^4*b^2))/(3*(a + b)^2*(b^5 - 2*a*b^4 + a^2*
b^3)) + (tan(c/2 + (d*x)/2)*(a^5*b + 2*a^6 + 12*a^2*b^4 - 4*a^3*b^3 - 6*a^4
b^2))/((a + b)(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)))/(d*(3*a*b^2 - tan(c
/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*
(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)
/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (a*atan(((a*((8*tan(c/2 + (d*x)/2
)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a
^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b
^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15
 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a
^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (a*((8*(16*a*b^20 - 4*b^21 + 12
*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^
7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b
^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 +
10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^
10 - a^11*b^9) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^
6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 4
8*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 12
0*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a
^14*b^8))/((b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 2
1*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^1
4 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a
^9*b^8 - a^10*b^7 - a^11*b^6)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6
 + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^1
2 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*(-(a + b)^7*(a - b
)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*1i)/(2*(b^18 - 7*a^2*b^1
6 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^
14*b^4)) + (a*((8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14
 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 16
0*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^1
2*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^1
2 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6
) - (a*((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 +
110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*

a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/((b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*1i)/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))/((16*(16*a*b^12 - 2*a^12*b + 4*a^13 + 48*a^2*b^11 - 64*a^3*b^10 - 64*a^4*b^9 + 110*a^5*b^8 + 66*a^6*b^7 - 110*a^7*b^6 - 34*a^8*b^5 + 70*a^9*b^4 + 11*a^10*b^3 - 26*a^11*b^2))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (a*((8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (a*((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/((b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)) - (a*((8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*

```
b^7 - a^11*b^6) - (a*((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 -
20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*
a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19
+ b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14
- 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (4*a*tan(
c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*
a^4*b^2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17
- 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12
 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/((b^18 - 7*a^2*b
^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 -
a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b
^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b
^6)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2))
/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*
b^8 + 7*a^12*b^6 - a^14*b^4)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6
+ 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12
 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4))))*(-(a + b)^7*(a - b
)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*1i)/(d*(b^18 - 7*a^2*b^1
6 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^
14*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.480 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=222

$$-\frac{b\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

```
[Out] -b*(3*a^2+2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(
7/2)/(a+b)^(7/2)/d-1/3*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x
+c))^3-1/6*a^2*(2*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^
2+1/6*a*(2*a^4-5*a^2*b^2+18*b^4)*sin(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*cos(d*x+
c))
```

Rubi [A] time = 0.34, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3021, 2754, 12, 2659, 205}

$$-\frac{b\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b*(3*a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(7/2)*(a + b)^(7/2)*d)) - (a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^3) - (a^2*(2*a^2 - 7*b^2)*Sin[c + d*x])/(6*b^
2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4)
*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{a^2-3ab\cos(c+dx)-\left(2a^2-3b^2\right)\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{2ab(}{}}{}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^4\right.}{6b^2\left(a\right.}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^4\right.}{6b^2\left(a\right.}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^4\right.}{6b^2\left(a\right.}$$

$$= -\frac{b\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{}{6b^2}$$

Mathematica [A] time = 1.23, size = 158, normalized size = 0.71

$$\frac{\frac{a\sin(c+dx)\left(4a^4+3ab\left(a^2+9b^2\right)\cos(c+dx)+11a^2b^2+\left(2a^4-5a^2b^2+18b^4\right)\cos^2(c+dx)\right)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3} - \frac{6b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-6*b*(3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(-a^2 + b^2)^(7/2) + (a*(4*a^4 + 11*a^2*b^2 + 3*a*b*(a^2 + 9*b^2)*Cos[c +
 d*x] + (2*a^4 - 5*a^2*b^2 + 18*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)
^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

fricas [A] time = 1.00, size = 893, normalized size = 4.02

$$\left[\frac{3\left(3a^5b+2a^3b^3+\left(3a^2b^4+2b^6\right)\cos(dx+c)^3+3\left(3a^3b^3+2ab^5\right)\cos(dx+c)^2+3\left(3a^4b^2+2a^2b^4\right)\cos(dx+\right.}{12\left(\left(a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11}\right)d\cos(dx+c)^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*a^5*b + 2*a^3*b^3 + (3*a^2*b^4 + 2*b^6)*cos(d*x + c)^3 + 3*(3*a
^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c))*
sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2
*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos
(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(4*a^7 + 7*a^5*b^2 - 11*a^3*b^
4 + (2*a^7 - 7*a^5*b^2 + 23*a^3*b^4 - 18*a*b^6)*cos(d*x + c)^2 + 3*(a^6*b +
 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 +
 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 +
6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*(3*a^5*b + 2*a^3*b^3 + (3*a^2*b^4 +
 2*b^6)*cos(d*x + c)^3 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 + 3*(3*a^4*
b^2 + 2*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)
/(sqrt(a^2 - b^2)*sin(d*x + c))) - (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4 + (2*a^7
 - 7*a^5*b^2 + 23*a^3*b^4 - 18*a*b^6)*cos(d*x + c)^2 + 3*(a^6*b + 8*a^4*b^3
 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7
 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6
- 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5
- 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a
^5*b^6 + a^3*b^8)*d)]
```

giac [A] time = 1.88, size = 399, normalized size = 1.80

$$\frac{\frac{3\left(3a^2b+2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{6a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^3b^2\tan\left(\frac{1}{2}dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(3*a^2*b + 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)
```

```
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (6*a^5*tan(1/2*d
*x + 1/2*c)^5 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b^2*tan(1/2*d*x + 1/
2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^
5 + 4*a^5*tan(1/2*d*x + 1/2*c)^3 + 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 36*a
*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*b*tan(1/2*
d*x + 1/2*c) + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*a^2*b^3*tan(1/2*d*x + 1/
2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

maple [B] time = 0.06, size = 776, normalized size = 3.50

$$\frac{2a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^3(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}+\frac{}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2+4/3/d*a^3/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3+12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*
a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2+2/d*a^3/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c)-3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+6/d/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)*b^2-3/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(
a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-2/d*b^3/(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(
a-b)/((a-b)*(a+b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.21, size = 378, normalized size = 1.70

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^3+9\,a\,b^2\right)}{3\,(a+b)^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^3+3\,a^2\,b+6\,a\,b^2\right)}{(a+b)^3\,(a-b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^3-3\,a^2\right.}{(a+b)\left(a^3-3\,a^2\,b+3\right.}}{d\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)+3\,a^2\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^4,x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^3*(9*a*b^2 + a^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2))
 + (tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 3*a^2*b + 2*a^3))/((a + b)^3*(a - b)) +
 (tan(c/2 + (d*x)/2)*(6*a*b^2 - 3*a^2*b + 2*a^3))/((a + b)*(3*a*b^2 - 3*a^2
*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3
*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) +
3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))
) - (b*atan((b*tan(c/2 + (d*x)/2)*(3*a^2 + 2*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3))/(2*(3*a^2*b + 2*b^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(3*a
^2 + 2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.481 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=206

$$\frac{a\left(a^2+4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\cdots$$

```
[Out] a*(a^2+4*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2
)/(a+b)^(7/2)/d-1/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a*(
a^2-6*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(a^4-10*a^2*b^
2-6*b^4)*sin(d*x+c)/b/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2790, 2754, 12, 2659, 205}

$$\frac{a\left(a^2+4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (a*(a^2 + 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Co
s[c + d*x])^3) + (a*(a^2 - 6*b^2)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b
*Cos[c + d*x])^2) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Sin[c + d*x])/(6*b*(a^2 - b
^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{3ab+\left(a^2-3b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2b(2\cdots}{\cdots}}{\cdots}$$

$$= -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4-\cdots\right.}{6b\left(a^2\cdots\right.}$$

$$= -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4-\cdots\right.}{6b\left(a^2\cdots\right.}$$

$$= -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4-\cdots\right.}{6b\left(a^2\cdots\right.}$$

$$= \frac{a\left(a^2+4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\cdots}{6b\left(a^2\cdots\right.}$$

Mathematica [A] time = 1.20, size = 162, normalized size = 0.79

$$\frac{\frac{6a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} + \frac{\sin(c+dx)\left(-13a^4b-2a^2b^3+b\left(a^4-10a^2b^2-6b^4\right)\cos^2(c+dx)+3a\left(a^4-9a^2b^2-2b^4\right)\cos(c+dx)\right)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((6*a*(a^2 + 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(
-a^2 + b^2)^(7/2) + ((-13*a^4*b - 2*a^2*b^3 + 3*a*(a^4 - 9*a^2*b^2 - 2*b^4)
*Cos[c + d*x] + b*(a^4 - 10*a^2*b^2 - 6*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/
((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.92, size = 893, normalized size = 4.33

$$\left[\frac{3\left(a^{6}+4\,a^{4}b^{2}+\left(a^{3}b^{3}+4\,ab^{5}\right)\cos\left(dx+c\right)^{3}+3\left(a^{4}b^{2}+4\,a^{2}b^{4}\right)\cos\left(dx+c\right)^{2}+3\left(a^{5}b+4\,a^{3}b^{3}\right)\cos\left(dx+c\right)\right)\sqrt{-}}{12\left(\left(a^{8}b^{3}-4\,a^{6}b^{5}+6\,a^{4}b^{7}-4\,a^{2}b^{9}+b^{11}\right)d\cos\left(dx+c\right)^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^6 + 4*a^4*b^2 + (a^3*b^3 + 4*a*b^5)*cos(d*x + c)^3 + 3*(a^4*b^2
 + 4*a^2*b^4)*cos(d*x + c)^2 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c))*sqrt(-a^
2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5 - (
a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7)*cos(d*x + c)^2 - 3*(a^7 - 10*a^5*b^
2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5
+ 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 +
 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^
7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(a^6 + 4*a^4*b^2 + (a^3*b^3 + 4*a*b
^5)*cos(d*x + c)^3 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^2 + 3*(a^5*b + 4*
a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a
^2 - b^2)*sin(d*x + c))) - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5 - (a^6*b - 11
*a^4*b^3 + 4*a^2*b^5 + 6*b^7)*cos(d*x + c)^2 - 3*(a^7 - 10*a^5*b^2 + 7*a^3*
b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^
7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6
 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5
 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*
a^5*b^6 + a^3*b^8)*d)]
```

giac [B] time = 1.12, size = 427, normalized size = 2.07

$$-\frac{\frac{3\left(a^{3}+4\,ab^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}+\frac{3\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-27\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2))
```

```
)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (3*a^5*tan(1/2*d*
x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1
/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^
5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 + 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 16*a^2
*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*tan(1/2
*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2*c) + 6*b
^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

maple [B] time = 0.05, size = 930, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -1/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(
a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2-2/d/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5*b^3-28/3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-4/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3+1/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-6/d*a^2*b/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2-2/d/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c)*b^3+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)
*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+4/d*a*b^
2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.15, size = 381, normalized size = 1.85

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2+4\,b^2\right)(2\,a-2\,b)\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,(a-b)^{7/2}\left(a^3+4\,a\,b^2\right)}\right)\left(a^2+4\,b^2\right)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}-\frac{\tan\left(\frac{c}{2}+\cdots\right.}{d\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b
+ a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a*b^2 + a^3)))*(a^2 + 4*b^2
))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((tan(c/2 + (d*x)/2)^5*(2*a*b^2 + 6*a^
2*b + a^3 + 2*b^3))/((a + b)^3*(a - b)) + (4*tan(c/2 + (d*x)/2)^3*(7*a^2*b
+ 3*b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) - (tan(c/2 + (d*x)/2)*(2*a*b^2
- 6*a^2*b + a^3 - 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*
a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2
+ (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + ta
n(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.482 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=192

$$-\frac{b\left(4a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\ }{3a\ldots}$$

```
[Out] -b*(4*a^2+b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/
2)/(a+b)^(7/2)/d+1/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*(2*a^2
+3*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*a*(2*a^2+13*b^2)*si
n(d*x+c)/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2754, 12, 2659, 205}

$$-\frac{b\left(4a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\ }{3a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b*(4*a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(7/2)*(a + b)^(7/2)*d)) + (a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos
[c + d*x])^3) + ((2*a^2 + 3*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Co
s[c + d*x])^2) + (a*(2*a^2 + 13*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{3b-2a\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{-10ab+\left(2a^2\cdots\right.}{(a+b\,\mathrm{co}\cdots}}{6\left(a^{\cdots}\right.}$$

$$= \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2+\cdots\right.}{6\left(a^2-b^2\right)^3\cdots}$$

$$= \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2+\cdots\right.}{6\left(a^2-b^2\right)^3\cdots}$$

$$= \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2+\cdots\right.}{6\left(a^2-b^2\right)^3\cdots}$$

$$= -\frac{b\left(4a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{(2\cdots}{6\left(a^2-\cdots\right.}$$

Mathematica [A] time = 1.06, size = 164, normalized size = 0.85

$$\frac{\frac{\sin(c+dx)\left(6a^5+10a^3b^2+ab^2\left(2a^2+13b^2\right)\cos^2(c+dx)-3b\left(-2a^4-9a^2b^2+b^4\right)\cos(c+dx)-ab^4\right)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}-\frac{6b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-6*b*(4*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(7/2) + ((6*a^5 + 10*a^3*b^2 - a*b^4 - 3*b*(-2*a^4 - 9*a^2*b^2
 + b^4)*Cos[c + d*x] + a*b^2*(2*a^2 + 13*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])
/((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.70, size = 891, normalized size = 4.64

$$\left[\frac{3\left(4a^5b+a^3b^3+\left(4a^2b^4+b^6\right)\cos\left(dx+c\right)^3+3\left(4a^3b^3+ab^5\right)\cos\left(dx+c\right)^2+3\left(4a^4b^2+a^2b^4\right)\cos\left(dx+c\right)\right)\sqrt{\cdots}}{12\left(\left(a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11}\right)d\cos\left(dx+c\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(4*a^5*b + a^3*b^3 + (4*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(4*a^3*b
^3 + a*b^5)*cos(d*x + c)^2 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sqrt(-a^
2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^
6 + (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4
*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5
+ 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 +
 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^
7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*(4*a^5*b + a^3*b^3 + (4*a^2*b^4 +
b^6)*cos(d*x + c)^3 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c)^2 + 3*(4*a^4*b^2 +
 a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(
a^2 - b^2)*sin(d*x + c))) - (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6 + (2*a^
5*b^2 + 11*a^3*b^4 - 13*a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4*b^3 - 10
*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b
^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^
6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^
```

```
5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4
*a^5*b^6 + a^3*b^8)*d)]
```

giac [B] time = 0.95, size = 427, normalized size = 2.22

$$\frac{3\left(4\,a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (6*a^5*tan(1/2*d*x
 + 1/2*c)^5 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x + 1/2
*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^4*tan(1/2*d*x + 1/2*c)^5
 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*a^5*tan(1/2*d*x + 1/2*c)^3 + 16*a^3*b^
2*tan(1/2*d*x + 1/2*c)^3 - 28*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*
d*x + 1/2*c) + 6*a^4*b*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1/2*
c) + 27*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a*b^4*tan(1/2*d*x + 1/2*c) - 3*b^
5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x
 + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

maple [B] time = 0.05, size = 931, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2+1/d/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*b^3+4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+28/3/d/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*b^2+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-2/d*a^2*b/(a*
```

```
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c)+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2-1/d/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c)*b^3-4/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d*b^3
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.25, size = 382, normalized size = 1.99

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^3+2\,a^2\,b+6\,a\,b^2+b^3\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}+\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^3+7\,a\,b^2\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3-\dots\right.}{\left(a+b\right)\,\left(a^3-3\,a^2\dots\right.}}{d\,\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)+3\,a^2\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 2*a^2*b + 2*a^3 + b^3))/((a + b)^3*(a - b
)) + (4*tan(c/2 + (d*x)/2)^3*(7*a*b^2 + 3*a^3))/(3*(a + b)^2*(a^2 - 2*a*b +
 b^2)) + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^2*b + 2*a^3 - b^3))/((a + b)*(3
*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2
 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a
^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
 + a^3 - b^3))) - (b*atan((b*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a - 2*b)*(
3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a^2*b + b
^3)))*(4*a^2 + b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.483 $\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$\frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-\frac{5ab\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{}{3d}$$

```
[Out] a*(2*a^2+3*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7
/2)/(a+b)^(7/2)/d-1/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^3-5/6*a*b*s
in(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*sin(d*x+c)/
(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 205}

$$\frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-\frac{5ab\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(7/2)*(a + b)^(7/2)*d) - (b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^3) - (5*a*b*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])
^2) - (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Cos[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx = -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{-3a+2b\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{2(3a^2+2b}{(a+}}{6}$$

$$= -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{b\left(11a\right.}{6\left(a^2-b^2\right.}$$

$$= -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{b\left(11a\right.}{6\left(a^2-b^2\right.}$$

$$= -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{b\left(11a\right.}{6\left(a^2-b^2\right.}$$

$$= \frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{}{6\left(a^2-\right.}$$

Mathematica [A] time = 0.92, size = 159, normalized size = 0.86

$$\frac{\frac{6a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - \frac{b\sin(c+dx)\left(18a^4+b^2\left(11a^2+4b^2\right)\cos^2(c+dx)+3ab\left(9a^2+b^2\right)\cos(c+dx)-5a^2b^2+2b^4\right)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])
/(-a^2 + b^2)^(7/2) - (b*(18*a^4 - 5*a^2*b^2 + 2*b^4 + 3*a*b*(9*a^2 + b^2)*
Cos[c + d*x] + b^2*(11*a^2 + 4*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^
3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

fricas [B] time = 1.02, size = 895, normalized size = 4.86

$$\left[\frac{3\left(2a^6+3a^4b^2+\left(2a^3b^3+3ab^5\right)\cos\left(dx+c\right)^3+3\left(2a^4b^2+3a^2b^4\right)\cos\left(dx+c\right)^2+3\left(2a^5b+3a^3b^3\right)\cos\left(dx+\right.\right.}{12\left(\left(a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11}\right)d\cos\left(dx+c\right)^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*a^6 + 3*a^4*b^2 + (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + 3*(2*a
^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*
sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2
*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos
(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(18*a^6*b - 23*a^4*b^3 + 7*a^2
*b^5 - 2*b^7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b
^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 +
 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 +
6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(2*a^6 + 3*a^4*b^2 + (2*a^3*b^3 + 3*
a*b^5)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + 3*(2*a^5
*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
(sqrt(a^2 - b^2)*sin(d*x + c))) - (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^
7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b^2 - 8*a^3*
b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7
- 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^
5*b^6 + a^3*b^8)*d)]
```

giac [B] time = 0.75, size = 399, normalized size = 2.17

$$-\frac{\frac{3\left(2a^3+3ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{18a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-27a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^2b^3\tan\left(\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2
```

```
)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (18*a^4*b*tan(1
/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^3*tan(1/2*d
*x + 1/2*c)^5 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*b^5*tan(1/2*d*x + 1/2*c)
^5 + 36*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 32*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 -
4*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*t
an(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x
+ 1/2*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

maple [B] time = 0.05, size = 776, normalized size = 4.22

$$-\frac{6a^2b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^3(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}-\frac{}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2
*c)^5*b^2-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(
a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3-12/d*a^2*b/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3-4/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)
^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*a^2*b/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c)+3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2-2/d/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)*b^3+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+3/d*a*b^2/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*
(a-b)/((a-b)*(a+b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.12, size = 378, normalized size = 2.05

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^2+3\,b^2\right)\left(2\,a-2\,b\right)\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\left(2\,a^3+3\,a\,b^2\right)\sqrt{a+b}\,\left(a-b\right)^{7/2}}\right)\left(2\,a^2+3\,b^2\right)}{d\left(a+b\right)^{7/2}\left(a-b\right)^{7/2}}-\frac{4 \cdots}{d\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-3\,a^3+3\,a^2\,b+3\,a \cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*
b + a^3 - b^3))/(2*(3*a*b^2 + 2*a^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(2*a^2 +
 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(9*a^2*
b + b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)^5*(3*a*b^
2 + 6*a^2*b + 2*b^3))/((a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)*(6*a^2*b -
3*a*b^2 + 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*a*b^2 -
tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/
2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 +
(d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.484 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=251

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{b\left(8a^6-8a^4b^2+\right.}{}$$

```
[Out] -b*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/
(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+arctanh(sin(d*x+c))/a^4/d+1/3*b^
2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b^2*(8*a^2-3*b^2)*sin(d*x
+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*b^2*(26*a^4-17*a^2*b^2+6*b^4)*
sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.79, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{b\left(-8a^4b^2+7a^2b^4+8a^6-2b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-17a^2b^2+26a^4+6b^4\right)\sin(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{b^2\left(8a^2\right.}{6a^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d)) + ArcTanh[Sin[c
+ d*x]]/(a^4*d) + (b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^3) + (b^2*(8*a^2 - 3*b^2)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos
[c + d*x])^2) + (b^2*(26*a^4 - 17*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(6*a^3*(a^
2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3\left(a^2-b^2\right)-3ab\cos(c+dx)+2b^2\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(6\left(a^2-\right.\right.}{}}{}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{b^2\left(26a\right.}{6a^3\left(a\right.}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{b^2\left(26a\right.}{6a^3\left(a\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^4 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\sin}{6a^2\left(a^2-b^2\right)^2 d(a+b\,c}$$

$$= -\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \frac{\tanh^{-1}(\sin(c+dx))}{a^4 d} +$$

Mathematica [A] time = 3.07, size = 274, normalized size = 1.09

$$\frac{\frac{2a^3b^2\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^3} + \frac{a^2b^2\left(8a^2-3b^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + \frac{ab^2\left(26a^4-17a^2b^2+6b^4\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))} + \frac{6b\left(-8a^6+8a^4b^2-7a^2b^4+2b^6\right)\tanh^{-1}\left(\frac{(a-b)\,t}{}\right.}{\left(b^2-a^2\right)^{7/2}}}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a^3*b^2*Si
n[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + (a^2*b^2*(8*a^2 - 3*
b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + (a*b^2*(2
```

```
6*a^4 - 17*a^2*b^2 + 6*b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Cos[c
 + d*x])))/(6*a^4*d)
```

fricas [B] time = 5.13, size = 1815, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7 + (8*a^6*b^4 - 8*a^4
*b^6 + 7*a^2*b^8 - 2*b^10)*cos(d*x + c)^3 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^
3*b^7 - 2*a*b^9)*cos(d*x + c)^2 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 - 2*
a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 6*(a^11 -
 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4
*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^
6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5
- 4*a^4*b^7 + a^2*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*(a^11 - 4*a^
9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7
- 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4
*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a
^4*b^7 + a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(36*a^9*b^2 - 68
*a^7*b^4 + 43*a^5*b^6 - 11*a^3*b^8 + (26*a^7*b^4 - 43*a^5*b^6 + 23*a^3*b^8
- 6*a*b^10)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^
9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*
b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4
*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5
 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^4 -
4*a^9*b^6 + a^7*b^8)*d), -1/6*(3*(8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b
^7 + (8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10)*cos(d*x + c)^3 + 3*(8*a^7
*b^3 - 8*a^5*b^5 + 7*a^3*b^7 - 2*a*b^9)*cos(d*x + c)^2 + 3*(8*a^8*b^2 - 8*a
^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*co
s(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*(a^11 - 4*a^9*b^2 + 6*a
^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9
 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 +
a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^
2*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*(a^11 - 4*a^9*b^2 + 6*a^7*b^
4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^1
0)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9
)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*
b^6 - 11*a^3*b^8 + (26*a^7*b^4 - 43*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10)*cos(d*
x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)*cos(d*x + c))*s
```

```
in(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d*
cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^1
0)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6
*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^
8)*d)]
```

giac [B] time = 1.66, size = 554, normalized size = 2.21

$$\frac{3\left(8a^6b-8a^4b^3+7a^2b^5-2b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^{10}-3a^8b^2+3a^6b^4-a^4b^6\right)\sqrt{a^2-b^2}}+\frac{36a^6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-60a^5b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a
^2 - b^2)) + (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*x +
1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x + 1/2*
c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*c)^5 +
 6*b^8*tan(1/2*d*x + 1/2*c)^5 + 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 116*a^4
*b^4*tan(1/2*d*x + 1/2*c)^3 + 56*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*b^8*ta
n(1/2*d*x + 1/2*c)^3 + 36*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*a^5*b^3*tan(1/2
*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*a^3*b^5*tan(1/2*d*x + 1
/2*c) - 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*a*b^7*tan(1/2*d*x + 1/2*c) + 6*
b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*log(abs(tan(1/
2*d*x + 1/2*c) + 1))/a^4 - 3*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4)/d
```

maple [B] time = 0.10, size = 1377, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a
^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2+4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c
)^5*b^3-6/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-
b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-1/d*b^5/a^2/(a*tan(1/2*d*
```

```
x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5+2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+24/d/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b
+b^2)*tan(1/2*d*x+1/2*c)^3*b^2-44/3/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
+4/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a
*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+12/d/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*b^2-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^3-6/d*b^4/a/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c)+1/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+2/d*b^6/a^3/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)-8/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b)
)^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+8/d*b^3/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)
/((a-b)*(a+b))^(1/2))-7/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*b^7/a^4/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*
(a-b)/((a-b)*(a+b))^(1/2))-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^4*ln(tan(
1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.51, size = 7235, normalized size = 28.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^4),x)
```

```
[Out] - (atan((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11
 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 +
```

110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a
^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 +
 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (8*tan(c/2 + (d*x)/
2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*
a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120
*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 -
a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 1
0*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))/a^4 - (8*tan(c/2 + (d
x)/2)(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 +
 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120
*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b
^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^1
2*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))*1i)/a^4 - ((((8*(16*a^20*b
- 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*
b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4
 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^
11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4
 - 5*a^17*b^3 - 5*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14
+ 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 +
 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 -
48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^
8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 -
 5*a^14*b^3 - 5*a^15*b^2)))/a^4 + (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b
- 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^
9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 4
8*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9
 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^
14*b^3 - 5*a^15*b^2))*1i)/a^4)/((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a
^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^1
4*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^
2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^
13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2)
 - (8*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12
 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^
7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/
(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10
*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))
/a^4 - (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^
2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b
^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/
(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7
- 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))/a^4 +
 (((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^1
1*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*
b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 -

a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*
b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^2
1*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10
+ 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5
+ 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11
- a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^
5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))/a^4 + (8*tan(c/2 + (d*x)/2)*(4
*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*
b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5
- 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7
*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 1
0*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))/a^4 - (16*(16*a^12*b - 2*a*b^12 + 4*
b^13 - 26*a^2*b^11 + 11*a^3*b^10 + 70*a^4*b^9 - 34*a^5*b^8 - 110*a^6*b^7 +
66*a^7*b^6 + 110*a^8*b^5 - 64*a^9*b^4 - 64*a^10*b^3 + 48*a^11*b^2))/(a^19*b
+ a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10
*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2)))*2i)/(a^4
*d) - ((tan(c/2 + (d*x)/2)^5*(2*b^6 - a*b^5 - 6*a^2*b^4 + 4*a^3*b^3 + 12*a^
4*b^2))/((a^3*b - a^4)*(a + b)^3) - (4*tan(c/2 + (d*x)/2)^3*(3*b^6 - 11*a^2
*b^4 + 18*a^4*b^2))/(3*(a + b)^2*(a^5 - 2*a^4*b + a^3*b^2)) + (tan(c/2 + (d
x)/2)(a*b^5 + 2*b^6 - 6*a^2*b^4 - 4*a^3*b^3 + 12*a^4*b^2))/((a + b)*(3*a^
5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*
b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b -
3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^
2*b + a^3 - b^3))) - (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b
- 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^
9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 4
8*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9
+ 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^
14*b^3 - 5*a^15*b^2) - (b*(-(a + b)^7*(a - b)^7)^(1/2)*((8*(16*a^20*b - 4*a
^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 -
30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64
*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^
9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*
a^17*b^3 - 5*a^18*b^2) - (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/
2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b
^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14
*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b
^3 - 8*a^20*b^2))/((a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^
8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)*(a^16*b + a^17 - a^6*b^11 - a^7
*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 1
0*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*
b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^1
2*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2
*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*1i)/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8
*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)) + (b*((8*tan

(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a
^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*
b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^1
7 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6
 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (b*(-(a + b)^7*(a
 - b)^7)^(1/2)*((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*
b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^
6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20
 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b
^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (4*b*tan(c/2 +
(d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^
2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*
a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120
*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/((a^18 - a^4*b^14 + 7*
a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b
^2)*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*
b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*
(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 -
 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(-(a
 + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*1i)/(2*(a^
18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a
^14*b^4 - 7*a^16*b^2)))/((16*(16*a^12*b - 2*a*b^12 + 4*b^13 - 26*a^2*b^11 +
 11*a^3*b^10 + 70*a^4*b^9 - 34*a^5*b^8 - 110*a^6*b^7 + 66*a^7*b^6 + 110*a^8
*b^5 - 64*a^9*b^4 - 64*a^10*b^3 + 48*a^11*b^2))/(a^19*b + a^20 - a^9*b^11 -
 a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*
b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (b*((8*tan(c/2 + (d*x)/2)*(4
*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*
b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5
 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7
*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 1
0*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) - (b*(-(a + b)^7*(a - b)^7)^(1/2)*((8
*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^1
0 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 -
 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10
*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 +
 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (4*b*tan(c/2 + (d*x)/2)*(-(a + b)
^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*(8*a^21*b - 8*a
^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^
13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^1
8*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/((a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*
b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)*(a^16*b + a^17
 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6
+ 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*(8*a^6 - 2*b^6 + 7
*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*
a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(-(a + b)^7*(a - b)^7)

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^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6
*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)
) - (b*((8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a
^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*
b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))
/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7
 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (b*
(-(a + b)^7*(a - b)^7)^(1/2)*((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b
^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^
7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/
(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b
^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (
4*b*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*
b^4 - 8*a^4*b^2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^
11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*
a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/((a^18 -
 a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*
b^4 - 7*a^16*b^2)*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*
b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 -
5*a^15*b^2)))*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14
+ 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^
16*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b
^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12
*b^6 + 21*a^14*b^4 - 7*a^16*b^2))))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2
*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*1i)/(d*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8
*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**4, x)
```

3.485 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=308

$$-\frac{4b\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{b^2\left(9a^2-4b^2\right)\tan(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{\left(6a^6-65a^4\ldots\right.}{\ldots}$$

```
[Out] b^2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2
*c)/(a+b)^(1/2))/a^5/(a-b)^(7/2)/(a+b)^(7/2)/d-4*b*arctanh(sin(d*x+c))/a^5/
d+1/6*(6*a^6-65*a^4*b^2+68*a^2*b^4-24*b^6)*tan(d*x+c)/a^4/(a^2-b^2)^3/d+1/3
*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b^2*(9*a^2-4*b^2)*tan(
d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/2*b^2*(12*a^4-11*a^2*b^2+4*b^
4)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 1.27, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{b^2\left(-35a^4b^2+28a^2b^4+20a^6-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-65a^4b^2+68a^2b^4+6a^6-24b^6\right)\tan(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) - (4*b*ArcTanh
[Sin[c + d*x]])/(a^5*d) + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Tan[c
 + d*x])/(6*a^4*(a^2 - b^2)^3*d) + (b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x])^3) + (b^2*(9*a^2 - 4*b^2)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])^2) + (b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Tan[c +
 d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3a^2-4b^2-3ab\cos(c+dx)+3b^2\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(9a^2-4b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(6a^4-\right.}{}}{}$$

$$= \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(9a^2-4b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{b^2\left(12a\right.}{2a^3\left(a\right.}$$

$$= \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} +$$

$$= \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} +$$

$$= -\frac{4b\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{}{3a\left(a^2-\right.}$$

$$= \frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b\tanh^{-1}(\sin(c+d\right.}{a^5d}$$

Mathematica [A] time = 6.23, size = 416, normalized size = 1.35

$$\frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} - \frac{4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{\sin\left(\frac{1}{2}(c+\right.}{a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[((a - b)*Tan[(c +
 d*x)/2])/Sqrt[-a^2 + b^2]])/(a^5*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d)) + (4*b
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a^5*d) - (4*b*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]])/(a^5*d) + Sin[(c + d*x)/2]/(a^4*d*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])) - (b^3*Sin[c + d*x])/(3*a^2*(a - b)*(a + b)*d*(a + b*Cos[c + d
*x])^3) + (-11*a^2*b^3*Sin[c + d*x] + 6*b^5*Sin[c + d*x])/(6*a^3*(a - b)^2*
(a + b)^2*d*(a + b*Cos[c + d*x])^2) + (-47*a^4*b^3*Sin[c + d*x] + 50*a^2*b^
5*Sin[c + d*x] - 18*b^7*Sin[c + d*x])/(6*a^4*(a - b)^3*(a + b)^3*d*(a + b*C
os[c + d*x]))
```

fricas [B] time = 6.32, size = 2048, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11)*cos(d*x + c)^4 +
 3*(20*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c)^3 + 3*(20
*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + (20*a^9*b^
2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a
*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + a^2)) + 24*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 +
b^12)*cos(d*x + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b
^11)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2
*b^10)*cos(d*x + c)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b
^9)*cos(d*x + c))*log(sin(d*x + c) + 1) - 24*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*
b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^
7 - 4*a^3*b^9 + a*b^11)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^
6 - 4*a^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5
- 4*a^5*b^7 + a^3*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(6*a^12 - 2
4*a^10*b^2 + 36*a^8*b^4 - 24*a^6*b^6 + 6*a^4*b^8 + (6*a^9*b^3 - 71*a^7*b^5
+ 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11)*cos(d*x + c)^3 + 3*(6*a^10*b^2 - 59
*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*cos(d*x + c)^2 + (18*a^1
1*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c))*s
in(d*x + c))/((a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d*
cos(d*x + c)^4 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^
10)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^
7*b^9)*d*cos(d*x + c)^2 + (a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^
8*b^8)*d*cos(d*x + c)), 1/6*(3*((20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b
^11)*cos(d*x + c)^4 + 3*(20*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*c
os(d*x + c)^3 + 3*(20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*
x + c)^2 + (20*a^9*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c))
```

```
*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)
)) - 12*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)
^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cos(d*x + c)^
3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*cos(d*x + c
)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*cos(d*x + c))*
log(sin(d*x + c) + 1) + 12*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 +
 b^12)*cos(d*x + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*
b^11)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^
2*b^10)*cos(d*x + c)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*
b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (6*a^12 - 24*a^10*b^2 + 36*a^8*
b^4 - 24*a^6*b^6 + 6*a^4*b^8 + (6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a
^3*b^9 + 24*a*b^11)*cos(d*x + c)^3 + 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b
^6 - 77*a^4*b^8 + 20*a^2*b^10)*cos(d*x + c)^2 + (18*a^11*b - 132*a^9*b^3 +
239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c))*sin(d*x + c))/((a^13*
b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d*cos(d*x + c)^4 + 3*(
a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(d*x + c)^3
 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x + c
)^2 + (a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c
))]
```

giac [B] time = 1.63, size = 587, normalized size = 1.91

$$-\frac{\frac{3\left(20a^6b^2-35a^4b^4+28a^2b^6-8b^8\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^{11}-3a^9b^2+3a^7b^4-a^5b^6\right)\sqrt{a^2-b^2}}+\frac{60a^6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-105a^5b^4\tan\left(\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*
sqrt(a^2 - b^2)) + (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan(1/2
*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(1/2*d
*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*d*x +
1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 + 120*a^6*b^3*tan(1/2*d*x + 1/2*c)
^3 - 236*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 + 152*a^2*b^7*tan(1/2*d*x + 1/2*c)^
3 - 36*b^9*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*tan(1/2*d*x + 1/2*c) + 105*a
^5*b^4*tan(1/2*d*x + 1/2*c) - 24*a^4*b^5*tan(1/2*d*x + 1/2*c) - 117*a^3*b^6
*tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*a*b^8*tan(1/2*
d*x + 1/2*c) + 18*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4
- a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3)
 + 12*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 12*b*log(abs(tan(1/2*d*x +
```

```
 1/2*c) - 1))/a^5 + 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^
4))/d
```

maple [B] time = 0.12, size = 1429, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -20/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3-5/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5+18/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/d*b^6/a^3/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5-6/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-40/d/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a^2-2*a*b+b^2)/(a^2
+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+116/3/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3-12/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-20/d/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*b^3+5/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+18/d*b^5/a^2/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)-2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-6/d*b^7/a^
4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+
3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((
a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-35/d
*b^4/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x
+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1
/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)+4/d
*b/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)-4/d*b/a^5*ln
(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.95, size = 7490, normalized size = 24.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^4),x)
```

```
[Out] (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 +
768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9
 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4
- 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^1
0*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4
- 5*a^16*b^3 - 5*a^17*b^2) - (4*b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^1
3 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^
8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 -
20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*
b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 -
5*a^21*b^2) - (32*b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^1
3 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b
^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3
 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*
a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b
^3 - 5*a^17*b^2))))/a^5)*4i)/a^5 + (b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 12
8*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 26
00*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 7
68*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a
^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 +
10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (4*b*((16*(8*a^23*b
- 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 +
 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95
*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b
^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 1
0*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (32*b*tan(c/2 + (d*x)/2)*(8*a^23*b
- 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 +
 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 +
 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 -
a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^
5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5)*4i)/a^5)/((32*(128*b^16
```

- 64*a*b^15 - 832*a^2*b^14 + 400*a^3*b^13 + 2288*a^4*b^12 - 1088*a^5*b^11 - 3472*a^6*b^10 + 1602*a^7*b^9 + 3088*a^8*b^8 - 1280*a^9*b^7 - 1520*a^10*b^6 + 480*a^11*b^5 + 320*a^12*b^4))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (4*b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (4*b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (32*b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5))/a^5 + (4*b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (4*b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (32*b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5))/a^5))*8i)/(a^5*d) - ((tan(c/2 + (d*x)/2)^3*(12*a*b^7 - 18*a^8 - 72*b^8 + 236*a^2*b^6 - 47*a^3*b^5 - 273*a^4*b^4 + 60*a^5*b^3 + 72*a^6*b^2))/(3*a^4*(a + b)^2*(a - b)^3) - (tan(c/2 + (d*x)/2)^5*(12*a*b^7 + 18*a^8 + 72*b^8 - 236*a^2*b^6 - 47*a^3*b^5 + 273*a^4*b^4 + 60*a^5*b^3 - 72*a^6*b^2))/(3*a^4*(a + b)^3*(a - b)^2) + (tan(c/2 + (d*x)/2)*(4*a*b^6 - 2*a^6*b - 2*a^7 + 8*b^7 - 24*a^2*b^5 - 11*a^3*b^4 + 26*a^4*b^3 + 6*a^5*b^2))/(a^4*(a + b)*(a - b)^3) + (tan(c/2 + (d*x)/2)^7*(4*a*b^6 + 2*a^6*b - 2*a^7 - 8*b^7 + 24*a^2*b^5 - 11*a^3*b^4 - 26*a^4*b^3 + 6*a^5*b^2))/(a^4*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x)/2)^4*(6*a^2*b - 6*b^3) - tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3) - tan(c/2 + (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3)

+ a^3 + b^3 - tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (b^2*atan(((b^2*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*1i)/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)) + (b^2*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*1i)/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*

a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))/((32*(128*b^16 - 64*a*b^15 - 832*a^2
*b^14 + 400*a^3*b^13 + 2288*a^4*b^12 - 1088*a^5*b^11 - 3472*a^6*b^10 + 1602
*a^7*b^9 + 3088*a^8*b^8 - 1280*a^9*b^7 - 1520*a^10*b^6 + 480*a^11*b^5 + 320
*a^12*b^4))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^
8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*
a^21*b^2) - (b^2*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^
14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^
7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12
*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 +
5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15
*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^1
1*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^
16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b
^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*
a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b
^3 - 5*a^21*b^2) - (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(
20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*
b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^1
6*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*
b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b
^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^
9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5
+ 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20
*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 -
21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*(-(a
+ b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^
19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a
^15*b^4 - 7*a^17*b^2)) + (b^2*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15
- 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b
^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*
b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11
- a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*
b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (b^2*((16*(8*a^23*b - 8*a^10
*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15
*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^
4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*
a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b
^4 - 5*a^20*b^3 - 5*a^21*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a -
b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b
^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*
b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b
^4 - 48*a^21*b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^1
0 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 -
a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 +
10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*(-(a + b)^7*(a - b)

```
^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 +
 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^1
7*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4
*b^2))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^
13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6
- 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*1i)/(d*(a^19 - a^5*b^14 + 7*a^7*b^12 - 2
1*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**4, x)
```

3.486 $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=264

$$\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}+\frac{2a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(8a^4+1\right.}{}$$

```
[Out] -8/35*a*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*cos(d*x+c)*(a+b*cos(d*x
+c))^(3/2)*sin(d*x+c)/b/d+2/105*(8*a^2+25*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^
(1/2)/b^2/d+2/105*a*(8*a^2+19*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+
c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/105*(8*a^4+17*a^2*b^2-25*b
^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*co
s(d*x+c))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}-\frac{2\left(17a^2b^2+8a^4-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(8a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*a*(8*a^2 + 19*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^4 + 17*
a^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(105*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*a^2 + 25*b^2)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^2*d) - (8*a*(a + b*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^
(3/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
```

```
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \frac{2\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7bd} + \frac{2\int\sqrt{a+b\cos(c+dx)}}{}$$

$$= -\frac{8a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35b^2d} + \frac{2\cos(c+dx)(a+b\cos(c+dx))}{7bd}$$

$$= \frac{2\left(8a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^2d} - \frac{8a(a+b\cos(c+dx))^{3/}}{35b^2d}$$

$$= \frac{2\left(8a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^2d} - \frac{8a(a+b\cos(c+dx))^{3/}}{35b^2d}$$

$$= \frac{2\left(8a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^2d} - \frac{8a(a+b\cos(c+dx))^{3/}}{35b^2d}$$

$$= \frac{2a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(8a^4+17a^2b^2\right.}{10}$$

Mathematica [A] time = 1.14, size = 214, normalized size = 0.81

$$\frac{-4\left(8a^4+17a^2b^2-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b\sin(c+dx)\left(-16a^3+\left(145b^3-4a^2b\right)\cos(c+dx)+\right.}{210b^3d\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (4*a*(8*a^3 + 8*a^2*b + 19*a*b^2 + 19*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(8*a^4 + 17*a^2*b^2 - 25*b^4)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] +
b*(-16*a^3 + 136*a*b^2 + (-4*a^2*b + 145*b^3)*Cos[c + d*x] + 36*a*b^2*Cos[2
*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*b^3*d*Sqrt[a + b*
Cos[c + d*x]])
```

fricas [F] time = 1.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

maple [B] time = 1.09, size = 827, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos
(1/2*d*x+1/2*c)^9*b^4+144*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)
^7*b^4-4*cos(1/2*d*x+1/2*c)^5*a^2*b^2-288*cos(1/2*d*x+1/2*c)^5*a*b^3+640*co
s(1/2*d*x+1/2*c)^5*b^4-8*cos(1/2*d*x+1/2*c)^3*a^3*b+6*cos(1/2*d*x+1/2*c)^3*
a^2*b^2+230*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4-8*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
```

```
(1/2))*b^4+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-8*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+19*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^2*b^2-19*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^3+8*cos(1/2*d*x+1/2*c)*a^3*b-2*cos(1/2*d*x+1/2*c)*a^2*b^2-86*cos(
1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/b^3/(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/
2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.487 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=207

$$\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(2a^2-9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)(}{}$$

```
[Out] 2/5*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d-4/15*a*sin(d*x+c)*(a+b*cos(d*x+c)
)^(1/2)/b/d-2/15*(2*a^2-9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^
(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+4/15*a*(a^2-b^2)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a
+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(2a^2-9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(2*a^2 - 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(a^2 - b^2)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(1
5*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(15*b*d) + (2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2\int\left(\frac{3b}{2}-a\cos(c+dx)\right)\sqrt{a+b\,\mathrm{co}}}{5b}$$

$$= -\frac{4a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+d}{5bd}$$

$$= -\frac{4a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+d}{5bd}$$

$$= -\frac{4a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+d}{5bd}$$

$$= \frac{2\left(9-\frac{2a^2}{b^2}\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\,\mathrm{co}}{a}}}{15b^2d\sqrt{a-}}$$

Mathematica [A] time = 0.86, size = 180, normalized size = 0.87

$$\frac{b\sin(c+dx)\left(2a^2+8ab\cos(c+dx)+3b^2\cos(2(c+dx))+3b^2\right)+4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(2*a^3 + 2*a^2*b - 9*a*b^2 - 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 4*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(2*a^2 + 3*b^2 +
8*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*b^2*d*Sqrt[a
 + b*Cos[c + d*x]])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

maple [B] time = 0.82, size = 665, normalized size = 3.21

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+16\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-48\left(\cos^5\left(\frac{d}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1
/2*d*x+1/2*c)^7*b^3+16*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*b
^3+2*cos(1/2*d*x+1/2*c)^3*a^2*b-24*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d*
x+1/2*c)^3*b^3+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^3+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-2*cos(1/2*d*x+1/2*c)*a^2*b+8*c
os(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/b^2/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^2 \sqrt{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos(c+dx)}\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**2, x)
```

3.488 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=162

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a
+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^2-b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*Sqrt[a + b*Cos[
c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{b}{2}+\frac{1}{2}a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{a\int \sqrt{a+b\cos(c+dx)}\,dx}{3b} - \frac{(a^2-b^2)}{}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos}{a}}}{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c\right.}{3bd\sqrt{a+b\cos(c+dx}}$$

Mathematica [A] time = 0.55, size = 137, normalized size = 0.85

$$\frac{-2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2b\sin(c+dx)(a+b\cos(c+dx))+2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c-\right.}{3bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*b*d*Sq
rt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

maple [B] time = 0.74, size = 452, normalized size = 2.79

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\left(\cos^3\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/2*c)
*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cos\left(c + dx\right)}\,\cos\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x), x)
```

3.489 $\int \sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+
b))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2655, 2653}

$$\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$= \frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 1.00

$$\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]],x]
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.00, size = 170, normalized size = 2.98

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b}\ c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(a-b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x)), x)
```

3.490 $\int \sqrt{a+b\cos(c+dx)} \sec(c+dx)\,dx$

Optimal. Leaf size=118

$$\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*
x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2
)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2803, 2663, 2661, 2807, 2805}

$$\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
```

```
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\,dx = a\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + b\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\left(a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} + \frac{\left(b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+}{a+b}}}}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 2.18, size = 81, normalized size = 0.69

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(bF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*EllipticF[(c + d*x)/2, (2*b)/(a +
b)] + a*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c + d
*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

maple [A] time = 1.09, size = 194, normalized size = 1.64

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b-EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos\left(c+dx\right)}\,\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x), x)
```

3.491 $\int \sqrt{a+b\cos(c+dx)}\,\sec^2(c+dx)\,dx$

Optimal. Leaf size=197

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{b\sqrt{\frac{a+b}{}}}{}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b
))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b
*cos(d*x+c))^(1/2)+b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.50, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2796, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{b\sqrt{\frac{a+b}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] -((Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])) + (a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (b*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d
*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\ \sec^2(c+dx)\,dx = \frac{\sqrt{a+b\cos(c+dx)}\ \tan(c+dx)}{d} + \int \frac{\left(\frac{b}{2}-\frac{1}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\ \tan(c+dx)}{d} - \frac{1}{2}\int \sqrt{a+b\cos(c+dx)}\ dx - \frac{\int \frac{\left(-\frac{b^2}{2}\right.}{}}{}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\ \tan(c+dx)}{d} + \frac{1}{2}a\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx + \frac{1}{2}b\int$$

$$= -\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{a+b\cos(c+dx)}\ \tan(c+dx}{d}$$

$$= -\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 7.33, size = 307, normalized size = 1.56

$$\frac{4\tan(c+dx)\sqrt{a+b\cos(c+dx)} + \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh\right.\right.\right.}{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] ((2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(
a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*E
llipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
+ d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(
-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

maple [B] time = 0.83, size = 622, normalized size = 3.16

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-2a-2b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a-2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
```

```
1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b-EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b)*s
in(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos\left(c+dx\right)}\,\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.492 $\int \sqrt{a+b\cos(c+dx)}\,\sec^3(c+dx)\,dx$

Optimal. Leaf size=262

$$\frac{\left(4a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -1/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+
c))/(a+b))^(1/2)+3/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*b*(a+b*cos(
d*x+c))^(1/2)*tan(d*x+c)/a/d+1/2*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+
c)/d
```

Rubi [A] time = 0.73, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2796, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] -(b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) +
((4*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2
, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*x]]) + (b*Sqrt[a + b*Cos[c +
d*x]]*Tan[c + d*x])/(4*a*d) + (Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c
+ d*x])/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,\sec^3(c+dx)\,dx = \frac{\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{\left(\frac{b}{2}+a\cos(c+dx)+\right.}{\sqrt{a+}}$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad} + \frac{\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(}{2d}$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad} + \frac{\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(}{2d}$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad} + \frac{\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(}{2d}$$

$$= -\frac{b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\sqrt{a+b\cos(c+dx)}\,\tan(c+}{4ad}$$

$$= -\frac{b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 6.49, size = 515, normalized size = 1.97

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{b\tan(c+dx)}{4a}+\frac{1}{2}\tan(c+dx)\sec(c+dx)\right)}{d} + \frac{\frac{2(8a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2ib^2\sin(c+dx)\cos(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] ((8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c +
d*x]] + ((2*I)*b^2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c +
 d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
```

```
*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos
[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c +
d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d
*x])^2)))/(16*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((b*Tan[c + d*x])/(4*a) + (S
ec[c + d*x]*Tan[c + d*x])/2))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

maple [B] time = 0.91, size = 977, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(12*a*b+8*b^2)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-4*a^2-6*a*b-2*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))*a^2+EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)*sin(1/2*d*x
+1/2*c)^4-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))*a^2+EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)*sin(1
```

```
/2*d*x+1/2*c)^2+3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))*b^2)/a/(2*cos(1/2*d*x+1/2*c)^2-1)^2/(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos\left(c+dx\right)}\,\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.493 $\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=314

$$\frac{2\left(8a^2+49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}+\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315b^2d}-\frac{2a\left(8a^4+31a\right.}{}$$

```
[Out] 2/315*(8*a^2+49*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d-8/63*a*(a+b*co
s(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/9*cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b/d+2/315*a*(8*a^2+39*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2
/315*(8*a^4+33*a^2*b^2+147*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))
^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/315*a*(8*a^4+31*a^2*b^2-39*b^
4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos
(d*x+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2+49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}+\frac{2a\left(8a^2+39b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315b^2d}-\frac{2a\left(31a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*
a*(8*a^4 + 31*a^2*b^2 - 39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*
(8*a^2 + 39*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^2*d) + (2*(8
*a^2 + 49*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^2*d) - (8*a*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b^2*d) + (2*Cos[c + d*x]*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{2\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9bd} + \frac{2\int(a+b\cos(c+d\ldots}{\ldots}$$

$$= -\frac{8a(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63b^2d} + \frac{2\cos(c+dx)(a+b\cos(c+d\ldots}{9bd}$$

$$= \frac{2\left(8a^2+49b^2\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315b^2d} - \frac{8a(a+b\cos(c+d\ldots}{63b\ldots}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a\ldots}{\ldots}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a\ldots}{\ldots}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a\ldots}{\ldots}$$

$$= \frac{2\left(8a^4+33a^2b^2+147b^4\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\ldots}{\ldots}$$

Mathematica [A] time = 1.36, size = 262, normalized size = 0.83

$$\frac{-8a\left(8a^4+31a^2b^2-39b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b\sin(c+dx)\left(-32a^4+\left(1606ab^3-8a^3b\right)\cos(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*(8*a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 8*a*(8
*a^4 + 31*a^2*b^2 - 39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)] + b*(-32*a^4 + 916*a^2*b^2 + 301*b^4 + (-8*a^3*b
+ 1606*a*b^3)*Cos[c + d*x] + 4*(53*a^2*b^2 + 84*b^4)*Cos[2*(c + d*x)] + 170
*a*b^3*Cos[3*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)])*Sin[c + d*x])/(1260*b^3*
d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4+a\cos\left(dx+c\right)^3\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

maple [B] time = 0.88, size = 995, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x)
```

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1360*a*b^4+2240*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-424*a^2*b^3-2040*a*b^4-2072*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-4*a^3*b^2+424*a^2*b^3+1568*a*b^4+952*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(8*a^4*b+2*a^3*b^2-282*a^2*b^3-444*a*b^4-168*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-31*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+33*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-33*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^3/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.494 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=258

$$-\frac{2\left(6a^2-25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{4a\left(3a^2-41b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\left(6a^4-\right.}{}$$

```
[Out] -4/35*a*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*(a+b*cos(d*x+c))^(5/2)*si
n(d*x+c)/b/d-2/105*(6*a^2-25*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-4/1
05*a*(3*a^2-41*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d
/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/105*(6*a^4-31*a^2*b^2+25*b^4)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(6a^2-25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}+\frac{2\left(-31a^2b^2+6a^4+25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-4*a*(3*a^2 - 41*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(6*a^4 - 31
*a^2*b^2 + 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(105*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(6*a^2 - 25*b^2
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b*d) - (4*a*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(35*b*d) + (2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*
x])/(7*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2\int\left(\frac{5b}{2}-a\cos(c+dx)\right)(a+b\,\mathrm{c}}{7b}$$

$$= -\frac{4a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd} + \frac{2(a+b\cos(c+dx))^{5/2}\sin(c}{7bd}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105bd} - \frac{4a(a+b\cos(c+d}{35l}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105bd} - \frac{4a(a+b\cos(c+d}{35l}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105bd} - \frac{4a(a+b\cos(c+d}{35l}$$

$$= \frac{4a\left(41-\frac{3a^2}{b^2}\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(6a^4-31a^2b^2\right.}{1}$$

Mathematica [A] time = 1.15, size = 214, normalized size = 0.83

$$\frac{4\left(6a^4-31a^2b^2+25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b\sin(c+dx)\left(12a^3+b\left(108a^2+145b^2\right)\cos(c+dx)+\right.}{210b^2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(3*a^3 + 3*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 4*(6*a^4 - 31*a^2*b^2 + 25*b^4)
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] +
 b*(12*a^3 + 178*a*b^2 + b*(108*a^2 + 145*b^2)*Cos[c + d*x] + 78*a*b^2*Cos[
2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*b^2*d*Sqrt[a + b
*Cos[c + d*x]])
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.70, size = 827, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos
(1/2*d*x+1/2*c)^9*b^4+312*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)
^7*b^4+108*cos(1/2*d*x+1/2*c)^5*a^2*b^2-624*cos(1/2*d*x+1/2*c)^5*a*b^3+640*
cos(1/2*d*x+1/2*c)^5*b^4+6*cos(1/2*d*x+1/2*c)^3*a^3*b-162*cos(1/2*d*x+1/2*c
)^3*a^2*b^2+440*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4+6*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-31*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b^4-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+6*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+82*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-82*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^3-6*cos(1/2*d*x+1/2*c)*a^3*b+54*cos(1/2*d*x+1/2*c)*a^2*b^2-12
8*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/b^2/(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*
d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c+d\,x)^2\,(a+b\,\cos (c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.495 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=199

$$-\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)(a}{}$$

```
[Out] 2/5*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/5*a*sin(d*x+c)*(a+b*cos(d*x+c))^(
1/2)/d+2/5*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/
d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/5*a*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))
*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(5*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(a^2 - b^2)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(5*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d
) + (2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int \left(\frac{3b}{2} + \frac{3}{2}a\cos(c+dx)\right)\sqrt{a+$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

$$= \frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\,c}{}}}{5bd\sqrt{a+}}$$

Mathematica [A] time = 0.78, size = 174, normalized size = 0.87

$$\frac{b\sin(c+dx)\left(4a^2+6ab\cos(c+dx)+b^2\cos(2(c+dx))+b^2\right)-2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2\,(}{5bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(4*a^2 + b^2 + 6*a*b*C
os[c + d*x] + b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(5*b*d*Sqrt[a + b*Cos[c +
 d*x]])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^2 + a\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

maple [B] time = 0.76, size = 663, normalized size = 3.33

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(8\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+12\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-16\left(\cos^5\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*cos(1/2
*d*x+1/2*c)^7*b^3+12*cos(1/2*d*x+1/2*c)^5*a*b^2-16*cos(1/2*d*x+1/2*c)^5*b^3
+4*cos(1/2*d*x+1/2*c)^3*a^2*b-18*cos(1/2*d*x+1/2*c)^3*a*b^2+10*cos(1/2*d*x+
1/2*c)^3*b^3-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^3-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-4*cos(1/2*d*x+1/2*c)*a^2*b+6*cos(1/2*d*x
+1/2*c)*a*b^2-2*cos(1/2*d*x+1/2*c)*b^3)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^{\frac{3}{2}}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral((a + b*cos(c + d*x))**(3/2)*cos(c + d*x), x)
```

3.496 $\int (a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=157

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{8a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+8/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^2-b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/
2)
```

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2656, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{8a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\,dx = \frac{2b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}\left(3a^2+b^2\right)+2ab\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}(4a)\int \sqrt{a+b\cos(c+dx)}\,dx + \frac{1}{3}\left(-a^2+b\right.$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(4a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}{3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{8a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{3d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.55, size = 134, normalized size = 0.85

$$\frac{-2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2b\sin(c+dx)(a+b\cos(c+dx))+8a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c\right.}{3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt
[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.00, size = 450, normalized size = 2.87

$$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\cos^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^{2}+2\left(\cos^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\left(\cos^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b^2+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/
2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2), x)
```

3.497 $\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

Optimal. Leaf size=179

$$\frac{2a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(
a+b))^(1/2)+2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)
/d/(a+b*cos(d*x+c))^(1/2)+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x
+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2804, 2655, 2653, 2803, 2663, 2661, 2807, 2805}

$$\frac{2a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2804

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = a\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx + b\int \sqrt{a+b\cos(c+dx)}\,dx$$

$$= a^2\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + (ab)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx + \frac{\left(b\sqrt{a+\right.}{}$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+}{\sqrt{\frac{a}{a+b}+\frac{b\,c}{}}}}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 2.30, size = 107, normalized size = 0.60

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\left(bF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(a + b)*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + a*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2,
(c + d*x)/2, (2*b)/(a + b)])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 3.56, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

maple [A] time = 0.72, size = 249, normalized size = 1.39

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

```
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))^{\frac{3}{2}} \sec(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2)*sec(c + d*x), x)
```

3.498 $\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{\left(a^2+2b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}-\frac{a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a
+b))^(1/2)+(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/d/(a+b*cos(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*co
s(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a*(a+b*cos(d*x+c))^(1/2)*ta
n(d*x+c)/d
```

Rubi [**A**] time = 0.54, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2799, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2+2b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}-\frac{a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] -((a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2 + 2*b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + (3*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \frac{a\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} + \int \frac{\left(\frac{3ab}{2}+b^2\cos(c+dx)-\frac{1}{2}ab\cos\right.}{\sqrt{a+b\cos(c}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} - \frac{1}{2}a\int\sqrt{a+b\cos(c+dx)}\,dx - \frac{\int}{}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} + \frac{1}{2}(3ab)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a\sqrt{a+b\cos(c+dx)}\,\tan(c}{d}$$

$$= -\frac{a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(a^2+2b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F}{d\sqrt{a+b\cos(c-}}$$

Mathematica [C] time = 11.18, size = 363, normalized size = 1.74

$$\frac{4a\tan(c+dx)\sqrt{a+b\cos(c+dx)}+b\left(-\frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)\right|\frac{a+}{a-}\right.\right.}{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (b*((8*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/Sqrt[a + b*Cos[c + d*x]] + (10*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a
 + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*S
qrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticP
i[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)])))/(b^2*Sqrt[-(a + b)^(-1)])) + 4*a*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

maple [B] time = 0.78, size = 740, normalized size = 3.54

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4ab\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^2-2ab\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a^2-2*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+
2*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a*b-3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b)*sin(1
/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+
2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a-3*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.499 $\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=255

$$\frac{\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4d}+\frac{7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+a\right.}{4d\sqrt{a+b\cos(c+dx}}$$

```
[Out] -5/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c)
)/(a+b))^(1/2)+7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/
2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5/4*b*(a+b*cos(
d*x+c))^(1/2)*tan(d*x+c)/d+1/2*a*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+
c)/d
```

Rubi [A] time = 0.76, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2799, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4d}+\frac{7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+a\right.}{4d\sqrt{a+b\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (-5*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (7*a*b*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]])
+ ((4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (5*b*Sqrt[a + b*Cos[
c + d*x]]*Tan[c + d*x])/(4*d) + (a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Ta
n[c + d*x])/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \frac{a\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{\left(\frac{5ab}{2}+\left(a^2+2b^2\right)\right.}{}$$

$$= \frac{5b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\,\mathrm{t}}{2d}$$

$$= \frac{5b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\,\mathrm{t}}{2d}$$

$$= \frac{5b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\,\mathrm{t}}{2d}$$

$$= -\frac{5b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{5b\sqrt{a+b\cos(c+dx)}\,\tan(}{4d}$$

$$= -\frac{5b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{4d\sqrt{a+b\cos(c+dx}}$$

Mathematica [C] time = 6.45, size = 508, normalized size = 1.99

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}a\tan(c+dx)\sec(c+dx)+\frac{5}{4}b\tan(c+dx)\right)}{d} + \frac{\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{10ib^2\sin(c}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] ((8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*
x]] + ((10*I)*b^2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c +
d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
```

```
x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[
c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d
*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*
x])^2)))/(16*d) + (Sqrt[a + b*Cos[c + d*x]]*((5*b*Tan[c + d*x])/4 + (a*Sec[
c + d*x]*Tan[c + d*x])/2))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

maple [B] time = 0.87, size = 980, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x)
```

```
[Out] -1/4*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-40*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(28*a*b+40*b^2)*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+(-4*a^2-14*a*b-10*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*(7*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-5*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+5*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))*a^2-3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)*s
in(1/2*d*x+1/2*c)^4-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*(7*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a*b-5*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+5*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))*a^2-3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
```

```
1/2))*b^2)*sin(1/2*d*x+1/2*c)^2+7*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a+5*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.500 $\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=371

$$\frac{2\left(8a^2+81b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2+67b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{693b^2d}+\frac{2\left(8a^4+57\right.}{}$$

```
[Out] 2/693*a*(8*a^2+67*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/693*(8*a^2
+81*b^2)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d-8/99*a*(a+b*cos(d*x+c))^(7
/2)*sin(d*x+c)/b^2/d+2/11*cos(d*x+c)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d+
2/693*(8*a^4+57*a^2*b^2+135*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/
693*a*(8*a^4+51*a^2*b^2+741*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c)
)^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/693*(8*a^6+49*a^4*b^2+78*a^2
*b^4-135*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3
/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2+81b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2+67b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{693b^2d}+\frac{2\left(57a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (
2*(8*a^6 + 49*a^4*b^2 + 78*a^2*b^4 - 135*b^6)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(693*b^2*d) + (2*a*(8*a^2 + 67*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(693*b^2*d) + (2*(8*a^2 + 81*b^2)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(693*b^2*d) - (8*a*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^
2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
```

```
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{2\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{11bd} + \frac{2\int(a+b\cos(c+dx)}{}$$

$$= -\frac{8a(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{99b^2d} + \frac{2\cos(c+dx)(a+b\cos(c+dx}{11bd}$$

$$= \frac{2\left(8a^2+81b^2\right)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{693b^2d} - \frac{8a(a+b\cos(c+dx}{99b^2}$$

$$= \frac{2a\left(8a^2+67b^2\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{693b^2d} + \frac{2\left(8a^2+81b^2\right)(a}{}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{693b^2d} + \frac{2a\left(8a^2+\right.}{}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{693b^2d} + \frac{2a\left(8a^2+\right.}{}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{693b^2d} + \frac{2a\left(8a^2+\right.}{}$$

$$= \frac{2a\left(8a^4+51a^2b^2+741b^4\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{693b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(8\right.}{}$$

Mathematica [A] time = 1.21, size = 268, normalized size = 0.72

$$\frac{16\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b\left(2a^4b+663a^2b^3+135b^5\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\left(8a^4+51a^2b^2+741b^4\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(2*a^4*b + 663*a^2*b^3 + 135*b^5)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*(8*a^4 + 51*a^2*b^2 + 741*b^4)*(
(a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])) - b*(a + b*Cos[c + d*x])*((64*a^4 - 3732*a^2*b^2 - 2610*b^4)
*Sin[c + d*x] - b*(4*(6*a^3 + 619*a*b^2)*Sin[2*(c + d*x)] + b*((452*a^2 + 5
```

```
13*b^2)*Sin[3*(c + d*x)] + 7*b*(46*a*Sin[4*(c + d*x)] + 9*b*Sin[5*(c + d*x)
])))))/(5544*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^5+2\,ab\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3\right)\sqrt{b\cos\left(dx+c\right)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*s
qrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

maple [B] time = 0.99, size = 1140, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/693*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4032*b^
6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-7168*a*b^5-10080*b^6)*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(4384*a^2*b^4+14336*a*b^5+11376*b^6)*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-928*a^3*b^3-6576*a^2*b^4-13232*a*b^5-6
984*b^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-4*a^4*b^2+928*a^3*b^3+50
24*a^2*b^4+6064*a*b^5+2772*b^6)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(8*
a^5*b+2*a^4*b^2-642*a^3*b^3-1416*a^2*b^4-1338*a*b^5-558*b^6)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a^6-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b+
51*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-51*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
```

```
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3+741*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4-741*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a^6-49*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a^4*b^2-78*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2*b^4+135*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^6)/b^3/
(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.501 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=308

$$-\frac{2\left(10a^2-49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}-\frac{4a\left(5a^2-57b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315bd}+\frac{4a\left(5a^4-6\right.}{}$$

```
[Out] -2/315*(10*a^2-49*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d-4/63*a*(a+b*co
s(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/9*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d-4/
315*a*(5*a^2-57*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-2/315*(10*a^4-27
9*a^2*b^2-147*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/
((a+b*cos(d*x+c))/(a+b))^(1/2)+4/315*a*(5*a^4-62*a^2*b^2+57*b^4)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*
(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2
)
```

Rubi [A] time = 0.51, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(10a^2-49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}-\frac{4a\left(5a^2-57b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315bd}+\frac{4a\left(-62a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(10*a^4 - 279*a^2*b^2 - 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(4*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4
*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) - (2*(
10*a^2 - 49*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d) - (4*a*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2*(a + b*Cos[c + d*x])
^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{2(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{2\int \left(\frac{7b}{2} - a\cos(c+dx)\right)(a+b\,c}{9b}$$

$$= -\frac{4a(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd} + \frac{2(a+b\cos(c+dx))^{7/2}\sin(c}{9bd}$$

$$= -\frac{2\left(10a^2-49b^2\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315bd} - \frac{4a(a+b\cos(c}{}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)(}{}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)(}{}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)(}{}$$

$$= -\frac{2\left(10a^4-279a^2b^2-147b^4\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} +$$

Mathematica [A] time = 1.38, size = 263, normalized size = 0.85

$$\frac{16a\left(5a^4-62a^2b^2+57b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b\sin(c+dx)\left(40a^4+4ab\left(160a^2+619b^2\right)\cos(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2),x]

[Out] (-8*(10*a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 1
6*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)] + b*(40*a^4 + 1984*a^2*b^2 + 301*b^4 + 4*a*
b*(160*a^2 + 619*b^2)*Cos[c + d*x] + 8*(85*a^2*b^2 + 42*b^4)*Cos[2*(c + d*x
)] + 260*a*b^3*Cos[3*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)])*Sin[c + d*x])/(1
260*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)^4+2\,ab\cos(dx+c)^3+a^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*s
qrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

maple [B] time = 0.95, size = 995, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b
^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2080*a*b^4+2240*b^5)*sin(1/2*d
*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1360*a^2*b^3-3120*a*b^4-2072*b^5)*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(320*a^3*b^2+1360*a^2*b^3+2408*a*b^4+952*b^
5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*a^4*b-160*a^3*b^2-666*a^2*b
^3-684*a*b^4-168*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-124*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+114*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a*b^4-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^5+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^4*b+279*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-2
```

```
79*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.502 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=249

$$\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}+\frac{2a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{21bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(3a^4+2a\right.}{}$$

```
[Out] 2/7*a*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(a+b*cos(d*x+c))^(5/2)*sin(d*
x+c)/d+2/21*(3*a^2+5*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/21*a*(3*a^2
+29*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)-2/21*(3*a^4+2*a^2*b^2-5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}-\frac{2\left(2a^2b^2+3a^4-5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{21bd\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(3a^2+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*(3*a^2 + 29*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(21*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a^4 + 2*a^2*
b^2 - 5*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b
)/(a + b)])/(21*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(3*a^2 + 5*b^2)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(a + b*Cos[c + d*x])^(3/2)*Sin
[c + d*x])/(7*d) + (2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2}{7}\int\left(\frac{5b}{2}+\frac{5}{2}a\cos(c+dx)\right)(a+b$$

$$= \frac{2a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx}{7d}$$

$$= \frac{2\left(3a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a(a+b\cos(c+dx))^{3/2}}{7d}$$

$$= \frac{2\left(3a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a(a+b\cos(c+dx))^{3/2}}{7d}$$

$$= \frac{2\left(3a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a(a+b\cos(c+dx))^{3/2}}{7d}$$

$$= \frac{2a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{21bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(3a^4+2a^2b^2\right.}{2}$$

Mathematica [A] time = 0.88, size = 214, normalized size = 0.86

$$\frac{-4\left(3a^4+2a^2b^2-5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b\sin(c+dx)\left(36a^3+b\left(72a^2+29b^2\right)\cos(c+dx)+24a\right.}{42bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*a*(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(3*a^4 + 2*a^2*b^2 - 5*b^4)*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*
(36*a^3 + 44*a*b^2 + b*(72*a^2 + 29*b^2)*Cos[c + d*x] + 24*a*b^2*Cos[2*(c +
 d*x)] + 3*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(42*b*d*Sqrt[a + b*Cos[c + d
*x]])
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^3+2\,ab\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(b*cos(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.85, size = 827, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/21*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*cos(1
/2*d*x+1/2*c)^9*b^4+96*cos(1/2*d*x+1/2*c)^7*a*b^3-120*cos(1/2*d*x+1/2*c)^7*
b^4+72*cos(1/2*d*x+1/2*c)^5*a^2*b^2-192*cos(1/2*d*x+1/2*c)^5*a*b^3+128*cos(
1/2*d*x+1/2*c)^5*b^4+18*cos(1/2*d*x+1/2*c)^3*a^3*b-108*cos(1/2*d*x+1/2*c)^3
*a^2*b^2+130*cos(1/2*d*x+1/2*c)^3*a*b^3-72*cos(1/2*d*x+1/2*c)^3*b^4-3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^2*b^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*b^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+29*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2*b^2-29*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a*b^3-18*cos(1/2*d*x+1/2*c)*a^3*b+36*cos(1/2*d*x+1/2*c)*a^2*b^2-34*cos(
1/2*d*x+1/2*c)*a*b^3+16*cos(1/2*d*x+1/2*c)*b^4)/b/(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.503 $\int (a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=197

$$-\frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2b\sin(c}{}$$

```
[Out] 2/5*b*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+16/15*a*b*sin(d*x+c)*(a+b*cos(d*x
+c))^(1/2)/d+2/15*(23*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c)
)^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-16/15*a*(a^2-b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+
b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2b\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d
*Sqrt[a + b*Cos[c + d*x]]) + (16*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(15*d) + (2*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\,dx = \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}\left(5a^2+3b^2\right)+4\right.$$

$$= \frac{16ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{4}{15}\int$$

$$= \frac{16ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} - \frac{1}{15}\left(8\right.$$

$$= \frac{16ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{((23}{}$$

$$= \frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{15d\sqrt{a+b\cos(c}}$$

Mathematica [A] time = 0.77, size = 177, normalized size = 0.90

$$\frac{b\sin(c+dx)\left(22a^2+28ab\cos(c+dx)+3b^2\cos(2(c+dx))+3b^2\right)-16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{15d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 16*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2
 + 28*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*d*Sqrt[a
 + b*Cos[c + d*x]])
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.86, size = 662, normalized size = 3.36

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+56\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-48\left(\cos^5\left(\frac{d}{}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1
/2*d*x+1/2*c)^7*b^3+56*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*b
^3+22*cos(1/2*d*x+1/2*c)^3*a^2*b-84*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d
*x+1/2*c)^3*b^3-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+8*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+23*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^3-23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-22*cos(1/2*d*x+1/2*c)*a^2*b
+28*cos(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.504 $\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

Optimal. Leaf size=222

$$\frac{2a^3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos}}{3d}$$

```
[Out] 2/3*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+14/3*a*b*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*b*(2*a^2+b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*
x+c))^(1/2)+2*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticP
i(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1
/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2793, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (14*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*b*(2*a^2 + b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*C
os[c + d*x]]) + (2*a^3*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
+ d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{2b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int\frac{\left(\frac{3a^3}{2}+\frac{1}{2}b\left(9a^2+b^2\right)\cos(c+\right.}{\sqrt{a+}}$$

$$=\frac{2b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{2\int\frac{\left(-\frac{3a^3b}{2}-\frac{1}{2}b^2\left(2a^2+b^2\right)\cos(c+dx)\right)\sec(c+}{\sqrt{a+b\cos(c+dx)}}}{3b}$$

$$=\frac{2b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}+a^3\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx+\frac{1}{3}($$

$$=\frac{14ab\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2b^2\sqrt{a+b\cos(c+dx)}\,\sin}{3d}$$

$$=\frac{14ab\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2b\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{3d\sqrt{a+b\cos(c}}$$

Mathematica [C] time = 1.76, size = 379, normalized size = 1.71

$$\frac{4b(9a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2a(6a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] ((4*b*(9*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*a*(6*a^2 + 7*b^2)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[
a + b*Cos[c + d*x]] + ((14*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt
[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*Ar
cSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(
-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
+ b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[
a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/Sqrt[-(a + b)^(-1)] + 4*b^2*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

fricas [F] time = 2.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

maple [A] time = 0.77, size = 528, normalized size = 2.38

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-6\left(\cos^3\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^3+2*cos(1/2*d*x+1/2*c)^3*a*b^2-6*cos(1/2*d*x+1/2*c)^3*b^3+2
*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2*b-7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-3*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*cos(1/2*d*x+1/2*c)*a*b
^2+2*cos(1/2*d*x+1/2*c)*b^3)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c),x)
[Out] Timed out
```

3.505 $\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=222

$$\frac{a(a^2+4b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(a^2-2b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a^2\tan(c+dx)\sqrt{a}}{d}$$

```
[Out] -(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(
d*x+c))/(a+b))^(1/2)+a*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x
+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5*a^2*b*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(
1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a^2*(a+b*cos(
d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.59, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2792, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{a(a^2+4b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(a^2-2b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a^2\tan(c+dx)\sqrt{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] -(((a^2 - 2*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (a*(a^2 + 4*b^2)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
 b*Cos[c + d*x]]) + (5*a^2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*Sqrt[a
+ b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \frac{a^2\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} + \int \frac{\left(\frac{5a^2b}{2}+3ab^2\cos(c+dx)-\frac{1}{2}b\left(\right.\right.}{\sqrt{a+b}}$$

$$= \frac{a^2\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} - \frac{\int \frac{\left(-\frac{5}{2}a^2b^2-\frac{1}{2}ab\left(a^2+4b^2\right)\cos(c+dx)\right)\sec(c+}{\sqrt{a+b\cos(c+dx)}}}{b}$$

$$= \frac{a^2\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} + \frac{1}{2}\left(5a^2b\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,d$$

$$= -\frac{\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a^2\sqrt{a+b\cos(c+}}{d}$$

$$= -\frac{\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a\left(a^2+4b^2\right)\sqrt{\frac{a+b}{}}}{d\sqrt{a+}}$$

Mathematica [C] time = 2.19, size = 390, normalized size = 1.76

$$\frac{\frac{2b(9a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(a^2-2b^2)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] ((24*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(9*a^2 + 2*b^2)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos
[c + d*x]] + ((2*I)*(a^2 - 2*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*
Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
+ b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*S
qrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4
*a^2*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

maple [B] time = 0.84, size = 960, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*b*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a^3-2*a^2*b)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3+4*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^2*b+2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2
-2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-5*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+4*b^2*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^3+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2*b+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-5*a^2*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.506 $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=270

$$\frac{b\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a^2\tan(c+dx)}{}$$

```
[Out] -9/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+
c))/(a+b))^(1/2)+1/4*b*(11*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(
d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*a*(4*a^2+15*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/
2)+9/4*a*b*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*(a+b*cos(
d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.88, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2792, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a^2\tan(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] (-9*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (b*(11*a^2 + 8*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*
Cos[c + d*x]]) + (a*(4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (9
*a*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a^2*Sqrt[a + b*Cos[c +
 d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \frac{a^2\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \frac{\left(\frac{9a^2b}{2}+a\left(a^2+\right.\right.}{}$$

$$=\frac{9ab\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d}+\frac{a^2\sqrt{a+b\cos(c+dx)}\,\sec(c+a}{2d}$$

$$=\frac{9ab\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d}+\frac{a^2\sqrt{a+b\cos(c+dx)}\,\sec(c+a}{2d}$$

$$=\frac{9ab\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d}+\frac{a^2\sqrt{a+b\cos(c+dx)}\,\sec(c+a}{2d}$$

$$=-\frac{9ab\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{9ab\sqrt{a+b\cos(c+dx)}\,\mathrm{t}}{4d}$$

$$=-\frac{9ab\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{b\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(}{a+b}}}{4d\sqrt{a+b\,\mathrm{co}}}$$

Mathematica [C] time = 2.65, size = 395, normalized size = 1.46

$$\frac{\frac{4b\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{a\left(8a^2+21b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+2a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] ((4*b*(a^2 + 4*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (a*(8*a^2 + 21*b^2)*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a
 + b*Cos[c + d*x]] - ((9*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2
*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
```

```
+ b*Cos[c + d*x]]], (a + b)/(a - b)]]))/Sqrt[-(a + b)^(-1)] + 2*a*Sqrt[a +
b*Cos[c + d*x]]*(2*a + 9*b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(8*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx + c) + a)^{\frac{5}{2}} \sec(dx + c)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

maple [B] time = 0.96, size = 1134, normalized size = 4.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x)
```

```
[Out] -1/4*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-72*a*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(44*a^2*b+72*a*b^2)*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+(-4*a^3-22*a^2*b-18*a*b^2)*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(11*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b+8*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-4*Elliptic
Pi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-15*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))*a*b^2-9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^2*b+9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2)*s
in(1/2*d*x+1/2*c)^4-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(11*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^2*b+8*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-4*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-15*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b^2-9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a^2*b+9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2)
*sin(1/2*d*x+1/2*c)^2+11*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
```

```
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2*b+8*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-15*b^2*a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-9*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a*b^2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2/(-2*sin(1/2*d*x+
1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2
*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
[Out] Timed out
```

3.507 $\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

Optimal. Leaf size=323

$$\frac{(16a^2+33b^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{a(16a^2+59b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}-\frac{(16a^2+33b^2)\sqrt{}}{}$$

```
[Out] -1/24*(16*a^2+33*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/(
(a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*a*(16*a^2+59*b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5/8*b*(4*a^2+b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos
(d*x+c))^(1/2)+1/24*(16*a^2+33*b^2)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+13/
12*a*b*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/3*a^2*sec(d*x+c)^2*
(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 1.17, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2792, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(16a^2+33b^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{a(16a^2+59b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}-\frac{(16a^2+33b^2)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] -((16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (a*(16*a^2 + 59*b^2)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(2
4*d*Sqrt[a + b*Cos[c + d*x]]) + (5*b*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c
 + d*x]]) + ((16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*d
) + (13*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d) + (a
^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \frac{a^2\sqrt{a+b\cos(c+dx)}\,\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int\frac{\left(\frac{13a^2b}{2}+a\left(2a^2\right.\right.}{}$$

$$=\frac{13ab\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{12d}+\frac{a^2\sqrt{a+b\cos(c+}}{}$$

$$=\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{24d}+\frac{13ab\sqrt{a+b\cos(c+}}{}$$

$$=\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{24d}+\frac{13ab\sqrt{a+b\cos(c+}}{}$$

$$=\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{24d}+\frac{13ab\sqrt{a+b\cos(c+}}{}$$

$$=-\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(16a^2+33b^2\right.}{}$$

$$=-\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a\left(16a^2+59\right.}{2}$$

Mathematica [C] time = 4.12, size = 434, normalized size = 1.34

$$\frac{\frac{2b\left(104a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(\left(8a^2+\frac{33b^2}{2}\right)\sin(2(c+dx))+8a^2\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] ((104*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)
/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(104*a^2 - 3*b^2)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*
Cos[c + d*x]] - ((2*I)*(16*a^2 + 33*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
+ b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)*E
```

```
llipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
+ d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(
-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*(26*a*b*Sin[c + d*x] + (8
*a^2 + (33*b^2)/2)*Sin[2*(c + d*x)] + 8*a^2*Tan[c + d*x]))/(96*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

maple [B] time = 1.11, size = 1742, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x)
```

```
[Out] -1/24*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((256*a^2
*b+528*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-128*a^3-384*a^2*b-472
*a*b^2-792*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(128*a^3+328*a^2*b+
472*a*b^2+396*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-48*a^3-100*a^2
*b-118*a*b^2-66*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-8*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+59*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^3+16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-33
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+33*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-60*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))*a^2*b-15*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
```

```
))*b^3)*sin(1/2*d*x+1/2*c)^6+12*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^3+59*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
^2-16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+16*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-33*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a*b^2+33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*b^3-60*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b-15*Ellip
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^3)*sin(1/2*d*x+1/2*c)^4-6*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(16*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+59*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-16*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^3+16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b-33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+33*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-60*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b-15*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b
/(a-b))^(1/2))*b^3)*sin(1/2*d*x+1/2*c)^2+16*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^3+59*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-16*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3+16*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-33*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+33*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-60*a^2*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-15*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)^3/(-2*sin(1/2*d*x+1/2
*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*
x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
[Out] Timed out
```

3.508 $\int (a+b\cos(c+dx))^{7/2}\,dx$

Optimal. Leaf size=246

$$\frac{2b\left(71a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}+\frac{32a\left(11a^2+13b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(7\right.}{}$$

```
[Out] 24/35*a*b*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*b*(a+b*cos(d*x+c))^(5/2)*
sin(d*x+c)/d+2/105*b*(71*a^2+25*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+32
/105*a*(11*a^2+13*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/
((a+b*cos(d*x+c))/(a+b))^(1/2)-2/105*(71*a^4-46*a^2*b^2-25*b^4)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(71a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(-46a^2b^2+71a^4-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{a+b\cos(c+dx)}}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (32*a*(11*a^2 + 13*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)])/(105*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(71*a^4 - 46*a
^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*b)/(a + b)])/(105*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(71*a^2 + 25*b^2)*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (24*a*b*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*d) + (2*b*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x]
)/(7*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{7/2}\,dx = \frac{2b(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2}{7}\int (a+b\cos(c+dx))^{3/2}\left(\frac{1}{2}\left(7a^2+5b^2\right)+\right.$$

$$= \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2b(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{4}{35}$$

$$= \frac{2b\left(71a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105d} + \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(}{35d}$$

$$= \frac{2b\left(71a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105d} + \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(}{35d}$$

$$= \frac{2b\left(71a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105d} + \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(}{35d}$$

$$= \frac{32a\left(11a^2+13b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(71a^4-46a^2b^2-25b^4\right.}{105d\sqrt{a}}$$

Mathematica [A] time = 1.10, size = 211, normalized size = 0.86

$$\frac{-4\left(71a^4-46a^2b^2-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b\sin(c+dx)\left(488a^3+b\left(752a^2+145b^2\right)\cos(c+a\right.}{210}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (64*a*(11*a^3 + 11*a^2*b + 13*a*b^2 + 13*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(71*a^4 - 46*a^2*b^2 - 25*b
^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
] + b*(488*a^3 + 262*a*b^2 + b*(752*a^2 + 145*b^2)*Cos[c + d*x] + 162*a*b^2
*Cos[2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*d*Sqrt[a +
b*Cos[c + d*x]])
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sqrt{b\cos(dx+c)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sqrt(b*cos(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.92, size = 824, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos
(1/2*d*x+1/2*c)^9*b^4+648*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)
^7*b^4+752*cos(1/2*d*x+1/2*c)^5*a^2*b^2-1296*cos(1/2*d*x+1/2*c)^5*a*b^3+640
*cos(1/2*d*x+1/2*c)^5*b^4+244*cos(1/2*d*x+1/2*c)^3*a^3*b-1128*cos(1/2*d*x+1
/2*c)^3*a^2*b^2+860*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4
-71*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+46*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b^4+176*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4
-176*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+208*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-208*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b^3-244*cos(1/2*d*x+1/2*c)*a^3*b+376*cos(1/2*d*x+1/2
*c)*a^2*b^2-212*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\cos(c + d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.509 $\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{59F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{60\sqrt{7}\,d}+\frac{47E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}\,d}+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}-\frac{3\sin(c+dx)(4\cos(c+dx)}{70d}$$

```
[Out] -3/70*(3+4*cos(d*x+c))^(3/2)*sin(d*x+c)/d+1/14*cos(d*x+c)*(3+4*cos(d*x+c))^
(3/2)*sin(d*x+c)/d+47/140*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+59/420*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/
d*7^(1/2)+59/105*sin(d*x+c)*(3+4*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3023, 2753, 2752, 2661, 2653}

$$\frac{59F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{60\sqrt{7}\,d}+\frac{47E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}\,d}+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}-\frac{3\sin(c+dx)(4\cos(c+dx)}{70d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (47*EllipticE[(c + d*x)/2, 8/7])/(20*Sqrt[7]*d) + (59*EllipticF[(c + d*x)/2
, 8/7])/(60*Sqrt[7]*d) + (59*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(105*d)
 - (3*(3 + 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(70*d) + (Cos[c + d*x]*(3 +
4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(14*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx &= \frac{\cos(c+dx)(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{14d}+\frac{1}{14}\int \sqrt{3+4\cos(c+dx}\\
&= -\frac{3(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{70d}+\frac{\cos(c+dx)(3+4\cos(c+dx))^{3/2}}{14d}\\
&= \frac{59\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{105d}-\frac{3(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{70d}\\
&= \frac{59\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{105d}-\frac{3(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{70d}\\
&= \frac{47E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}\,d}+\frac{59F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{60\sqrt{7}\,d}+\frac{59\sqrt{3+4\cos(c+dx)}\,\sin(c+}{105d}
\end{aligned}
$$

Mathematica [A] time = 0.25, size = 92, normalized size = 0.67

$$
\frac{59\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+141\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+(212\sin(c+dx)+9\sin(2(c+dx))+30\sin(3(c+dx)))\sqrt{4\cos(}}{420d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (141*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + 59*Sqrt[7]*EllipticF[(c + d*x)/2
, 8/7] + Sqrt[3 + 4*Cos[c + d*x]]*(212*Sin[c + d*x] + 9*Sin[2*(c + d*x)] +
30*Sin[3*(c + d*x)]))/(420*d)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$
\text{integral}\left(\sqrt{4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^{3},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^{3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

maple [A] time = 0.65, size = 275, normalized size = 1.99

$$-\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(7680\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-14976\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/420*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7680*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-14976*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+12344*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+413*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^
(1/2))-141*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-4480*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\sqrt{4\cos\left(c+d\,x\right)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.510 $\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{5d}$$

```
[Out] 1/10*(3+4*cos(d*x+c))^(3/2)*sin(d*x+c)/d+21/20*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/
20*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2/7*14^(1/2))/d*7^(1/2)-1/5*sin(d*x+c)*(3+4*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2791, 2753, 2752, 2661, 2653}

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(20*d) - (Sqrt[7]*EllipticF[(c + d
*x)/2, 8/7])/(20*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(5*d) + ((3 +
 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx &= \frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}+\frac{1}{10}\int(6-3\cos(c+dx))\sqrt{3+4\cos(\\ &= -\frac{\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{5d}+\frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}+\\ &= -\frac{\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{5d}+\frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}-\\ &= \frac{21\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}-\frac{\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}-\frac{\sqrt{3+4\cos(c+dx)}\,\sin(c}{5d}\end{aligned}$$

Mathematica [A] time = 0.14, size = 81, normalized size = 0.77

$$\frac{-\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+21\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+2(\sin(c+dx)+2\sin(2(c+dx)))\sqrt{4\cos(c+dx)+3}}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[3 + 4*Cos[c + d*x]],x]
```

[Out] (21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] - Sqrt[7]*EllipticF[(c + d*x)/2, 8/7] + 2*Sqrt[3 + 4*Cos[c + d*x]]*(Sin[c + d*x] + 2*Sin[2*(c + d*x)]))/(20*d)

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)

maple [A] time = 0.66, size = 253, normalized size = 2.41

$$-\frac{\sqrt{\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-256\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+384\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x)

[Out] -1/20*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-256*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+384*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-140*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,\sqrt{4\,\cos(c+d\,x)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos(c+dx)+3}\,\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*cos(c + d*x)**2, x)
```

3.511 $\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{6d}+\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

```
[Out] 1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+2/3*sin(d*x+c)*
(3+4*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2753, 2752, 2661, 2653}

$$\frac{\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{6d}+\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) + (Sqrt[7]*EllipticF[(c + d*x)/
2, 8/7])/(6*d) + (2*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \frac{2\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{2+\frac{3}{2}\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{4}\int \sqrt{3+4\cos(c+dx)}\,dx + \frac{7}{12}\int$$
$$= \frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d} + \frac{\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{6d} + \frac{2\sqrt{3+4\cos(c+dx)}\,\sin(c+}{3d}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 0.88

$$\frac{\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+3\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+4\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (3*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + Sqrt[7]*EllipticF[(c + d*x)/2, 8/7
] + 4*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos(dx+c)+3}\,\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

maple [A] time = 0.62, size = 231, normalized size = 2.96

$$-\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(64\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{8\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{6\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/6*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x
+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2*2^(1/2))-56*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-8*sin(1/2*d*x+1/
2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos(dx+c)+3}\,\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\sqrt{4\cos(c+d\,x)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2),x)
[Out] int(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos(c+dx)+3}\,\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*cos(c + d*x), x)
```

3.512 $\int \sqrt{3+4\cos(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2653}

$$\frac{2\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{3+4\cos(c+dx)}\,dx = \frac{2\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{2\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{4\cos\left(dx+c\right)+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{4\cos\left(dx+c\right)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*cos(d*x + c) + 3), x)
```

maple [B] time = 0.42, size = 137, normalized size = 5.96

$$\frac{2\sqrt{\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\sqrt{}\right.}{\sqrt{-8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*cos(d*x+c))^(1/2),x)
[Out] 2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2*2^(1/2))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{4\cos\left(dx+c\right)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{4\cos(c+dx)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(c + d*x) + 3)^(1/2),x)
[Out] int((4*cos(c + d*x) + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos(c+dx)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(4*cos(c + d*x) + 3), x)
```

3.513 $\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{8F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}+\frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

```
[Out] 8/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)+6/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2803, 2661, 2805}

$$\frac{8F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}+\frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (8*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d) + (6*EllipticPi[2, (c + d*x)/2,
 8/7])/(Sqrt[7]*d)
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{3+4\cos(c+dx)}\,\sec(c+dx)\,dx = 3\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx + 4\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx$$
$$= \frac{8F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d} + \frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Mathematica [A] time = 0.05, size = 41, normalized size = 0.85

$$\frac{8F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x],x]
[Out] (8*EllipticF[(c + d*x)/2, 8/7] + 6*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7
]*d)
```

fricas [F] time = 2.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{4\cos\left(dx+c\right)+3}\,\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos\left(dx+c\right)+3}\,\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

maple [A] time = 0.55, size = 158, normalized size = 3.29

$$-\frac{2\sqrt{\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(4\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(4*EllipticF(cos(1/2*d*x+1/2
*c),2*2^(1/2))-3*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/(-8*sin(1/2*d*
x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\,\cos\left(dx+c\right)+3}\,\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{4\,\cos\left(c+d\,x\right)+3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x),x)
```

```
[Out] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos\left(c+dx\right)+3}\,\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x), x)
```

3.514 $\int \sqrt{3+4\cos(c+dx)}\,\sec^2(c+dx)\,dx$

Optimal. Leaf size=95

$$\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}+\frac{\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)}{d}$$

```
[Out] 3/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)+4/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/
2))/d*7^(1/2)+(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2796, 3060, 2653, 3002, 2661, 2805}

$$\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}+\frac{\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] -((Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d) + (3*EllipticF[(c + d*x)/2, 8/7]
)/(Sqrt[7]*d) + (4*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d) + (Sqrt[3 +
 4*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx = \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(2-2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}$$

$$= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d} - \frac{1}{4}\int \frac{(-8-6\cos(c+dx))\sec(c+dx}{\sqrt{3+4\cos(c+dx)}}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d} + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d} + \frac{3}{2}\int \frac{}{\sqrt{3+4}}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d} + \frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d} + \frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d} + \frac{\sqrt{3}}{}$$

Mathematica [C] time = 1.12, size = 157, normalized size = 1.65

$$\frac{6\sqrt{7}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right) + 21\sqrt{4\cos(c+dx)+3}\tan(c+dx) + \frac{i\sqrt{7}\sin(c+dx)\left(-12F\left(i\sinh^{-1}\left(\sqrt{4\cos(c+dx)+3}\right)|-\frac{1}{7}\right)+21E\left(i\sinh\right.\right.}{\sqrt{\sin}}}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (6*Sqrt[7]*EllipticPi[2, (c + d*x)/2, 8/7] + (I*Sqrt[7]*(21*EllipticE[I*Arc
Sinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*C
os[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]
]], -1/7])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2] + 21*Sqrt[3 + 4*Cos[c + d*x]]
*Tan[c + d*x])/(21*d)
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{4\cos(dx+c)+3}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos(dx+c)+3}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

maple [B] time = 0.80, size = 350, normalized size = 3.68

$$-\frac{\sqrt{-\left(-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}+\frac{3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-8\left(\cos^2\left(\right.\right.}}{\sqrt{-8\left(\sin^4\left(\right.\right.}}\right.}{\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-8*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2*2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2*2^(1/2))-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos\left(dx+c\right)+3}\,\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{4\cos\left(c+d\,x\right)+3}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^2,x)
[Out] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos(c+dx)+3}\,\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(3+4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x)**2, x)
```

3.515 $\int \sqrt{3+4\cos(c+dx)}\,\sec^3(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{5\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d}+\frac{\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)}{3d}+\frac{\sqrt{4\cos(c+dx)+}}{}$$

```
[Out] 3/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)+5/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-1/3*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*1
4^(1/2))/d*7^(1/2)+1/3*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*sec(d*x+c)*(
3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.36, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2796, 3055, 3059, 2653, 3002, 2661, 2805}

$$\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{5\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d}+\frac{\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)}{3d}+\frac{\sqrt{4\cos(c+dx)+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) + (3*EllipticF[(c + d*x)/2, 8/
7])/(Sqrt[7]*d) + (5*EllipticPi[2, (c + d*x)/2, 8/7])/(3*Sqrt[7]*d) + (Sqrt
[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{3+4\cos(c+dx)}\,\sec^3(c+dx)\,dx = \frac{\sqrt{3+4\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{(2+3\cos(c+dx)+}{\sqrt{3+}}$$

$$= \frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\,\sec(c+dx)\tan(c}{2d}$$

$$= \frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\,\sec(c+dx)\tan(c}{2d}$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c}{}$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} + \frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d} + \frac{5\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d} + \frac{\sqrt{3+}}{}$$

Mathematica [C] time = 1.27, size = 194, normalized size = 1.44

$$\frac{\frac{12F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}} + \frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}} + (2\cos(c+dx)+3)\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)\sec(c+dx) + \frac{2i\sin(c+dx)\left(-12F\left(i\right.\right.}{}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] ((12*EllipticF[(c + d*x)/2, 8/7])/Sqrt[7] + (6*EllipticPi[2, (c + d*x)/2, 8
/7])/Sqrt[7] + (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]]
, -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*Ellip
ticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt
[7]*Sqrt[Sin[c + d*x]^2]) + (3 + 2*Cos[c + d*x])*Sqrt[3 + 4*Cos[c + d*x]]*S
ec[c + d*x]*Tan[c + d*x])/(6*d)
```

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{4\cos(dx+c)+3}\,\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4 \cos(dx+c)+3}\,\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

maple [B] time = 0.95, size = 408, normalized size = 3.02

$$-\frac{\sqrt{-\left(-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}\cdots\right.\right.}}{3\left(2\left(\cos^2\left(\frac{dx}{2}\cdots\right.\right.\right.}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-8*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/
2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)^2-2/3*cos(1/2*d*x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))+1/3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-5/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1
/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(
1/2)))/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4 \cos(dx+c)+3}\,\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{4\,\cos\left(c+d\,x\right)+3}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^3,x)
[Out] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\cos\left(c+dx\right)+3}\,\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(3+4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x)**3, x)
```

3.516 $\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$

Optimal. Leaf size=140

$$-\frac{59F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{60\sqrt{7}d}-\frac{47E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20\sqrt{7}d}-\frac{\sin(c+dx)\cos(c+dx)(3-4\cos(c+dx))^{3/2}}{14d}-\frac{3\sin(c+dx)(3}{}$$

```
[Out] -3/70*(3-4*cos(d*x+c))^(3/2)*sin(d*x+c)/d-1/14*(3-4*cos(d*x+c))^(3/2)*cos(d
*x+c)*sin(d*x+c)/d+47/140*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*E
llipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+59/420*(sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/
d*7^(1/2)+59/105*sin(d*x+c)*(3-4*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3023, 2753, 2752, 2662, 2654}

$$-\frac{59F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{60\sqrt{7}d}-\frac{47E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20\sqrt{7}d}-\frac{\sin(c+dx)\cos(c+dx)(3-4\cos(c+dx))^{3/2}}{14d}-\frac{3\sin(c+dx)(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^3,x]
```

```
[Out] (-47*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (59*EllipticF[(c +
Pi + d*x)/2, 8/7])/(60*Sqrt[7]*d) + (59*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*
x])/(105*d) - (3*(3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(70*d) - ((3 - 4*
Cos[c + d*x])^(3/2)*Cos[c + d*x]*Sin[c + d*x])/(14*d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\,\cos^3(c+dx)\,dx = -\frac{(3-4\cos(c+dx))^{3/2}\cos(c+dx)\sin(c+dx)}{14d} - \frac{1}{14}\int \sqrt{3-4\cos(c+$$

$$= -\frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} - \frac{(3-4\cos(c+dx))^{3/2}\cos(c+d}{14d}$$

$$= \frac{59\sqrt{3-4\cos(c+dx)}\,\sin(c+dx)}{105d} - \frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx}{70d}$$

$$= \frac{59\sqrt{3-4\cos(c+dx)}\,\sin(c+dx)}{105d} - \frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx}{70d}$$

$$= -\frac{47E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{20\sqrt{7}\,d} - \frac{59F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{60\sqrt{7}\,d} + \frac{59\sqrt{3-4\cos(c+}{105}$$

Mathematica [A] time = 0.21, size = 114, normalized size = 0.81

$$\frac{654\sin(c+dx)-511\sin(2(c+dx))+108\sin(3(c+dx))-60\sin(4(c+dx))-413\sqrt{4\cos(c+dx)-3}\,F\left(\frac{1}{2}(c+\right.}{420d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^3,x]
```

```
[Out] (141*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] - 413*Sqrt[-3 + 4*
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 654*Sin[c + d*x] - 511*Sin[2*(c +
 d*x)] + 108*Sin[3*(c + d*x)] - 60*Sin[4*(c + d*x)])/(420*d*Sqrt[3 - 4*Cos[
c + d*x]])
```

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-4\cos(dx+c)+3}\,\cos(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\cos(dx+c)+3}\,\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

maple [A] time = 0.75, size = 276, normalized size = 1.97

$$\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(7680\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8064\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/420*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7680*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-8064*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+5432*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+59*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14
^(1/2))+141*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-568*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\cos(dx+c)+3}\,\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^3\,\sqrt{3-4\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.517 $\int \sqrt{3-4\cos(c+dx)}\,\cos^2(c+dx)\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}+\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}+\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{5d}$$

```
[Out] -1/10*(3-4*cos(d*x+c))^(3/2)*sin(d*x+c)/d-21/20*(sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1
/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)+1/5*sin(d*x+c)*(3-4*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2791, 2753, 2752, 2662, 2654}

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}+\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}+\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^2,x]
```

```
[Out] (21*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*d) - (Sqrt[7]*EllipticF[(
c + Pi + d*x)/2, 8/7])/(20*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(5*
d) - ((3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-4\cos(c+dx)}\,\cos^2(c+dx)\,dx &= -\frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}-\frac{1}{10}\int \sqrt{3-4\cos(c+dx)}\,(-6-3\\ &= \frac{\sqrt{3-4\cos(c+dx)}\,\sin(c+dx)}{5d}-\frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}-\\ &= \frac{\sqrt{3-4\cos(c+dx)}\,\sin(c+dx)}{5d}-\frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}-\\ &= \frac{21\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{20d}-\frac{\sqrt{7}\,F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{20d}+\frac{\sqrt{3-4\cos(c+}}{5}\end{aligned}$$

Mathematica [A] time = 0.21, size = 104, normalized size = 0.97

$$-\frac{14\sin(c+dx)-16\sin(2(c+dx))+8\sin(3(c+dx))+7\sqrt{4\cos(c+dx)-3}\,F\left(\frac{1}{2}(c+dx)\middle|8\right)+21\sqrt{4\cos(c+d}}{20d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^2,x]
```

```
[Out] -1/20*(21*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 7*Sqrt[-3 +
 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 14*Sin[c + d*x] - 16*Sin[2*(c
+ d*x)] + 8*Sin[3*(c + d*x)])/(d*Sqrt[3 - 4*Cos[c + d*x]])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

maple [A] time = 0.72, size = 253, normalized size = 2.36

$$\frac{\sqrt{-\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-256\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/20*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-256*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2/7*14^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/
2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-12*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\sqrt{3-4\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 - 4\cos{\left(c + dx \right)}}\cos^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(3-4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*cos(c + d*x)**2, x)
```

3.518 $\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{6d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{2d}+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}$$

```
[Out] 1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)+1/6*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+2/3*sin(d*x+c)*
(3-4*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2753, 2752, 2662, 2654}

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{6d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{2d}+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) - (Sqrt[7]*EllipticF[(c +
 Pi + d*x)/2, 8/7])/(6*d) + (2*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx = \frac{2\sqrt{3-4\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{-2+\frac{3}{2}\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{3-4\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{1}{4}\int \sqrt{3-4\cos(c+dx)}\,dx-\frac{7}{12}\int$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{2d}-\frac{\sqrt{7}\,F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{6d}+\frac{2\sqrt{3-4\cos(c+}{3d}$$

Mathematica [A] time = 0.10, size = 94, normalized size = 1.18

$$\frac{12\sin(c+dx)-8\sin(2(c+dx))-7\sqrt{4\cos(c+dx)-3}\,F\left(\frac{1}{2}(c+dx)\middle|8\right)+3\sqrt{4\cos(c+dx)-3}\,E\left(\frac{1}{2}(c+dx)\middle|8\right)}{6d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x],x]
```

```
[Out] (3*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] - 7*Sqrt[-3 + 4*Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 8] + 12*Sin[c + d*x] - 8*Sin[2*(c + d*x)])
/(6*d*Sqrt[3 - 4*Cos[c + d*x]])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-4\cos(dx+c)+3}\,\cos(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

maple [A] time = 0.74, size = 231, normalized size = 2.89

$$\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(64\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{6\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+
1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2/7*14^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(8*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1
/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + d\,x) \sqrt{3 - 4 \cos(c + d\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 - 4\cos(c + dx)}\,\cos(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*cos(c + d*x), x)
```

3.519 $\int \sqrt{3-4\cos(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{2\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}$$

[Out] -2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2654}

$$\frac{2\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] (2*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\,dx=\frac{2\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{d}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.83

$$-\frac{2\sqrt{4\cos(c+dx)-3}\,E\left(\frac{1}{2}(c+dx)\middle|8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]],x]

```
[Out] (-2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos[
c + d*x]])
```

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-4\,\cos\left(dx+c\right)+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-4\,\cos\left(dx+c\right)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3), x)
```

maple [B] time = 0.46, size = 138, normalized size = 5.75

$$-\frac{2\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\,\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2/7*14^(1/2))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-4\,\cos\left(dx+c\right)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{3-4\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 4*cos(c + d*x))^(1/2),x)
[Out] int((3 - 4*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3-4\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(3 - 4*cos(c + d*x)), x)
```

3.520 $\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=50

$$-\frac{8F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{6\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

```
[Out] 8/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)+6/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2803, 2662, 2806}

$$-\frac{8F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{6\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (-8*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d) - (6*EllipticPi[2, (c + P
i + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
```

```
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx = 3\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx - 4\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx$$
$$= -\frac{8F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d} - \frac{6\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 1.22

$$\frac{2\sqrt{4\cos(c+dx)-3}\left(3\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)-4F\left(\frac{1}{2}(c+dx)\middle|8\right)\right)}{d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*(-4*EllipticF[(c + d*x)/2, 8] + 3*EllipticPi[2
, (c + d*x)/2, 8]))/(d*Sqrt[3 - 4*Cos[c + d*x]])
```

fricas [F] time = 2.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-4\cos(dx+c)+3}\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\cos(dx+c)+3}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

maple [A] time = 0.80, size = 159, normalized size = 3.18

$$\frac{2\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\,\left(4\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/7*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*(4*EllipticF(cos(1/2*d*x+1
/2*c),2/7*14^(1/2))+3*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/(8*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/
2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\,\cos\left(dx+c\right)+3}\,\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{3-4\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3-4\cos\left(c+dx\right)}\,\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x), x)
```

3.521 $\int \sqrt{3-4\cos(c+dx)}\,\sec^2(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{3F\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{\sqrt{7}\,d}+\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{d}$$

```
[Out] -3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)-4/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)+(sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1
/2))/d*7^(1/2)+(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2796, 3060, 2654, 3002, 2662, 2806}

$$\frac{3F\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{\sqrt{7}\,d}+\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] -((Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d) + (3*EllipticF[(c + Pi + d*
x)/2, 8/7])/(Sqrt[7]*d) + (4*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\,\sec^2(c+dx)\,dx = \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{d} + \int \frac{\left(-2+2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{d} + \frac{1}{4}\int \frac{(-8+6\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{d} + \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{d} + \frac{3}{2}\int \frac{}{\sqrt{3}}$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{d} + \frac{3F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d} + \frac{4\Pi\left(2;\frac{1}{2}(c+\pi+dx\right.}{\sqrt{7}\,d}$$

Mathematica [C] time = 1.47, size = 178, normalized size = 1.82

$$\frac{21\sqrt{3-4\cos(c+dx)}\,\tan(c+dx) - \frac{42\sqrt{4\cos(c+dx)-3}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}} - \frac{i\sqrt{7}\,\sin(c+dx)\left(-12F\left(i\sinh^{-1}\left(\sqrt{3-4\cos(c+dx)}\right)|-\frac{1}{7}\right)+21E\left(i\,s\right.\right.}{\sqrt{}}}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] ((-42*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*C
os[c + d*x]] - (I*Sqrt[7]*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]]
, -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*Ellip
ticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/Sqrt[
Sin[c + d*x]^2] + 21*Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(21*d)
```

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-4\cos(dx+c)+3}\,\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\cos(dx+c)+3}\,\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

maple [B] time = 0.88, size = 351, normalized size = 3.58

$$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}+\frac{3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}\cdots\right.\right.}}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}\cdots\right.\right.}}\cdots\right.}{\sin\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)+3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2
)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2
-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2/7*14^(1/2))+4/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*
d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/sin(1/2*d*x+1/2*c)/(-8*cos(1/
2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-4\cos\left(dx+c\right)+3}\,\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{3-4\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
[Out] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 - 4\cos(c + dx)}\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(3-4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.522 $\int \sqrt{3-4\cos(c+dx)}\,\sec^3(c+dx)\,dx$

Optimal. Leaf size=138

$$-\frac{3F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}+\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{5\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}\,d}-\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d}+\frac{\sqrt{\cdot}}{}$$

```
[Out] 3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-1/3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*1
4^(1/2))/d*7^(1/2)-1/3*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*sec(d*x+c)*(
3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.37, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2796, 3055, 3059, 2654, 3002, 2662, 2806}

$$-\frac{3F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}+\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{5\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}\,d}-\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d}+\frac{\sqrt{\cdot}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) - (3*EllipticF[(c + Pi + d
*x)/2, 8/7])/(Sqrt[7]*d) - (5*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3*Sqrt
[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 - 4*Cos[c
+ d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\,\sec^3(c+dx)\,dx = \frac{\sqrt{3-4\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{(-2+3\cos(c+dx}{\sqrt{3}}$$

$$= -\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\,\sec(c+dx)\tan}{2d}$$

$$= -\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\,\sec(c+dx)\tan}{2d}$$

$$= \frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} - \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos}}{}$$

$$= \frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} - \frac{3F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d} - \frac{5\Pi\left(2;\frac{1}{2}(c+\pi+dx\right.}{3\sqrt{7}\,d}$$

Mathematica [C] time = 1.88, size = 237, normalized size = 1.72

$$\frac{-\frac{12\sqrt{4\cos(c+dx)-3}\,F\left(\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}} + \frac{6\sqrt{4\cos(c+dx)-3}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}} - \sqrt{3-4\cos(c+dx)}\,(2\cos(c+dx)-3)\tan(c+dx)\,\mathrm{se}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] ((-12*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c
 + d*x]] + (6*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt
[3 - 4*Cos[c + d*x]] + (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c
+ d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] -
 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x
])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) - Sqrt[3 - 4*Cos[c + d*x]]*(-3 + 2*Cos[c
+ d*x])*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

fricas [F] time = 1.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-4\cos(dx+c)+3}\,\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\cos(dx+c)+3}\,\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

maple [B] time = 1.04, size = 408, normalized size = 2.96

$$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}+\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-(\cdots}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\cdots}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2
*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^2+2/3*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*s
in(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-5/21*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)
))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\cos(dx+c)+3}\,\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3-4\ \cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
[Out] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 - 4 \cos{\left(c + d x \right)}} \sec^{3}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(3-4*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.523 $\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=215

$$-\frac{2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{8a\sin(c+$$

```
[Out] -8/15*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/5*cos(d*x+c)*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)/b/d+2/15*(8*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+
b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*a*(8*a^2+7*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos
(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2793, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{8a\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(8*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(8*a^2 + 7*b^2
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (8*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(15*b^2*d) + (2*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(5*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5bd} + \frac{2\int \frac{a+\frac{3}{2}b\cos(c+dx)-2a\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{5b}$$

$$= -\frac{8a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5bd} +$$

$$= -\frac{8a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5bd} -$$

$$= -\frac{8a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5bd} +$$

$$= \frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F}{15b^3d\sqrt{a+b\cos(c-}}$$

Mathematica [A] time = 0.91, size = 182, normalized size = 0.85

$$\frac{b\sin(c+dx)\left(-8a^2-2ab\cos(c+dx)+3b^2\cos(2(c+dx))+3b^2\right)-2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(8*a^3 + 8*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(8*a^2 + 7*b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-8*a^2 + 3*b^
2 - 2*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*b^3*d*Sq
rt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.82, size = 665, normalized size = 3.09

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\,b^2-48\left(\cos^5\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1
/2*d*x+1/2*c)^7*b^3-4*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*b^
3-8*cos(1/2*d*x+1/2*c)^3*a^2*b+6*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d*x+
1/2*c)^3*b^3-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-7*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+8*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^3-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+8*cos(1/2*d*x+1/2*c)*a^2*b-2*cos
(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/b^3/(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/
2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.524 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=165

$$\frac{2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

```
[Out] 2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-4/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(2*a^2+b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d
*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2791, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^
2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*d*Sqrt[a +
 b*Cos[c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{b}{2}-a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{3b}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{1}{3}\left(1+\frac{2a^2}{b^2}\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx - \frac{(2a)\int}{}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{\left(2a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{3b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{4a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(1+\frac{2a^2}{b^2}\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\right.}{3d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.62, size = 137, normalized size = 0.83

$$\frac{2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2b\sin(c+dx)(a+b\cos(c+dx))-4a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c\right.}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-4*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + 2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*b^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.94, size = 453, normalized size = 2.75

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2+2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+
1/2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+
b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

mupad [B] time = 0.56, size = 116, normalized size = 0.70

$$\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)\left(2a^2+b^2\right)-2a\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|\frac{2b}{a+b}\right)(a+b)\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*sin(c + d*x)*(a + b*cos(c + d*x))^(1/2))/(3*b*d) + (2*((a + b*cos(c + d*
x))/(a + b))^(1/2)*(ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*(2*a^2 + b^2) -
 2*a*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b)))/(3*b^2*d*(a + b*cos(
c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

3.525 $\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=122

$$\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(
a+b))^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b
/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\int \sqrt{a+b\cos(c+dx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{\left(a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{b\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 2.42, size = 86, normalized size = 0.70

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)
/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sqrt[a + b*Cos[c
 + d*x]])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.77, size = 220, normalized size = 1.80

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [B] time = 0.66, size = 80, normalized size = 0.66

$$\frac{2\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,(a+b)-a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\right)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{b\,d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d*
x)/2, (2*b)/(a + b)))*((a + b*cos(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*cos
(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

3.526 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+
c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2663, 2661}

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 1.00

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)
```

maple [C] time = 0.10, size = 75, normalized size = 1.32

$$\frac{2\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a+b}}\ \mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\frac{\sqrt{2}\ \sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)
```

mupad [B] time = 0.60, size = 52, normalized size = 0.91

$$\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\cos\left(c+d\,x\right)}{a+b}}}{d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*((a + b*cos(c + d*x))/(a + b))^(
1/2))/(d*(a + b*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cos(c + d*x)), x)
```

3.527 $\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d
*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2807, 2805}

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}$$
$$= \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.09, size = 58, normalized size = 1.00

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.60, size = 166, normalized size = 2.86

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(c+d\,x)\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

3.528 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=206

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{b\sqrt{\frac{a+b\cos\cdots}{\cdots}}\cdots}{a\cdots}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+c))/(a
+b))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b
*cos(d*x+c))^(1/2)-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

Rubi [A] time = 0.49, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2802, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{b\sqrt{\frac{a+b\cos\cdots}{\cdots}}\cdots}{a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + (Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a
*d*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(a*d
)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} + \frac{\int \frac{\left(-\frac{b}{2}-\frac{1}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} - \frac{\int \sqrt{a+b\cos(c+dx)}\,dx}{2a} - \frac{\int \frac{\left(\frac{b^2}{2}-\frac{1}{2}ab\cos(c+dx)\right)\sec(c}{\sqrt{a+b\cos(c+dx)}}}{ab}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} + \frac{1}{2}\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx - \frac{b\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,d}{2a}$$

$$= -\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} + \frac{\sqrt{\frac{a+b\cos}{a+}}}{2}$$

$$= -\frac{\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} - \frac{b\sqrt{\frac{a+b\cos}{a}}}{a}$$

Mathematica [C] time = 8.50, size = 310, normalized size = 1.50

$$\frac{4\tan(c+dx)\sqrt{a+b\cos(c+dx)} - \frac{6b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sin\right.\right.\right.}{}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((-6*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/
(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*
EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/
(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
 + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^
(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 1.16, size = 532, normalized size = 2.58

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{a\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))+1/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/a*b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

3.529 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=268

$$\frac{\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{3b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 3/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/d/((a+b*cos(d*x
+c))/(a+b))^(1/2)-1/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^
(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)-3/4*b*(a
+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a^2/d+1/2*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)
*tan(d*x+c)/a/d
```

Rubi [A] time = 0.71, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2+3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{3b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (3*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2
*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (b*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*x]])
 + ((4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d
*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) - (3*b*Sqrt[a + b
*Cos[c + d*x]]*Tan[c + d*x])/(4*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-\frac{3b}{2}+a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)\right)\sec^2(c+d}{\sqrt{a+b\cos(c+dx)}}}{2a}$$

$$= -\frac{3b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2ad}$$

$$= -\frac{3b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2ad}$$

$$= -\frac{3b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2ad}$$

$$= \frac{3b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{3b\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \sqrt{a$$

$$= \frac{3b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}} + (4+$$

Mathematica [C] time = 6.40, size = 518, normalized size = 1.93

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\tan(c+dx)\sec(c+dx)}{2a} - \frac{3b\tan(c+dx)}{4a^2}\right)}{d} + \frac{\frac{2(8a^2+9b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{6ib^2\sin(c+dx)\cos(2(c+dx))}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 + 9*b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c +
d*x]] - ((6*I)*b^2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c +
 d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
```

```
*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos
[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c +
d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d
*x])^2)))/(16*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((-3*b*Tan[c + d*x])/(4*a^
2) + (Sec[c + d*x]*Tan[c + d*x])/(2*a)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.99, size = 710, normalized size = 2.65

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{a\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}+\frac{3b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\cdots}}{2\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/a*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)^2+3/2*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/4*b
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
```

```
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/4/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-3/4*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a
-b))^(1/2))-3/4/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)/sin(1/2*d*x+1
/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**3/sqrt(a + b*cos(c + d*x)), x)
```

3.530 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{8a\left(4a^2+b^2\right)\sqrt{\frac{a+b\cos}{a}}}{5b^4d\sqrt{a+}}$$

```
[Out] -2*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2/5*a*(
8*a^2-3*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d+2/5*(6*a^2-b
^2)*cos(d*x+c)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d+2/5*(16*a^
4-8*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/(a
^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-8/5*a*(4*a^2+b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+
b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2792, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2a\left(8a^2-3b^2\right)\sin(c}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(5*b^4*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) - (8*a*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(5*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c +
d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a*(8*a
^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^3*(a^2 - b^2)*d) +
(2*(6*a^2 - b^2)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2
*(a^2 - b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)\left(2a^2-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}\left(6a^2-b^2\right)\cos^2(c+dx)\right.}{\sqrt{a+b\cos(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(6a^2-b^2\right)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\ \mathrm{s}}{5b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\ \sin(c+dx}{5b^3\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\ \sin(c+dx}{5b^3\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\ \sin(c+dx}{5b^3\left(a^2-b^2\right)d}$$

$$= \frac{2\left(16a^4-8a^2b^2-3b^4\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5b^4\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{8a\left(4a^2+b^2\right)\sqrt{\frac{a+}{}}}{5b^4d\sqrt{a}}$$

Mathematica [A] time = 1.31, size = 242, normalized size = 0.74

$$\frac{-b\sin(c+dx)\left(16a^4+4ab\left(a^2-b^2\right)\cos(c+dx)-7a^2b^2+\left(b^4-a^2b^2\right)\cos(2(c+dx))+b^4\right)-8a\left(4a^4-3a^2b^2-\right.}{5b^4d(a-b)(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(16*a^5 + 16*a^4*b - 8*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4 - 3*b^5)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 8*a*(4*a^4
 - 3*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)] - b*(16*a^4 - 7*a^2*b^2 + b^4 + 4*a*b*(a^2 - b^2)*Cos[c +
 d*x] + (-(a^2*b^2) + b^4)*Cos[2*(c + d*x)])*Sin[c + d*x])/(5*(a - b)*b^4*(
a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\ \cos(dx+c)^4}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^4/(b^2*cos(d*x + c)^2 + 2*a*
b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.13, size = 1285, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5*(-8*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(
a^2-b^2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(a^3-a^2*b-a*b^2+b^3)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*b*(8*a^4+2*a^3*b-4*a^2*b^2-2*a*b^3+b^4)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-16*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+12*(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+4*(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a*b^4+16*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-16*(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-8*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
```

```
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^3*b^2+8*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-3*(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+3*(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^5)/b^4/(a-b)/(a+b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{4}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^4}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.531 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)}+\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c\right.}{3b^3d\sqrt{a+b\cos(c+dx}}$$

```
[Out] -2*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*(4*a^
2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d-2/3*a*(8*a^2-5*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d/((a+b*c
os(d*x+c))/(a+b))^(1/2)+2/3*(8*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*co
s(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)}+\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c\right.}{3b^3d\sqrt{a+b\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(8*a^2 - 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(8
*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)
/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c +
 d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{a^2-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}\left(4a^2-b^2\right)\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} -$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} -$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} -$$

$$= -\frac{2a\left(8a^2-5b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{3b^3d\sqrt{a+b\cos(}}$$

Mathematica [A] time = 0.88, size = 197, normalized size = 0.77

$$\frac{2\left(8a^4-7a^2b^2-b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2b\sin(c+dx)\left(-4a^3+\left(b^3-a^2b\right)\cos(c+dx)+ab^2\right)-2a\left(\right.}{3b^3d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 2*(8*a^4 - 7*a^2*b^2 - b^4)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - 2*b*(
-4*a^3 + a*b^2 + (-(a^2*b) + b^3)*Cos[c + d*x])*Sin[c + d*x])/(3*(a - b)*b^
3*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^3}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3/(b^2*cos(d*x + c)^2 + 2*a*
b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.00, size = 984, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(4*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(a
^2-b^2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*a^3+a^2*b-a*b^2-b^3)*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)+8*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-7
*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b^4-8*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+8*(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+5*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^2*b^2-5*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
```

```
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^3/(a-b
)/(a+b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx + c\right)^{3}}{\left(b\cos\left(dx + c\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^3}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.532 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{b^2d\sqrt{a+b\cos(c+}}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(2*a^2-b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/((a+b*cos(d*x+c
))/(a+b))^(1/2)-4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/
2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2790, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{b^2d\sqrt{a+b\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b^2*d*S
qrt[a + b*Cos[c + d*x]]) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{ab}{2}+\frac{1}{2}\left(2a^2-b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(2a)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b^2} + \frac{\left(2a^2-b^2\right)\int \sqrt{a+}}{b^2\left(a^2\right.}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b}{}}}{b^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\right.}{b^2d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.70, size = 159, normalized size = 0.85

$$\frac{2\left(2a^3+2a^2b-ab^2-b^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2a\left(2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+ab\right.}{b^2d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*b*Sin[c + d*x]))/((
a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2/(b^2*cos(d*x + c)^2 + 2*a*
b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.94, size = 530, normalized size = 2.85

$$4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a^3-4b^2a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\right.\right.}{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-2*b^2*a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-2*a^
2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/b^2/(a-b)/(a+b)/sin(1/2*d*x+1/
2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c + dx\right)}{\left(a + b\cos\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**2/(a + b*cos(c + d*x))**(3/2), x)
```

3.533 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x
+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*(a
^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
 d*x]]) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{b}{2}+\frac{1}{2}a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$
$$= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} - \frac{a\int \sqrt{a+b\cos(c+dx)}\,dx}{b(a^2-b^2)}$$
$$= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{\left(a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} +$$
$$= -\frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}} + \frac{}{(a^2}$$

Mathematica [A] time = 0.53, size = 137, normalized size = 0.81

$$\frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right) + 2ab\sin(c+dx) - 2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)] + 2*a*b*Sin[c + d*x])/((a - b)*b*(a + b)*d*Sqrt[a
 + b*Cos[c + d*x]])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)/(b^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)

maple [A] time = 0.98, size = 373, normalized size = 2.19

$$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a^2-b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{d}{2}\right.\right.}{a-b}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)

[Out] -2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-2*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/b/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\left(a+b\cos(c+dx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

3.534 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2664, 21, 2655, 2653}

$$\frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2 -
b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx &= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2} \\
&= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\int \sqrt{a+b\cos(c+dx)}\,dx}{a^2-b^2} \\
&= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&= \frac{2\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 83, normalized size = 0.78

$$\frac{2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2b\sin(c+dx)}{d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/
(a + b)] - 2*b*Sin[c + d*x])/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

maple [A] time = 0.79, size = 217, normalized size = 2.05

$$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{a-b}}}{(a-b)(a+b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\, \cos(c + d\, x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\cos(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(-3/2), x)
```

3.535 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*b*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b
))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/
2)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*
cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2802, 3059, 2655, 2653, 12, 2807, 2805}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*(a
^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos
[c + d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{1}{2}\left(a^2-b^2\right)-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}b^2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int -\frac{b\left(a^2-b^2\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}\,dx}{ab\left(a^2-b^2\right)} - \frac{b\int \sqrt{a+b\cos(c+}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{a+b\cos(c+dx)}\right)\int}{a\left(a^2-b^2\right)\sqrt{}}$$

$$= -\frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} +$$

$$= -\frac{2b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 5.19, size = 402, normalized size = 2.28

$$\frac{\frac{4b^2\sin(c+dx)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} - \frac{\frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - 2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+}{b-a}}}{}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-(((-4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)
/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(2*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[
c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Co
s[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[
-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ellipt
icF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
```

```
)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
 + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b)))
 + (4*b^2*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(2*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.92, size = 376, normalized size = 2.14

$$\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\;b\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-2b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{a-b}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2))*b^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/a/(a-b)/
(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

3.536 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{b(a^2-3b^2)\sin(c+dx)}{a^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{(a^2-3b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{a^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] b*(a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-(a^2-3*b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((a+b*co
s(d*x+c))/(a+b))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)-3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*co
s(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+tan(d*x+c)/a/d/(a+b*cos
(d*x+c))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2802, 3056, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b(a^2-3b^2)\sin(c+dx)}{a^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{(a^2-3b^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{a^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a
 + b*Cos[c + d*x]]) - (3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(a^2 -
3*b^2)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + Tan[c +
 d*x]/(a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{3b}{2}+\frac{1}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{a}$$

$$= \frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(-\frac{3}{4}b\left(a^2-b^2\right)+\frac{1}{2}ab^2 c\right.}{}}{}$$

$$= \frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\left(\frac{3}{4}b^2\left(a^2-b^2\right)-\frac{1}{4}ab\left(a^2\right.\right.}{\sqrt{a+b}}}{a^2b\left(\right.}$$

$$= \frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{2a}$$

$$= -\frac{\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+\right.}}$$

$$= -\frac{\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{ad\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 4.15, size = 441, normalized size = 1.59

$$\frac{\frac{4\tan(c+dx)\left(a^3+b\left(a^2-3b^2\right)\cos(c+dx)-ab^2\right)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} - \frac{b\left(\frac{2\left(7a^2-9b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i\left(a^2-3b^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}\left(2a\right.}{}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-((b*((-8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(7*a^2 - 9*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*C
os[c + d*x]] - ((2*I)*(a^2 - 3*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))
]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*Ellipti
cE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)
```

```
] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b^2*Sqrt[-(a + b)^(-1)])
))/((a - b)*(a + b))) + (4*(a^3 - a*b^2 + b*(a^2 - 3*b^2)*Cos[c + d*x])*Tan
[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 2.02, size = 894, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2/a^2/s
in(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2/a^2*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))+2/a*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
```

```
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/
a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**(3/2), x)
```

3.537 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$-\frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{5b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{b\left(7a^2-15b^2\right)}{4a}$$

```
[Out] -1/4*b^2*(7*a^2-15*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+1
/4*b*(7*a^2-15*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^3/(
a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-5/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2+15*b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b
*cos(d*x+c))^(1/2)-5/4*b*tan(d*x+c)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/2*sec(d*
x+c)*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos}{a+}}}{4a^3d\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (b*(7*a^2 - 15*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (5*b*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*
a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*
Cos[c + d*x]]) - (b^2*(7*a^2 - 15*b^2)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]]) - (5*b*Tan[c + d*x])/(4*a^2*d*Sqrt[a + b*Cos[c + d
*x]]) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{5b}{2}+a\cos(c+dx)+\frac{3}{2}b\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{2a}$$

$$= -\frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}\left(4a^2+15b^2\right)+\frac{3}{2}ab\cos(c+dx)\right.}{(a+b\cos(c}}{2a}$$

$$= -\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan}{2ad\sqrt{a+b\cos}}$$

$$= -\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan}{2ad\sqrt{a+b\cos}}$$

$$= -\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan}{2ad\sqrt{a+b\cos}}$$

$$= \frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b^2\left(7a^2-15b^2\right)\sin(c+}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(}}$$

$$= \frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{5b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+d\right.}{4a^2d\sqrt{a+b\cos(c+d}}$$

Mathematica [C] time = 6.48, size = 597, normalized size = 1.73

$$\frac{\sqrt{a+b\cos(c+dx)}\left(-\frac{7b\tan(c+dx)}{4a^3}+\frac{\tan(c+dx)\sec(c+dx)}{2a^2}+\frac{2b^4\sin(c+dx)}{a^3\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d} - \frac{\frac{2\left(4a^3b-20ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{\sqrt{a+b\cos(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -1/16*((2*(4*a^3*b - 20*a*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^4 + 29*a^2
*b^2 - 45*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2
```

```
, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(7*a^2*b^2 - 15*b^4)*Sq
rt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[
2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((
a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2
 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(a^3*(-a +
b)*(a + b)*d) + (Sqrt[a + b*Cos[c + d*x]]*((2*b^4*Sin[c + d*x])/(a^3*(a^2 -
 b^2)*(a + b*Cos[c + d*x])) - (7*b*Tan[c + d*x])/(4*a^3) + (Sec[c + d*x]*Ta
n[c + d*x])/(2*a^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx + c)^3}{(b \cos (dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 2.39, size = 1542, normalized size = 4.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(-1/2/a
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
```

```
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)-2/
a^3*b^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*b^2/a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2/a^2*b*(-1/a*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\left(a+b\cos(c+dx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**(3/2), x)
```

3.538 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=436

$$-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{8a^2\left(2a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{4a\left(32a^4-49a^2b^2+7b^4\right)\sin(c}{15b^4d\left(a^2-\right.}$$

```
[Out] -2/3*a^2*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a
^2*(2*a^2-3*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
^(1/2)-4/15*a*(32*a^4-49*a^2*b^2+7*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b
^4/(a^2-b^2)^2/d+2/15*(48*a^4-71*a^2*b^2+3*b^4)*cos(d*x+c)*sin(d*x+c)*(a+b*
cos(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d+2/15*(128*a^6-212*a^4*b^2+55*a^2*b^4+9*
b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^5/(a^2-b^2)^2/d/((
a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*a*(128*a^4-116*a^2*b^2-17*b^4)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*
(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^5/(a^2-b^2)/d/(a+b*cos(d*
x+c))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2792, 3047, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{8a^2\left(2a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-71a^2b^2+48a^4+3b^4\right)\sin(c}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(128*a^6 - 212*a^4*b^2 + 55*a^2*b^4 + 9*b^6)*Sqrt[a + b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - (2*a*(128*a^4 - 116*a^2*b^2 - 17*b^4)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (8*a^2*(2*a^2 - 3*b^2)*Cos[c
+ d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
4*a*(32*a^4 - 49*a^2*b^2 + 7*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1
5*b^4*(a^2 - b^2)^2*d) + (2*(48*a^4 - 71*a^2*b^2 + 3*b^4)*Cos[c + d*x]*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\left(3a^2-\frac{3}{2}ab\cos(c+dx)-\frac{1}{2}\left(8a^2-3b^2\right)\cos^2(c}{(a+b\cos(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} +$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} +$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} -$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} -$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} -$$

$$= \frac{2\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^5\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\,(12}{}$$

Mathematica [A] time = 1.99, size = 272, normalized size = 0.62

$$\frac{b\left(\frac{10a^5\sin(c+dx)}{a^2-b^2} - \frac{10a^4\left(11a^2-15b^2\right)\sin(c+dx)(a+b\cos(c+dx))}{\left(a^2-b^2\right)^2} - 28a\sin(c+dx)(a+b\cos(c+dx))^2 + 3b\sin(2(c+dx))(a+b}{15b^5d(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*((128*a^6 - 212*a^4*b^2 + 55*a^2*b
^4 + 9*b^6)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-128*a^5 + 128*a^4*b
 + 116*a^3*b^2 - 116*a^2*b^3 + 17*a*b^4 - 17*b^5)*EllipticF[(c + d*x)/2, (2
```

```
*b)/(a + b)]))/(a - b)^2 + b*((10*a^5*Sin[c + d*x])/(a^2 - b^2) - (10*a^4*(
11*a^2 - 15*b^2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*a*(a
 + b*Cos[c + d*x])^2*Sin[c + d*x] + 3*b*(a + b*Cos[c + d*x])^2*Sin[2*(c + d
*x)]))/(15*b^5*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 2.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{5}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^5/(b^3*cos(d*x + c)^3 + 3*a*
b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{5}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 4.37, size = 1684, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16/b^2*(-1/
10/b*cos(1/2*d*x+1/2*c)^3*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-8/b
```

```
^3*(2*a+3*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)+1/6/b*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/1
2/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2/b^5*(3*a^2+4*a*b+3*b^2)*(a-b)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))
-2*(4*a^3+3*a^2*b+2*a*b^2+b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+10*a^4
/b^5/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2/b^5*a^5*(
1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d
*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2
*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^5}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^5}{(a+b\cos(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.539 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=345

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}+\frac{2\left(16a^4-16a^2b^2-b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\ldots}{3b^4d\left(a^2-b^2\right)\sqrt{a+\ldots}}$$

```
[Out] -2/3*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4/3*a
^3*(3*a^2-5*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(2
*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d-8/3*a*(4*a^4-7*
a^2*b^2+2*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/(a^2-b
^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(16*a^4-16*a^2*b^2-b^4)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/(a^2-b^2)/d/(a+b*cos(
d*x+c))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2792, 3031, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+\ldots}}{3b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]) + (2*(16*a^4 - 16*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[
c + d*x]]) - (2*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^(3/2)) + (4*a^3*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3*b^3*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos(c+dx)\left(2a^2-\frac{3}{2}ab\cos(c+dx)-\frac{3}{2}\left(2a^2-b^2\right)\cos^2(c\cdots}{(a+b\cos(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{4\int \cdots}{\cdots}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{2(2\cdots}{\cdots}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{2(2\cdots}{\cdots}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{2(2\cdots}{\cdots}$$

$$= -\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(16a^4-16a^2b^{2}\cdots\right.}{3b^4\left(a\cdots\right.}$$

Mathematica [A] time = 1.64, size = 237, normalized size = 0.69

$$\frac{2\left(\frac{b\sin(c+dx)\left(16a^6-25a^4b^2+\left(b^3-a^2b\right)^2\cos(2(c+dx))+4ab\left(5a^4-8a^2b^2+b^4\right)\cos(c+dx)+b^6\right)}{2\left(a^2-b^2\right)^2} + \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(16a^5-16a^4b-16a^3b^2+16a^2b^3-ab^4\cdots\right.\right.}{\cdots}\right.}{3b^4d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(-4*(4*a^5 - 7*a^3*b^2 + 2*a*b^4)
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (16*a^5 - 16*a^4*b - 16*a^3*b^2 +
16*a^2*b^3 - a*b^4 + b^5)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2
 + (b*(16*a^6 - 25*a^4*b^2 + b^6 + 4*a*b*(5*a^4 - 8*a^2*b^2 + b^4)*Cos[c +
d*x] + (-(a^2*b) + b^3)^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)
))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^4}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^4/(b^3*cos(d*x + c)^3 + 3*a*
b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 3.38, size = 1291, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8/b^2*(-1/6
/b*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)+1/6/b*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/12/b^2*(-2*a+6*b)*(
a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))))+4/b^4*(a+b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*
d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(3*a^2+2*a*b+b^2)/b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-8*a^3/b^4/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1
```

```
/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)+2/b^4*a^4*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2
*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)
/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b
)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.540 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx$

Optimal. Leaf size=281

$$-\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2a\left(8a^2-9b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{3b^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+d}}$$

```
[Out] -2/3*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a^2
*(a^2-2*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(8*a^4
-15*a^2*b^2+3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a
^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*a*(8*a^2-9*b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b
/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/(a^2-b^2)/d/(a+b*cos(d*x+
c))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3021, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2a\left(8a^2-9b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{3b^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]) - (2*a*(8*a^2 - 9*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) -
 (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)) - (8*a^2*(a^2 - 2*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a +
 b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{a^2-\frac{3}{2}ab\cos(c+dx)-\frac{1}{2}\left(4a^2-3b^2\right)\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{1}{2}}{}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{(a\,(8}{}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{((8a^4}{}$$

$$= \frac{2\left(8a^4-15a^2b^2+3b^4\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\left(8a^2-9b^2\right)\sqrt{\frac{a+}{}}}{3b^3\left(a^2-b^2\right)}$$

Mathematica [A] time = 1.27, size = 188, normalized size = 0.67

$$\frac{2\left(\frac{a^2b\sin(c+dx)\left(-4a^3+\left(9b^3-5a^2b\right)\cos(c+dx)+8ab^2\right)}{\left(a^2-b^2\right)^2} + \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(8a^4-15a^2b^2+3b^4\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\left(-8a^3+8a^2b+9ab^2-9b^3\right)F\left(\frac{1}{2}(c+dx)|\right.\right.}{(a-b)^2}\right.}{3b^3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((8*a^4 - 15*a^2*b^2 + 3*b^4)*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-8*a^3 + 8*a^2*b + 9*a*b^2 - 9*b^3)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2 + (a^2*b*(-4*a^3 + 8*a*b
^2 + (-5*a^2*b + 9*b^3)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^3*
d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{3}}{b^{3}\cos(dx+c)^{3}+3\,ab^{2}\cos(dx+c)^{2}+3\,a^{2}b\cos(dx+c)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3/(b^3*cos(d*x + c)^3 + 3*a*
b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(dx+c)^{3}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.88, size = 907, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+6*a^2
/b^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2/b^3*a^3*(
1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d
*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2
*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(
```

```
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.541 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\cdots\right.}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+d\cdots}}$$

```
[Out] -2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4/3*a*(a^2-3*b^2)*
sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-4/3*a*(a^2-3*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((a+b*cos(d*x+
c))/(a+b))^(1/2)+2/3*(2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x
+c))/(a+b))^(1/2)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2790, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\cdots\right.}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-4*a*(a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2
*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Sin[c
+ d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (4*a*(a^2 - 3*b^2)
*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3ab}{2}+\frac{1}{2}\left(2a^2-3b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{4\int \cdots}{\cdots}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2a \cdots\right.}{\cdots}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2a \cdots\right.}{\cdots}$$

$$= -\frac{4a\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\cdots}{3b^2\left(a^2-b^2\right)d\sqrt{a+\cdots}}$$

Mathematica [A] time = 1.15, size = 175, normalized size = 0.67

$$\frac{2\left(\frac{ab\sin(c+dx)\left(a^3+2b\left(a^2-3b^2\right)\cos(c+dx)-5ab^2\right)}{\left(a^2-b^2\right)^2}-\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(2\left(a^3-3ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+\left(-2a^3+2a^2b+3ab^2-3b^3\right)F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)\right)}{(a-b)^2}\right)}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(2*(a^3 - 3*a*b^2)*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)] + (-2*a^3 + 2*a^2*b + 3*a*b^2 - 3*b^3)*EllipticF[
(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (a*b*(a^3 - 5*a*b^2 + 2*b*(a^2 -
 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*Cos[c +
 d*x])^(3/2))
```

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^2}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2/(b^3*cos(d*x + c)^3 + 3*a*
b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.85, size = 846, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-4/b^2*a/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)+2/b^2*a^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2
*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)
/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b
)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.542 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=243

$$\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2+\right.}{}$$

```
[Out] 2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*(a^2+3*b^2)*sin(d*x
+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1
/2)+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/(a^2-
b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*
(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2 + 3*b^2)*Sin[c + d*x])/(3*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3b}{2}-\frac{1}{2}a\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{-ab-\frac{1}{4}(}{\sqrt{}}}{3}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{a\int \frac{}{\sqrt{a+b\,c}}}{3b\left(a^2\right.}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{\left(\left(a^2+3b\right.\right.}{}$$

$$= -\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{3b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+}}$$

Mathematica [A] time = 1.00, size = 154, normalized size = 0.63

$$\frac{2\left(\frac{\sin(c+dx)\left(b\left(a^2+3b^2\right)\cos(c+dx)+2a\left(a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a(b-a)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{b(a-b)^2}\right)}{3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2 + 3*b^2)*EllipticE[(c + d
*x)/2, (2*b)/(a + b)] + a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/
((a - b)^2*b)) + ((2*a*(a^2 + b^2) + b*(a^2 + 3*b^2)*Cos[c + d*x])*Sin[c +
d*x])/(a^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)/(b^3*cos(d*x + c)^3 + 3*a*b^
2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.73, size = 742, normalized size = 3.05

$$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\frac{2b\left(\sin^{2}\left(\frac{dx}{2}\cdots\right.\right.}{a-b}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b/sin(1/2
*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2
*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*a/b*(1/6/b/(a-b)/(a+b
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b
)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
```

```
*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin
(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx + c)}{(b\cos(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + d\,x)}{(a + b\,\cos(c + d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c + dx)}{(a + b\cos(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*cos(c + d*x))**(5/2), x)
```

3.543 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{8a\sqrt{a}}{}$$

```
[Out] -2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a*b*sin(d*x+c)/(a^
2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+8/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos
(d*x+c))^(1/2)/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))
^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{8a\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^
2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^2 - b^2)*d*Sqrt[
a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{1}{4}(3\cdots}{\cdots}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{(4a)\int\cdots}{\cdots}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left(4a\sqrt{a\cdots}\right.}{\cdots}$$

$$= \frac{8a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\cdots}{3(\cdots}$$

Mathematica [A] time = 0.93, size = 158, normalized size = 0.71

$$\frac{2b\sin(c+dx)\left(-5a^2-4ab\cos(c+dx)+b^2\right)-2(a-b)(a+b)^2\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+8a(a+b)^2\left(\cdots\right.}{3d(a-b)^2(a+b)^2(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (8*a*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*EllipticE[(c + d*x)/2,
(2*b)/(a + b)] - 2*(a - b)*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*E
llipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cos[c + d*
x])*Sin[c + d*x])/(3*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 1.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c
)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)
```

maple [A] time = 1.66, size = 489, normalized size = 2.21

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3b(a-b)(a+b)\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{a-b}{2b}\right)^2}+\frac{16b(\cdots}{3(a-b)^2(a+b)^2\sqrt{-(\cdots}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/3/b/(a-b)
/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+16/3*b*sin(1/2*d*x+1/2*c)^
2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-8/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(-5/2), x)
```

3.544 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=320

$$\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2b\,(}{}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*b^2*(7*a^2-3*b^
2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-2/3*b*(7*a^2-3*b^2)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((a+b*c
os(d*x+c))/(a+b))^(1/2)+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/
(a+b))^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^
(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2b\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*(7*a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos
[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)) + (2*b^2*(7*a^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}\left(a^2-b^2\right)-\frac{3}{2}ab\cos(c+dx)+\frac{1}{2}b^2\cos^2(c+dx)\right)\sec(c+}{(a+b\cos(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{4\int}{}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{}{\sqrt{a}}}{}$$

$$= -\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(}$$

$$= -\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{3a\left(a^2-b^2\right)d\sqrt{a+b\cos}}$$

Mathematica [C] time = 4.80, size = 464, normalized size = 1.45

$$\frac{\frac{4b^2\sin(c+dx)\left(8a^3+b\left(7a^2-3b^2\right)\cos(c+dx)-4ab^2\right)}{\left(a^3-ab^2\right)^2(a+b\cos(c+dx))^{3/2}} + \frac{-\frac{8ab\left(3a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(3b^2-7a^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+}{a-b}}}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((-8*a*b*(3*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4 - 19*a^2*b^2 +
9*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-7*a^2 + 3*b^2)*Sqrt[-((b*(-1
+ Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c +
```

```
d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
 + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*
ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/
(a*Sqrt[-(a + b)^(-1)]))/(a^2*(a - b)^2*(a + b)^2) + (4*b^2*(8*a^3 - 4*a*b^
2 + b*(7*a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^3 - a*b^2)^2*(a + b*C
os[c + d*x])^(3/2)))/(6*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.72, size = 845, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^2*b/si
n(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*
x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2/a^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))-2/a*b*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*si
```

```
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2
*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)
/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b
)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**(5/2), x)
```

3.545 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=380

$$-\frac{5b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^3d\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2-5b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2-5b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+}$$

```
[Out] 1/3*b*(3*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+1/3*b
*(3*a^4-26*a^2*b^2+15*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1
/2)-1/3*(3*a^4-26*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c
))^(1/2)/a^3/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/3*(3*a^2-5*b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/(a^2-b^2)/d/
(a+b*cos(d*x+c))^(1/2)-5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/
(a+b))^(1/2)/a^3/d/(a+b*cos(d*x+c))^(1/2)+tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(
3/2)
```

Rubi [A] time = 1.10, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2802, 3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(-26a^2b^2+3a^4+15b^4\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2-5b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2-5b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+d\right.}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] -((3*a^4 - 26*a^2*b^2 + 15*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]) + ((3*a^2 - 5*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (5*
b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^3*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3
*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4 - 26*a^2*b^2 + 1
5*b^4)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + Tan
[c + d*x]/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
```

```
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(-\frac{5b}{2}+\frac{3}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{a}$$

$$= \frac{b\left(3a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(-\frac{15}{4}b\left(a^2-b^2\right)\right.}{}}{}$$

$$= \frac{b\left(3a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{}{ad(}$$

$$= \frac{b\left(3a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{}{ad(}$$

$$= \frac{b\left(3a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4-26a^2b^2+15b^4\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{}{ad(}$$

$$= -\frac{\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\left(3a^2-5b\right.}{3a^2\left(a^2-b^2\right)d(a}$$

$$= -\frac{\left(3a^4-26a^2b^2+15b^4\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(3a^2-5b^2\right)\sqrt{\frac{a+}{}}}{3a^2\left(a^2-b^2\right.}$$

Mathematica [C] time = 6.58, size = 638, normalized size = 1.68

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\tan(c+dx)}{a^3}-\frac{2b^3\sin(c+dx)}{3a^2(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{4(5a^2b^3\sin(c+dx)-3b^5\sin(c+dx))}{3a^3(a^2-b^2)^2(a+b\cos(c+dx))}\right)}{d}-\frac{b\left(\frac{2(20ab^3-36a^3b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F}{\sqrt{a+b\cos(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] -1/12*(b*((2*(-36*a^3*b + 20*a*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(33*a^4 -
86*a^2*b^2 + 45*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(3*a^4 - 26*a^2*b
^2 + 15*b^4)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])
/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d
*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^
2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2
))))/(a^3*(-a + b)^2*(a + b)^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((-2*b^3*Sin[
c + d*x])/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (4*(5*a^2*b^3*Sin[c
+ d*x] - 3*b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) +
Tan[c + d*x]/a^3))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 3.61, size = 1320, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2/a^3/s
in(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+4/a^3*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))+2/a^2*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/
2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2)))+2*b^2/a^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a
-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-
2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*
a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}{\left(c + dx \right)}}{\left(a + b \cos{\left(c + dx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**(5/2), x)
```

3.546 $\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=282

$$-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{15d\left(a^2-b^2\right)^3\sqrt{a+b\cos(c+dx)}}-\frac{16ab\sin(c+dx)}{15d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^{3/2}}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}-\frac{1}{1}$$

```
[Out] -2/5*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(5/2)-16/15*a*b*sin(d*x+c)/(
a^2-b^2)^2/d/(a+b*cos(d*x+c))^(3/2)-2/15*b*(23*a^2+9*b^2)*sin(d*x+c)/(a^2-b
^2)^3/d/(a+b*cos(d*x+c))^(1/2)+2/15*(23*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2
))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-16/1
5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/(a^2-b^2)^2/d
/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{15d\left(a^2-b^2\right)^3\sqrt{a+b\cos(c+dx)}}-\frac{16ab\sin(c+dx)}{15d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^{3/2}}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}-\frac{1}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-7/2),x]
```

```
[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*(a^2 - b^2)^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15
*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(5*(a^2 - b
^2)*d*(a + b*Cos[c + d*x])^(5/2)) - (16*a*b*Sin[c + d*x])/(15*(a^2 - b^2)^2
*d*(a + b*Cos[c + d*x])^(3/2)) - (2*b*(23*a^2 + 9*b^2)*Sin[c + d*x])/(15*(a
^2 - b^2)^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx = -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{5\left(a^2-b^2\right)}$$

$$= -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} + \frac{4\int}{}$$

$$= -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} - \frac{2}{15(}$$

$$= -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} - \frac{2}{15(}$$

$$= -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} - \frac{2}{15(}$$

$$= \frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^3 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{16a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{15\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

Mathematica [A] time = 1.42, size = 189, normalized size = 0.67

$$\frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{5/2}\left(\left(23a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+8a(b-a)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{(a-b)^3}+\frac{b\sin(c+dx)\left(34a^4+b^2\left(23a^2+9b^2\right)\cos^2(c+dx)+2ab\left(27a^2+5b^2\right)\cos(c\right.}{\left(b^2-a^2\right)^3}\right.}{15d(a+b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-7/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(c +
d*x)/2, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
))/(a - b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Cos[
c + d*x] + b^2*(23*a^2 + 9*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/(-a^2 + b^2)^
3))/(15*d*(a + b*Cos[c + d*x])^(5/2))
```

fricas [F] time = 2.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{b^4\cos\left(dx+c\right)^4+4\,ab^3\cos\left(dx+c\right)^3+6\,a^2b^2\cos\left(dx+c\right)^2+4\,a^3b\cos\left(dx+c\right)+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/(b^4*cos(d*x + c)^4 + 4*a*b^3*cos(d*x + c
)^3 + 6*a^2*b^2*cos(d*x + c)^2 + 4*a^3*b*cos(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-7/2), x)
```

maple [A] time = 2.36, size = 616, normalized size = 2.18

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{10b^2(a-b)(a+b)\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{a-b}{2b}\right)^3}+\frac{8a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\cdots}}{15b(a-b)^2\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/10/b^2/(a
-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^3+8/15*a/b/(a-b)^2/(a+b)^
2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+4/15*b*sin(1/2*d*x+1/2*c)^2/(a-
b)^3/(a+b)^3*cos(1/2*d*x+1/2*c)*(23*a^2+9*b^2)/(-(-2*cos(1/2*d*x+1/2*c)^2*b
-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*a^4*b-3
0*a^3*b^2-30*a^2*b^3+15*a*b^4+15*b^5)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2/1
5*(23*a^2+9*b^2)/(a-b)^2/(a+b)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*
d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x))^(7/2),x)
[Out] int(1/(a + b*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.547 $\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$

Optimal. Leaf size=111

$$-\frac{23F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}\,d}+\frac{9\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}-\frac{\sin(c+dx)\sqrt{4\cos(c+d}}{10d}$$

```
[Out] -23/140*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+9/20*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/10*sin(d
*x+c)*(3+4*cos(d*x+c))^(1/2)/d+1/10*cos(d*x+c)*sin(d*x+c)*(3+4*cos(d*x+c))^
(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2793, 3023, 2752, 2661, 2653}

$$-\frac{23F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}\,d}+\frac{9\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}-\frac{\sin(c+dx)\sqrt{4\cos(c+d}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (9*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(20*d) - (23*EllipticF[(c + d*x)/2,
 8/7])/(20*Sqrt[7]*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d) + (C
os[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{10d} + \frac{1}{10}\int \frac{3+6\cos(c+dx)-6\cos^2(c+}{\sqrt{3+4\cos(c+dx)}}$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{10d} + \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{10d} + \cdots$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{10d} + \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{10d} + \cdots$$

$$= \frac{9\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d} - \frac{23F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}\,d} - \frac{\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{10d} + \cdots$$

Mathematica [A] time = 0.18, size = 81, normalized size = 0.73

$$\frac{-23\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+63\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+7(\sin(2(c+dx))-2\sin(c+dx))\sqrt{4\cos(c+dx)+3}}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (63*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] - 23*Sqrt[7]*EllipticF[(c + d*x)/2,
 8/7] + 7*Sqrt[3 + 4*Cos[c + d*x]]*(-2*Sin[c + d*x] + Sin[2*(c + d*x)]))/(1
40*d)
```

fricas [F] time = 1.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{3}}{\sqrt{4\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{3}}{\sqrt{4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

maple [A] time = 0.76, size = 231, normalized size = 2.08

$$-\frac{\sqrt{\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-64\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+56\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{20\sqrt{-8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/20*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-64*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2*2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x
+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x
```

```
+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^3}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^3}{\sqrt{4\cos(c+d\,x)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(4*cos(c + d*x) + 3)^(1/2),x)
[Out] int(cos(c + d*x)^3/(4*cos(c + d*x) + 3)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(3+4*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.548 $\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{17F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{12\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{4d}+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

```
[Out] 17/84*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/6*sin(d*x+c
)*(3+4*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2791, 2752, 2661, 2653}

$$\frac{17F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{12\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{4d}+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(4*d) + (17*EllipticF[(c + d*x)/2, 8
/7])/(12*Sqrt[7]*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx &= \frac{\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{6d}+\frac{1}{6}\int \frac{2-3\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx\\ &= \frac{\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{6d}-\frac{1}{8}\int \sqrt{3+4\cos(c+dx)}\,dx+\frac{17}{24}\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\\ &= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{4d}+\frac{17F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{12\sqrt{7}\,d}+\frac{\sqrt{3+4\cos(c+dx)}\,\sin(c+dx)}{6d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 0.90

$$\frac{17\sqrt{7}\,F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)-21\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+14\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (-21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + 17*Sqrt[7]*EllipticF[(c + d*x)/2
, 8/7] + 14*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(84*d)
```

fricas [F] time = 1.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^2}{\sqrt{4\,\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^2}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

maple [A] time = 0.59, size = 231, normalized size = 2.96

$$-\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{8\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{12\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/12*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(32*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+17*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d
*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))+3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2*2^(1/2))-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-8*sin(1/2*d*x+
1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^2}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

mupad [B] time = 0.09, size = 78, normalized size = 1.00

$$\frac{\sin\left(c+d\,x\right)\,\sqrt{4\,\cos\left(c+d\,x\right)+3}}{6\,d}-\frac{\sqrt{\frac{4\,\cos(c+d\,x)}{7}+\frac{3}{7}}\,\left(42\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)-34\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)\right)}{24\,d\,\sqrt{4\,\cos\left(c+d\,x\right)+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(4*cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] (sin(c + d*x)*(4*cos(c + d*x) + 3)^(1/2))/(6*d) - (((4*cos(c + d*x))/7 + 3/
7)^(1/2)*(42*ellipticE(c/2 + (d*x)/2, 8/7) - 34*ellipticF(c/2 + (d*x)/2, 8/
7)))/(24*d*(4*cos(c + d*x) + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{4\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(4*cos(c + d*x) + 3), x)
```

3.549 $\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2\sqrt{7}\,d}$$

```
[Out] -3/14*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2752, 2661, 2653}

$$\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) - (3*EllipticF[(c + d*x)/2, 8/7
])/(2*Sqrt[7]*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{1}{4}\int \sqrt{3+4\cos(c+dx)}\,dx - \frac{3}{4}\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d} - \frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2\sqrt{7}\,d}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.84

$$\frac{7E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right) - 3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
[Out] (7*EllipticE[(c + d*x)/2, 8/7] - 3*EllipticF[(c + d*x)/2, 8/7])/(2*Sqrt[7]*
d)
```

fricas [F] time = 1.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)}{\sqrt{4\,\cos\left(dx+c\right)+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

maple [A] time = 0.61, size = 155, normalized size = 3.04

$$\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(3\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\right.\right.}{2\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/
2*c),2*2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x+1/
2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

mupad [B] time = 0.63, size = 54, normalized size = 1.06

$$\frac{\sqrt{\frac{4\cos(c+d\,x)}{7}+\frac{3}{7}}\,\left(7\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)-3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)\right)}{2\,d\,\sqrt{4\cos(c+d\,x)+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(4*cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] (((4*cos(c + d*x))/7 + 3/7)^(1/2)*(7*ellipticE(c/2 + (d*x)/2, 8/7) - 3*elli
pticF(c/2 + (d*x)/2, 8/7)))/(2*d*(4*cos(c + d*x) + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt{4\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)/sqrt(4*cos(c + d*x) + 3), x)
```

3.550 $\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=23

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

```
[Out] 2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2661}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{2F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 4*Cos[c + d*x]],x]
[Out] (2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{4\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(4*cos(d*x + c) + 3), x)
```

maple [C] time = 0.02, size = 23, normalized size = 1.00

$$\frac{2\sqrt{7}\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\frac{2\sqrt{14}}{7}\right)}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+4*cos(d*x+c))^(1/2),x)
[Out] 2/7/d*7^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2/7*14^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(4*cos(d*x + c) + 3), x)
```

mupad [B] time = 0.57, size = 39, normalized size = 1.70

$$\frac{2\sqrt{\frac{4\cos(c+d\,x)}{7}+\frac{3}{7}}\;\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{8}{7}\right)}{d\,\sqrt{4\,\cos\left(c+d\,x\right)+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] (2*((4*cos(c + d*x))/7 + 3/7)^(1/2)*ellipticF(c/2 + (d*x)/2, 8/7))/(d*(4*co
s(c + d*x) + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{4\cos\left(c+dx\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*cos(c + d*x) + 3), x)
```

3.551 $\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{2\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

```
[Out] 2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2,2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2805}

$$\frac{2\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{2\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 1.00

$$\frac{2\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

fricas [F] time = 2.12, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)}{\sqrt{4\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)}{\sqrt{4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

maple [B] time = 0.48, size = 138, normalized size = 5.75

$$\frac{2\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\;\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2))/sin(1/
2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)}{\sqrt{4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(c+d\,x)\,\sqrt{4\,\cos(c+d\,x)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{4\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)/sqrt(4*cos(c + d*x) + 3), x)
```

3.552 $\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d}+\frac{\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)}{3d}$$

```
[Out] 1/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/7*14^(1/2))/d*7^(1/2)-4/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-1/3*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*1
4^(1/2))/d*7^(1/2)+1/3*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.26, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2802, 3060, 2653, 3002, 2661, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d}+\frac{\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) + EllipticF[(c + d*x)/2, 8/7]/
(Sqrt[7]*d) - (4*EllipticPi[2, (c + d*x)/2, 8/7])/(3*Sqrt[7]*d) + (Sqrt[3 +
 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{3}\int \frac{\left(-2-2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} - \frac{1}{12}\int \frac{(8-6\cos(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx - \frac{1}{6}\int \sqrt{}$$
$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{2}\int \frac{1}{\sqrt{3+4\cos(c+dx)}}$$
$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} + \frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d} - \frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d} + \frac{\sqrt{3+4\cos(c+dx)}}{3d}$$

Mathematica [C] time = 1.18, size = 158, normalized size = 1.56

$$\frac{-\frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}} + \sqrt{4\cos(c+dx)+3}\,\tan(c+dx) + \frac{i\sin(c+dx)\left(-12F\left(i\sinh^{-1}\left(\sqrt{4\cos(c+dx)+3}\right)|-\frac{1}{7}\right)+21E\left(i\sinh^{-1}\left(\sqrt{4\cos(c+dx)+}\right.\right.\right.}{3\sqrt{7}\sqrt{\sin^2(c+dx)}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] ((-6*EllipticPi[2, (c + d*x)/2, 8/7])/Sqrt[7] + ((I/3)*(21*EllipticE[I*ArcS
inh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Co
s[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]
], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 + 4*Cos[c +
 d*x]]*Tan[c + d*x])/(3*d)
```

fricas [F] time = 2.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^2}{\sqrt{4\cos(dx+c)+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

maple [B] time = 0.84, size = 350, normalized size = 3.47

$$-\frac{\sqrt{-\left(-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-8\left(\cos^2\right.}}{\sqrt{-8\left(\sin^4\right.}}\right.}{\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-8*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3*cos(1/2*d*
x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2*2^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2*2^(1/2))+4/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\,\sqrt{4\cos(c+d\,x)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c + dx)}{\sqrt{4\cos(c + dx) + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(4*cos(c + d*x) + 3), x)
```

3.553 $\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=137

$$-\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d}+\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{7}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)}{3d}+\frac{\sqrt{4\cos(c+dx}}{}$$

```
[Out] -1/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/3*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2/7*
14^(1/2))/d*7^(1/2)-1/3*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/6*sec(d*x+c)*
(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.37, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2653, 3002, 2661, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d}+\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{7}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)}{3d}+\frac{\sqrt{4\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) - EllipticF[(c + d*x)/2, 8/7]/(
3*Sqrt[7]*d) + (Sqrt[7]*EllipticPi[2, (c + d*x)/2, 8/7])/(3*d) - (Sqrt[3 +
4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x
]*Tan[c + d*x])/(6*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{\sqrt{3+4\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}\int \frac{\left(-6+3\cos(c+dx)+2\cos^2(c\right.}{\sqrt{3+4\cos(c+}}$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{6d} +$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{6d} -$$

$$= \frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} - \frac{\sqrt{3+4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\,\sec(c}{6d}$$

$$= \frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} - \frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d} + \frac{\sqrt{7}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} - \frac{\sqrt{3+4\cos(c+d}}{3d}$$

Mathematica [C] time = 1.32, size = 195, normalized size = 1.42

$$\frac{\frac{4F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}} + \frac{18\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}} - (2\cos(c+dx)-1)\sqrt{4\cos(c+dx)+3}\,\tan(c+dx)\sec(c+dx) - \frac{2i\sin(c+dx)\left(-12F\left(\right.\right.}{}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] ((4*EllipticF[(c + d*x)/2, 8/7])/Sqrt[7] + (18*EllipticPi[2, (c + d*x)/2, 8
/7])/Sqrt[7] - (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]]
, -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*Ellip
ticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt
[7]*Sqrt[Sin[c + d*x]^2]) - (-1 + 2*Cos[c + d*x])*Sqrt[3 + 4*Cos[c + d*x]]*
Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

fricas [F] time = 2.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^3}{\sqrt{4\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^3}{\sqrt{4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

maple [B] time = 0.84, size = 408, normalized size = 2.98

$$-\frac{\sqrt{-\left(-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}+\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\cdots\right.\right.}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\cdots\right.\right.\right.}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-8*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/3*cos(1/2*d*
x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)^2+2/3*cos(1/2*d*x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^
4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-7/3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2
*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\,\sqrt{4\cos(c+d\,x)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{4\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(3+4*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**3/sqrt(4*cos(c + d*x) + 3), x)
```

3.554 $\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{23F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20\sqrt{7}\,d}-\frac{9\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\,\cos(c+dx)}{10d}-\frac{\sin(c+dx)\sqrt{3-4}}{10d}$$

```
[Out] -23/140*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d
*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+9/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/10*sin(d
*x+c)*(3-4*cos(d*x+c))^(1/2)/d-1/10*cos(d*x+c)*sin(d*x+c)*(3-4*cos(d*x+c))^
(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2793, 3023, 2752, 2662, 2654}

$$\frac{23F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20\sqrt{7}\,d}-\frac{9\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\,\cos(c+dx)}{10d}-\frac{\sin(c+dx)\sqrt{3-4}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (-9*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*d) + (23*EllipticF[(c + P
i + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/
(10*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]*Sin[c + d*x])/(10*d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{\sqrt{3-4\cos(c+dx)}\,\cos(c+dx)\sin(c+dx)}{10d} - \frac{1}{10}\int \frac{3-6\cos(c+dx)-6\cos^2(c

Mathematica [A] time = 0.17, size = 102, normalized size = 0.90

$$\frac{-4\sin(c+dx)+\sin(2(c+dx))+2\sin(3(c+dx))+23\sqrt{4\cos(c+dx)-3}\,F\left(\frac{1}{2}(c+dx)\middle|8\right)+9\sqrt{4\cos(c+dx)-3}}{20d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] (9*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 23*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] - 4*Sin[c + d*x] + Sin[2*(c + d*x)] + 2*Sin[3*(c + d*x)])/(20*d*Sqrt[3 - 4*Cos[c + d*x]])

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-4\cos(dx+c)+3}\,\cos(dx+c)^3}{4\cos(dx+c)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3/(4*cos(d*x + c) - 3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(dx+c)^3}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)

maple [A] time = 0.63, size = 254, normalized size = 2.25

$$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-448\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+504\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x)

```
[Out] -1/140*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-448*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*s
in(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-56*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^3}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^3}{\sqrt{3-4\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(3 - 4*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3/(3 - 4*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.555 $\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{17F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{12\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{4d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

```
[Out] -17/84*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*
x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/4*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/6*sin(d*x+
c)*(3-4*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2791, 2752, 2662, 2654}

$$\frac{17F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{12\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{4d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(4*d) + (17*EllipticF[(c + Pi +
 d*x)/2, 8/7])/(12*Sqrt[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(6*
d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx &= -\frac{\sqrt{3-4\cos(c+dx)}\,\sin(c+dx)}{6d}-\frac{1}{6}\int \frac{-2-3\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx\\
&= -\frac{\sqrt{3-4\cos(c+dx)}\,\sin(c+dx)}{6d}-\frac{1}{8}\int \sqrt{3-4\cos(c+dx)}\,dx+\frac{17}{24}\int \frac{1}{\sqrt{3-4\cos}}\\
&= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{4d}+\frac{17F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{12\sqrt{7}\,d}-\frac{\sqrt{3-4\cos(c+dx)}\,\sin(c+d}{6d}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 94, normalized size = 1.18

$$\frac{-6\sin(c+dx)+4\sin(2(c+dx))+17\sqrt{4\cos(c+dx)-3}\,F\left(\frac{1}{2}(c+dx)\middle|8\right)+3\sqrt{4\cos(c+dx)-3}\,E\left(\frac{1}{2}(c+dx)\middle|8\right.}{12d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (3*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 17*Sqrt[-3 + 4*Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 8] - 6*Sin[c + d*x] + 4*Sin[2*(c + d*x)])
/(12*d*Sqrt[3 - 4*Cos[c + d*x]])
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)^2}{4\,\cos\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2/(4*cos(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

maple [A] time = 0.68, size = 232, normalized size = 2.90

$$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(224\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\right.\right.}}{84\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/84*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(224*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+17*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/
2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-21*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2/7*14^(1/2))-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(8*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1
/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

mupad [B] time = 0.09, size = 78, normalized size = 0.98

$$\frac{\sqrt{4\cos(c+dx)-3}\,\left(6\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|8\right)+34\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|8\right)\right)}{24\,d\,\sqrt{3-4\cos(c+dx)}}-\frac{\sin(c+dx)\,\sqrt{3-4\cos(c+dx)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(3 - 4*cos(c + d*x))^(1/2),x)
```

```
[Out] ((4*cos(c + d*x) - 3)^(1/2)*(6*ellipticE(c/2 + (d*x)/2, 8) + 34*ellipticF(c
/2 + (d*x)/2, 8)))/(24*d*(3 - 4*cos(c + d*x))^(1/2)) - (sin(c + d*x)*(3 - 4
*cos(c + d*x))^(1/2))/(6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(3 - 4*cos(c + d*x)), x)
```

3.556 $\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=53

$$\frac{3F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}$$

```
[Out] -3/14*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2752, 2662, 2654}

$$\frac{3F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) + (3*EllipticF[(c + Pi +
d*x)/2, 8/7])/(2*Sqrt[7]*d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\left(\frac{1}{4}\int \sqrt{3-4\cos(c+dx)}\,dx\right)+\frac{3}{4}\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx$$
$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{2d}+\frac{3F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{2\sqrt{7}\,d}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 1.13

$$\frac{\sqrt{4\cos(c+dx)-3}\,\left(3F\left(\frac{1}{2}(c+dx)\middle|8\right)+E\left(\frac{1}{2}(c+dx)\middle|8\right)\right)}{2d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[-3 + 4*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 8] + 3*EllipticF[(c + d*
x)/2, 8]))/(2*d*Sqrt[3 - 4*Cos[c + d*x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-4\,\cos\left(dx+c\right)+3}\,\cos\left(dx+c\right)}{4\,\cos\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)/(4*cos(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

maple [A] time = 0.73, size = 158, normalized size = 2.98

$$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\,\left(3\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{14\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/14*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*(3*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))-7*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2
*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{\sqrt{-4\,\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

mupad [B] time = 0.14, size = 52, normalized size = 0.98

$$\frac{\sqrt{4\,\cos(c+d\,x)-3}\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|8\right)+3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|8\right)\right)}{2\,d\,\sqrt{3-4\,\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(3 - 4*cos(c + d*x))^(1/2),x)
```

```
[Out] ((4*cos(c + d*x) - 3)^(1/2)*(ellipticE(c/2 + (d*x)/2, 8) + 3*ellipticF(c/2
+ (d*x)/2, 8)))/(2*d*(3 - 4*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))**(1/2),x)

[Out] Integral(cos(c + d*x)/sqrt(3 - 4*cos(c + d*x)), x)
```

3.557 $\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{2F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

```
[Out] -2/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2662}

$$\frac{2F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx=\frac{2F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 1.83

$$\frac{2\sqrt{4\cos(c+dx)-3}\,F\left(\frac{1}{2}(c+dx)\middle|8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos[c
 + d*x]])
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-4\cos\left(dx+c\right)+3}}{4\cos\left(dx+c\right)-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)/(4*cos(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4\cos\left(dx+c\right)+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-4*cos(d*x + c) + 3), x)
```

maple [C] time = 0.07, size = 54, normalized size = 2.25

$$\frac{2\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\ \mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}|2\sqrt{2}\right)}{d\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)*(8*cos(1/2*d*x+1/2*c)^2-7)^(1/2)*Inve
rseJacobiAM(1/2*d*x+1/2*c,2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4\cos\left(dx+c\right)+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-4*cos(d*x + c) + 3), x)
```

mupad [B] time = 0.57, size = 39, normalized size = 1.62

$$\frac{2\,\sqrt{4\,\cos\left(c+d\,x\right)-3}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|8\right)}{d\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 4*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(4*cos(c + d*x) - 3)^(1/2)*ellipticF(c/2 + (d*x)/2, 8))/(d*(3 - 4*cos(c
+ d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3-4\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3 - 4*cos(c + d*x)), x)
```

3.558 $\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{2\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

```
[Out] 2/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticPi(cos(1/2*d*x+
1/2*c),2,2/7*14^(1/2))/d*7^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2806}

$$-\frac{2\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (-2*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{2\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 1.80

$$\frac{2\sqrt{4\cos(c+dx)-3}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/(d*Sqrt[3 - 4*C
os[c + d*x]])
```

fricas [F] time = 2.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)}{4\cos\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)/(4*cos(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

maple [B] time = 0.59, size = 139, normalized size = 5.56

$$\frac{2\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\ \text{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\right.}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/7*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/sin
(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{3-4\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\sqrt{3-4\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)/sqrt(3 - 4*cos(c + d*x)), x)
```

3.559 $\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=104

$$\frac{F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}\,d}+\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d}$$

```
[Out] -1/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+
1/2*c),2/7*14^(1/2))/d*7^(1/2)+4/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)+1/3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*
14^(1/2))/d*7^(1/2)+1/3*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2802, 3060, 2654, 3002, 2662, 2806}

$$\frac{F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}\,d}+\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) + EllipticF[(c + Pi + d*x
)/2, 8/7]/(Sqrt[7]*d) - (4*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3*Sqrt[7]
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{3}\int \frac{\left(2+2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{12}\int \frac{(8+6\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx - \frac{1}{6}\int \sqrt{}$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{2}\int \frac{1}{\sqrt{3-4\cos(c+d}}$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} + \frac{F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}\,d} - \frac{4\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d} + \frac{\sqrt{3-}}{}$$

Mathematica [C] time = 1.52, size = 179, normalized size = 1.72

$$\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx) + \frac{6\sqrt{4\cos(c+dx)-3}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}} - \frac{i\sin(c+dx)\left(-12F\left(i\sinh^{-1}\left(\sqrt{3-4\cos(c+dx)}\right)|-\frac{1}{7}\right)+21E\left(i\sinh^{-1}\left(\right.\right.\right.}{3\sqrt{7}\sqrt{\sin}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] ((6*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*Cos
[c + d*x]] - ((I/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7
] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[
-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sq
rt[Sin[c + d*x]^2]) + Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

fricas [F] time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-4\cos(dx+c)+3}\,\sec(dx+c)^2}{4\cos(dx+c)-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2/(4*cos(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

maple [B] time = 0.83, size = 351, normalized size = 3.38

$$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\cdots\right.\right.}}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\cdots\right.\right.}}\cdots\right.}{\sin\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3*cos(1/2*d*x
+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)+1/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1
/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2/7*14^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/
2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-4/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*s
in(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/sin(1/2*d*x+1/2*c)/(-8
*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\,\sqrt{3-4\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(3-4*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**2/sqrt(3 - 4*cos(c + d*x)), x)
```

3.560 $\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{7}\,\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d}+\frac{\sqrt{3}}{}$$

```
[Out] -1/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*
14^(1/2))/d*7^(1/2)+1/3*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/6*sec(d*x+c)*
(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.37, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2654, 3002, 2662, 2806}

$$\frac{F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}\,d}-\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{7}\,\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d}+\frac{\sqrt{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) + EllipticF[(c + Pi + d*x
)/2, 8/7]/(3*Sqrt[7]*d) - (Sqrt[7]*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 - 4*Cos[c + d
*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{\sqrt{3-4\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}\int \frac{\left(6+3\cos(c+dx)-2\cos^2(c+\right.}{\sqrt{3-4\cos(c+d}}$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{18}$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{72}$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\,\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}}{}$$

$$= -\frac{\sqrt{7}\,E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} + \frac{F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3\sqrt{7}\,d} - \frac{\sqrt{7}\,\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} + \sqrt{}$$

Mathematica [C] time = 1.90, size = 236, normalized size = 1.69

$$\frac{-\frac{4\sqrt{4\cos(c+dx)-3}\,F\left(\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}} + \frac{18\sqrt{4\cos(c+dx)-3}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}} + \sqrt{3-4\cos(c+dx)}\,(2\cos(c+dx)+1)\tan(c+dx)\,\mathrm{se}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] ((-4*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c
+ d*x]] + (18*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt
[3 - 4*Cos[c + d*x]] - (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c
+ d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] -
 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x
])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 - 4*Cos[c + d*x]]*(1 + 2*Cos[c +
 d*x])*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

fricas [F] time = 1.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-4\cos\left(dx+c\right)+3}\,\sec\left(dx+c\right)^{3}}{4\cos\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3/(4*cos(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{3}}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

maple [B] time = 1.06, size = 408, normalized size = 2.91

$$-\frac{\sqrt{-\left(8\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^{2}}-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\cdots\right)}}{3\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/3*cos(1/2*d*x
+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)^2-2/3*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-1/3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-1/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/
2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(
1/2)))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\,\sqrt{3-4\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(3-4*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**3/sqrt(3 - 4*cos(c + d*x)), x)
```

3.561 $\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10B\sin(c$$

```
[Out] 6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*A*cos(d*x+c)^(3/2)*sin(d*x+c)/d
+2/7*B*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*B*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2635, 2639, 2641}

$$\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10B\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*B*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx = A\int \cos^{\frac{5}{2}}(c+dx)\,dx + B\int \cos^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2B\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(3A)$$

$$= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10B\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2A\cos^{\frac{3}{2}}(c+dx}{5d}$$

$$= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10B\sqrt{\cos(c+dx)}\sin(c}{21d}$$

Mathematica [A] time = 0.53, size = 77, normalized size = 0.69

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+126AE\left(\frac{1}{2}(c+dx)\middle|2\right)+50BF\left(\frac{1}{2}(c+\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (126*A*EllipticE[(c + d*x)/2, 2] + 50*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(105*d)
```

fricas [F] time = 2.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^3 + A\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2), x)
```

maple [A] time = 0.71, size = 290, normalized size = 2.61

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A
-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.96, size = 87, normalized size = 0.78

$$-\frac{2\,A\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\,\frac{11}{4};\,\cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,B\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\,\frac{13}{4};\,\mathrm{co}\right.}{9\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] - (2*A*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*cos(c + d*x)^(9/2)*sin(c + d*x
)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.562 $\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] 6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2
/3*A*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2635, 2641, 2639}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*A*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx = A\int \cos^{\frac{3}{2}}(c+dx)\,dx + B\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}A\int$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2A\sqrt{\cos(c+dx)}\,\sin(c+}{3d}$$

Mathematica [A] time = 0.24, size = 66, normalized size = 0.76

$$\frac{2\left(\sin(c+dx)\sqrt{\cos(c+dx)}\,(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]),x]
[Out] (2*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

fricas [F] time = 1.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^2 + A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos\left(dx+c\right)+A)\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2), x)
```

maple [B] time = 0.74, size = 262, normalized size = 3.01

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.74, size = 80, normalized size = 0.92

$$\frac{2\,A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,A\,\sqrt{\cos(c+d\,x)}\ \sin(c+d\,x)}{3\,d}-\frac{2\,B\cos(c+d\,x)^{7/2}\ \sin(c+d\,x)\ {}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] (2*A*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*A*cos(c + d*x)^(1/2)*sin(c + d
*x))/(3*d) - (2*B*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.563 $\int \sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*B*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2639, 2635, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = A\int \sqrt{\cos(c+dx)}\,dx + B\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+}}$$
$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+}{3d}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 0.87

$$\frac{2\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
[Out] (2*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos
[c + d*x]]*Sin[c + d*x])))/(3*d)
```

fricas [F] time = 2.04, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.64, size = 229, normalized size = 3.75

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.65, size = 53, normalized size = 0.87

$$\frac{2\,A\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,B\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] (2*A*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*ellipticF(c/2 + (d*x)/2, 2))/(3*
d) + (2*B*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\sqrt{\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.564 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=35

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2748, 2641, 2639}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = A\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + B\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [A] time = 0.07, size = 35, normalized size = 1.00

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/d
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B\cos(dx+c)+A}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.69, size = 152, normalized size = 4.34

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.26, size = 33, normalized size = 0.94

$$\frac{2\,A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*A*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*ellipticE(c/2 + (d*x)/2, 2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.565 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] -2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2636, 2639, 2641}

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d + (2
*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= A\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - A\int \sqrt{\cos(c+dx)}\,dx\\ &= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 51, normalized size = 0.89

$$\frac{2\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + d*x)/2, 2] + (A*Sin[c
 + d*x])/Sqrt[Cos[c + d*x]]))/d
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.74, size = 148, normalized size = 2.60

$$-\frac{2\left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.97, size = 60, normalized size = 1.05

$$\frac{2\,B\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*B*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*sin(c + d*x)*hypergeom([-1/4, 1/
2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
[Out] Integral((A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.566 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*
B*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2636, 2641, 2639}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= A\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\ &= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}A\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - B\int \sqrt{\cos(c+dx)}\,dx \\ &= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.42, size = 65, normalized size = 0.78

$$\frac{\frac{2\sin(c+dx)(A+3B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} + 2AF\left(\frac{1}{2}(c+dx)\middle|2\right) - 6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2] + 2*A*EllipticF[(c + d*x)/2, 2] + (2*(A + 3
*B*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

[Out] integral((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)

maple [B] time = 1.59, size = 397, normalized size = 4.78

$$2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)

[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)

mupad [B] time = 1.23, size = 87, normalized size = 1.05

$$\frac{2\,A\,\sin\left(c+d\,x\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,\sin\left(c+d\,x\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*A*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c +
 d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*sin(c + d*x)*hypergeom([-1/4, 1/
2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.567 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6A\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] -6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)+
2/3*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2748, 2636, 2639, 2641}

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6A\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*A*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= A\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx+B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{5}(3A)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6A\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}-\frac{1}{5}(3A)\\ &= -\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\end{aligned}$$

Mathematica [A] time = 0.32, size = 95, normalized size = 0.86

$$\frac{9A\sin(2(c+dx))+6A\tan(c+dx)-18A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10B\sin(c+dx)+10B\cos^{\frac{3}{2}}(c+dx)F\left(\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-18*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*B*Cos[c + d*x]^(3/
2)*EllipticF[(c + d*x)/2, 2] + 10*B*Sin[c + d*x] + 9*A*Sin[2*(c + d*x)] + 6
*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)

maple [B] time = 1.83, size = 502, normalized size = 4.52

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2A\left(12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.46, size = 87, normalized size = 0.78

$$\frac{2\,A\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*A*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c
+ d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*sin(c + d*x)*hypergeom([-3/4, 1
/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.568 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{2\left(9a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(9a^2+7b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)\,c}{7d}$$

```
[Out] 2/15*(9*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(9*a^2+7*b^2)*c
os(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*b^2*
cos(d*x+c)^(7/2)*sin(d*x+c)/d+20/21*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2635, 2641, 3014, 2639}

$$\frac{2\left(9a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(9a^2+7b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)\,c}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(9*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a*b*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (20*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*(9*a^2 + 7*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a*b*Cos[c +
d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9
*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^{\frac{7}{2}}(c+dx)\,dx + \int \cos^{\frac{5}{2}}(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$

$$= \frac{4ab\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2b^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{7}(10ab)$$

$$= \frac{20ab\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2\left(9a^2+7b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{45d}$$

$$= \frac{2\left(9a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20ab\sqrt{\cos(c+}}{2}$$

Mathematica [A] time = 0.80, size = 113, normalized size = 0.71

$$\frac{84\left(9a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\,\left(7\left(36a^2+43b^2\right)\cos(c+dx)+5b(36a\cos(2(c+dx))\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (84*(9*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a^2 + 43*b^2)*Cos[c + d*x] + 5*b*(156*a
+ 36*a*Cos[2*(c + d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

fricas [F] time = 2.06, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos\left(dx+c\right)^4+2\,ab\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*s
qrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

maple [B] time = 0.80, size = 398, normalized size = 2.49

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(1440ab+2240b^2\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1440*a*b+2240*b^2)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-504*a^2-2160*a*b-2072*b^2)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(504*a^2+1680*a*b+952*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-126*a^2-480*a*b-168*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+150*a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.04, size = 135, normalized size = 0.84

$$-\frac{2\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,b^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\right.}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] - (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*b^2*cos(c + d*x)^(11/2)*sin(c
+ d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)
^(1/2)) - (4*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4
, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.569 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{2(7a^2+5b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{5d}$$

```
[Out] 12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a*b*cos(d*x+c)^(3
/2)*sin(d*x+c)/d+2/7*b^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(7*a^2+5*b^2)*s
in(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2635, 2639, 3014, 2641}

$$\frac{2(7a^2+5b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2 + 5*b^2)*EllipticF[(c
+ d*x)/2, 2])/(21*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (4*a*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b^2*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^{\frac{5}{2}}(c+dx)\,dx + \int \cos^{\frac{3}{2}}(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$

$$= \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2b^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(6ab)$$

$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{4ab\,c}{}$$

$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2\left(7a^2+5b^2\right)}{}$$

Mathematica [A] time = 0.63, size = 98, normalized size = 0.73

$$\frac{10\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(70a^2+84ab\cos(c+dx)+15b^2\cos(2(c+dx))+65b\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2)*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(70*a^2 + 65*b^2 + 84*a*b*Cos[c + d*x] + 15*b^2
*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

fricas [F] time = 1.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos\left(dx+c\right)^3+2\,ab\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

maple [B] time = 0.75, size = 362, normalized size = 2.68

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-336ab-360b^2\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-336*a*b-360*b^2)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*a^2+336*a*b+280*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-70*a^2-84*a*b-80*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+35*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.89, size = 128, normalized size = 0.95

$$\frac{2\left(a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}-\frac{2\,b^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right.\right.}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (2*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4,
 cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*a*b*cos(c + d*x)^(7/2)*
sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x
)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.570 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{2(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] 2/5*(5*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b^2*cos(d*x+c)^(3/2
)*sin(d*x+c)/d+4/3*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2635, 2641, 3014, 2639}

$$\frac{2(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (4*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b^2
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\,\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(2ab)$$
$$= \frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4ab\sqrt{\cos(c+}}{3}$$

Mathematica [A] time = 0.32, size = 79, normalized size = 0.78

$$\frac{6\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20abF\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\sin(c+dx)\sqrt{\cos(c+dx)}\,(10a+3b\cos(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (6*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*EllipticF[(c + d*x)/2
, 2] + 2*b*Sqrt[Cos[c + d*x]]*(10*a + 3*b*Cos[c + d*x])*Sin[c + d*x])/(15*d
)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.69, size = 321, normalized size = 3.18

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(40ab+24b^2\right)\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(40*a*b+24*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-20*a*b-6*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10
*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.01, size = 102, normalized size = 1.01

$$\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,b^2\,\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\right.}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (4*a*b*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*b^2*cos(c + d*x
)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin
(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.571 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b^2*sin(d*x+c)*cos(d*x+
c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2789, 2639, 3014, 2641}

$$\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx &= (2ab)\int \sqrt{\cos(c+dx)}\,dx + \int \frac{a^2+b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx \\ &= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\left(3a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}} \\ &= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.16, size = 64, normalized size = 0.89

$$\frac{2\left(\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+b^2\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2,
2] + b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2}{\sqrt{\cos\left(dx+c\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.80, size = 283, normalized size = 3.93

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(4b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*b^2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.94, size = 76, normalized size = 1.06

$$\frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b^2\,\sqrt{\cos\left(c+d\,x\right)}\ \sin\left(c+d\,x\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^2*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (2*b^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (4*a*b*ellipticE(c
/2 + (d*x)/2, 2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.572 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] -2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^
(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2789, 2641, 3012, 2639}

$$-\frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (4*a*b*EllipticF[(c + d*x)/2
, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= (2ab)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\left(a^2-b^2\right)\int \sqrt{\cos(c+dx)}\,dx\\ &= -\frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.30, size = 62, normalized size = 0.91

$$\frac{2\left(\left(b^2-a^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}}+2bF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*((-a^2 + b^2)*EllipticE[(c + d*x)/2, 2] + a*(2*b*EllipticF[(c + d*x)/2,
2] + (a*Sin[c + d*x])/Sqrt[Cos[c + d*x]])))/d
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/cos(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.69, size = 202, normalized size = 2.97

$$-\frac{2\left(2ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\cdots\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*(2*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-2*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.13, size = 81, normalized size = 1.19

$$\frac{2\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.573 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+4*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2636, 2639, 3012, 2641}

$$\frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2 + 3*b^2)*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a*b*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= (2ab)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (2ab)\int \sqrt{\cos(c+dx)}\,dx - \frac{1}{3}\left(-a^2-3b^2\right)\int \frac{}{\sqrt{}}\\ &= -\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(}{d\sqrt{\cos(}}\end{aligned}$$

Mathematica [A] time = 0.62, size = 73, normalized size = 0.77

$$\frac{2\left(\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)(a+6b\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2,
 2] + (a*(a + 6*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/cos(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

maple [B] time = 1.58, size = 514, normalized size = 5.41

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+6*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*
d*x+1/2*c)^2+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2-24*a*b*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
```

```
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*a^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2+12*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.24, size = 108, normalized size = 1.14

$$\frac{2b^2\,F\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2a^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{4\,a\,b\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*sin(c + d*x)*hypergeom([-3/4
, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2
)) + (4*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.574 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$-\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+}{3d\cos^{\frac{3}{2}}(c+}$$

```
[Out] -2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*sin(d*x+c)/d/c
os(d*x+c)^(5/2)+4/3*a*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(3*a^2+5*b^2)*sin
(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2636, 2641, 3012, 2639}

$$-\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+}{3d\cos^{\frac{3}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a^2 + 5*b^2)*Sin[c + d*x])
/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = (2ab)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(2ab)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - \frac{1}{5}\left(-3a^2-5b^2\right)$$

$$= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(3a^2+5b^2\right)\sin(c+d}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin}{3d\cos^{\frac{3}{2}}}$$

Mathematica [A] time = 0.40, size = 124, normalized size = 0.92

$$\frac{-6\left(3a^2+5b^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+9a^2\sin(2(c+dx))+6a^2\tan(c+dx)+20ab\sin(c+dx)+20ab\,\mathrm{co}}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*(3*a^2 + 5*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*C
os[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 9*a^2*S
in[2*(c + d*x)] + 15*b^2*Sin[2*(c + d*x)] + 6*a^2*Tan[c + d*x])/(15*d*Cos[c
 + d*x]^(3/2))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/cos(d*x + c)^(7/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c) + a)^2}{\cos(dx+c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

maple [B] time = 2.09, size = 660, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*a^2/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*b^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
```

```
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1)+4*a*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.38, size = 113, normalized size = 0.84

$$\frac{6\,a^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\;-\frac{1}{4};\;\cos(c+d\,x)^2\right)+30\,b^2\cos(c+d\,x)^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x)^2\right)}{15\,d\cos(c+d\,x)^{5/2}\sqrt{1-\cos(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^(7/2),x)
```

```
[Out] (6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*b^2*c
os(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) + 20
*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))
/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.575 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=194

$$\frac{2a(7a^2+15b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(27a^2+7b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b(27a^2+7b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+$$

```
[Out] 2/15*b*(27*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(7*a^2+15*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*
b*(27*a^2+7*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+40/63*a*b^2*cos(d*x+c)^(5/2)
*sin(d*x+c)/d+2/9*b^2*cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*sin(d*x+c)/d+2/21*a
*(7*a^2+15*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(7a^2+15b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(27a^2+7b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b(27a^2+7b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*b*(27*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*a^2 + 15*
b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a^2 + 7*b^2)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(45*d) + (40*a*b^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b
^2*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{2b^2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{3}{2}}(c+dx)\Big($$

$$= \frac{40ab^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b^2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$= \frac{40ab^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b^2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d}$$

$$= \frac{2a\left(7a^2+15b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2b\left(27a^2+7b^2\right)\cos^{\frac{3}{2}}(c+}{45d}$$

$$= \frac{2b\left(27a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2a\left(7a^2+15b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} +$$

Mathematica [A] time = 1.03, size = 137, normalized size = 0.71

$$\frac{60\left(7a^3+15ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(27a^2b+7b^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\,\left(5\left(84a^3+54\right.\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (84*(27*a^2*b + 7*b^3)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^3 + 15*a*b^2)*El
lipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a^2 + 43*b^2)*Cos[c
+ d*x] + 5*(84*a^3 + 234*a*b^2 + 54*a*b^2*Cos[2*(c + d*x)] + 7*b^3*Cos[3*(c
 + d*x)]))*Sin[c + d*x])/(630*d)
```

fricas [F] time = 1.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos\left(dx+c\right)^4+3\,ab^2\cos\left(dx+c\right)^3+3\,a^2b\cos\left(dx+c\right)^2+a^3\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c
)^2 + a^3*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

maple [B] time = 0.75, size = 470, normalized size = 2.42

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2160b^2a+2240b^3\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2160*a*b^2+2240*b^3)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*a^2*b-3240*a*b^2-2072*b^3)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^3+1512*a^2*b+2520*a*b^2+952*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^3-378*a^2*b-720*a*b^2-168*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*b^2
*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.05, size = 178, normalized size = 0.92

$$\frac{2a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3d}+\frac{2a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{2b^3\cos(c+dx)^{11/2}\,\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};\cos(c+dx\right.}{11d\sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c
 + d*x))/(3*d) - (2*b^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11
/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (6*a^2*b*cos(c
+ d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d
*(sin(c + d*x)^2)^(1/2)) - (2*a*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hyperge
om([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.576 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=159

$$\frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\left(21a^2+5b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{32}{}$$

```
[Out] 2/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/35*a
*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b^2*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c)
)*sin(d*x+c)/d+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3023, 2748, 2639, 2635, 2641}

$$\frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\left(21a^2+5b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{32}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*a*(5*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(21*a^2 + 5*b^
2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*d) + (32*a*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) + (2*b^2*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3\,dx = \frac{2b^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{\cos(c+dx)}\ \Big($$

$$= \frac{32ab^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2b^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\,\mathrm{s}}{7d}$$

$$= \frac{32ab^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2b^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\,\mathrm{s}}{7d}$$

$$= \frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(21a^2+5b^2\right)\sqrt{\cos(c+dx)}\ \sin(c}{21d}$$

$$= \frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2}{-}$$

Mathematica [A] time = 0.79, size = 110, normalized size = 0.69

$$\frac{42\left(5a^3+9ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10\left(21a^2b+5b^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\sqrt{\cos(c+dx)}\ \left(210a^2+126ab\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (42*(5*a^3 + 9*a*b^2)*EllipticE[(c + d*x)/2, 2] + 10*(21*a^2*b + 5*b^3)*Ell
ipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(210*a^2 + 65*b^2 + 126*a*b*C
os[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sqrt(cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.74, size = 421, normalized size = 2.65

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(240b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-504b^2a-360b^3\right)\left(\sin^6\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-504*a*b^2-360*b^3)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(420*a^2*b+504*a*b^2+280*b^3)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-210*a^2*b-126*a*b^2-80*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+105*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.94, size = 146, normalized size = 0.92

$$\frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,b\,\sqrt{\cos(c+d\,x)}\ \sin(c+d\,x)\right)}{d}-\frac{2\,b^3\cos(c+d\,x)^{9/2}\ \sin(c+d\,x)\ {}_2\mathrm{F}}{9\,d\,\sqrt{\sin(c+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^2*b*ellipticF(c/2 + (d*x)/2, 2) + a
^2*b*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (2*b^3*cos(c + d*x)^(9/2)*sin(c
+ d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(
1/2)) - (6*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4
, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.577 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=116

$$\frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}{5d}+$$

```
[Out] 6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a*b^2*sin(d*x
+c)*cos(d*x+c)^(1/2)/d+2/5*b^2*(a+b*cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A] time = 0.18, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2793, 3023, 2748, 2641, 2639}

$$\frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}{5d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (6*b*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(a^2 + b^2)*Elli
pticF[(c + d*x)/2, 2])/d + (8*a*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d)
+ (2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \frac{2b^2\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{1}{2}a\left(5a^2+b^2\right)+\frac{3}{2}b\left(5a^2+b\right.}{\sqrt{\mathrm{co}}}$$

$$= \frac{8ab^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d} + \frac{2b^2\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))\sin(c+dx)}{5d} +$$

$$= \frac{8ab^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d} + \frac{2b^2\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))\sin(c+dx)}{5d} +$$

$$= \frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8ab^2\sqrt{\cos(c+dx)}\,\mathrm{s}}{5d}$$

Mathematica [A] time = 0.40, size = 84, normalized size = 0.72

$$\frac{2\left(3\left(5a^2b+b^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+b^2\sin(c+dx)\sqrt{\cos(c+dx)}\,(5a+b\cos(c+dx)\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*(5*a^2*b + b^3)*EllipticE[(c + d*x)/2, 2] + 5*a*(a^2 + b^2)*EllipticF
[(c + d*x)/2, 2] + b^2*Sqrt[Cos[c + d*x]]*(5*a + b*Cos[c + d*x])*Sin[c + d*
x]))/(5*d)
```

fricas [F] time = 1.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\cos\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.68, size = 376, normalized size = 3.24

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-8b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20b^2a+8b^3\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*b^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*a*b^2+8*b^3)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-10*a*b^2-2*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5
*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+5*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.88, size = 125, normalized size = 1.08

$$\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d}-\frac{2\,b^3\cos\left(c+d\,x\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^2*b*ellipticE(c/2 + (d*x)/2, 2
))/d + (2*a*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*b^2*cos(c + d*x)^(1/2
)*sin(c + d*x))/d - (2*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2,
7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.578 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(}{}$$

```
[Out] -2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^2*(a+b*cos
(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*b*(3*a^2-b^2)*sin(d*x+c)*cos(d*x
+c)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2792, 3023, 2748, 2641, 2639}

$$\frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) - (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*d) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{2a^2b-\frac{1}{2}a\left(a^2-3b^2\right)\cos(c+dx)-\frac{1}{2}b\left(3a^2\right.}{\sqrt{\cos(c+dx)}}$$

$$=-\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4}{3}\int$$

$$=-\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\left(a\right.$$

$$=-\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)\sqrt{c}}{}$$

Mathematica [A] time = 0.56, size = 86, normalized size = 0.69

$$\frac{2\left(\frac{\sin(c+dx)\left(3a^3+b^3\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}-3\left(a^3-3ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(9a^2b+b^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(-3*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, 2] + (9*a^2*b + b^3)*Elliptic
F[(c + d*x)/2, 2] + ((3*a^3 + b^3*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c +
d*x]]))/(3*d)
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.80, size = 303, normalized size = 2.44

$$-\frac{2\left(4b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+9a^2b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*a^2*b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b^2-6*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cos\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.93, size = 124, normalized size = 1.00

$$\frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{6\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^3\,\sin\left(c+d\,x\right)\ {}_2}{d\,\sqrt{\cos\left(c+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (6*a*b^2*ellipticE(c/2 + (d*x)/
2, 2))/d + (6*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^3*cos(c + d*x)^(1
/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, c
os(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.579 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2b\sin(c+}{3d\sqrt{\cos(c+}}$$

```
[Out] -2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*(a+b*c
os(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)+16/3*a^2*b*sin(d*x+c)/d/cos(d*x+c)
^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2792, 3021, 2748, 2641, 2639}

$$\frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2b\sin(c+}{3d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (16*a^2*b*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x
]]) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{4a^2b+\frac{1}{2}a\left(a^2+9b^2\right)\cos(c+dx)-\frac{1}{2}b\left(a^2\right.}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{16a^2b\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4}{3}\int \frac{\frac{1}{4}a\left(a^2+9b^2\right)-\frac{3}{4}b\left(3\right.}{\sqrt{\cos(c}}$$

$$= \frac{16a^2b\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \left(b\left(3a^2-b^2\right)\right)\int \sqrt{\cos(c+}$$

$$= -\frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{16a^2b\sin(c+dx}{3d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 1.33, size = 85, normalized size = 0.71

$$\frac{2\left((3b^3-9a^2b)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left((a^2+9b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)(a+9b\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*((-9*a^2*b + 3*b^3)*EllipticE[(c + d*x)/2, 2] + a*((a^2 + 9*b^2)*Ellipti
cF[(c + d*x)/2, 2] + (a*(a + 9*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(
3/2))))/(3*d)
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

maple [B] time = 1.64, size = 631, normalized size = 5.26

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+18*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1
/2*d*x+1/2*c)^2+18*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-6*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2-36*a^2*b*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4-a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*b^2*a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+2*
a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*a^2*b*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.64, size = 128, normalized size = 1.07

$$\frac{2\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)b^3+3\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)b^2\right)}{d}+\frac{2\,a^3\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{6\,a^2\,b\,\sin(c+d\,x)\;{}_2}{d\,\sqrt{\cos(c+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*(b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a*b^2*ellipticF(c/2 + (d*x)/2, 2)))
/d + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*
cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*a^2*b*sin(c + d*x)*hypergeo
```

```
m([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)
^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.580 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6a\left(a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)(a+b\,\mathrm{cc}}{5d\cos^{\frac{5}{2}}(c+dx}$$

```
[Out] -6/5*a*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a^2*b*sin(d*
x+c)/d/cos(d*x+c)^(3/2)+2/5*a^2*(a+b*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(5
/2)+6/5*a*(a^2+5*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2792, 3021, 2748, 2636, 2639, 2641}

$$\frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6a\left(a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)(a+b\,\mathrm{cc}}{5d\cos^{\frac{5}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(a^2 + b^2)*Ell
ipticF[(c + d*x)/2, 2])/d + (8*a^2*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
 + (6*a*(a^2 + 5*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a^2*(a +
b*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{6a^2b+\frac{3}{2}a\left(a^2+5b^2\right)\cos(c+dx)+\frac{1}{2}b\left(a^2\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4}{15}\int \frac{\frac{9}{4}a\left(a^2+5b^2\right)+\frac{15}{4}b\left(\right.}{\cos^{\frac{3}{2}}(c}$$

$$= \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \left(b\left(a^2+b^2\right)\right)\int \frac{1}{\sqrt{\cos(c+d}}$$

$$= \frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\left(a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2a^2(}{}$$

$$= -\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.96, size = 125, normalized size = 0.84

$$\frac{3\left(a^3+5ab^2\right)\sin(2(c+dx))+2a^3\tan(c+dx)+10b\left(a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6a\left(a^2+5b^2\right)\cos^{\frac{3}{2}}(}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*(a^2 + 5*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*b*(a^
2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*b*Sin[c + d*
x] + 3*(a^3 + 5*a*b^2)*Sin[2*(c + d*x)] + 2*a^3*Tan[c + d*x])/(5*d*Cos[c +
d*x]^(3/2))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

maple [B] time = 2.07, size = 738, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*a
^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1
)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*b^2*a*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2
)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.74, size = 156, normalized size = 1.05

$$\frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{5\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{6\,a\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\right.\right.}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^3*sin(c + d*x)*hypergeom([-5/4
, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/
2)) + (6*a*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d
*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^2*b*sin(c + d*x)*hyperge
om([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2
)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.581 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{2a\left(5a^2+21b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2b\left(9a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(9a^2+5b^2\right)}{5d\sqrt{\cos(c}}$$

```
[Out] -2/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(5*a^2+21*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/35*
a^2*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)/d/cos(
d*x+c)^(3/2)+2/7*a^2*(a+b*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/5*b*(
9*a^2+5*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2792, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a\left(5a^2+21b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2b\left(9a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(9a^2+5b^2\right)}{5d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-2*b*(9*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*a^2 + 21*b
^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (32*a^2*b*Sin[c + d*x])/(35*d*Cos[c
 + d*x]^(5/2)) + (2*a*(5*a^2 + 21*b^2)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/
2)) + (2*b*(9*a^2 + 5*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a^2*
(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b+\frac{1}{2}a\left(5a^2+21b^2\right)\cos(c+dx)+\frac{1}{2}b}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4}{35}\int \frac{\frac{5}{4}a\left(5a^2+21b^2\right)+}{\text{co}}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{5}\left(b\left(9a^2+5b^2\right)\right)\int \frac{}{\text{cos}}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b\left(9a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2}{}$$

$$= -\frac{2b\left(9a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\left(5a^2+21b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{32a^2b\sin(c}{35d\cos^{\frac{5}{2}}(c}$$

Mathematica [A] time = 0.81, size = 177, normalized size = 0.91

$$\frac{25a^3\sin(2(c+dx))+30a^3\tan(c+dx)+10a\left(5a^2+21b^2\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-42b\left(9a^2+5b^2\right)\cos^{\frac{5}{2}}(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-42*b*(9*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*a*
(5*a^2 + 21*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a^2*b*S
in[c + d*x] + 378*a^2*b*Cos[c + d*x]^2*Sin[c + d*x] + 210*b^3*Cos[c + d*x]^
2*Sin[c + d*x] + 25*a^3*Sin[2*(c + d*x)] + 105*a*b^2*Sin[2*(c + d*x)] + 30*
a^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

maple [B] time = 2.60, size = 847, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-6/5*a^2*b/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1
/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*b^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1)+6*b^2*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))+2*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2
+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

mupad [B] time = 2.00, size = 147, normalized size = 0.76

$$\frac{\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+d\,x)^2\right)}{7}+2\,b^3\cos(c+d\,x)^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos(c+d\,x)^2\right)+\frac{6\,a^2\,b\,\cos(c+d}{}}{d\cos(c+d\,x)^{7/2}\sqrt{1-\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(9/2),x)
```

```
[Out] ((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + 2*b^
3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) +
 (6*a^2*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*
x)^2))/5 + 2*a*b^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4,
cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.582 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2a^3\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/b^2/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d-2*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2
^(1/2))/b^3/(a+b)/d+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d
```

Rubi [A] time = 0.39, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2 + b^2)*EllipticF[(c +
d*x)/2, 2])/(3*b^3*d) - (2*a^3*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
b^3*(a + b)*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
```

```
n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{a}{2}+\frac{1}{2}b\cos(c+dx)-\frac{3}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{3b}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{2\int \frac{-\frac{ab}{2}-\frac{1}{2}\left(3a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{3b^2} - \frac{a\int \sqrt{\cos(c+dx)}\,dx}{b^2}$$

$$= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{a^3\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{b^3} + \cdots$$

$$= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d} - \frac{2a^3\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a+b)d} + \frac{2\sqrt{c\cdots}}{\cdots}$$

Mathematica [A] time = 2.04, size = 158, normalized size = 1.41

$$\frac{-\frac{6\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b^2\sqrt{\sin^2(c+dx)}} - \frac{6a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\cdots}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (4*EllipticF[(c + d*x)/2, 2] - (6*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1
])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/(6*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.87, size = 516, normalized size = 4.61

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\left(4b^2a-4b^3\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2b^2a+2b^3\right)\left(\sin^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*a*b^2-4*b^
3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a*b^2+2*b^3)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.583 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2a^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^2/(a+b)
/d
```

Rubi [A] time = 0.16, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2804, 2639, 2803, 2641, 2805}

$$\frac{2a^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticF[(c + d*x)/2, 2])/(b^2*
d) + (2*a^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2804

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{\int \sqrt{\cos(c+dx)}\,dx}{b}-\frac{a\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2}+\frac{a^2\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{b^2}\\
&= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2a^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d}
\end{aligned}
$$

Mathematica [A] time = 0.30, size = 81, normalized size = 1.08

$$
-\frac{2\sin(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+a\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+bE\left(\sin^{-1}\left(\sqrt{\cos(\right.\right.}{b^2d\sqrt{\sin^2(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin
[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]]
, -1])*Sin[c + d*x])/(b^2*d*Sqrt[Sin[c + d*x]^2])
```

fricas [F] time = 152.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.79, size = 227, normalized size = 3.03

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{b^2\left(a-b\right)\sqrt{-2\left(\sin^4\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x)),x)
[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

3.584 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b/(a+b)/d
```

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2803, 2641, 2805}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/(b*(a + b)*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{b}$$
$$= \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.91

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b))/(b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.68, size = 188, normalized size = 3.55

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{(a-b)\,b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2)))/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos(c+d\,x)}}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.585 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=29

$$\frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*b/(a+b),2^(1/2))/(a+b)/d
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2805}

$$\frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx = \frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Mathematica [A] time = 0.08, size = 29, normalized size = 1.00

$$\frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.63, size = 150, normalized size = 5.17

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,\left(a+b\,\cos(c+dx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.586 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] -2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a+b)/d+2*sin(d*x+c)/a/d/cos(
d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2802, 3059, 2639, 12, 2805}

$$-\frac{2b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(a*(a + b)*d) + (2*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
```

```
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx &= \frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{2\int \frac{-\frac{b}{2}-\frac{1}{2}a\cos(c+dx)-\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{a} \\
&= \frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{a} - \frac{2\int \frac{b^2}{2\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{ab} \\
&= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{b\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{a} \\
&= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d} + \frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [B] time = 3.11, size = 195, normalized size = 2.53

$$-\frac{\frac{2\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{6b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\right|}{a+b}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] -1/2*((6*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*a*(2*Ell
ipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
 + b)))/b - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1
])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.94, size = 354, normalized size = 4.60

$$-\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/a/(a-b)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.587 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=128

$$\frac{2b^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2/(a+
b)/d+2/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-2*b*sin(d*x+c)/a^2/d/cos(d*x+c)^(1
/2)
```

Rubi [A] time = 0.55, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2b^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*
a*d) + (2*b^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d) +
(2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*b*Sin[c + d*x])/(a^2*d*Sqr
t[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-\frac{3b}{2}+\frac{1}{2}a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}\left(a^2+3b^2\right)+ab\cos(c+dx)+\frac{3}{4}b^2\cos}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx)}}{3a^2}$$

$$= \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}b\left(a^2+3b^2\right)-\frac{1}{4}ab^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{3a^2b}$$

$$= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{\cos(c}}}{3a}$$

$$= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2b^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d} + \cdots$$

Mathematica [A] time = 4.58, size = 210, normalized size = 1.64

$$\frac{\frac{2\left(2a^2+9b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+}\right.\right.\right.}{a\sqrt{\sin^2(c+dx)}}}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] ((2*(2*a^2 + 9*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*
a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2
```

```
, 2])/(a + b)) + (4*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)
 + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ellip
ticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), Ar
cSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/(6*a
^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 1.96, size = 452, normalized size = 3.53

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{a^2\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{2b}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^3/a^2/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-2/a^2*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
```

```
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos(c+d\,x)}^{5/2}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

3.588 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=245

$$-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(5a^2-2b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)}-\frac{a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(15a^4-\right.}{}$$

```
[Out] -a*(5*a^2-4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)/d+1/3*(15*a^4-16*a^2*b^2-2*b^4)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))/b^4/(a^2-b^2)/d-a^3*(5*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^4
/(a+b)^2/d-a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))+1
/3*(5*a^2-2*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)/d
```

Rubi [A] time = 0.70, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-16a^2b^2+15a^4-2b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}-\frac{a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{a^3\left(5a^2-7b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d(a-b)(a+b)^2}-\frac{}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((a*(5*a^2 - 4*b^2)*EllipticE[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)*d)) + ((15
*a^4 - 16*a^2*b^2 - 2*b^4)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d)
 - (a^3*(5*a^2 - 7*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)
*b^4*(a + b)^2*d) + ((5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^
2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(
a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a^2}{2}-ab\cos(c+dx)-\frac{1}{2}\left(5a^2-2b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{2\int \frac{-\frac{1}{4}a(5a\ldots}{\ldots}}{\ldots}$$

$$= \frac{\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{2\int \frac{\frac{1}{4}ab(5a\ldots}{\ldots}}{\ldots}$$

$$= -\frac{a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^{\frac{3}{2}}\ldots}{b\left(a^2-b^2\right)\ldots}$$

$$= -\frac{a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{\left(15a^4-16a^2b^2-2b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4\left(a^2-b^2\right)d} - \frac{a^3\left(5a^2\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.96, size = 266, normalized size = 1.09

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3a^3}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2\right) - \frac{\frac{2\left(5a^3-8ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{8\left(2a^2+b^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} + \ldots}{\ldots}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2 + (3*a^3)/((a^2 - b^2)*(a + b*Cos[c + d*x])))*Sin[
c + d*x] - ((2*(5*a^3 - 8*a*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
```

```
/(a + b) + (8*(2*a^2 + b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(5*a^2 - 4*b^2)*(-2*a*b*El
lipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt
[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c
+ d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))
/(12*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

maple [B] time = 2.59, size = 1070, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^2*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4/b^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*(3*a^2+2*a*b+b^2)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*a^3/b^3/(-2*a*b+2*b^2)*(sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))+2/b^4*a^4*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*
b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{7/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.589 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=185

$$\frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\,(a^2-b^2)}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\,(a^2-b^2)\,(a+b\cos(c+dx))}-\frac{a\,(3a^2-4b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\,(a^2-b^2)}+\frac{a^2\,(3a^2-5b^2)\,\Pi}{b^3d(a-}$$

```
[Out] (3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(3*a^2-4*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^
2-b^2)/d+a^2*(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^3/(a+b)^2/d-a^2*si
n(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.46, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2792, 3059, 2639, 3002, 2641, 2805}

$$-\frac{a\,(3a^2-4b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\,(a^2-b^2)}+\frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\,(a^2-b^2)}+\frac{a^2\,(3a^2-5b^2)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a-b)(a+b)^2}-\frac{a^2\sin(c-}{bd\,(a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - (a*(3*a^2
 - 4*b^2)*EllipticF[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)*d) + (a^2*(3*a^2 - 5*
b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) -
 (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{a^2}{2}-ab\cos(c+dx)-\frac{1}{2}(3a^2-2b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b(a^2-b^2)}$$

$$= -\frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{a^2b}{2}-\frac{1}{2}a(3a^2-4b^2)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b^2(a^2-b^2)} + \frac{(3a^2-2b^2)\int \cdots}{2b^2(a\cdots}$$

$$= \frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a^2-b^2)\,d} - \frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{(a^2(3a^2-5b^2))\cdots}{2\cdots}$$

$$= \frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a^2-b^2)\,d} - \frac{a(3a^2-4b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a^2-b^2)\,d} + \frac{a^2(3a^2-5b^2)\,\Pi\left(\cdots\right.}{(a-b)b^3(\cdots}$$

Mathematica [A] time = 1.85, size = 251, normalized size = 1.36

$$\frac{\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{(b^2-a^2)(a+b\cos(c+dx))} + \frac{\frac{2(a^2-2b^2)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2(3a^2-2b^2)\sin(c+dx)\left((b^2-2a^2)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\cdots\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x])
) + ((2*(a^2 - 2*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) +
4*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b)) + (2*(3*a^2 - 2*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*
a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x]
)/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

maple [B] time = 2.24, size = 815, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-12*a^2/b^2/(-2*a*b+2*b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2))-2/b^3*a^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^
2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.590 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=163

$$\frac{(a^2-2b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\,(a^2-b^2)}-\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\,(a^2-b^2)}-\frac{a\,(a^2-3b^2)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\,(a^2-b^2)\,(a+b\cos(c+dx))}$$

```
[Out] -a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/b/(a^2-b^2)/d+(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(a^2-3
*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*
x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^2/(a+b)^2/d+a*sin(d*x+c)*cos(d*x+c)^(1/
2)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.38, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2799, 3059, 2639, 3002, 2641, 2805}

$$\frac{(a^2-2b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\,(a^2-b^2)}-\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\,(a^2-b^2)}-\frac{a\,(a^2-3b^2)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\,(a^2-b^2)\,(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((a*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d)) + ((a^2 - 2*b^2)*Ellipti
cF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - (a*(a^2 - 3*b^2)*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2])/((a - b)*b^2*(a + b)^2*d) + (a*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{a}{2}+b\cos(c+dx)+\frac{1}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{-a^2+b^2}$$

$$= \frac{a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{ab}{2}+\frac{1}{2}\left(a^2-2b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{b\left(a^2-b^2\right)} - \frac{a\int\sqrt{\cos(c+dx)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(a\left(a^2-3b^2\right)\right)\int\frac{1}{\sqrt{\cos(c+dx)}\,(a+b}}{2b^2\left(a^2-b^2\right)}$$

$$= -\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(a^2-2b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{a\left(a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{(a-b)b^2(a+b)^2d}$$

Mathematica [A] time = 3.40, size = 194, normalized size = 1.19

$$\frac{\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} - \frac{\frac{2\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b^2\sqrt{\sin^2(c+dx)}} - \frac{10a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\right.}{a+b}}{(a-b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) -
 (8*EllipticF[(c + d*x)/2, 2] - (10*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2])/(a + b) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*
(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c +
 d*x]^2]))/((a - b)*(a + b)))/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

maple [B] time = 1.81, size = 794, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8/b*a
/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/b^2*a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*
a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.591 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{\left(a^2+b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))/(a^2-b^2)/d+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d-(a^2+b^2)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^
(1/2))/(a-b)/b/(a+b)^2/d-b*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos
(d*x+c))
```

Rubi [A] time = 0.40, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2796, 3059, 2639, 3002, 2641, 2805}

$$\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{\left(a^2+b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/((a^2 - b^2)*d) + (a*EllipticF[(c + d*x)/2, 2])/(
b*(a^2 - b^2)*d) - ((a^2 + b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
((a - b)*b*(a + b)^2*d) - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*
d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = -\frac{b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{b}{2}-a\cos(c+dx)-\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{-a^2+b^2}$$

$$= -\frac{b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)} + \frac{\int \frac{-\frac{b^2}{2}+\frac{1}{2}ab\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,d}{b\left(a^2-b^2\right)}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} - \frac{b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2b\left(a^2-b^2\right)} - \frac{\left(a^2+b^2\right)\int}{}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} + \frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} - \frac{\left(a^2+b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b(a+b)^2d} - \frac{b\sqrt{\cos(}}{\left(a^2-b^2\right)}$$

Mathematica [A] time = 3.70, size = 229, normalized size = 1.55

$$\frac{\frac{4b\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(b^2-a^2\right)(a+b\cos(c+dx))} - \frac{2\left(\frac{\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a\sqrt{\sin^2(c+dx)}} - \frac{b^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{a+b}\right.}{b(b-a)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x]))
- (2*(-((b^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*a*(2*E
llipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a + b)) + ((-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)
*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/
a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2])
))/(b*(-a + b)*(a + b)))/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

maple [B] time = 1.91, size = 713, normalized size = 4.82

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{2a\left(-\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/(-2*a*b+2*b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))-2/b*a*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/
(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a
*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.592 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2-b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] -b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/(a^2-b^2)/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/(a^2-b^2)/d+(3*a^2-b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b)
,2^(1/2))/a/(a-b)/(a+b)^2/d+b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(
a+b*cos(d*x+c))
```

Rubi [A] time = 0.44, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2802, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2-b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -((b*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - EllipticF[(c + d*x)/2,
 2]/((a^2 - b^2)*d) + ((3*a^2 - b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(a*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2-b^2\right)-ab\cos(c+dx)-\frac{1}{2}b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,d}{a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}b\left(2a^2-b^2\right)+\frac{1}{2}ab^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{ab\left(a^2-b^2\right)} - \frac{b\int}{}$$

$$= -\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{2\left(a^2-b^2\right)}$$

$$= -\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\right|}{a(a-b)(a+b)^2d}$$

Mathematica [A] time = 3.51, size = 238, normalized size = 1.52

$$\frac{\frac{4b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(4a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{2\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{c}\right.\right.\right.}{a\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x]))
 + ((2*(4*a^2 - 3*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) +
 8*a*(-EllipticF[(c + d*x)/2, 2] + (a*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2])/(a + b)) - (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c +
d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

maple [B] time = 1.36, size = 612, normalized size = 3.90

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a\left(a^2-b^2\right)\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\cdots\right.\right.}}{(a+b)a\sqrt{-2\left(\sin\cdots\right.}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^2/a/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b/a/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b/a/(a^2-b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-6*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a/(a^2-b^2)/(-2*a*b
+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.593 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} dx$

Optimal. Leaf size=217

$$\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{b\left(5a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}+\frac{b^2\sin(c+}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}$$

```
[Out] -(2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d+b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d-b*(5*a
^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/
2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2/(a-b)/(a+b)^2/d+(2*a^2-3*b^2)*sin(d*x+c
)/a^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)+b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*
x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{b\left(5a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}+\frac{b^2\sin(c+}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -(((2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d)) + (b*Ell
ipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - (b*(5*a^2 - 3*b^2)*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2 - 3*b^2)
*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b^2*Sin[c + d*x])/
(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2-3b^2\right)-ab\cos(c+dx}{\cos^{\frac{3}{2}}(c+dx)(a+b}}{a\left(a^2-\right.}$$

$$= \frac{\left(2a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c}$$

$$= \frac{\left(2a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c}$$

$$= -\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(2a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{}{a\left(a^2-\right.}$$

$$= -\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{b\left(5a^2-3b^2\right)\Pi\left(\right.}{a^2(a-b}$$

Mathematica [A] time = 3.14, size = 278, normalized size = 1.28

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{b^3\sin(c+dx)}{\left(b^2-a^2\right)(a+b\cos(c+dx))}+2\tan(c+dx)\right)-\frac{\frac{\left(8ab^2-4a^3\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}+\frac{2\left(9b^3-10a^2b\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{a+b}}{}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (-(((2*(-10*a^2*b + 9*b^3)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a +
b) + ((-4*a^3 + 8*a*b^2)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b - (2*(2*a^2 - 3*b^2)*(-2*a*b*Ellip
ticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d
*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b))) +
 4*Sqrt[Cos[c + d*x]]*((b^3*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x]
)) + 2*Tan[c + d*x]))/(4*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 2.49, size = 874, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/a^2*b^2/(-2*a
*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))+2/a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c
)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/a*b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
```

```
*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^
2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.594 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=281

$$\frac{(2a^2-5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{(2a^2-5b^2)\sin(c+dx)}{3a^2d(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)}+\frac{b(4a^2-\cdots}{\cdots}$$

```
[Out] b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*a^2-5*b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2
/(a^2-b^2)/d+b^2*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+1/
3*(2*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)+b^2*sin(d*x+c)/
a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-b*(4*a^2-5*b^2)*sin(d*x+c)/
a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.00, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(2a^2-5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(a^2-b^2)}+\frac{b(4a^2-5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)}+\frac{b^2(7a^2-5b^2)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}+\frac{\cdots}{ad(a^2-b^2)\,\mathrm{c}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + ((2*a^2
 - 5*b^2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*(a^2 - b^2)*d) + (b^2*(7*a^2 -
5*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d)
 + ((2*a^2 - 5*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
- (b*(4*a^2 - 5*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) +
 (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2-5b^2\right)-ab\cos(c+dx)+\ldots}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos\ldots}}{a\left(a^2-b^2\ldots}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+\ldots}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(4a^2-5b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{\ldots}{a\left(a^2-b^2\ldots}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(4a^2-5b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{\ldots}{a\left(a^2-b^2\ldots}$$

$$= \frac{b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(4a^2\ldots}{a^3\left(a^2-\ldots}$$

$$= \frac{b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\left(7a^2\ldots}{\ldots}$$

Mathematica [A] time = 3.57, size = 294, normalized size = 1.05

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{3b^4\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))}+2\tan(c+dx)(a\sec(c+dx)-6b)\right)+\frac{\frac{8(7a^3-10ab^2)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}}{}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (((2*(4*a^4 + 44*a^2*b^2 - 45*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(a + b) + (8*(7*a^3 - 10*a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(4*a^2 - 5*b^2)*(-2
*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt
[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a +
 b)) + 4*Sqrt[Cos[c + d*x]]*((3*b^4*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c
 + d*x])) + 2*(-6*b + a*Sec[c + d*x])*Tan[c + d*x]))/(12*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 3.36, size = 1008, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*b^3/a^3/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-4/a^3*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))+2*b^2/a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2
)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.595 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=346

$$-\frac{a^2\left(7a^2-13b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{a\left(35a^4-65a^2b^2+24b^4\right)E\left(\frac{1}{2}(c+\right.}{4b^4d\left(a^2-b^2\right)^2}$$

```
[Out] -1/4*a*(35*a^4-65*a^2*b^2+24*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^4/(a^2-b^2)^2/d+1/12*(105*a^
6-223*a^4*b^2+128*a^2*b^4+8*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^5/(a^2-b^2)^2/d-1/4*a^3*(35*a
^4-86*a^2*b^2+63*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^5/(a+b)^3/d-1/2*a^2*c
os(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*a^2*(7*a^2-
13*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))+1/12
*(35*a^4-61*a^2*b^2+8*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d
```

Rubi [A] time = 1.04, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2792, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-223a^4b^2+128a^2b^4+105a^6+8b^6\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^5d\left(a^2-b^2\right)^2}-\frac{a\left(-65a^2b^2+35a^4+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{a^3\left(-86a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -(a*(35*a^4 - 65*a^2*b^2 + 24*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 -
 b^2)^2*d) + ((105*a^6 - 223*a^4*b^2 + 128*a^2*b^4 + 8*b^6)*EllipticF[(c +
d*x)/2, 2])/(12*b^5*(a^2 - b^2)^2*d) - (a^3*(35*a^4 - 86*a^2*b^2 + 63*b^4)*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^5*(a + b)^3*d) +
((35*a^4 - 61*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*b^3*(a^
2 - b^2)^2*d) - (a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x])^2) - (a^2*(7*a^2 - 13*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
```

```
b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a^2}{2}-2ab\cos(c+dx)-\frac{1}{2}\left(7a^2-4b^2\right)\cos^2(c+dx}{(a+b\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(7a^2-13b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int}{}$$

$$= \frac{\left(35a^4-61a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx)}$$

$$= \frac{\left(35a^4-61a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx)}$$

$$= -\frac{a\left(35a^4-65a^2b^2+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d} + \frac{\left(35a^4-61a^2b^2+8b^4\right)\sqrt{\cos(c+dx}}{12b^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a\left(35a^4-65a^2b^2+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d} + \frac{\left(105a^6-223a^4b^2+128a^2b^4+8b^6\right)}{12b^5\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 3.23, size = 354, normalized size = 1.02

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(35a^6-57a^4b^2+4\left(b^3-a^2b\right)^2\cos(2(c+dx))+ab\left(49a^4-83a^2b^2+16b^4\right)\cos(c+dx)+4b^6\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{\frac{2\left(35a^5-73a^3b^2+56ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(35*a^6 - 57*a^4*b^2 + 4*b^6 + a*b*(49*a^4 - 83*a^2*
b^2 + 16*b^4)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*Cos[2*(c + d*x)])*Sin[c +
 d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(35*a^5 - 73*a^3*b^2 +
56*a*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(7*a^4 -
 14*a^2*b^2 - 2*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b
)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(35*a^4 - 65*a^2*b^2 + 24*b^4)*(-
```

```
2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[Arc
Sin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqr
t[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2
*(a + b)^2))/(48*b^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (dx + c)^{\frac{9}{2}}}{(b \cos (dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

maple [B] time = 3.69, size = 2194, normalized size = 6.34

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^3*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/b^4*(3*a+2*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(6*a^2+3*a*b+b^2)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/b^5*a^5*(-1/2*b^2/a/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1
```

/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+40/b^4*a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+10/b^5*a^4*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{9/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)/(a + b*cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(9/2)/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

3.596 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=282

$$-\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{a^2(5a^2-11b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{4b^2d(a^2-b^2)^2(a+b\cos(c+dx))}-\frac{3a(5a^4-11a^2b^2+8b^4)F\left(\frac{1}{2}(c+\right.}{4b^4d(a^2-b^2)^2}$$

```
[Out] 1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d-3/4*a*(5*a^4-11*
a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))/b^4/(a^2-b^2)^2/d+1/4*a^2*(15*a^4-38*a^2*b^2+35*b
^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^4/(a+b)^3/d-1/2*a^2*cos(d*x+c)^(3/2)*si
n(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*a^2*(5*a^2-11*b^2)*sin(d*x+c)
*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.77, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3047, 3059, 2639, 3002, 2641, 2805}

$$-\frac{3a(-11a^2b^2+5a^4+8b^4)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d(a^2-b^2)^2}+\frac{(-29a^2b^2+15a^4+8b^4)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a^2-b^2)^2}+\frac{a^2(-38a^2b^2+15a^4+3}{4b^4d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2
)^2*d) - (3*a*(5*a^4 - 11*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(4*b^
4*(a^2 - b^2)^2*d) + (a^2*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/(
a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^4*(a + b)^3*d) - (a^2*Cos[c + d*x]^
(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(5*a^
2 - 11*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*
Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
```

```
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a^2}{2}-2ab\cos(c+dx)-\frac{1}{2}\left(5a^2-4b^2\right)\cos^2(c+d}{(a+b\cos(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))} + \cdots$$

$$= -\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))} - \cdots$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d} - \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2(5\cdots}{4\cdots}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d} - \frac{3a\left(5a^4-11a^2b^2+8b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2d}\cdots$$

Mathematica [A] time = 3.01, size = 309, normalized size = 1.10

$$\frac{\frac{\frac{8\left(a^3-4ab^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{\left(5a^4-7a^2b^2+8b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a\cdots\right)}{ab^2\sqrt{\sin^2(c+\cdots}}}{(a-b)^2(a+b)^2}}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*a^2*Sqrt[Cos[c + d*x]]*(5*a^3 - 11*a*b^2 + b*(7*a^2 - 13*b^2)*Cos[c +
d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((5*a^4 - 7*a
^2*b^2 + 8*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(a^
3 - 4*a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2]))/(a + b) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*(-2*a*b*Ellip
ticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d
*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^
2))/(8*b^2*d)
```

fricas [F] time = 160.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(7/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 +
3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

maple [B] time = 3.52, size = 1935, normalized size = 6.86

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
```

EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)+2/b^4*a^4*(-1/2*b^2/a/(a^2-b^2)*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a
+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2
*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-24*a^2/b^3/(-
2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))-8/b^4*a^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(
a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b

```
/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{7/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

3.597 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=264

$$-\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{3a\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{(3$$

```
[Out] -3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(3*a^4-5*a^2*b^2+8*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))/b^3/(a^2-b^2)^2/d-3/4*a*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1
/2))/(a-b)^2/b^3/(a+b)^3/d-1/2*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/
d/(a+b*cos(d*x+c))^2+3/4*a*(a^2-3*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b
^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.78, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-5a^2b^2+3a^4+8b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{3a\left(-2a^2b^2+a^4+5b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{4b^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-3*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((
3*a^4 - 5*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*
d) - (3*a*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*(a - b)^2*b^3*(a + b)^3*d) - (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{a^2}{2}-2ab\cos(c+dx)-\frac{1}{2}\left(3a^2-4b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int}{}$$

$$= -\frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int}{}$$

$$= -\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} - \frac{a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a\left(a^2-3b^2\right.}{4b\left(a^2-\right.}$$

$$= -\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(3a^4-5a^2b^2+8b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{3a\left(a^4-\right.}{}$$

Mathematica [A] time = 2.17, size = 284, normalized size = 1.08

$$\frac{\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3+3b\left(a^2-3b^2\right)\cos(c+dx)-7ab^2\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{\frac{2\left(a^3+5ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{16\left(a^2+2b^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} + \frac{6\left(a^2-3b\right.}{}}{}}{16bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((4*a*Sqrt[Cos[c + d*x]]*(a^3 - 7*a*b^2 + 3*b*(a^2 - 3*b^2)*Cos[c + d*x])*S
in[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(a^3 + 5*a*b^2)*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (16*(a^2 + 2*b^2)*((a +
 b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
))/(a + b) + (6*(a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]],
 -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^
2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*S
qrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

maple [B] time = 3.08, size = 1914, normalized size = 7.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/b^3
*a^3*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2
```

```
)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(
a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/
(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
(a-b),2^(1/2)))+12/b^2*a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+6/b^3*a^2*(-b^2/a/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a
^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

3.598 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=244

$$\frac{a\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{\left(a^2+5b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a\sin(c+dx}{2d\left(a^2-b^2\right)(a}$$

```
[Out] -1/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)^2/d+1/4*a*(a^2-7*b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^
2/(a^2-b^2)^2/d-1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^
2/(a+b)^3/d+1/2*a*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^
2+1/4*(a^2+5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c)
)
```

Rubi [A] time = 0.66, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2799, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{a\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}-\frac{\left(-10a^2b^2+a^4-3b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a-b)^2(a+b)^3}+\frac{\left(a^2+5\right.}{4d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (a*(a^2
- 7*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((a^4 - 10*a^
2*b^2 - 3*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^2*
(a + b)^3*d) + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2) + ((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*(a^2
- b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-\frac{a}{2}+2b\cos(c+dx)-\frac{1}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int -\frac{3}{4}a\left(a^2 \ldots\right.}{\ldots}$$

$$= \frac{a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{3}{4}ab\left(a^2 \ldots\right.}{\ldots}$$

$$= -\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+5b^2\right)\sqrt{\cos \ldots}}{4\left(a^2-b^2\right)^2 d \ldots}$$

$$= -\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{a\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} - \frac{\left(a^4-10a^2b^2-3b^4\right) \ldots}{4(a-b)^2 \ldots}$$

Mathematica [A] time = 2.07, size = 272, normalized size = 1.11

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(a^2+5b^2\right)\cos(c+dx)+3a\left(a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{-\frac{2\left(5a^2+b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\left(a^2+5b^2\right)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2 \ldots\right)}{ab^2\sqrt{\sin \ldots}}}{\ldots}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(3*a*(a^2 + b^2) + b*(a^2 + 5*b^2)*Cos[c + d*x])*Sin
[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((-2*(5*a^2 + b^2)*Elli
pticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*a*(2*EllipticF[(c + d*x
)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2*(a^
2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*
EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a
), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^
2]))/((a - b)^2*(a + b)^2))/(16*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

maple [B] time = 3.05, size = 1836, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*a^2*(-1/2
*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2
-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-
```

```
b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/
a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*
a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*
b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2)))-4/b/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-4/b^2*a*(-b^2/a/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2
*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos \left(c+d\,x\right)^{3/2}}{{\left(a+b\,\cos \left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.599 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=250

$$\frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}-\frac{b\left(5a^2+b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4ad\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{b\sin(c+d}{2d\left(a^2-b^2\right)(}$$

```
[Out] 1/4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)^2/d+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-
b^2)^2/d-1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a-b)^2/b/(a+b)
^3/d-1/2*b*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*b
*(5*a^2+b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.68, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2796, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}-\frac{\left(10a^2b^2+3a^4-b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4abd(a-b)^2(a+b)^3}-\frac{b\left(5a^2+b\right.}{4ad\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + (3*(a^2 +
 b^2)*EllipticF[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) - ((3*a^4 + 10*a^2*b
^2 - b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b*(a +
b)^3*d) - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) - (b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 -
 b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = -\frac{b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{b}{2}-2a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{1}{4}b \cdots}{\cdots}}{\cdots}$$

$$= -\frac{b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{4} \cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} - \frac{b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(5a^2+b^2\right)\sqrt{\cos \cdots}}{4a\left(a^2-b^2\right)^2 d \cdots}$$

$$= \frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} - \frac{\left(3a^4+10a^2b^2-b^4\right)\Pi \cdots}{4a(a-b)^2 b \cdots}$$

Mathematica [A] time = 3.07, size = 291, normalized size = 1.16

$$\frac{\frac{\frac{2\left(3b^3-9a^2b\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8a\left(2a^2+b^2\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}+\frac{2\left(5a^2+b^2\right)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cdots}\right.\right.\right.}{ab\sqrt{\sin^2(c+dx)}\cdots}}{(a-b)^2(a+b)^2}\cdots}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-4*b*Sqrt[Cos[c + d*x]]*(7*a^3 - a*b^2 + b*(5*a^2 + b^2)*Cos[c + d*x])*Si
n[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*b + 3*b^3)
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^2 + b^2)*(2
*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a + b)))/b + (2*(5*a^2 + b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 +
 b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*
b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

maple [B] time = 2.96, size = 1736, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b*a*(-1/2*b^
2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
```

```
^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2
/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
))+2/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/
a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.600 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$-\frac{(7a^2-b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}-\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b^2\sin\cdots}{2ad\left(a^2\cdots\right.}$$

```
[Out] -3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*(7*a^2-b^2)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/
a/(a^2-b^2)^2/d+3/4*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2/(a-b)^2
/(a+b)^3/d+1/2*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c
))^2+3/4*b^2*(3*a^2-b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b
*cos(d*x+c))
```

Rubi [A] time = 0.77, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{(7a^2-b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}-\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{3\left(-2a^2b^2+5a^4+b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a-b)^2(a+b)^3}+\frac{3b^2\cdots}{4\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] (-3*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((
7*a^2 - b^2)*EllipticF[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + (3*(5*a^4 -
 2*a^2*b^2 + b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)
^2*(a + b)^3*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^2) + (3*b^2*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2-3b^2\right)-2ab\cos(c+dx)+\frac{1}{2}b^2\cos^2(c}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sin}{4a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sin}{4a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= -\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}$$

$$= -\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} - \frac{\left(7a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} + \frac{3\left(5a\right.}{}$$

Mathematica [A] time = 3.00, size = 301, normalized size = 1.15

$$\frac{\frac{4b^2\sin(c+dx)\sqrt{\cos(c+dx)}\left(11a^3+\left(9a^2b-3b^3\right)\cos(c+dx)-5ab^2\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} + \frac{\frac{16\left(ab^2-4a^3\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} - \frac{6\left(3a^2-b^2\right)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a}\right.\right.}{}}{}}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*(11*a^3 - 5*a*b^2 + (9*a^2*b - 3*b^3)*Cos[c + d*
x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(16*a^4 - 19
*a^2*b^2 + 9*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*
(-4*a^3 + a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]))/(a + b) - (6*(3*a^2 - b^2)*(-2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos \left(dx+c\right)+a\right)^{3} \sqrt{\cos \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

maple [B] time = 1.92, size = 1176, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-b^2/a/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/2*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2*b+a-b)-7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a+b)/(a^2-b^2)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/
```

```
4/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*b^2-9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*b^3/a^2/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-3/4*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/2*a^2/(a^2-b^2)^2
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2))-3/2/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.601 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=328

$$\frac{b\left(11a^2-5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}+\frac{b^2\sin(c+d}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(}$$

```
[Out] -1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*b*(11*a^2-5
*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*b*(35*a^4-38*a^2*b^2+15*b^4)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/
(a+b),2^(1/2))/a^3/(a-b)^2/(a+b)^3/d+1/4*(8*a^4-29*a^2*b^2+15*b^4)*sin(d*x+
c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b
*cos(d*x+c))^2/cos(d*x+c)^(1/2)+1/4*b^2*(11*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-
b^2)^2/d/(a+b*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{b\left(11a^2-5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2b^2+8a^4+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{b\left(-38a^2b^2+35a^4+15b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\right.}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] -((8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^
2)^2*d) + (b*(11*a^2 - 5*b^2)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)
^2*d) - (b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Si
n[c + d*x])/(4*a^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b^2*Sin[c + d*x])
/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b^2*(11*a
^2 - 5*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*
Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2-5b^2\right)-2ab\cos(c}{\cos^{\frac{3}{2}}(c+dx)(a\cdot}}{2a\left(a^2\right.}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2} + \frac{b^2\left(11a^2\right.}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\mathrm{co}}}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+}$$

$$= -\frac{\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-29a^2b^2+15b^4\right)\sin}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c}}$$

$$= -\frac{\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{b\left(11a^2-5b^2\right)F\left(\frac{1}{2}(c+dx\right.}{4a^2\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 3.62, size = 334, normalized size = 1.02

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{b^3\sin(c+dx)\left(-15a^3+\left(7b^3-13a^2b\right)\cos(c+dx)+9ab^2\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+8\tan(c+dx)\right)-\frac{\frac{2\left(56a^4b-95a^2b^3+45b^5\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8a\left(2a^4-10a\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] (-(((2*(56*a^4*b - 95*a^2*b^3 + 45*b^5)*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b) + (8*a*(2*a^4 - 10*a^2*b^2 + 5*b^4)*(2*EllipticF[(c + d*x)/
2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(8
*a^4 - 29*a^2*b^2 + 15*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -
1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)
*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqr
t[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + 4*Sqrt[Cos[c + d*x]]*((b^3*(-1
5*a^3 + 9*a*b^2 + (-13*a^2*b + 7*b^3)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b
^2)^2*(a + b*Cos[c + d*x])^2) + 8*Tan[c + d*x]))/(16*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 3.71, size = 1992, normalized size = 6.07

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*b*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+4*b^2/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/a^2*b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+

```
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(
a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(
-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.602 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=395

$$\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+\frac{b\left(24a^4-65a\right.}{4}$$

```
[Out] 1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*a^4-6
1*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*b^2*(63*a^4-86*a^2*b^2+3
5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d
*x+1/2*c),2*b/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b)^3/d+1/12*(8*a^4-61*a^2*b^2+3
5*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)+1/2*b^2*sin(d*x+c)/a/(
a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2+1/4*b^2*(13*a^2-7*b^2)*sin(d
*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-1/4*b*(24*a^4-65*
a^2*b^2+35*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.35, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-61a^2b^2+8a^4+35b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-65a^2b^2+24a^4+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b^2\left(-86a^2b^2+63a^4+3\right.}{4a^4d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 -
b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*EllipticF[(c + d*x)/2, 2])/(12*a
^3*(a^2 - b^2)^2*d) + (b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/
(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 61*a^2*
b^2 + 35*b^4)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)) - (
b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(
a + b*Cos[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sin[c + d*x])/(4*a^2*(a^2 -
b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2-7b^2\right)-2ab\cos(c+\ldots}{\cos^{\frac{5}{2}}(c+dx)(a+b\ldots}}{2a\left(a^2-\ldots\right.}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{b^2\left(13a^2-\ldots\right.}{4a^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}\ldots}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\ldots}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+\ldots}{4a^4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+\ldots}{4a^4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}$$

$$= \frac{b\left(24a^4-65a^2b^2+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin\ldots}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c\ldots}$$

$$= \frac{b\left(24a^4-65a^2b^2+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)F\left(\ldots\right.}{12a^3\left(a^2-b^2\right)^2\ldots}$$

Mathematica [A] time = 5.96, size = 349, normalized size = 0.88

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{3b^4\sin(c+dx)\left(19a^3+b\left(17a^2-11b^2\right)\cos(c+dx)-13ab^2\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+8\tan(c+dx)(a\sec(c+dx)-9b)\right)+\frac{16\left(20a^5-64a^3b^2+35ab^4\right)\ldots}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] (((2*(16*a^6 + 328*a^4*b^2 - 641*a^2*b^4 + 315*b^6)*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2])/(a + b) + (16*(20*a^5 - 64*a^3*b^2 + 35*a*b^4)*((a + b)
```

```
*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/
(a + b) + (6*(24*a^4 - 65*a^2*b^2 + 35*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[C
os[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1]
+ (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c
+ d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2) + 4*Sqrt[Cos[c + d*
x]]*((3*b^4*(19*a^3 - 13*a*b^2 + b*(17*a^2 - 11*b^2)*Cos[c + d*x])*Sin[c +
d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + 8*(-9*b + a*Sec[c + d*x])*Ta
n[c + d*x]))/(48*a^4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 5.45, size = 2128, normalized size = 5.39

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2/a^2*(-1/2
*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2
-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-
b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
```

),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-12*b^3/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-6/a^4*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4*b^2/a^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

3.603 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=438

$$\frac{\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\sin(c+dx)\surd$$

```
[Out] 1/4*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1/2*sin(d*x+c)
*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/4*(a-b)*cot(d*x+c)*EllipticE((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
+1/4*(a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/4*(a^2-4*b^2)*cot(d*x+c)*EllipticPi((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/b^2/d
```

Rubi [A] time = 0.92, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2821, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\sin(c+dx)\surd$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (Sqrt[a
+ b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (Sqrt[a + b]*(a
^2 - 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) + (a*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{\int \frac{\frac{ab}{2}+b^2\cos(c+dx)+\frac{1}{2}ab\cos}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c}}}{2b}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{2d}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{2d}$$

$$= \frac{\sqrt{a+b}\,(a^2-4b^2)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{4b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(}{a+}}}{4bd}$$

Mathematica [C] time = 17.59, size = 1152, normalized size = 2.63

$$-\frac{12a^2\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+d}{a}}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-12*a^
2*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
```

```
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]) - 16*a*b*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])) + 2*a*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a
- b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*C
os[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*d)
```

fricas [F] time = 88.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.27, size = 1233, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*(-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+2*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*cos(d*x+c)^4*b^2-2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+8*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x
+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*s
in(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b*sin(d*x+c)+3*cos(d*x+c)^3*a*b+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*
b-2*cos(d*x+c)^2*b^2-a^2*cos(d*x+c)-2*a*b*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2
)/cos(d*x+c)^(1/2)/sin(d*x+c)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos\left(c+dx\right)}\,\cos^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2), x)
```

3.604 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=371

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\cdots\right.}{d}$$

```
[Out] sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(a-b)*cot(d*x+c)*Ellip
ticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a/d+cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/d-a*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A] time = 0.57, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2821, 3054, 2809, 12, 2801, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\cdots\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[a
+ b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (a*Sqrt[a + b]*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rule 3054

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\int \frac{-\frac{ab}{2}+\frac{1}{2}ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}a\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \frac{\int -\cdots}{\cdots}$$

$$= -\frac{a\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec\cdots}{a+\cdots}}\cdots}{bd}$$

$$= -\frac{a\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec\cdots}{a+\cdots}}\cdots}{bd}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec\cdots}{a\cdots}}\cdots}{ad}$$

Mathematica [A] time = 7.06, size = 314, normalized size = 0.85

$$\frac{\sqrt{\cos(c+dx)}\left(-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}} + \frac{2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}} + \frac{4a\sqrt{\frac{a\cdots}{(a+b\cdots}}\cdots}{\cdots}\cdots\right)}{2d\sqrt{a+b\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])] - (4*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (4*a*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + b*Sec[(c + d*x)/2]*Sin[(3
*(c + d*x))/2] + 2*a*Tan[(c + d*x)/2] - b*Tan[(c + d*x)/2]))/(2*d*Sqrt[a +
b*Cos[c + d*x]])
```

fricas [F] time = 3.07, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.27, size = 801, normalized size = 2.16

$$-\frac{-2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
```

```
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+
2*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)+cos(d*x+c)^3*b+a*cos(d*x+c)^2-cos(d*x+c)^2*b-a*cos(d*x+c))/(a+b*cos
(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.605 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{2\csc(c+dx)\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right)|-\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

```
[Out] -2*(a+b*cos(d*x+c))*csc(d*x+c)*EllipticPi((a+b)^(1/2)*cos(d*x+c)^(1/2)/(a+b
*cos(d*x+c))^(1/2),b/(a+b),((-a+b)/(a+b))^(1/2))*(a*(1-cos(d*x+c))/(a+b*cos
(d*x+c)))^(1/2)*(a*(1+cos(d*x+c))/(a+b*cos(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2811}

$$-\frac{2\csc(c+dx)\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right)|-\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[(a*(1 - Cos[c + d*x]))/(a + b*Cos[c + d*x])]*Sqrt[(a*(1 + Cos[c +
d*x]))/(a + b*Cos[c + d*x])]*(a + b*Cos[c + d*x])*Csc[c + d*x]*EllipticPi[b
/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]]]
, -((a - b)/(a + b))])/(Sqrt[a + b]*d)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(1+\cos(c+dx))}{a+b\cos(c+dx)}}\,(a+b\cos(c+dx))\csc(c+dx)\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+}}\right.\right.}{\sqrt{a+b}\,d}$$

Mathematica [A] time = 1.28, size = 137, normalized size = 1.01

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left((a-b)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2b\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b}{a}\right.\right.}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*b*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.22, size = 197, normalized size = 1.46

$$-\frac{2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)a-\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)b+2b\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx}{\sin(dx+c}\right.\right.}{d\sqrt{a+b\cos(dx+c)}\,\cos(dx+c)^{\frac{3}{2}}(-1+\cos(dx+c))^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(a+b*cos(d*x+c))^(1/2)*(EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*b*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2)))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*s
in(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\cos(c+d\,x)}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.606 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=229

$$\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\,\cot}{}$$

```
[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a/d-2*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.26, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2795, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\,\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b
)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + (-a+b)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec}{a-}}}{ad}$$

Mathematica [A] time = 3.46, size = 203, normalized size = 0.89

$$-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\cos(c+dx)}\left(-\sin(c+dx)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}-\sqrt{\cos(c+dx)}\,\sqrt{\cos(c+dx)+1}\,F\left(\sin^{-1}\right.\right.}{d\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c}{(a+b)(\cos(c-}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] -((Sqrt[a + b*Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(Sqrt[Cos[c + d*x]]*Sqrt[1 +
 Cos[c + d*x]]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - Sqrt
[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)] - Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
```

```
Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]))
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

maple [B] time = 0.25, size = 789, normalized size = 3.45

$$-\frac{2\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a+\mathrm{EllipticF}\left(\frac{-1+}{\mathrm{si}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
```

```
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c
)^2*b+a*cos(d*x+c)-b*cos(d*x+c)-a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*
x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cos\left(c+dx\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.607 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=271

$$\frac{2b(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2\sin(c+dx)\sqrt{a+}}{3d\cos^{\frac{3}{2}}(c}$$

```
[Out] 2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/3*(a-b)*b*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/a^2/d+2/3*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a
+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.40, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2796, 2998, 2816, 2994}

$$\frac{2b(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2\sin(c+dx)\sqrt{a+}}{3d\cos^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d) + (2*
(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
```

```
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{b}{2}+\frac{1}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2(a-b)b\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\mathrm{se}}{a}}}{3a^2d}$$

Mathematica [A] time = 7.87, size = 247, normalized size = 0.91

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(2a^2+2a(a+2b)\cos(c+dx)+b(a+b)\cos(2(c+dx))+ab+b^2\right)+2a(a+b)\sqrt{\cos(c+dx)+1}}{}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]

[Out] (-2*b*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (2*a^2 + a*b + b^2 + 2*a*(a + 2*b)*Cos[c + d*x] + b*(a + b)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(3*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]])

fricas [F] time = 1.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)

maple [B] time = 0.30, size = 880, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*a^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2
+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*a^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+cos(d*x+c)^3*a
*b+cos(d*x+c)^3*b^2+cos(d*x+c)^2*a^2+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-2*a*
b*cos(d*x+c)-a^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\cos(c+d\,x)}}{{\cos(c+d\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(5/2), x)
```

3.608 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=329

$$-\frac{2(a-b)\sqrt{a+b}\,(9a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d}+\frac{2(a-b)\sqrt{\ldots}}{}$$

```
[Out] 2/5*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/15*b*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*a^2-2*b^2)*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/a^3/d-2/15*(a-b)*(9*a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```

Rubi [A] time = 0.65, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2796, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,\left(9a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a-b)\sqrt{\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(15*a^2*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c +
d*x]^(5/2)) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d
*x]^(3/2))
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[c + f*x])^(n
```

```
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{b}{2}+\frac{3}{2}a\cos(c+dx)+b\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}(9a^2-2b}{\cos^{\frac{3}{2}}(c+dx}}{}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)} - \frac{((a-b)(9a}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(9a^2-2b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{15a^3d}$$

Mathematica [A] time = 13.29, size = 453, normalized size = 1.38

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\left(\frac{2\sec(c+dx)\left(9a^2\sin(c+dx)-2b^2\sin(c+dx)\right)}{15a^2}+\frac{2b\tan(c+dx)\sec(c+dx)}{15a}+\frac{2}{5}\tan(c+dx)\sec^2(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*
(9*a^2 + 7*a*b - 2*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (9*a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[
c + d*x])^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
```

```
Cos[c + d*x]]*((2*Sec[c + d*x]*(9*a^2*Sin[c + d*x] - 2*b^2*Sin[c + d*x]))/(
15*a^2) + (2*b*Sec[c + d*x]*Tan[c + d*x])/(15*a) + (2*Sec[c + d*x]^2*Tan[c
+ d*x])/5))/d
```

fricas [F] time = 2.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

maple [B] time = 0.26, size = 1555, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(cos(d*x+c)^2*a*b^2-9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*a^3+2*cos(d*x+c)^3*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+7
*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b-2*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*cos(d*x+c)^
```

```
2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2-2*cos(d*x+c)^4*b^3+9*cos(d*x+c)^3*a^3-4*cos(d*x+c)*a^2*b+9*cos(d*x+
c)^4*a^2*b+cos(d*x+c)^4*a*b^2-5*cos(d*x+c)^3*a^2*b-2*cos(d*x+c)^3*a*b^2+2*c
os(d*x+c)^3*b^3-6*cos(d*x+c)^2*a^3-9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3+2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^3+9*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+
c)/cos(d*x+c)^(5/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.609 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=389

$$\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b(a-b)\sqrt{a+b}\,(19a^2+8b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{105a^4d}$$

```
[Out] 2/7*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/35*b*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/105*(25*a^2-4*b^2)*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/105*(a-b)*b*(19*a^2+8*b^2)
*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^4/d+2/105*(a-b)*(25*a^2+6*a*b+8*b^2)*cot(d*x+c)*Ellip
ticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a^3/d
```

Rubi [A] time = 0.92, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2796, 3055, 2998, 2816, 2994}

$$\frac{2(25a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,(25a^2+6ab+8b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(}{}}}{105a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(19*a^2 + 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 + 6*a*b + 8*b^2)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(105*a^3*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{b}{2}+\frac{5}{2}a\cos(c+dx)+2b\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}(25a^2-4b\cdots}{c\cdots}}{\cdots}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2-4b\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2-4b\cdots\right.}{\cdots}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(19a^2+8b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(\cdots}{a+\cdots}}\cdots}{105a^4d}$$

Mathematica [C] time = 6.23, size = 1304, normalized size = 3.35

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
```

```
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-19
*a^3*b - 8*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])) + 2*(-19*a^2*b^2 - 8*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a -
 b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Co
s[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(25*a^2*Sin[c + d*x] - 4*b^2*Sin[c + d*x
]))/(105*a^2) + (2*Sec[c + d*x]*(19*a^2*b*Sin[c + d*x] + 8*b^3*Sin[c + d*x]
))/(105*a^3) + (2*b*Sec[c + d*x]^2*Tan[c + d*x])/(35*a) + (2*Sec[c + d*x]^3
*Tan[c + d*x])/7))/d
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

maple [B] time = 0.31, size = 1826, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(25*cos(d*x+c)^5*a^3*b+25*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4-15*a^4+19*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*
a^3*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^4*sin(d*x+c)*a^2*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-19*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-19*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^
4*sin(d*x+c)*a^2*b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+19*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b+2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x
+c)*a^2*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-19*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*
b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)^3*sin(d*x+c)*a*b^3-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4+25*cos(d*x+c)^3*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
```

```
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^4+19*cos(d*x+c)^5*a^2*b^2-4*cos(d*x+c)^5*a*b^3+19*cos(d*x+c)^4*a^3*b-
20*cos(d*x+c)^4*a^2*b^2+8*cos(d*x+c)^4*a*b^3-26*cos(d*x+c)^3*a^3*b-4*cos(d*
x+c)^3*a*b^3+cos(d*x+c)^2*a^2*b^2-18*cos(d*x+c)*a^3*b+25*cos(d*x+c)^4*a^4-1
0*cos(d*x+c)^2*a^4+8*cos(d*x+c)^5*b^4-8*cos(d*x+c)^4*b^4)/(a+b*cos(d*x+c))^
(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2),x)
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

3.610 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=508

$$\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(3a^2+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1}{a-b}}}{24abd}$$

```
[Out] 1/3*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/24*(3*a^2+16*b^2
)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1/4*a*sin(d*x+c)*c
os(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24*(a-b)*(3*a^2+16*b^2)*cot(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a/b/d+1/24*(a+2*b)*(3*a+8*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/8*a*(a^2-1
2*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] time = 1.26, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2821, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(3a^2+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1}{a-b}}}{24abd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
24*a*b*d) + (Sqrt[a + b]*(a + 2*b)*(3*a + 8*b)*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(24*b*d) + (a*Sqrt[a + b]*(a^2 - 12*b^2)*Cot[c + d*x]*EllipticPi[(a
+ b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(8*b^2*d) + ((3*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + (a*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{ab}{2}+\right.}{}$$

$$= \frac{a\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{\sqrt{\cos(c+dx)}\,(a+}{}$$

$$= \frac{\left(3a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} + \frac{a\sqrt{\cos(c+dx)}\,\sqrt{a+}}{}$$

$$= \frac{\left(3a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} + \frac{a\sqrt{\cos(c+dx)}\,\sqrt{a+}}{}$$

$$= \frac{a\sqrt{a+b}\,\left(a^2-12b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{8b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(3a^2+16b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{24abd}$$

Mathematica [C] time = 19.59, size = 1189, normalized size = 2.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((-4*a*(17*a^2 + 16*b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 208*a^2*b*((Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
```

```
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2 + 16*b^2)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((7*a*Sin[c + d*x])/12
 + (b*Sin[2*(c + d*x)])/6))/d
```

fricas [F] time = 3.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.26, size = 1683, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/24/d*(6*cos(d*x+c)^2*a*b^2+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+6*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-16*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+3*
a^3*cos(d*x+c)+16*cos(d*x+c)*a*b^2+3*cos(d*x+c)^2*a^2*b-8*cos(d*x+c)^5*b^3+
16*cos(d*x+c)^2*b^3+14*cos(d*x+c)*a^2*b-22*cos(d*x+c)^4*a*b^2-17*cos(d*x+c)
^3*a^2*b-8*cos(d*x+c)^3*b^3-3*cos(d*x+c)^2*a^3-3*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-16*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*b^3-72*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a*b^2*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b*sin(d*x+c)-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+52*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-72*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*
b^2-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b-16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-14*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+52*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.611 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=433

$$-\frac{\sqrt{a+b}\,\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{\sin(c+ d}{2}$$

```
[Out] 1/2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+3/4*a*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-5/4*(a-b)*cot(d*x+c)*EllipticE((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d+1/4
*(5*a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(3*a^2+4*b^2)*cot(d*x+c)*EllipticPi((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/b/d
```

Rubi [A] time = 1.17, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2821, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\,\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{\sin(c+ d}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-5*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) + (Sqrt[a
+ b]*(5*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(3
*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (3*a
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + ((a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
```

```
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\,dx = \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(-\frac{ab}{2}+b^2\cos(c+dx)+\frac{3}{2}a\right.}{\cos^{\frac{3}{2}}(c+dx)}}{2b}$$

$$= -\frac{a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx}{2d\sqrt{\cos(c+dx)}}$$

$$= \frac{3a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx}{2d\sqrt{\cos(c+dx)}}$$

$$= \frac{3a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx}{2d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,\left(3a^2+4b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{4bd}$$

$$= -\frac{5(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{4d}$$

Mathematica [A] time = 12.27, size = 437, normalized size = 1.01

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{-4\left(4a^2-ab+2b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+12a^2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+10}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*b*(a + b*Cos[c + d*x])*Sin[c + d*x] + (10*a*(a + b)*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(4*a^2 - a*b + 2*b^2)*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)] + 12*a^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 16*
b^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1,
```

```
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 5*a*b*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 10*a^2*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - 5*a*b*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]))/(8*d
*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 3.99, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.19, size = 1421, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(8*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-5*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-
5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b-6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
```

```
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+8*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+4*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c
)-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin
(d*x+c)-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a^2*sin(d*x+c)-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*cos(d*x+c)^4*b^2-7*cos(d*x+c)^3*a*b-5*c
os(d*x+c)^2*a^2+5*cos(d*x+c)^2*a*b+2*cos(d*x+c)^2*b^2+5*a^2*cos(d*x+c)+2*a*
b*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x)), x)
```

3.612 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=375

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,(2a+b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{d}$$

```
[Out] b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(a-b)*b*cot(d*x+c)*E
llipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a/d+(2*a+b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-3*a*cot(d*x+c)*EllipticPi((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 0.64, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2821, 3053, 2809, 2998, 2816, 2994}

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,(2a+b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (3*a*Sqrt[a + b]*Co
t[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (b*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
```

```
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \int \frac{-\frac{ab}{2}+a^2\cos(c+dx)+\frac{3}{2}ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}(3ab)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{-}{\cos^{\frac{3}{2}}(c}$$

$$= -\frac{3a\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{d}$$

$$= -\frac{(a-b)b\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1-}{}}}{ad}$$

Mathematica [A] time = 7.57, size = 339, normalized size = 0.90

$$-\frac{\sqrt{\cos(c+dx)}\,\sec^2\left(\frac{1}{2}(c+dx)\right)\left(b\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))+4a(a-2b)\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a - 2*b)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 12*a*b*
```

```
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(d*Sqrt[a + b*Cos[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)))
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

maple [B] time = 0.27, size = 1003, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/d/(a+b*cos(d*x+c))^(1/2)*(2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-4*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b+EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
```

```
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+6*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+cos(d*
x+c)^3*b^2+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-a*b*cos(d*x+c))/cos(d*x+c)^(1/
2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cos(c+dx))^{\frac{3}{2}}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2)/sqrt(cos(c + d*x)), x)
```

3.613 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=337

$$-\frac{2(a-2b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+b}\,c}{}$$

```
[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/d-2*(a-2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*b*cot(d*x+c)*
EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+
c))/(a-b))^(1/2)/d
```

Rubi [A] time = 0.47, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2798, 2809, 2998, 2816, 2994}

$$-\frac{2(a-2b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+b}\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*(a - 2*b)*
Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*b*Sqrt[a + b]*Cot[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
```

```
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a\int \frac{a+2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + b^2\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{a(}}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{a(1}}{d}$$

Mathematica [A] time = 13.13, size = 357, normalized size = 1.06

$$\frac{\cos(c+dx)\left(\frac{2(a^2+2ab-b^2)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\,F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}} - 2a^2\tan\left(\frac{1}{2}(c+dx)\right) + \frac{4b^2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\,\Pi\left(-1;\sin\right.}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(a + b*Cos[c + d*x])*Sin[c + d*x] + Cos[c + d*x]*((-2*a*(a + b)*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (2*(a
^2 + 2*a*b - b^2)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])] + (4*b^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])] - a*b*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] -
 2*a^2*Tan[(c + d*x)/2] + a*b*Tan[(c + d*x)/2]))/(d*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

maple [B] time = 0.22, size = 1183, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*(2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*b^2+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+cos(d*x+c)^2*a
```

```
*b+a^2*cos(d*x+c)-a*b*cos(d*x+c)-a^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos
(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
[Out] Integral((a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/2), x)
```

3.614 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=277

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-3b)(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{3ad}$$

```
[Out] 2/3*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+8/3*(a-b)*b*cot(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/d+2/3*(a-3*b)*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.44, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2799, 2998, 2816, 2994}

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-3b)(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (8*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2*(a
 - 3*b)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2
*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{2ab+\frac{1}{2}\left(a^2+3b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}((a-3b)(a-b))\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos}}$$

$$= \frac{8(a-b)b\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{3ad}$$

Mathematica [A] time = 4.98, size = 256, normalized size = 0.92

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(\left(a^2+4ab+3b^2\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}\,F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-4b\tan\left(\frac{1}{2}(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((2*(a + b*Cos[c + d*x])*(a + 4*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^
(3/2) + 2*Sqrt[Cos[(c + d*x)/2]^2]*(-4*b*(a + b)*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] + (a^2 + 4*a*b + 3*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 4*b*(
a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2])
)/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

maple [B] time = 0.26, size = 1075, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*a^2+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+3*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-4*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+
c)*a*b-4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*b^2+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-
4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b-4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+cos(d*x+c)^3*a*b+4*cos(d*x+c)^3*b^2+cos(d
*x+c)^2*a^2+4*cos(d*x+c)^2*a*b-4*cos(d*x+c)^2*b^2-5*a*b*cos(d*x+c)-a^2)/(a+
b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
[Out] Integral((a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(5/2), x)
```

3.615 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{2(a-b)\sqrt{a+b}\,(3a^2+b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{5a^2d}+\frac{4b\sin(c}{}$$

```
[Out] 2/5*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+4/5*b*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/5*(a-b)*(3*a^2+b^2)*cot(d*x+c)
*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b
))^(1/2)/a^2/d-2/5*(a-b)*(3*a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.66, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2799, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,(3a^2+b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{5a^2d}+\frac{4b\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*
a^2*d) - (2*(a - b)*(3*a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(5*a*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(
5/2)) + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)
)
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
```

```
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{3ab+\frac{1}{2}\left(3a^2+5b^2\right)\cos(c+dx)+ab\,\mathrm{co}}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{3}{4}a(3}{\cos^{\frac{3}{2}}}}{}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{5}((a-b$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(3a^2+b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(}{a+}}}{5a^2d}$$

Mathematica [A] time = 13.52, size = 443, normalized size = 1.36

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)\left(3a^2\sin(c+dx)+b^2\sin(c+dx)\right)}{5a}+\frac{2}{5}a\tan(c+dx)\sec^2(c+dx)+\frac{4}{5}b\tan(c+d\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(3*a
^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))/(5*a*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^(
3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
```

```
]]*((2*Sec[c + d*x]*(3*a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(5*a) + (4*b*S
ec[c + d*x]*Tan[c + d*x])/5 + (2*a*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

maple [B] time = 0.22, size = 1539, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d*(3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3+4*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+cos(d*x+c)^3*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*cos(d*
x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3-3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b-cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(
```

```
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-cos(d*x+c)^3*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+3*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3+4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*co
s(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+3*cos(d*x+c)^4*a^2*b+2
*cos(d*x+c)^4*a*b^2+cos(d*x+c)^4*b^3+3*cos(d*x+c)^3*a^3+cos(d*x+c)^3*a*b^2-
cos(d*x+c)^3*b^3-2*cos(d*x+c)^2*a^3-3*cos(d*x+c)^2*a*b^2-3*cos(d*x+c)*a^2*b
-a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2),x)
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.616 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=387

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^2d}$$

```
[Out] 2/7*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+16/35*b*sin(d*x+
c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/105*(25*a^2+3*b^2)*sin(d*x+c
)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+4/105*(a-b)*b*(41*a^2-3*b^2)*
cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a^3/d+2/105*(a-b)*(25*a^2-57*a*b-6*b^2)*cot(d*x+c)*Ellip
ticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a^2/d
```

Rubi [A] time = 0.95, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2799, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^3*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 - 57*a*b - 6*b^2)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(7*d*Cos[c + d*x]^(7/2)) + (16*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{4ab+\frac{1}{2}\left(5a^2+7b^2\right)\cos(c+dx)+2ab\,\mathrm{c}}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+d}}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}a}{}}{}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a\right.}{}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a\right.}{}$$

$$= \frac{4(a-b)b\sqrt{a+b}\,\left(41a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{105a^3d}$$

Mathematica [C] time = 6.26, size = 1302, normalized size = 3.36

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
```

```
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-82
*a^3*b + 6*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])) + 2*(-82*a^2*b^2 + 6*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a -
 b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Co
s[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(25*a^2*Sin[c + d*x] + 3*b^2*Sin[c + d*x
]))/(105*a) + (4*Sec[c + d*x]*(41*a^2*b*Sin[c + d*x] - 3*b^3*Sin[c + d*x]))
/(105*a^2) + (16*b*Sec[c + d*x]^2*Tan[c + d*x])/35 + (2*a*Sec[c + d*x]^3*Ta
n[c + d*x])/7))/d
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

maple [B] time = 0.27, size = 1827, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(25*cos(d*x+c)^5*a^3*b+25*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4-15*a^4+82*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*
a^3*b+51*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^4*sin(d*x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-82*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-82
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^4*sin(d*x+c)*a^2*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+82*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b+51*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d
*x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-82*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-82*(cos(d*x+c)/(1+
```

```
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^
2*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^3*sin(d*x+c)*a*b^3+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4+25*cos(d*x+c)^3*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4+82*cos(d*x+c)^5*a^2*b^2+3*cos(d*x+c)^5*a*b^3+82*cos(d*x+c)^4*a^3*
b-55*cos(d*x+c)^4*a^2*b^2-6*cos(d*x+c)^4*a*b^3-68*cos(d*x+c)^3*a^3*b+3*cos(
d*x+c)^3*a*b^3-27*cos(d*x+c)^2*a^2*b^2-39*cos(d*x+c)*a^3*b+25*cos(d*x+c)^4*
a^4-10*cos(d*x+c)^2*a^4-6*cos(d*x+c)^5*b^4+6*cos(d*x+c)^4*b^4)/(a+b*cos(d*x
+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2),x)
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

3.617 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=454

$$\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\ (}{}$$

```
[Out] 2/9*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+20/63*b*sin(d*x+
c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*a^2+3*b^2)*sin(d*x+c
)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+8/315*b*(22*a^2-b^2)*sin(d*x+
c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*a^4+33*a^
2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d-2/315*(a-b)*(147*a^3-39*a^2*b-6*a*b^2
-8*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A] time = 1.32, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2799, 3055, 2998, 2816, 2994}

$$\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{5}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\ (}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 33*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(315*a^4*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 39*a^2*b - 6*
a*b^2 - 8*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*a*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (20*b*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*a^2 + 3*b^
2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(5/2)) + (8
*b*(22*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Cos[c +
 d*x]^(3/2))
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{5ab+\frac{1}{2}\left(7a^2+9b^2\right)\cos(c+dx)+3ab\ldots}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+d\ldots}}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{4\int \ldots}{\ldots}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a\ldots\right.}{\ldots}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a\ldots\right.}{\ldots}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a\ldots\right.}{\ldots}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(147a^4+33a^2b^2+8b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\ldots\right.}{315a^4d}$$

Mathematica [C] time = 6.30, size = 1368, normalized size = 3.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] -1/315*((-4*a*(-39*a^4*b + 31*a^2*b^3 + 8*b^5)*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- 4*a*(147*a^5 + 33*a^3*b^2 + 8*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*b + 33*a^2*b^3 + 8*b^5)*((I*Cos[(
c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/S
qrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*
((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^3*d) + (
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(49*a^2*Sin[
c + d*x] + 3*b^2*Sin[c + d*x]))/(315*a) + (8*Sec[c + d*x]^2*(22*a^2*b*Sin[c
 + d*x] - b^3*Sin[c + d*x]))/(315*a^2) + (2*Sec[c + d*x]*(147*a^4*Sin[c + d
*x] + 33*a^2*b^2*Sin[c + d*x] + 8*b^4*Sin[c + d*x]))/(315*a^3) + (20*b*Sec[
c + d*x]^3*Tan[c + d*x])/63 + (2*a*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

maple [B] time = 0.42, size = 2503, normalized size = 5.51

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x)
[Out] -2/315/d*(cos(d*x+c)^3*a^2*b^3+147*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-10*cos(d*x+c)^5*a^4*b+3
3*cos(d*x+c)^5*a^3*b^2-34*cos(d*x+c)^5*a^2*b^3+8*cos(d*x+c)^5*a*b^4-68*cos(
d*x+c)^4*a^3*b^2-4*cos(d*x+c)^4*a*b^4-52*cos(d*x+c)^3*a^4*b-53*cos(d*x+c)^2
*a^3*b^2-85*cos(d*x+c)*a^4*b+147*cos(d*x+c)^6*a^4*b+88*cos(d*x+c)^6*a^3*b^2
+33*cos(d*x+c)^6*a^2*b^3-4*cos(d*x+c)^6*a*b^4-35*a^5+8*cos(d*x+c)^6*b^5+147
*cos(d*x+c)^5*a^5-8*cos(d*x+c)^5*b^5-98*cos(d*x+c)^4*a^5-14*cos(d*x+c)^2*a^
5+8*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^4-147*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*cos(d*x+c)^5*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-33*co
s(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b^3-8*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+186*cos(d*x+c)^4*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+33*cos(d*x+c
)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
```

```
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b^2+2*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*cos(d*x+c)^4*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
4*b-33*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b^2-33*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-8*cos(d*x+c)^4*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+18
6*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^4*b+33*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+2*cos(d*x+c)^5*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-147*cos
(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^5-8*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+147*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*cos(d*x+c)^4*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-
8*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^5)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\cos(c + d x))^{3/2}}{\cos(c + d x)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(11/2),x)
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(11/2),x)
[Out] Timed out
```

3.618 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=506

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,(33a^2+26ab+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+d}{a-b}}}{24d}$$

```
[Out] 1/3*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+1/24*(33*a^2+1
6*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+13/12*a*b*sin(d
*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24*(a-b)*(33*a^2+16*b^2)*
cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a/d+1/24*(33*a^2+26*a*b+16*b^2)*cot(d*x+c)*EllipticF((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-5/8
*a*(a^2+4*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A] time = 1.36, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2793, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,(33a^2+26ab+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+d}{a-b}}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(33*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(24*a*d) + (Sqrt[a + b]*(33*a^2 + 26*a*b + 16*b^2)*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(24*d) - (5*a*Sqrt[a + b]*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(8*b*d) + ((33*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (13*a*b*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b^2*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}\,dx = \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\sqrt{\cos(c+dx}}{}$$

$$= \frac{13ab\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{12d} + \frac{b^2\cos^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{\left(33a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} + \frac{13ab\sqrt{\cos(c+dx)}}{}$$

$$= \frac{\left(33a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} + \frac{13ab\sqrt{\cos(c+dx)}}{}$$

$$= -\frac{5a\sqrt{a+b}\,\left(a^2+4b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\right.}{8bd}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(33a^2+16b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{24ad}$$

Mathematica [C] time = 19.06, size = 1203, normalized size = 2.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((-4*a*(59*a^2*b + 16*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(48*a^3 + 76*a
*b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[
```

```
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*
(33*a^2*b + 16*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE
[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d
*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Se
c[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt
[Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((1
3*a*b*Sin[c + d*x])/12 + (b^2*Sin[2*(c + d*x)])/6))/d
```

fricas [F] time = 2.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sqrt(cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.25, size = 1866, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x)
```

[Out] -1/24/d*(-18*cos(d*x+c)^2*a*b^2-48*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+30*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-33*a^3*cos(d*x+c)-16*cos(d*x+c)*a*b^2-33*cos(d*x+c)^2*a^2*b+8*cos(d*x+c)^5*b^3-16*cos(d*x+c)^2*b^3-26*cos(d*x+c)*a^2*b+34*cos(d*x+c)^4*a*b^2+59*cos(d*x+c)^3*a^2*b+8*cos(d*x+c)^3*b^3+33*cos(d*x+c)^2*a^3+33*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+120*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+26*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-76*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-48*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+120*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+33*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+26*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.619 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=443

$$\frac{\sqrt{a+b}\,\left(8a^2+9ab+2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\,(}{}$$

```
[Out] 9/4*a*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b^2*sin(d*
x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-9/4*(a-b)*b*cot(d*x+c)*Ellip
ticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/d+1/4*(8*a^2+9*a*b+2*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a
+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(15*a^2+4*b^2)*cot(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 1.00, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2793, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\,\left(8a^2+9ab+2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-9*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) + (Sqrt[
a + b]*(8*a^2 + 9*a*b + 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (
Sqrt[a + b]*(15*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(4*d) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*
x]]) + (b^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\frac{1}{2}a\left(4a^2+b^2\right)+b\left(6a^2+b^2\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{9ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+a}{2d}$$

$$= \frac{9ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+a}{2d}$$

$$= -\frac{\sqrt{a+b}\left(15a^2+4b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{4d}$$

$$= -\frac{9(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(}}{4d}$$

Mathematica [A] time = 6.62, size = 329, normalized size = 0.74

$$\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(2b\left(15a^2+4b^2\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2\left(4a^3-12\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x] + Sqrt[Cos[(c +
 d*x)/2]^2]*(9*a*b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*(4*a^3 -
12*a^2*b + a*b^2 - 2*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*b*(15*a^2
+ 4*b^2)*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 9*a*b*(a + b*Cos[c + d*x
])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(4*d*Sqrt[a + b
*Cos[c + d*x]])
```

fricas [F] time = 3.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)

maple [B] time = 0.21, size = 1629, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)

[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(30*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-24*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)^4*b^3+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(

```
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x
+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*s
in(d*x+c)-24*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+11*cos(d*x+c)^3*a*b^2+9*co
s(d*x+c)^2*a^2*b-9*cos(d*x+c)^2*a*b^2-2*cos(d*x+c)^2*b^3-9*cos(d*x+c)*a^2*b
-2*cos(d*x+c)*a*b^2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{5/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
[Out] Timed out
```

3.620 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=445

$$-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(2a^2-6ab-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\Big(}{d}$$

```
[Out] 2*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(2*a^2-b^2)*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+(a-b)*(2*a^2-b^2)*cot(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a/d-(2*a^2-6*a*b-b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-5*a*b*cot(d*x+c)*Elli
pticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/d
```

Rubi [A] time = 1.01, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2792, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(2a^2-6ab-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\Big(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
- (5*a*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*
a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*a^2
 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{3a^2b}{2} - \frac{1}{2}a\left(a^2-3b^2\right)\cos(c+dx) - \frac{1}{2}b\,(}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{5ab\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{d}$$

$$= \frac{(a-b)\sqrt{a+b}\,\left(2a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{ad}$$

Mathematica [C] time = 18.19, size = 1185, normalized size = 2.66

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*
a*(-4*a^2*b - b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(2*a^3 - 6*a*b^2)*((Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(2*a^2*b -
```

```
 b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
 + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]
)))/(2*d)
```

fricas [F] time = 56.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^{2} \cos\left(dx + c\right)^{2} + 2\, ab \cos\left(dx + c\right) + a^{2}\right)\sqrt{b \cos\left(dx + c\right) + a}}{\cos\left(dx + c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \cos\left(dx + c\right) + a\right)^{\frac{5}{2}}}{\cos\left(dx + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

maple [B] time = 0.20, size = 1626, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-10
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*a*b^2-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
2+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin
(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b^3*sin(d*x+c)-10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-cos(d*x+c)^3*b
^3-2*cos(d*x+c)^2*a^2*b-cos(d*x+c)^2*a*b^2+cos(d*x+c)^2*b^3-2*a^3*cos(d*x+c
)+2*cos(d*x+c)*a^2*b+cos(d*x+c)*a*b^2+2*a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x
+c)/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.621 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=392

$$\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d}+\frac{2a^2\sin(c}{}$$

```
[Out] 2/3*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+14/3*(a-b)*b*c
ot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/d+2/3*(a^2-7*a*b+9*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*b^2*cot(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 0.74, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2792, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d}+\frac{2a^2\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (14*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) + (2*Sqr
t[a + b]*(a^2 - 7*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (
2*b^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a^2
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{7a^2b}{2}+\frac{1}{2}a\left(a^2+9b^2\right)\cos(c+dx)+\frac{3}{2}b^3}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{7a^2b}{2}+\frac{1}{2}a\left(a^2+9b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx +$$

$$= -\frac{2b^2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{d}$$

$$= \frac{14(a-b)b\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{3d}$$

Mathematica [A] time = 7.04, size = 328, normalized size = 0.84

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(\left(a^3+7a^2b+9ab^2-3b^3\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+6b^3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((2*a*(a + b*Cos[c + d*x])*(a + 7*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x
]^(3/2) + 2*Sqrt[Cos[(c + d*x)/2]^2]*(-7*a*b*(a + b)*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)] + (a^3 + 7*a^2*b + 9*a*b^2 - 3*b^3)*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 7*a*b*(a + b*Cos
[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(3*d*Sq
rt[a + b*Cos[c + d*x]])
```

fricas [F] time = 61.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

maple [B] time = 0.28, size = 1485, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3+7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
```

```
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)*b^3+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-7*cos(d*x+c)^2*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2
+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^3+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*b^3-7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+cos(d*x+c)^3*a^2*b+7
*cos(d*x+c)^3*a*b^2+cos(d*x+c)^2*a^3+7*cos(d*x+c)^2*a^2*b-7*cos(d*x+c)^2*a*
b^2-8*cos(d*x+c)*a^2*b-a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \cos(c + d x))^{5/2}}{\cos(c + d x)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

3.622 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=338

$$-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad}$$

```
[Out] 2/5*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+22/15*a*b*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*a^2+23*b^2)*
cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a/d-2/15*(a-b)*(9*a^2-8*a*b+15*b^2)*cot(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a
/d
```

Rubi [A] time = 0.76, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2792, 3055, 2998, 2816, 2994}

$$-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(15*a*d) - (2*(a - b)*Sqrt[a + b]*(9*a^2 - 8*a*b + 15*b^2)*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(15*a*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(5*d*Cos[c + d*x]^(5/2)) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(15*d*Cos[c + d*x]^(3/2))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
```

```
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{11a^2b}{2} + \frac{3}{2}a\left(a^2+5b^2\right)\cos(c+dx) + \frac{1}{2}b}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{15}($$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(9a^2+23b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{15ad}$$

Mathematica [A] time = 11.80, size = 427, normalized size = 1.26

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\left(\frac{2}{15}\sec(c+dx)\left(9a^2\sin(c+dx)+23b^2\sin(c+dx)\right)+\frac{2}{5}a^2\tan(c+dx)\sec^2(c+}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt
[1 + Cos[c + d*x]]*((9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^
```

```
3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (9*a^2 + 23*b^2)
*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*
Tan[(c + d*x)/2]))/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(9*a^2*Sin[c + d
*x] + 23*b^2*Sin[c + d*x]))/15 + (22*a*b*Sec[c + d*x]*Tan[c + d*x])/15 + (2
*a^2*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)
```

maple [B] time = 0.22, size = 1750, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-34*cos(d*x+c)^2*a*b^2+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3-9*cos(d*x+c)^3*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*a^
3-23*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
```

```
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^2+17*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+23*cos(d*x+c)^3*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b-23*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+23*cos(d*x+c)^4*b^3+9*cos(d*x+c)^3*a^3-
14*cos(d*x+c)*a^2*b+9*cos(d*x+c)^4*a^2*b+11*cos(d*x+c)^4*a*b^2+5*cos(d*x+c)
^3*a^2*b+23*cos(d*x+c)^3*a*b^2-23*cos(d*x+c)^3*b^3-6*cos(d*x+c)^2*a^3+15*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*
sin(d*x+c)*b^3-9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+17*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+23*cos(d*x+c)^2*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^2-23*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*b^3+9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-23*cos(d*x+c
)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^3+9*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\cos(c + d\,x))^{5/2}}{\cos(c + d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2),x)
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

3.623 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=387

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,\left(5a^2-24ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(}{}}}{21ad}$$

```
[Out] 2/7*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+6/7*a*b*sin(d*
x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/21*(5*a^2+9*b^2)*sin(d*x+c
)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/21*(a-b)*b*(29*a^2+3*b^2)*cot
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/a^2/d+2/21*(a-b)*(5*a^2-24*a*b+3*b^2)*cot(d*x+c)*EllipticF(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/
d
```

Rubi [A] time = 1.04, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,\left(5a^2-24ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(}{}}}{21ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(21*a^2*d) + (2*(a - b)*Sqrt[a + b]*(5*a^2 - 24*a*b + 3*b^2)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(21*a*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(7*d*Cos[c + d*x]^(5/2)) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(21*d*Cos[c + d*x]^(3/2))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{15a^2b}{2}+\frac{1}{2}a\left(5a^2+21b^2\right)\cos(c+dx)+}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\,c}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{5}{4}}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(5a\right.}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(5a\right.}{}$$

$$= \frac{2(a-b)b\sqrt{a+b}\,\left(29a^2+3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{21a^2d}$$

Mathematica [C] time = 6.30, size = 1302, normalized size = 3.36

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
```

```
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-29*a
^3*b - 3*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b
), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]])) + 2*(-29*a^2*b^2 - 3*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(21*a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]*((2*Sec[c + d*x]^2*(5*a^2*Sin[c + d*x] + 9*b^2*Sin[c + d*x]))/21
 + (2*Sec[c + d*x]*(29*a^2*b*Sin[c + d*x] + 3*b^3*Sin[c + d*x]))/(21*a) + (
6*a*b*Sec[c + d*x]^2*Tan[c + d*x])/7 + (2*a^2*Sec[c + d*x]^3*Tan[c + d*x])/
7))/d
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)
```

maple [B] time = 0.28, size = 1827, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d*(5*cos(d*x+c)^5*a^3*b+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4-3*a^4+29*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*
b+27*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^4*sin(d*x+c)*a^2*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-29*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-29*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*s
in(d*x+c)*a^2*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b+27*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c
)*a^2*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)^3*sin(d*x+c)*a*b^3-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-29*(cos(d*x+c)/(1+cos(
```

```
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^
2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*sin(d*x+c)*a*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4+5*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*cos(d*x+c)^3
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^4+29*cos(d*x+c)^5*a^2*b^2+9*cos(d*x+c)^5*a*b^3+29*cos(d*x+c)^4*a^3*b-11*
cos(d*x+c)^4*a^2*b^2+3*cos(d*x+c)^4*a*b^3-22*cos(d*x+c)^3*a^3*b-12*cos(d*x+
c)^3*a*b^3-18*cos(d*x+c)^2*a^2*b^2-12*cos(d*x+c)*a^3*b+5*cos(d*x+c)^4*a^4-2
*cos(d*x+c)^2*a^4+3*cos(d*x+c)^5*b^4-3*cos(d*x+c)^4*b^4)/(a+b*cos(d*x+c))^(
1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.624 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=454

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+\ldots}{9d\ldots}$$

```
[Out] 2/9*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+38/63*a*b*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*a^2+75*b^2)*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/315*b*(163*a^2+5*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*a^4+2
79*a^2*b^2-10*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-2/315*(a-b)*(147*a^3-114*a^2*b+
165*a*b^2+10*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```

Rubi [A] time = 1.41, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b +
 165*a*b^2 + 10*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a^
2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (38*a*b
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*
a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5
/2)) + (2*b*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a
*d*Cos[c + d*x]^(3/2))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{19a^2b}{2} + \frac{1}{2}a\left(7a^2+27b^2\right)\cos(c+dx) + \cdots}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos\cdots}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{4\int\cdots}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(4\cdots\right.}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(4\cdots\right.}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(4\cdots\right.}{\cdots}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4+279a^2b^2-10b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|\cdots\right.}{315a^3d}$$

Mathematica [C] time = 6.31, size = 1368, normalized size = 3.01

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] -1/315*((-4*a*(-114*a^4*b + 124*a^2*b^3 - 10*b^5)*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - 4*a*(147*a^5 + 279*a^3*b^2 - 10*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
 (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*b + 279*a^2*b^3 - 10*b^5)*((
I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)])
+ (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))
)/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(a^2
*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(49*a
^2*Sin[c + d*x] + 75*b^2*Sin[c + d*x]))/315 + (2*Sec[c + d*x]^2*(163*a^2*b*
Sin[c + d*x] + 5*b^3*Sin[c + d*x]))/(315*a) + (2*Sec[c + d*x]*(147*a^4*Sin[
c + d*x] + 279*a^2*b^2*Sin[c + d*x] - 10*b^4*Sin[c + d*x]))/(315*a^2) + (38
*a*b*Sec[c + d*x]^3*Tan[c + d*x])/63 + (2*a^2*Sec[c + d*x]^4*Tan[c + d*x])/
9))/d
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)
```

maple [B] time = 0.38, size = 2504, normalized size = 5.52

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(-80*cos(d*x+c)^3*a^2*b^3+147*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+65*cos(d*x+c)^5*a^4
*b+279*cos(d*x+c)^5*a^3*b^2-199*cos(d*x+c)^5*a^2*b^3-10*cos(d*x+c)^5*a*b^4-
272*cos(d*x+c)^4*a^3*b^2+5*cos(d*x+c)^4*a*b^4-82*cos(d*x+c)^3*a^4*b-170*cos
(d*x+c)^2*a^3*b^2-130*cos(d*x+c)*a^4*b+147*cos(d*x+c)^6*a^4*b+163*cos(d*x+c
)^6*a^3*b^2+279*cos(d*x+c)^6*a^2*b^3+5*cos(d*x+c)^6*a*b^4-35*a^5-10*cos(d*x
+c)^6*b^5+147*cos(d*x+c)^5*a^5+10*cos(d*x+c)^5*b^5-98*cos(d*x+c)^4*a^5-14*c
os(d*x+c)^2*a^5-10*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279*cos(d*x+c)
^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3*b^2-279*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+261*cos(d*x+c)^4*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
```

```
a^4*b+279*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3*b^2+155*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-10*cos(d*x+c)^4*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^4-147*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^4*b-279*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-279*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3
+10*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^4+261*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*cos(d*x+c)^5*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+155*
cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^3-147*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+10*cos(d*x+c)^5*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+147*cos(d*x+c
)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^5-147*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^5+10*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5)/(a+b*cos(d*x+c))^(1/2)/si
n(d*x+c)/cos(d*x+c)^(9/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2),x)
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
[Out] Timed out
```

3.625 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2\left(81a^2+113b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693d\cos^{\frac{7}{2}}(c+dx)}+\frac{2b\left(229a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sin(c+d}{11d\,c}$$

```
[Out] 2/11*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(11/2)+46/99*a*b*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+2/693*(81*a^2+113*b^2)*s
in(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/693*b*(229*a^2+3*b^2)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/693*(135*a^4+205*
a^2*b^2-4*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/6
93*(a-b)*b*(741*a^4+51*a^2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/693*(a-b)*
(135*a^4-606*a^3*b+57*a^2*b^2+6*a*b^3+8*b^4)*cot(d*x+c)*EllipticF((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A] time = 1.77, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(81a^2+113b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693d\cos^{\frac{7}{2}}(c+dx)}+\frac{2b\left(229a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(205a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(741*a^4 + 51*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(693*a^4*d) + (2*(a - b)*Sqrt[a + b]*(135*a^4 - 606*a^3*b +
 57*a^2*b^2 + 6*a*b^3 + 8*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^3*
d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2
)) + (46*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2
)) + (2*(81*a^2 + 113*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Co
s[c + d*x]^(7/2)) + (2*b*(229*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
```

```
 d*x])/(693*a*d*Cos[c + d*x]^(5/2)) + (2*(135*a^4 + 205*a^2*b^2 - 4*b^4)*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{\frac{23a^2b}{2}+\frac{3}{2}a\left(3a^2+11b^2\right)\cos(c+dx)+}{\cos^{\frac{11}{2}}(c+dx)\sqrt{a+b}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{4\int}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(741a^4+51a^2b^2+8b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\right.}{693a^4d}$$

Mathematica [C] time = 6.35, size = 1431, normalized size = 2.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] ((-4*a*(135*a^6 - 78*a^4*b^2 - 49*a^2*b^4 - 8*b^6)*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - 4*a*(-741*a^5*b - 51*a^3*b^3 - 8*a*b^5)*((Sqrt[((a + b)*Cot[(c + d*x)
```

```
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-741*a^4*b^2 - 51*a^2*b^4 - 8*b^6)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(
693*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^
4*(81*a^2*Sin[c + d*x] + 113*b^2*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^3*(22
9*a^2*b*Sin[c + d*x] + 3*b^3*Sin[c + d*x]))/(693*a) + (2*Sec[c + d*x]^2*(13
5*a^4*Sin[c + d*x] + 205*a^2*b^2*Sin[c + d*x] - 4*b^4*Sin[c + d*x]))/(693*a
^2) + (2*Sec[c + d*x]*(741*a^4*b*Sin[c + d*x] + 51*a^2*b^3*Sin[c + d*x] + 8
*b^5*Sin[c + d*x]))/(693*a^3) + (46*a*b*Sec[c + d*x]^4*Tan[c + d*x])/99 + (
2*a^2*Sec[c + d*x]^5*Tan[c + d*x])/11))/d
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(13/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/2), x)
```

maple [B] time = 0.59, size = 2789, normalized size = 5.34

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -2/693/d*(-4*cos(d*x+c)^7*a*b^5+741*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b+663*cos(d*x+c)^6*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4
*b^2+51*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b^3+2*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4+8*cos(d*x+c)^6*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^5-74
1*cos(d*x+c)^6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^5*b-741*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^4*b^2-51*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^3*b^3-51*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*
sin(d*x+c)*a^2*b^4-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a*b^5+741*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^5*b+8*cos(d*x+c)
^7*b^6-8*cos(d*x+c)^6*b^6+135*cos(d*x+c)^6*a^6-54*cos(d*x+c)^4*a^6-18*cos(d
*x+c)^2*a^6+135*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^6*sin(d*x+c)*a^6-63*a^6+663*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^4*b^2+51*(cos(d*
```

```
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d
*x+c)*a^3*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^5*sin(d*x+c)*a^2*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a*b^5-741*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*
a^5*b-741*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^5*sin(d*x+c)*a^4*b^2-51*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^3*b^3-51*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^2*
b^4-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)^5*sin(d*x+c)*a*b^5+741*cos(d*x+c)^6*a^5*b-307*cos(d*x+c)^6*a^4*b^2+51
*cos(d*x+c)^6*a^3*b^3-52*cos(d*x+c)^6*a^2*b^4+8*cos(d*x+c)^6*a*b^5-566*cos(
d*x+c)^5*a^5*b-140*cos(d*x+c)^5*a^3*b^3-4*cos(d*x+c)^5*a*b^5-160*cos(d*x+c)
^4*a^4*b^2+cos(d*x+c)^4*a^2*b^4-86*cos(d*x+c)^3*a^5*b-116*cos(d*x+c)^3*a^3*
b^3-274*cos(d*x+c)^2*a^4*b^2-224*cos(d*x+c)*a^5*b+135*cos(d*x+c)^7*a^5*b+74
1*cos(d*x+c)^7*a^4*b^2+205*cos(d*x+c)^7*a^3*b^3+51*cos(d*x+c)^7*a^2*b^4-8*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6
*sin(d*x+c)*b^6+135*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^6-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*b^6)/(a+b*cos(d*x+c))^(
1/2)/sin(d*x+c)/cos(d*x+c)^(11/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(13/2),x)
[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(13/2),x)
[Out] Timed out
```

3.626 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=379

$$\frac{a\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{\sin(c+dx)\sqrt{a+b\cos}}{bd\sqrt{\cos(c+dx}}$$

```
[Out] sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-(a-b)*cot(d*x+c)*Ell
ipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/a/b/d+cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+a*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] time = 0.74, antiderivative size = 414, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2820, 2809, 3003, 2993, 12, 2801, 2816, 2994}

$$\frac{a\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+d}}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqrt[
a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (a*Sqrt[a + b]*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*Sin[c + d*x])/(b*d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2820

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> -Dist[(a*d)/(2*b), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a
 + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[(Sqrt[d*Sin[e + f*x]]*(a + 2
*b*Sin[e + f*x]))/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
```

```
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{a\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b}$$

$$= \frac{a\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec}{a-}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec}{a-}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec}{a-}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec}{a-}}}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+s}{}}}{abd}$$

Mathematica [C] time = 4.46, size = 479, normalized size = 1.26

$$\frac{\sqrt{\cos(c+dx)}\left(2a\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\,\tan\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\,\tan\left(\frac{1}{2}(c+dx)\right)+b\sqrt{\frac{a-b}{a+b}}\,\sin\left(\frac{3}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((2*I)*(a - b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))] - (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -
((a + b)/(a - b))] + (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(
c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a
- b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sq
```

```
rt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])
)/(2*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a
 + b*Cos[c + d*x]])
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.25, size = 622, normalized size = 1.64

$$-\frac{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{}{(1}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x
```

```
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*a*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)+cos(d*x+c)^3
*b+a*cos(d*x+c)^2-cos(d*x+c)^2*b-a*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)
```

3.627 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2809}

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=-\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec}{a-}}}{bd}$$

Mathematica [A] time = 0.92, size = 130, normalized size = 1.12

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 65.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.25, size = 159, normalized size = 1.37

$$-\frac{2\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\left(\sin^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{}{(1+}}}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-2*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/(a+b*cos(d*x+c))^(1
/2)*sin(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

3.628 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] 2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2816}

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad}$$

Mathematica [A] time = 1.40, size = 170, normalized size = 1.56

$$-\frac{4(a+b)\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{a-b}}\sqrt{\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a}}F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\cos(c+dx)}{a(\cos(c+dx)-1)}}\right)|\frac{2a}{a-b}\right)}{ad\sqrt{a+b\cos(c+dx)}\left(-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (-4*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[-(((a + b)*Cot[(c + d*x)/2]^2)/(a - b))
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(a - b
)])/(a*d*Sqrt[a + b*Cos[c + d*x]]*(-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a))^(3/2))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^2+a\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 + a*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 0.21, size = 123, normalized size = 1.13

$$-\frac{2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(\sin^4(dx+c)\right)}{d\sqrt{a+b\cos(dx+c)}\cos(dx+c)^{\frac{3}{2}}(-1+\cos(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(a+b*cos(d*x+c))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.629 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=224

$$\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\,\cot(c+dx)\cdots}{}$$

```
[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.23, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2801, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\,\cot(c+dx)\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt
[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = -\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{1+\cos(c+dx}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a+}}}{a^2d}$$

Mathematica [A] time = 4.96, size = 211, normalized size = 0.94

$$\frac{2\left(\tan\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))+a\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+\right.\right.\right.}{ad\sqrt{\cos(c+dx)}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*(-((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]) + a*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + (a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(a*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 2.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^3 + a*
cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.24, size = 612, normalized size = 2.73

$$-\frac{2\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\;\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)\,a-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\;\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*b+a*cos(d*x
+c)-b*cos(d*x+c)-a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\,\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.630 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=274

$$-\frac{4b(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d}+\frac{2\sqrt{a+b}\,(a+2b)\,c}{}$$

```
[Out] 2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)-4/3*(a-b)*b*cot(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^3/d+2/3*(a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```

Rubi [A] time = 0.40, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2802, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\,(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{4b(a-b)\sqrt{a+b}\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*d) + (2
*Sqrt[a + b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d) + (2*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-b+\frac{1}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(2b)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a} + -$$

$$= -\frac{4(a-b)b\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{3a^3d}$$

Mathematica [A] time = 13.97, size = 371, normalized size = 1.35

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\left(\frac{2\tan(c+dx)\sec(c+dx)}{3a}-\frac{4b\tan(c+dx)}{3a^2}\right)}{d}+\frac{16\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\,\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (16*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[C
os[c + d*x]*Sec[(c + d*x)/2]^2]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a - 2*b)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c + d*x]*(a
+ b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Cos[c + d*
x]^(3/2)*(1 + Cos[c + d*x])^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-4*b*Tan[c + d*x])/(3*a^2) + (2*Sec[c + d
*x]*Tan[c + d*x])/(3*a)))/d
```

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b\cos(dx+c)^4+a\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^4 + a*
cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 0.32, size = 883, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*a^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+2*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*b^2+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+co
s(d*x+c)^3*a*b-2*cos(d*x+c)^3*b^2+cos(d*x+c)^2*a^2-2*cos(d*x+c)^2*a*b+2*cos
(d*x+c)^2*b^2+a*b*cos(d*x+c)-a^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x
+c)^(3/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{5/2}\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(5/2)), x)
```

3.631 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=465

$$-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{\left(3a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{}$$

```
[Out] -2*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+(3*
a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)
-(3*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a/b^2/d/(a+b)^(1/2)+(3*a+b)*cot(d*x+c)*EllipticF((a+b*c
os(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)+3*
a*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A] time = 0.99, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2792, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{\left(3a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt[a + b]*d))
+ ((3*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d) + (3*a*
Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d) - (2*a^2
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]
) + ((3*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*
d*Sqrt[Cos[c + d*x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{a^2}{2}-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}\left(3a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \cdots$$

$$= -\frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \cdots$$

$$= \frac{3a\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\mathrm{se}\cdots}{\cdots}}}{b^3d}$$

$$= -\frac{\left(3a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c\cdots}{a-b}}}{ab^2\sqrt{a+b}\,d}$$

Mathematica [C] time = 6.22, size = 1201, normalized size = 2.58

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(-a^2 + b^2)*d*Sqrt[a + b*Cos[c
+ d*x]]) + ((-4*a*(a^2 - b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 8*a^2*b*((Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
```

```
x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2 - b^2)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
2*(a - b)*b*(a + b)*d)
```

fricas [F] time = 3.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{5}{2}}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.22, size = 1661, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(cos(d*x+c)^2*a*b^2+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+6*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+3*a^3*cos(
d*x+c)-cos(d*x+c)*a*b^2+3*cos(d*x+c)^2*a^2*b-cos(d*x+c)^2*b^3-2*cos(d*x+c)*
a^2*b-cos(d*x+c)^3*a^2*b+cos(d*x+c)^3*b^3-3*cos(d*x+c)^2*a^3-3*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^3-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*a*b^2*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-6*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*a*b^2-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/b^2/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.632 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\right.}{b^2d}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*c
ot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/b/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2
))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/b^2/d
```

Rubi [A] time = 0.50, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2797, 2809, 2794, 2795, 2816, 2994}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d) - (2*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b^2*d) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
```

```
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2797

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(3/2), x_Symbol] :> Dist[d/b, Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin
[e + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Sin[e + f*x]]/(a + b*Sin[e +
f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} - \frac{a\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+s}{}}}{b^2d}$$

$$= -\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+s}{}}}{b^2d}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}\,d} - \frac{2\,c}{}$$

Mathematica [C] time = 17.73, size = 985, normalized size = 2.55

$$\frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(2a\frac{i\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\cos(c+dx)}\,E\left(i\sinh^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)|-\frac{2a}{-a-b}\right)\sec(c+dx)}{b\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,\sqrt{\frac{(a+b\cos(c+dx))\sec(c+dx)}{a+b}}} + \frac{2a\left(\frac{a\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx\right.}{b-a}}}{}\right)}{}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x
]]) - (-4*a*b*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
```

```
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])) + 2*a*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh
[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x]
)/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d
*x]])))/((a - b)*(a + b)*d)
```

fricas [F] time = 81.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{3}{2}}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.25, size = 1206, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*a^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+cos(d*
x+c)^2*a^2-cos(d*x+c)^2*a*b-a^2*cos(d*x+c)+a*b*cos(d*x+c))/(a+b*cos(d*x+c))
^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/b/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(3/2), x)
```

3.633 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=266

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{ad\sqrt{a+b}}$$

```
[Out] 2*a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)-2*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)
/a/d/(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2794, 2795, 2816, 2994}

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) + (2*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} - \frac{\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx}{a+b} - \frac{a\int}{}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} +$$

Mathematica [A] time = 4.82, size = 196, normalized size = 0.74

$$\frac{2\left((a-b)\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)-(a+b)\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b}{a}\right.\right.}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*((a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (a
+ b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a - b)*Sq
rt[Cos[c + d*x]]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]
)
```

fricas [F] time = 1.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.21, size = 809, normalized size = 3.04

$$2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)\,a+2\,\text{EllipticF}\left(\frac{-1+\cos}{\sin(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-a*cos(d*x+
c)^2+cos(d*x+c)^2*b+a*cos(d*x+c)-b*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/cos(d
*x+c)^(1/2)/sin(d*x+c)/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}}{{\left(a+b\,\cos(c+d\,x)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/2), x)
```

3.634 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c\cdots}}{\sqrt{a+b}\sqrt{\cos(\cdots}}\right.\right.}{a^2d\sqrt{a+b}}$$

```
[Out] -2*b*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*b*cot
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a^2/d/(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2800, 2998, 2816, 2994}

$$-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c\cdots}}{\sqrt{a+b}\sqrt{\cos(\cdots}}\right.\right.}{a^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*b*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*b*Sin[c + d*x])/((a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2800

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[(2*b*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(b + a
*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c}}}{a^2-b^2}$$

$$= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(}}}{a+b}$$

$$= \frac{2b\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{a^2\sqrt{a+b}\,d}$$

Mathematica [A] time = 5.79, size = 202, normalized size = 0.76

$$\frac{2\left(b(b-a)\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)+a(a+b)\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right.\right.\right.\right.}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(-(b*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])
+ a*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(
-a + b)*Sqrt[Cos[c + d*x]]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*Sqrt[a + b*C
os[c + d*x]])
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^2\cos\left(dx+c\right)^3+2\,ab\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^3 +
2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.23, size = 830, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(-EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-a*b*cos(d*x
+c)+cos(d*x+c)*b^2)/cos(d*x+c)^(1/2)/sin(d*x+c)/(a+b)/(a-b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.635 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=285

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{a^2d\sqrt{a+b}}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*(a
^2-2*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+
c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-2*(a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2802, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a + b]*d) - (
2*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*b^2
*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
```

```
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2-2b^2\right)-\frac{1}{2}ab\cos(}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{(a+2b)\int \frac{}{\sqrt{\cos(c+dx)}}}{a(a+b}$$

$$= \frac{2\left(a^2-2b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{a^3\sqrt{a+b}\,d}$$

Mathematica [C] time = 6.28, size = 1233, normalized size = 4.33

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(2*a^2*b - 2*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(a^3 - 2*a*b^2)*
((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(a^2*b
 - 2*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh
[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x]
)/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(
```

```
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d
*x]])))/(a^2*(-a + b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]*((-2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*Tan
[c + d*x])/a^2))/d
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^{2}\cos\left(dx+c\right)^{4}+2\,ab\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^4 +
2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.25, size = 1452, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-cos(d*x+c)*sin(d*x
```

```
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+
c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*
sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*b^3*sin(d*x+c)+cos(d*x+c)^2*a^2*b+cos(d*x+c)^2*a*b^2-2*cos(d*x+c)^2*b^3+a
^3*cos(d*x+c)-cos(d*x+c)*a^2*b-2*cos(d*x+c)*a*b^2+2*cos(d*x+c)*b^3-a^3+b^2*
a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/a^2/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\cos(c + dx))^{\frac{3}{2}}\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*cos(c + d*x))**(3/2)*cos(c + d*x)**(3/2)), x)
```

3.636 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=357

$$\frac{2(a+2b)(a+4b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{a+b}}+\frac{2b^2 s}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)+2/3*
(a^2-4*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^(3
/2)-2/3*b*(5*a^2-8*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)+2/3*(a+2*b)*(a+4*b)*cot(d*
x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)
/a^3/d/(a+b)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2802, 3055, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}-\frac{2b\left(5a^2-8b^2\right)\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (-2*b*(5*a^2 - 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]
*d) + (2*(a + 2*b)*(a + 4*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt
[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[
a + b*Cos[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2-4b^2\right)-\frac{1}{2}ab\cos(c+}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}}{a\left(a^2-\right.}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\,\mathrm{c}}}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\,\mathrm{c}}}{3a^2\left(a^2-b^2\right)}$$

$$= -\frac{2b\left(5a^2-8b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{3a^4\sqrt{a+b}\,d}$$

Mathematica [C] time = 6.36, size = 1269, normalized size = 3.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(a^4 + 7*a^2*b^2 - 8*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(5*a^3*b
 - 8*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
```

```
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
) + 2*(5*a^2*b^2 - 8*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec
[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*
x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
b*Sqrt[Cos[c + d*x]])))/(3*a^3*(a - b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]*((2*b^4*Sin[c + d*x])/(a^3*(a^2 - b^2)*(a + b*Cos[c +
 d*x])) - (10*b*Tan[c + d*x])/(3*a^3) + (2*Sec[c + d*x]*Tan[c + d*x])/(3*a^
2)))/d
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^2\cos\left(dx+c\right)^5+2\,ab\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^5 +
2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 0.22, size = 1781, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(-8*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*b^4-a^4-5*cos(d*x+c)^3*a^2*b^2-5*cos(d*x+c)^2*a^3*b
+8*cos(d*x+c)^2*a*b^3-4*cos(d*x+c)*a*b^3+8*cos(d*x+c)^3*b^4-8*cos(d*x+c)^2*
b^4+a^2*b^2+cos(d*x+c)^3*a^3*b-4*cos(d*x+c)^3*a*b^3+4*cos(d*x+c)^2*a^2*b^2+
4*cos(d*x+c)*a^3*b-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^3+5*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*cos(d*x
+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^3-5*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
3+5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3*b+5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+cos(d*x+c)^2*a^4+cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4
-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
```

```
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^4+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^4)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x
+c)^(3/2)/(a+b)/(a-b)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.637 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=433

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^2d\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)}-\frac{2(3a+4b)\left(a^2+4b^2\right)}{}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2)+2/5*
(a^2-6*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^(5
/2)-2/5*b*(3*a^2-8*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/c
os(d*x+c)^(3/2)+2/5*(3*a^4+8*a^2*b^2-16*b^4)*cot(d*x+c)*EllipticE((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/d/(a+b)^(1/2)-2/5*(
3*a+4*b)*(a^2+4*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2
)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2802, 3055, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^3d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(a^2-6b^2\right)\sin(c+}{5a^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^5*S
qrt[a + b]*d) - (2*(3*a + 4*b)*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(5*a^4*Sqrt[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c +
d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^2 - 6*b^2)*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)) - (2*b*(3*a^2
- 8*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d*Cos[c
+ d*x]^(3/2))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2-6b^2\right)-\frac{1}{2}ab\cos(c+\cdots}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+\cdots}}}{a\left(a^2-\cdots\right.}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\,\mathrm{c}\cdots}}{5a^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\,\mathrm{c}\cdots}}{5a^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\,\mathrm{c}\cdots}}{5a^2\left(a^2-b^2\right)\cdots}$$

$$= \frac{2\left(3a^4+8a^2b^2-16b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\cdots}{5a^5\sqrt{a+b}\,d}$$

Mathematica [C]   time = 6.38, size = 1314, normalized size = 3.03

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((a^2 + 4*b^2)*((-4*a*(4*a^2*b - 4*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(
3*a^3 - 4*a*b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/
b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]])) + 2*(3*a^2*b - 4*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*
Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
 d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(b*Sqrt[Cos[c + d*x]]))))/(5*a^4*(-a + b)*(a + b)*d) + (Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]*((-2*b^5*Sin[c + d*x])/(a^4*(a^2 - b^2)*(a + b*C
os[c + d*x])) + (2*Sec[c + d*x]*(3*a^2*Sin[c + d*x] + 11*b^2*Sin[c + d*x]))
/(5*a^4) - (6*b*Sec[c + d*x]*Tan[c + d*x])/(5*a^3) + (2*Sec[c + d*x]^2*Tan[
c + d*x])/(5*a^2)))/d
```

fricas [F] time = 1.35, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^2\cos\left(dx+c\right)^6+2\,ab\cos\left(dx+c\right)^5+a^2\cos\left(dx+c\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^6 +
2*a*b*cos(d*x + c)^5 + a^2*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 0.27, size = 2478, normalized size = 5.72

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d*(-6*cos(d*x+c)^3*a^2*b^3+3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-16*cos(d*x+c)^4*b^5+3*co
s(d*x+c)^3*a^5+16*cos(d*x+c)^3*b^5+3*cos(d*x+c)^4*a^4*b-3*cos(d*x+c)^4*a^3*
b^2+8*cos(d*x+c)^4*a*b^4-5*cos(d*x+c)^3*a^4*b-6*cos(d*x+c)^2*a^3*b^2+2*cos(
d*x+c)*a^4*b-a^5+8*cos(d*x+c)^4*a^2*b^3+8*cos(d*x+c)^3*a^3*b^2-16*cos(d*x+c
)^3*a*b^4+8*cos(d*x+c)^2*a*b^4-2*cos(d*x+c)*a^2*b^3+a^3*b^2-2*cos(d*x+c)^2*
a^5-3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^5+16*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+3*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-3*cos(d*x+c)^
2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^5+16*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*b^5-8*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+16*cos(d*x+c)^3*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^4*b+8*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
```

```
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-16*cos(d*x+c)^
2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^4-3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^4*b-8*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-8*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+
16*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^4-cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+8*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-4*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^3-16*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-8*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)/(a+b)/(a-b)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{7/2}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.638 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=497

$$-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+ab-6b^2\right)\cot($$

```
[Out] -2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-2
/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos
(d*x+c))^(1/2)+2/3*(3*a^2-7*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-2/3*(3*a^2+
a*b-6*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-2*cot(d*x+c)*EllipticPi((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3
/d
```

Rubi [A] time = 1.06, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2792, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+ab-6b^2\right)\cot($$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^2 - 7*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^2*(a + b
)^(3/2)*d) - (2*(3*a^2 + a*b - 6*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^2*(a + b)^(3/2)*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) - (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(
```

```
a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d
*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{a^2}{2}-\frac{3}{2}ab\cos(c+dx)-\frac{3}{2}\left(a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{b^2} - \frac{2\int \frac{\frac{a^2b}{2}+\left(-\frac{3ab^2}{2}+\frac{3}{2}a\left(a^2-b^2 \right.\right.}{\sqrt{\cos(c+dx)}\,(a+b\cos}}{3b^2\left(a^2-b\right.}$$

$$= -\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+s}{}}}{b^3d}$$

$$= -\frac{2\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+s}{}}}{b^3d}$$

$$= \frac{2\left(3a^2-7b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(}{a-}}}{3(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [C] time = 6.30, size = 1282, normalized size = 2.58

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*a^2*Sin[c + d*x])/(3*b*(-a
^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(3*a^3*Sin[c + d*x] - 7*a*b^2*Sin[c
+ d*x]))/(3*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(a^3 - a*b^
2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-(a^2*b) - 3*b^3)*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
```

```
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^3 - 7*a*b^2)*((I*Cos
[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*
a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b +
 (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*(a - b
)^2*b*(a + b)^2*d)
```

fricas [F] time = 62.14, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{5}{2}}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.27, size = 3911, normalized size = 7.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)

[Out] -2/3/d*(6*cos(d*x+c)^2*a^4*b+8*cos(d*x+c)^3*a^2*b^3+6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*b^5-14*cos(d*x+c)^2*a^2*b^3-7*cos(d*x+c)*a^3*b^2-4*cos(d*x+c)^3*a^4*b+4*cos(d*x+c)^2*a^3*b^2-2*cos(d*x+c)*a^4*b+3*a^5*cos(d*x+c)+3*cos(d*x+c)^3*a^3*b^2-7*cos(d*x+c)^3*a*b^4+7*cos(d*x+c)^2*a*b^4+6*cos(d*x+c)*a^2*b^3+6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)-3*cos(d*x+c)^2*a^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^5+6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^5+6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*b^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^5-12*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)-9*(cos(d*x+c)/(1+cos

(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^4+4
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*a^3*b^2+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^4+6*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^4*b-6*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*a^4*b+14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)*a^2*b^3-12*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2*b^3+6*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*
a^4*b-12*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*sin(d*x+c)*cos(d*x+c)*a^3*b^2-12*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*a^2*b^3+6*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*
a*b^4+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*a^4*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b^2+2*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b^3-6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-3*cos(d*x+c)^2*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
^2+7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^3+7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(

```
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-7*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^3)/(a+b*c
os(d*x+c))^(3/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/(a-b)^2/(a+b)^2/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.639 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=342

$$-\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(a-3b)\cot(c+dx)\sqrt{\frac{a(1}{}}}{}$$

```
[Out] 2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a*
b*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+8/3*b*co
t(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/a/(a-b)/(a+b)^(3/2)/d+2/3*(a-3*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d
```

Rubi [A] time = 0.61, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2799, 2993, 2998, 2816, 2994}

$$-\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(a-3b)\cot(c+dx)\sqrt{\frac{a(1}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (8*b*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) + (2*
(a - 3*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d)
+ (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
```

```
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a}{2}+\frac{3}{2}b\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+d}}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+d}}$$

$$= \frac{8b\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} +$$

Mathematica [A] time = 6.38, size = 277, normalized size = 0.81

$$\frac{2\left(\sin(c+dx)\sqrt{\cos(c+dx)}\,\left(a^3+3ab^2+4b^3\cos(c+dx)\right)-\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\,(a+b\cos(c+dx))\left(-\left(a^2+4ab\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(Sqrt[Cos[c + d*x]]*(a^3 + 3*a*b^2 + 4*b^3*Cos[c + d*x])*Sin[c + d*x] -
Sqrt[Cos[(c + d*x)/2]^2]*(a + b*Cos[c + d*x])*(4*b*(a + b)*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] - (a^2 + 4*a*b + 3*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)] + 4*b*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c
+ d*x)/2])))/(3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.30, size = 1782, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(8*cos(d*x+c)^2*a*b^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-a^3*cos(d*x+c)-3*cos(d*x+c)*a*b
^2-4*cos(d*x+c)^2*a^2*b-4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*cos(d*x+c)^2*b^3+cos(d*x+c)^
3*a^3+4*cos(d*x+c)*a^2*b-5*cos(d*x+c)^3*a*b^2+4*cos(d*x+c)^3*b^3+3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*
x+c)*b^3+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*b^3+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*cos(d*x+c)^2*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
```

```
^2*b*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-4*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^2+5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)/(a+b*cos(d
*x+c))^(3/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/(a-b)^2/(a+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(5/2), x)
```

3.640 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a\cdots}{}}}{\cdots}$$

```
[Out] -2/3*b*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*(
3*a^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
-2/3*(3*a^2+b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d+2/3*(3*a-b)*cot(d*x+c)*Elli
pticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1
/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/
(a+b)^(3/2)/d
```

Rubi [A] time = 0.64, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2796, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a\cdots}{}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(a - b)*(a + b)
^(3/2)*d) + (2*(3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a
 + b)^(3/2)*d) - (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a
+ b*Cos[c + d*x])^(3/2)) + (2*(3*a^2 + b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
```

```
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{b}{2}-\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+}}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+}}$$

$$= -\frac{2\left(3a^2+b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(1+s}{}}}{3a^2(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.25, size = 1273, normalized size = 3.55

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*b*Sin[c + d*x])/(3*(a^2 -
 b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*b*Sin[c + d*x] + b^3*Sin[c + d*x]
))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(-(a^2*b) + b^3)*S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3 + a*b^2)*((Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Co
```

```
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2*b + b^3)*((I*Cos[(c + d*x
)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos
[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a*(a - b)^2*(a
+ b)^2*d)
```

fricas [F] time = 2.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.21, size = 2417, normalized size = 6.73

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

[Out] -2/3/d*(cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-3*cos(d*x+c)^3*a^2*b^2-6*cos(d*x+c)^2*a^3*b-2*cos(d*x+c)^2*a*b^3-cos(d*x+c)*a^2*b^2-cos(d*x+c)^3*b^4+cos(d*x+c)^2*b^4+2*cos(d*x+c)^3*a^3*b+2*cos(d*x+c)^3*a*b^3+4*cos(d*x+c)^2*a^2*b^2+4*cos(d*x+c)*a^3*b+sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3

```
*cos(d*x+c)^2*a^4-3*a^4*cos(d*x+c)+3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-3*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^4)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)/sin(d*x+c)/(a-b)^2/(a+b)^2/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(5/2), x)
```

3.641 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=381

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2-3ab-2b^2\right)\cot(c$$

```
[Out] 2/3*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-4/
3*b*(3*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)
)^(1/2)+4/3*b*(3*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d+2/3*(3*a^2-3*a*b-2
*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d
```

Rubi [A] time = 0.73, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2802, 2993, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2-3ab-2b^2\right)\cot(c$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (4*b*(3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b
)^(3/2)*d) + (2*(3*a^2 - 3*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^2*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (4*b*(3*a^2 - b^2)*Sin[c +
d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
```

```
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2-2b^2\right)-\frac{3}{2}ab\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,d}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4b\left(3a^2-b^2\right)\sin(}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4b\left(3a^2-b^2\right)\sin(}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}$$

$$= \frac{4b\left(3a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a-}}}{3a^3(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.28, size = 1296, normalized size = 3.40

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*b^2*Sin[c + d*x])/(3*a*(a^
2 - b^2)*(a + b*Cos[c + d*x])^2) + (4*(3*a^2*b^2*Sin[c + d*x] - b^4*Sin[c +
 d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3*a^4 - 5*
a^2*b^2 + 2*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6*a^3*b + 2*a*b^3)*((S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
```

```
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^2*b
^2 + 2*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSi
nh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*
x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
 d*x]])))/(3*a^2*(a - b)^2*(a + b)^2*d)
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^3\cos\left(dx+c\right)^4+3\,ab^2\cos\left(dx+c\right)^3+3\,a^2b\cos\left(dx+c\right)^2+a^3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^4 +
3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.29, size = 2743, normalized size = 7.20

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d/(a+b*cos(d*x+c))^(3/2)*(6*cos(d*x+c)^2*a^4*b-6*cos(d*x+c)^3*a^2*b^3-2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*sin(
d*x+c)+2*cos(d*x+c)^3*b^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^5+4*cos(d*x+c)^2*
a^2*b^3+7*cos(d*x+c)*a^3*b^2-12*cos(d*x+c)^2*a^3*b^2-6*cos(d*x+c)*a^4*b-3*c
os(d*x+c)*a*b^4-2*cos(d*x+c)^2*b^5+5*cos(d*x+c)^3*a^3*b^2-cos(d*x+c)^3*a*b^
4+4*cos(d*x+c)^2*a*b^4+2*cos(d*x+c)*a^2*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)-(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)+2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)
+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*si
n(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*b^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b^3*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^4+12*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b^2-4*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a*b^4+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)*a^4*b+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^3-9*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4*b-7*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)*a^3*b^2-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
```

```
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-6*cos(d*x+c)^2*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b^2-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^3+2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+6*cos(d*x+c)^2*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+6*c
os(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b^3-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^3)/cos(d*x+c)^(1/
2)/sin(d*x+c)/a^2/(a+b)^2/(a-b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cos(c+dx))^{\frac{5}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))**(5/2)*sqrt(cos(c + d*x))), x)
```

3.642 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{8b^2(2a^2-b^2)\sin(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2(3a^4-15\cdots}{\cdots}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+8/
3*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*
x+c))^(1/2)+2/3*(3*a^4-15*a^2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d-2/3
*(3*a^3+9*a^2*b-6*a*b^2-8*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d
```

Rubi [A] time = 0.81, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2802, 3055, 2998, 2816, 2994}

$$\frac{8b^2(2a^2-b^2)\sin(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2(9a^2b+3\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(
a - b)*(a + b)^(3/2)*d) - (2*(3*a^3 + 9*a^2*b - 6*a*b^2 - 8*b^3)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (8
*b^2*(2*a^2 - b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2-4b^2\right)-\frac{3}{2}ab\cdots}{\cos^{\frac{3}{2}}(c+dx)\cdots}}{3a\left(\cdots\right.}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{8b^2\left(2\cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cdots}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{8b^2\left(2\cdots\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cdots}}$$

$$= \frac{2\left(3a^4-15a^2b^2+8b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\cdots}}{3a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.40, size = 1321, normalized size = 3.32

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -1/3*((-4*a*(9*a^4*b - 17*a^2*b^3 + 8*b^5)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*
a*(3*a^5 - 15*a^3*b^2 + 8*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
```

```
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])) + 2*(3*a^4*b - 15*a^2*b^3 + 8*b^5)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^3*(a - b)^2*(a +
 b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*b^3*Sin[c + d*
x])/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(9*a^2*b^3*Sin[c + d*x]
 - 5*b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*Tan
[c + d*x])/a^3))/d
```

fricas [F] time = 2.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{5}+3\,ab^{2}\cos\left(dx+c\right)^{4}+3\,a^{2}b\cos\left(dx+c\right)^{3}+a^{3}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^5 +
3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 0.27, size = 3693, normalized size = 9.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^4*b^2*sin(d*x+c)+8*cos(d*x+c)^3*b^6-8*cos(d*x+c)^2*b^6+3*cos(d*x+c)*a^6+
6*b^2*a^4-3*b^4*a^2-3*a^6+3*cos(d*x+c)^3*a^4*b^2-15*cos(d*x+c)^3*a^2*b^4-4*
cos(d*x+c)^3*a*b^5+6*cos(d*x+c)^2*a^5*b-30*cos(d*x+c)^2*a^3*b^3+10*cos(d*x+
c)^2*a^2*b^4+16*cos(d*x+c)^2*a*b^5-15*cos(d*x+c)*a^4*b^2+22*cos(d*x+c)*a^3*
b^3+8*cos(d*x+c)*a^2*b^4-12*cos(d*x+c)*a*b^5+3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)+8*cos(d*x+c)^3*a^3
*b^3+6*cos(d*x+c)^2*a^4*b^2-6*cos(d*x+c)*a^5*b+2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)+8*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)-3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b+
15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*a^4*b^2+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*a^3*b^3-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^4*sin(d*x+c)-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^6+3*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6
-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^6-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^4-16*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+
c)*a*b^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*sin(d*x+c)*a^5*b-21*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b^2-13*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3+10*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
sin(d*x+c)*a^2*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^5-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b+12*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*
b^2+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^3*b^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5*b-6*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b^2-15*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*
sin(d*x+c)*a^3*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^5-3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+
c)*a^5*b-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*a^4*b^2+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^3+15*(cos(d*x+c)/(1+c

```
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2
*b^4-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)^2*sin(d*x+c)*a*b^5)/(a+b*cos(d*x+c))^(3/2)/sin(d*x+c)/cos(d*x+c)^(1/
2)/a^3/(a+b)^2/(a-b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.643 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=473

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}-\frac{8b\left(2a^4-7a\right.}{}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2)+4/
3*b^2*(5*a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*cos(
d*x+c))^(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/
a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*cot(d*x+c)
*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^5
/(a-b)/(a+b)^(3/2)/d+2/3*(a^4+9*a^3*b+16*a^2*b^2-12*a*b^3-16*b^4)*cot(d*x+c
)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
4/(a-b)/(a+b)^(3/2)/d
```

Rubi [A] time = 1.17, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2802, 3055, 2998, 2816, 2994}

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-13a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5
*(a - b)*(a + b)^(3/2)*d) + (2*(a^4 + 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*
b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (
2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^(3/2)) + (4*b^2*(5*a^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Co
s[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^
(3/2))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}\left(a^2-2b^2\right)-\frac{3}{2}ab}{\cos^{\frac{5}{2}}(c+dx)}}{3a}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{3a^5(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.49, size = 1351, normalized size = 2.86

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((-4*a*(a^6 + 15*a^4*b^2 - 32*a^2*b^4 + 16*b^6)*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - 4*a*(8*a^5*b - 28*a^3*b^3 + 16*a*b^5)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*b^2 - 28*a^2*b^4 + 16*b^6)*((I*Co
s[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2
]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2
*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b
+ (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^4*(
a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*b^
4*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (8*(3*a^2*b^4*
Sin[c + d*x] - 2*b^6*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x
])) - (16*b*Tan[c + d*x])/(3*a^4) + (2*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))
/d
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^6+3\,ab^2\cos(dx+c)^5+3\,a^2b\cos(dx+c)^4+a^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^6 +
3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 0.32, size = 4189, normalized size = 8.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^3*sin(d*x+c)*b^7+cos(d*x+c)^4*a^5*b^2-16*cos(d*x+c)^2*a^2*b^5+24
*cos(d*x+c)^2*a*b^6+6*cos(d*x+c)*a^6*b-12*cos(d*x+c)*a^4*b^3+6*cos(d*x+c)*a
^2*b^5-8*cos(d*x+c)^4*a^4*b^3-13*cos(d*x+c)^4*a^3*b^4+28*cos(d*x+c)^4*a^2*b
^5+8*cos(d*x+c)^4*a*b^6+2*cos(d*x+c)^3*a^6*b-16*cos(d*x+c)^3*a^5*b^2-8*cos(
d*x+c)^3*a^4*b^3+56*cos(d*x+c)^3*a^3*b^4-18*cos(d*x+c)^3*a^2*b^5-32*cos(d*x
+c)^3*a*b^6-8*cos(d*x+c)^2*a^6*b+13*cos(d*x+c)^2*a^5*b^2+28*cos(d*x+c)^2*a^
4*b^3-42*cos(d*x+c)^2*a^3*b^4+cos(d*x+c)^2*a^7-16*cos(d*x+c)^4*b^7+16*cos(d
*x+c)^3*b^7-a^7+2*a^5*b^2-a^3*b^4+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^7+16*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^7+(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(
d*x+c)*a^7+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^3*sin(d*x+c)*a^6*b-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5*b^2+7*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4*b^3
+28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*sin(d*x+c)*a^3*b^4-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
```

)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^5-16*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^6+8*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*s
in(d*x+c)*a^5*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4*b^3-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b^4-28*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d
*x+c)*a^2*b^5+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^6-7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*b-cos(d*x+c)^2*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b
^2+35*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4*b^3+24*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^4-20*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5-
16*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^6+8*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^6*b+16*cos(d*x+c)^2*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*b^2-20*cos(d
*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a^4*b^3-56*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^4-12*cos(d*x+c)^2*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^5+32*cos(d*x+
c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*a*b^6-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*a^6*b+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos

```
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b^2+28*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3-4*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3*b^4-16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5+8*cos(d*x+c)*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^6*b+8*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^
5*b^2-28*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*a^4*b^3-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^4+16*cos(d*x+c)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^5+16*
cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*a*b^6)/(a+b*cos(d*x+c))^(3/2)/sin(d*x+c)/cos(d*x+c)^(3/2)/(a-b)^
2/(a+b)^2/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{5/2}\,(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.644 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d}$$

```
[Out] 2/5*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),1/5*5^(1/2))/d*5^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2813}

$$\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/5])/(Sqrt[5]*d)
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = \frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Mathematica [B] time = 3.66, size = 131, normalized size = 4.09

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)+2}\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+d\right.\right.\right.}{d\sqrt{\frac{-3\cos(c+dx)-2}{\cos(c+dx)-1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2]
, -4])/(d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]
/(-1 + Cos[c + d*x])])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{3\,\cos\left(dx+c\right)+2}\,\sqrt{\cos\left(dx+c\right)}}{3\,\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 + 2*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,\cos\left(dx+c\right)+2}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.20, size = 115, normalized size = 3.59

$$-\frac{\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\sin^{4}\left(dx+c\right)\right)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\;\operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)}{5d\sqrt{2+3\cos\left(dx+c\right)}\;\cos\left(dx+c\right)^{\frac{3}{2}}\left(-1+\cos\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(2+3*cos(d*x+c))^(1/2)*sin
(d*x+c)^4*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),1/5*5^(1/2))/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \cos(dx+c)+2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\sqrt{3\cos(c+d\,x)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2)^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\cos(c+dx)+2}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(3*cos(c + d*x) + 2)*sqrt(cos(c + d*x))), x)
```

3.645 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=25

$$\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)\middle|5\right)}{d}$$

```
[Out] 2*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),5^(1/2))/d
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2813}

$$\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5])/d
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\middle|5\right)}{d}$$

Mathematica [B] time = 0.94, size = 156, normalized size = 6.24

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\left((3\cos(c+dx)-2)\csc^2\left(\frac{1}{2}(\right.\right.}}{\sqrt{5}\,d\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((-
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]/2], 4/5]*Sin[(c + d*x)/2]^4)/
(Sqrt[5]*d*Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

fricas [F] time = 1.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{3\cos\left(dx+c\right)-2}\,\sqrt{\cos\left(dx+c\right)}}{3\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 - 2*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\cos\left(dx+c\right)-2}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.19, size = 107, normalized size = 4.28

$$-\frac{2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\sin^{4}\left(dx+c\right)\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\;\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)}{d\sqrt{-2+3\cos\left(dx+c\right)}\;\cos\left(dx+c\right)^{\frac{3}{2}}\left(-1+\cos\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(-2+3*cos(d*x+c))^(1/2)*sin(d*x+c)^4
*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),5^(1/2))/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\cos\left(dx+c\right)-2}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\sqrt{3\,\cos(c+d\,x)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2)^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\cos(c+dx)-2}\,\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(3*cos(c + d*x) - 2)*sqrt(cos(c + d*x))), x)
```

3.646 $\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\sqrt{-\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{\cos(c+dx)}}$$

```
[Out] -2/5*EllipticF(sin(d*x+c)/(1-cos(d*x+c)),1/5*5^(1/2))*(-cos(d*x+c))^(1/2)/d
*5^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2814, 2813}

$$-\frac{2\sqrt{-\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])],
1/5])/(Sqrt[5]*d*Sqrt[Cos[c + d*x]])
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rule 2814

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{-\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{\cos(c+dx)}}$$

Mathematica [B] time = 1.10, size = 143, normalized size = 2.55

$$-\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{(2-3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c\right.}}{d\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*
Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[2 - 3*Co
s[c + d*x]]*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)^2-2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 -
2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

[Out] integrate(1/(sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)

maple [B] time = 0.18, size = 119, normalized size = 2.12

$$\frac{2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{2-3\cos\left(dx+c\right)}\,\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)\left(\sin^{4}\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^{\frac{3}{2}}\left(-2+3\cos\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)

[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(2-3*cos(d*x+c))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))/(-1+cos(d*x+c))^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,\cos\left(dx+c\right)+2}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{2-3\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^(1/2)*(2 - 3*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(2 - 3*cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3\cos\left(c+dx\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)

[Out] Integral(1/(sqrt(2 - 3*cos(c + d*x))*sqrt(cos(c + d*x))), x)

3.647 $\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=49

$$-\frac{2\sqrt{-\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*EllipticF(sin(d*x+c)/(1-cos(d*x+c)),5^(1/2))*(-cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2814, 2813}

$$-\frac{2\sqrt{-\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])],
5])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rule 2814

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{-\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [B] time = 1.52, size = 153, normalized size = 3.12

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(\right.}}{\sqrt{5}\,d\sqrt{-3\cos(c+dx)-2}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
5/2]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 4/5]*Sin[(c + d*x)/2]^4)/(Sqr
t[5]*d*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)^2+2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 +
2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.19, size = 132, normalized size = 2.69

$$\frac{\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{-2-3\cos\left(dx+c\right)}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{5\sin(dx+c)},\sqrt{5}\right)\left(\sin^{4}\left(dx+c\right)\right)\sqrt{5}}{5d\cos\left(dx+c\right)^{\frac{3}{2}}\left(2+3\cos\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/5/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(-2-3*cos(d*x+c))^(1/2)*10^
(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*5^(1/2)*(-1+cos
(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))
/(-1+cos(d*x+c))^2*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\,\cos\left(dx+c\right)-2}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{-3\,\cos\left(c+d\,x\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\cos\left(c+dx\right)-2}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*cos(c + d*x) - 2)*sqrt(cos(c + d*x))), x)
```

3.648 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] 2*cot(d*x+c)*EllipticF(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2),
I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2815}

$$\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

Mathematica [B] time = 1.05, size = 140, normalized size = 2.41

$$\frac{4\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)+3}\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(c+dx)\right)}}{\sqrt{6}}\right)\middle|6\right)}{d\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[-Cot[(c + d*x)/2]^2]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/S
qrt[6]], 6])/(d*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(3 + 2*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2])
```

fricas [F] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)^2+3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 + 3*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.22, size = 116, normalized size = 2.00

$$-\frac{\sqrt{2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\ \text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\left(\sin^4(dx+c)\right)}{5d\sqrt{3+2\cos(dx+c)}\ \cos(dx+c)^{\frac{3}{2}}(-1+\cos(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(3+2*cos(d*x+c))^(1/2)*10^
(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\cos\left(dx+c\right)+3}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{2\cos\left(c+d\,x\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) + 3)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\cos\left(c+dx\right)+3}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*cos(c + d*x) + 3)*sqrt(cos(c + d*x))), x)
```

3.649 $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)\middle|-\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] 2/5*cot(d*x+c)*EllipticF((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/5*I*5^(1
/2))*(-tan(d*x+c)^2)^(1/2)/d*5^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2815}

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)\middle|-\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]
]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d)
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)\middle|-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

Mathematica [B] time = 1.04, size = 144, normalized size = 2.40

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{(3-2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(3 - 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*S
qrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]/(-1 + Cos[c + d*x])]/Sqrt[3]], 6]*Sin[(c + d*x)/2]^4)/(d*Sqrt[
3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,\cos\left(dx+c\right)+3}\,\sqrt{\cos\left(dx+c\right)}}{2\,\cos\left(dx+c\right)^{2}-3\,\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 -
3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,\cos\left(dx+c\right)+3}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 0.19, size = 125, normalized size = 2.08

$$\frac{\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{3-2\cos\left(dx+c\right)}\,\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\;\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},i\sqrt{5}\right)\left(\sin^4\left(dx+c\right)\right)}{d\cos\left(dx+c\right)^{\frac{3}{2}}\left(-3+2\cos\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(3-2*cos(d*x+c))^(1/2)*(-2*(-
3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
I*5^(1/2))*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))/(-1+cos(d*x+c))^
2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\cos\left(dx+c\right)+3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{3-2\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(3 - 2*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3-2\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3 - 2*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.650 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{2\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

```
[Out] -2/5*csc(d*x+c)*EllipticF((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*I
*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-tan(d*x+c)^2)^(1/2)/d*5^(1
/2)
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2817, 2815}

$$-\frac{2\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/
(Sqrt[5]*d)
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rule 2817

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[-(d*Sin[e + f*x])]/Sqrt[d*Sin[e + f*x]]
, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[-(d*Sin[e + f*x])]), x], x] /; FreeQ
[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right.\right.}{\sqrt{5}\,d}$$

Mathematica [A] time = 1.26, size = 144, normalized size = 1.71

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{2\cos(c+dx)-3}{\cos(c+dx)-1}}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{2\cos(c+dx)-3}{\cos(c+dx)-1}}}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)-1}}\sqrt{2\cos(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt[
-Cot[(c + d*x)/2]^2]*EllipticF[ArcSin[Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[
c + d*x])]/Sqrt[3]], 6/5]*Tan[(c + d*x)/2])/(Sqrt[5]*d*Sqrt[Cos[c + d*x]/(-
1 + Cos[c + d*x])]*Sqrt[-3 + 2*Cos[c + d*x]])
```

fricas [F] time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\cos(dx+c)-3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)^2-3\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 - 3*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\cos(dx+c)-3}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 0.20, size = 123, normalized size = 1.46

$$\frac{i\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\sin^{4}(dx+c)\right)\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\;\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\sqrt{5}}{5d\sqrt{-3+2\cos(dx+c)}\;\cos(dx+c)^{\frac{3}{2}}\left(-1+\cos(dx+c)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5*I/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(-3+2*cos(d*x+c))^(1/2)*s
in(d*x+c)^4*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos
(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))
^2*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\cos(dx+c)-3}\,\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\sqrt{2\cos(c+d\,x)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) - 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\cos(c+dx)-3}\,\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(2*cos(c + d*x) - 3)*sqrt(cos(c + d*x))), x)
```

3.651 $\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] -2*csc(d*x+c)*EllipticF(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^(
1/2),I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-tan(d*x+c)^2)^(1/2)/
d
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2817, 2815}

$$-\frac{2\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d
*x]^2])/d
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rule 2817

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[-(d*Sin[e + f*x])]/Sqrt[d*Sin[e + f*x]]
, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[-(d*Sin[e + f*x])]), x], x] /; FreeQ
[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\right.}{d}$$

Mathematica [A] time = 1.06, size = 153, normalized size = 1.87

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(c\right.}}{\sqrt{5}\,d\sqrt{-2\cos(c+dx)-3}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
5/3]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 6/5]*Sin[(c + d*x)/2]^4)/(Sqr
t[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\cos(dx+c)-3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)^2+3\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 +
3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\cos(dx+c)-3}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 0.20, size = 137, normalized size = 1.67

$$-\frac{i\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{-3-2\cos\left(dx+c\right)}\,\left(\sin^4\left(dx+c\right)\right)\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\;\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)},i\sqrt{5}\right)}{5d\cos\left(dx+c\right)^{\frac{3}{2}}\left(3+2\cos\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/5*I/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(-3-2*cos(d*x+c))^(1/2)*
sin(d*x+c)^4*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5
*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))/cos(d*x+c)^(3/2)/(3+2*cos(
d*x+c))/(-1+cos(d*x+c))^2*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\,\cos\left(dx+c\right)-3}\,\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{-2\,\cos\left(c+d\,x\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\cos\left(c+dx\right)-3}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-2*cos(c + d*x) - 3)*sqrt(cos(c + d*x))), x)
```

3.652 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

```
[Out] 2/5*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),1/5*5^(1/2))*cos(d*x+c)^(1/2)/d*5^(
1/2)/(-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2814, 2813}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/
5])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rule 2814

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

Mathematica [B] time = 0.61, size = 150, normalized size = 2.78

$$-\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\right.}}{d\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[
(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sq
rt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)+2}}{3\cos(dx+c)^2+2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)/(3*cos(d*x + c)^2 +
2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)), x)
```

maple [B] time = 0.19, size = 122, normalized size = 2.26

$$\frac{\mathrm{EllipticF}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{5\sin(dx+c)},\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\sin^2(dx+c)\right)\sqrt{5}}{5d\sqrt{2+3\cos(dx+c)}\,\left(-1+\cos(dx+c)\right)\sqrt{-\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)/(2+3*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(-cos(d*x+c))^(1/2
)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\,\sqrt{3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-\cos(c+d\,x)}\,\sqrt{3\cos(c+d\,x)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) + 2)^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\,\sqrt{3\cos(c+dx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(3*cos(c + d*x) + 2)), x)
```

3.653 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)\middle|5\right)}{d\sqrt{-\cos(c+dx)}}$$

```
[Out] 2*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),5^(1/2))*cos(d*x+c)^(1/2)/d/(-cos(d*x
+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2814, 2813}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)\middle|5\right)}{d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5]
)/(d*Sqrt[-Cos[c + d*x]])
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rule 2814

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\middle|5\right)}{d\sqrt{-\cos(c+dx)}}$$

Mathematica [B] time = 0.40, size = 158, normalized size = 3.36

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\left((3\cos(c+dx)-2)\csc^2\left(\frac{1}{2}(\right.\right.}}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((-
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]/2], 4/5]*Sin[(c + d*x)/2]^4)/
(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)-2}}{3\cos(dx+c)^2-2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)/(3*cos(d*x + c)^2 -
2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)), x)
```

maple [B] time = 0.19, size = 109, normalized size = 2.32

$$\frac{2\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\sin^2(dx+c)\right)}{d\sqrt{-2+3\cos(dx+c)}\,\left(-1+\cos(dx+c)\right)\sqrt{-\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*((-2+3*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*s
in(d*x+c)^2/(-1+cos(d*x+c))/(-cos(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\,\sqrt{3\,\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-\cos(c+d\,x)}\,\sqrt{3\,\cos(c+d\,x)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) - 2)^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) - 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\,\sqrt{3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(3*cos(c + d*x) - 2)), x)
```

3.654 $\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

Optimal. Leaf size=34

$$-\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d}$$

```
[Out] -2/5*EllipticF(sin(d*x+c)/(1-cos(d*x+c)),1/5*5^(1/2))/d*5^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2813}

$$-\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (-2*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])], 1/5])/(Sqrt[5]*d)
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Mathematica [B] time = 0.55, size = 145, normalized size = 4.26

$$-\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{(2-3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(\right.}}{d\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*
Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[2 - 3*Co
s[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\cos\left(dx+c\right)+2}}{3\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)/(3*cos(d*x + c)^2 -
2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\cos\left(dx+c\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)), x)
```

maple [B] time = 0.16, size = 121, normalized size = 3.56

$$-\frac{2\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2-3\cos\left(dx+c\right)}\,\left(\sin^{2}\left(dx+c\right)\right)}{d\left(3\left(\cos^{2}\left(dx+c\right)\right)-5\cos\left(dx+c\right)+2\right)\sqrt{-\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*((-2+3*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+c))^(1/2)*s
in(d*x+c)^2/(3*cos(d*x+c)^2-5*cos(d*x+c)+2)/(-cos(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\cos\left(dx+c\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{2-3\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-cos(c + d*x))^(1/2)*(2 - 3*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(2 - 3*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\,\sqrt{2-3\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2 - 3*cos(c + d*x))), x)
```

3.655 $\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$-\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d}$$

```
[Out] -2*EllipticF(sin(d*x+c)/(1-cos(d*x+c)),5^(1/2))/d
```

Rubi [A] time = 0.06, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2813}

$$-\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (-2*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])], 5])/d
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d}$$

Mathematica [B] time = 0.49, size = 155, normalized size = 5.74

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c\right.}}{\sqrt{5}\,d\sqrt{-3\cos(c+dx)-2}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
5/2]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 4/5]*Sin[(c + d*x)/2]^4)/(Sqr
t[5]*d*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)-2}}{3\cos(dx+c)^2+2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)/(3*cos(d*x + c)^2 +
2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)), x)
```

maple [B] time = 0.18, size = 129, normalized size = 4.78

$$-\frac{\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{\sqrt{5}}{5}\right)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-2-3\cos(dx+c)}\,\left(\sin^2(dx+c)\right)}{5d\left(3\left(\cos^2(dx+c)\right)-\cos(dx+c)-2\right)\sqrt{-\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))*10^(1/2)*((2+3*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-
2-3*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(3*cos(d*x+c)^2-cos(d*x+c)-2)/(-cos(d*x+
c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\,\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{-3\,\cos\left(c+d\,x\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-cos(c + d*x))^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\,\sqrt{-3\cos\left(c+dx\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(-3*cos(c + d*x) - 2)), x)
```

3.656 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d\sqrt{-\cos(c+dx)}}$$

```
[Out] 2*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticF(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/
cos(d*x+c)^(1/2),I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d/(-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2817, 2815}

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]
]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/(d*Sqrt[-Cos[c
+ d*x]])
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rule 2817

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[-(d*Sin[e + f*x])]/Sqrt[d*Sin[e + f*x]]
, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[-(d*Sin[e + f*x])]), x], x] /; FreeQ
[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$
$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{-\tan^2(c+dx)}}{d\sqrt{-\cos(c+dx)}}$$

Mathematica [A] time = 0.61, size = 154, normalized size = 1.92

$$-\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2\cos(c+dx)+3)\csc^2\left(\right.}}{d\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[-Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt
[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/Sqrt[6]], 6]*Sin[(c + d*x)/2]^4)
/(d*Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)+3}}{2\cos(dx+c)^2+3\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)/(2*cos(d*x + c)^2 +
3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)), x)
```

maple [A] time = 0.16, size = 127, normalized size = 1.59

$$-\frac{i \,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)}, i\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\sin^2\left(dx+c\right)\right)\sqrt{5}}{5d\sqrt{3+2\cos\left(dx+c\right)}\,\left(-1+\cos\left(dx+c\right)\right)\sqrt{-\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*2^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)/(3+2*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(-cos(d*x+c
))^(1/2)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\,\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{2\,\cos\left(c+d\,x\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) + 3)^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\,\sqrt{2\cos\left(c+dx\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2*cos(c + d*x) + 3)), x)
```

3.657 $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

```
[Out] 2/5*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticF((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)
^(1/2),1/5*I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d*5^(1/2)/(-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2817, 2815}

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 - 2*Cos[c + d*x]
]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d*Sqrt[-Cos[c
+ d*x]])
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rule 2817

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[-(d*Sin[e + f*x])]/Sqrt[d*Sin[e + f*x]]
, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[-(d*Sin[e + f*x])]), x], x] /; FreeQ
[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$
$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{-\tan^2(c+}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

Mathematica [A] time = 0.48, size = 146, normalized size = 1.78

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{(3-2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c}{d\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(3 - 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*S
qrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]/(-1 + Cos[c + d*x])]/Sqrt[3]], 6]*Sin[(c + d*x)/2]^4)/(d*Sqrt[
3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}}{2\cos(dx+c)^2-3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)/(2*cos(d*x + c)^2 -
3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)), x)
```

maple [A] time = 0.13, size = 107, normalized size = 1.30

$$\frac{2i\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)}, \frac{i\sqrt{5}}{5}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{3-2\cos\left(dx+c\right)}\,\sqrt{5}}{5d\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\sqrt{-\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x)
```

```
[Out] 2/5*I/d*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\,\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-\cos\left(c+d\,x\right)}\,\sqrt{3-2\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-cos(c + d*x))^(1/2)*(3 - 2*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\,\sqrt{3-2\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(3 - 2*cos(c + d*x))), x)
```

3.658 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=62

$$-\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

```
[Out] -2/5*cot(d*x+c)*EllipticF((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*I
*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d*5^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2815}

$$-\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d
*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d)
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = -\frac{2\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}\,d}$$

Mathematica [B] time = 0.69, size = 160, normalized size = 2.58

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\cot(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\left((2\cos(c+dx)-3)\csc^2\right.}}{\sqrt{5}d(-\cos(c+dx))^{3/2}\sqrt{2\cos(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[-Cot[(c + d*x)/2]^2]*Cot[c + d*x]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)
/2]^2)]*Sqrt[-((-3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*EllipticF[ArcSin[
Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[c + d*x])]/Sqrt[3]], 6/5]*Sin[(c + d*x
)/2]^4)/(Sqrt[5]*d*(-Cos[c + d*x])^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
```

fricas [F] time = 1.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-\cos\left(dx+c\right)}\sqrt{2\cos\left(dx+c\right)-3}}{2\cos\left(dx+c\right)^2-3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)/(2*cos(d*x + c)^2 -
3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{2\cos\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)), x)
```

maple [A] time = 0.12, size = 98, normalized size = 1.58

$$\frac{2\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},i\sqrt{5}\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-3+2\cos\left(dx+c\right)}}{d\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\sqrt{-\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))/(-2*(-3+2*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3+2*cos(d*
x+c))^(1/2)/(-cos(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-\cos(c+d\,x)}\,\sqrt{2\,\cos(c+d\,x)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) - 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2*cos(c + d*x) - 3)), x)
```

3.659 $\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] -2*cot(d*x+c)*EllipticF(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^(
1/2),I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2815}

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-
Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

Mathematica [B] time = 0.46, size = 155, normalized size = 2.58

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(\right.}}{\sqrt{5}\,d\sqrt{-2\cos(c+dx)-3}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
5/3]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 6/5]*Sin[(c + d*x)/2]^4)/(Sqr
t[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

fricas [F] time = 1.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\,\cos\left(dx+c\right)-3}}{2\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)/(2*cos(d*x + c)^2 +
3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\,\cos\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)), x)
```

maple [B] time = 0.17, size = 128, normalized size = 2.13

$$-\frac{\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-3-2\cos\left(dx+c\right)}\,\left(\sin^2\left(dx+c\right)\right)}{5d\left(2\left(\cos^2\left(dx+c\right)\right)+\cos\left(dx+c\right)-3\right)\sqrt{-\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*10^(1/2)*((3+2*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(-3-2*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(2*cos(d*x+c)^2+cos(d*x+c)-3)/(-cos(d*
x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-\cos(c+d\,x)}\,\sqrt{-2\cos(c+d\,x)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-cos(c + d*x))^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2\cos(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(-2*cos(c + d*x) - 3)), x)
```

3.660 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

```
[Out] -4/3*cot(d*x+c)*EllipticPi(1/5*(2+3*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1
/2),5/3,5^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2809}

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*S
qrt[Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(3*
d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=-\frac{4\cot(c+dx)\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\,\sqrt{1-\sec(c+dx)}}{3d}$$

Mathematica [B] time = 2.84, size = 175, normalized size = 2.27

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)+2}\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\left(3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+\right.\right.\right.\right.}{3d\sqrt{\frac{-3\cos(c+dx)-2}{\cos(c+dx)-1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*Csc
[c + d*x]*(3*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]
/2], -4] - 5*EllipticPi[-2/3, ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x
)/2]^2]/2], -4]))/(3*d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt
[Cos[c + d*x]/(-1 + Cos[c + d*x])])
```

fricas [F] time = 1.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

maple [B] time = 0.21, size = 142, normalized size = 1.84

$$-\frac{\sqrt{10}\,\sqrt{2}\,\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\frac{\sqrt{5}}{5}\right)\right)\left(\sin^2\left(dx+c\right)\right)\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos}{1+\cos}}}{5d\sqrt{2+3\cos\left(dx+c\right)}\,\left(-1+\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*10^(1/2)*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))-
2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,1/5*5^(1/2)))*sin(d*x+c)^2*((2+3
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(2+3*c
os(d*x+c))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx + c\right)}}{\sqrt{3\,\cos\left(dx + c\right) + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c + d\,x\right)}}{\sqrt{3\,\cos\left(c + d\,x\right) + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) + 2)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c + dx\right)}}{\sqrt{3\cos\left(c + dx\right) + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(3*cos(c + d*x) + 2), x)
```

3.661 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}\,d}$$

```
[Out] -4/15*cot(d*x+c)*EllipticPi((-2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/3,1/
5*5^(1/2))*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2809}

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos[
c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]
*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx=-\frac{4\cot(c+dx)\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}{3\sqrt{5}\,d}$$

Mathematica [A] time = 0.69, size = 140, normalized size = 1.87

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{3\cos(c+dx)-2}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{1}{5}\right)-2\Pi\left(-\frac{1}{5};\sin^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}\right.\right.\right.\right.}{\sqrt{5}d\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(-2 + 3*C
os[c + d*x])/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Sqrt[5]*Tan[(c + d*x)/2]
], 1/5] - 2*EllipticPi[-1/5, ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5]))/(Sqrt
[5]*d*Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

fricas [F] time = 2.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

maple [B] time = 0.18, size = 132, normalized size = 1.76

$$-\frac{2\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{5}\right)\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\sin^2(dx+\right.}{d\sqrt{-2+3\cos(dx+c)}\,(-1+\cos(dx+c))\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,5^(1/2)))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(
d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{3\cos(c+d\,x)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) - 2)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(3*cos(c + d*x) - 2), x)
```

3.662 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

```
[Out] -4/15*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi((2-3*cos(d*x+c))^(1/2)/(-cos(d
*x+c))^(1/2),1/3,1/5*5^(1/2))*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*
5^(1/2)/(-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2810, 2809}

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[2 - 3*Cos[c
 + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c
+ d*x]])/(3*Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$
$$= -\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+s}}{3\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

Mathematica [A] time = 1.77, size = 145, normalized size = 1.46

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{-\frac{(2-3\cos(c+dx))^2}{(\cos(c+dx)+1)^2}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|5\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{d\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{\frac{2-3\cos(c+dx)}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[-((2 - 3*Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Tan[(c + d*x)/2]],
 5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], 5]))/(d*Sqrt[2 - 3*Cos[c +
 d*x]]*Sqrt[Cos[c + d*x]]*Sqrt[(2 - 3*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c) - 2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

maple [A] time = 0.19, size = 144, normalized size = 1.45

$$\frac{2\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{5}\right)\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2-3\cos(d}}{d\left(3\left(\cos^2(dx+c)\right)-5\cos(dx+c)+2\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,5^(1/2)))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(3*cos(d*x
+c)^2-5*cos(d*x+c)+2)/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\,\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{2-3\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(2 - 3*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(2 - 3*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(2-3*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(cos(c + d*x))/sqrt(2 - 3*cos(c + d*x)), x)
```

3.663 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{3d\sqrt{-\cos(c+dx)}}$$

```
[Out] -4/3*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi(1/5*(-2-3*cos(d*x+c))^(1/2)*5^(
1/2)/(-cos(d*x+c))^(1/2),5/3,5^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c))^
(1/2)/d/(-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2810, 2809}

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{3d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[-2 - 3*Cos[
c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1
 - Sec[c + d*x]])/(3*d*Sqrt[-Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$

$$= -\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1}}{3d\sqrt{-\cos(c+dx)}}$$

Mathematica [A] time = 2.05, size = 155, normalized size = 1.53

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{-\frac{(3\cos(c+dx)+2)^2}{(\cos(c+dx)+1)^2}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{1}{5}\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{\sqrt{5}\,d\sqrt{-3\cos(c+dx)-2}\sqrt{\cos(c+dx)}\sqrt{-\frac{3\cos(c+dx)+2}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-((2 + 3*
Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]*(EllipticF[ArcSin[Tan[(c + d*x)/2]],
 1/5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], 1/5]))/(Sqrt[5]*d*Sqrt[-
2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Sqrt[-((2 + 3*Cos[c + d*x])/(1 + Cos
[c + d*x]))])
```

fricas [F] time = 1.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c) + 2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

maple [A] time = 0.18, size = 161, normalized size = 1.59

$$\frac{\sqrt{2}\,\sqrt{10}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{5\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{5\sin(dx+c)},-5,\sqrt{5}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{2+3\cos(dx+c}{1+\cos(dx+c)}}}{5d\left(3\left(\cos^2(dx+c)\right)-\cos(dx+c)-2\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*2^(1/2)*10^(1/2)*(EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^
(1/2))-2*EllipticPi(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),-5,5^(1/2)))*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(-2-
3*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(3*cos(d*x+c)^2-cos(d*x+c)-2)/cos(d*x+c)^(
1/2)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\,\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{-3\,\cos(c+d\,x)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(-3*cos(c + d*x) - 2), x)
```

3.664 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] -3*cot(d*x+c)*EllipticPi(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2
),5/2,I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2808}

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

```
[Out] (-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*S
qrt[Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx=-\frac{3\cot(c+dx)\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d}$$

Mathematica [A] time = 1.44, size = 115, normalized size = 1.58

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)+3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{1}{5}\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c\right.\right.\right.\right.}{\sqrt{5}d\sqrt{(3\cos(c+dx)+\cos(2(c+dx))+1)\sec^4\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*(EllipticF[ArcSin[Tan[(c +
d*x)/2]], -1/5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], -1/5])*Sec[(c
+ d*x)/2]^2)/(Sqrt[5]*d*Sqrt[(1 + 3*Cos[c + d*x] + Cos[2*(c + d*x)])*Sec[(c
 + d*x)/2]^4])
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

maple [B] time = 0.18, size = 144, normalized size = 1.97

$$-\frac{\sqrt{10}\,\sqrt{2}\,\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\frac{i\sqrt{5}}{5}\right)\right)\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\sin^2\right.}{5d\sqrt{3+2\cos(dx+c)}\,\left(-1+\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*10^(1/2)*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2)
)-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,1/5*I*5^(1/2)))*((3+2*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(3+2*cos(d*x+c)
)^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{2\cos(c+d\,x)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(2*cos(c + d*x) + 3), x)
```

3.665 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

```
[Out] 3/5*cot(d*x+c)*EllipticPi((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),-1/2,1/5*
I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2808}

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] (3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c
 + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \frac{3\cot(c+dx)\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}{\sqrt{5}\,d}$$

Mathematica [A] time = 0.84, size = 117, normalized size = 1.56

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{3-2\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-5\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{d\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[(3 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])]*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], -5] -
2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], -5]))/(d*Sqrt[3 - 2*Cos[c + d*x]
]*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c) - 3)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

maple [B] time = 0.18, size = 153, normalized size = 2.04

$$\frac{\sqrt{2}\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},i\sqrt{5}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,i\sqrt{5}\right)\right)\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{3-}}{d\left(2\left(\cos^2(dx+c)\right)-5\cos(dx+c)+3\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))-2*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,I*5^(1/2)))*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(3-2*cos(d*x+c))^(1/2)*sin(d*x+
c)^2/(2*cos(d*x+c)^2-5*cos(d*x+c)+3)/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx + c\right)}}{\sqrt{-2\,\cos\left(dx + c\right) + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{3-2\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(3 - 2*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(3 - 2*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos{\left(c + dx \right)}}}{\sqrt{3 - 2\cos{\left(c + dx \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(3 - 2*cos(c + d*x)), x)
```

3.666 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

```
[Out] 3/5*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi((-3+2*cos(d*x+c))^(1/2)/(-cos(d*
x+c))^(1/2),-1/2,1/5*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
*5^(1/2)/(-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2810, 2808}

$$\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] (3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[
c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c
 + d*x]])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$
$$= \frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1}}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}}$$

Mathematica [C] time = 0.98, size = 135, normalized size = 1.36

$$-\frac{2i\sqrt{2\cos(c+dx)-3}\sqrt{\frac{\cos(c+dx)}{5\cos(c+dx)+5}}\left(F\left(i\sinh^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{1}{5}\right)-2\Pi\left(\frac{1}{5};i\sinh^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{d\sqrt{\cos(c+dx)}\sqrt{\frac{3-2\cos(c+dx)}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[-3 + 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(5 + 5*Cos[c + d*x])]*(
EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5] - 2*EllipticPi[1/5, I*
ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[(3 -
2*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)

maple [A] time = 0.18, size = 158, normalized size = 1.60

$$-\frac{i\sqrt{2}\,\left(2\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{1}{5},\frac{i\sqrt{5}}{5}\right)-\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}}{5d\sqrt{-3+2\cos(dx+c)}\,\left(-1+\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x)

[Out] -1/5*I/d*2^(1/2)*(2*EllipticPi(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5,1/5
*I*5^(1/2))-EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2)))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)/(-3+2*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)*5^
(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{2\,\cos\left(c+d\,x\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) - 3)^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) - 3)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{2\cos\left(c+dx\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(cos(c + d*x))/sqrt(2*cos(c + d*x) - 3), x)

3.667 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$-\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d\sqrt{-\cos(c+dx)}}$$

```
[Out] -3*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/
2)/(-cos(d*x+c))^(1/2),5/2,I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(
1/2)/d/(-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2810, 2808}

$$-\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] (-3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[-3 - 2*Cos[
c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1
 + Sec[c + d*x]])/(d*Sqrt[-Cos[c + d*x]])
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$
$$= -\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{}}{d\sqrt{-\cos(c+dx)}}$$

Mathematica [A] time = 0.60, size = 113, normalized size = 1.16

$$-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(2\cos(c+dx)+3)\sec^4\left(\frac{1}{2}(c+dx)\right)}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-\frac{1}{5}\right)-2\Pi\left(\right.\right.}{\sqrt{5}\,d\sqrt{-2\cos(c+dx)-3}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*(EllipticF[ArcSin[Tan[(c + d*x)/2]], -1/5] - 2*Ellip
ticPi[-1, ArcSin[Tan[(c + d*x)/2]], -1/5])*Sqrt[Cos[c + d*x]*(3 + 2*Cos[c +
 d*x])*Sec[(c + d*x)/2]^4])/(Sqrt[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c
 + d*x]])
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\cos(dx+c)-3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c) + 3)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

maple [A] time = 0.18, size = 168, normalized size = 1.73

$$-\frac{i\sqrt{2}\,\sqrt{10}\,\left(\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)}, i\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)}, 5, i\sqrt{5}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{3+2\cos(dx+}{1+\cos(dx+}}}{5d\left(2\left(\cos^2(dx+c)\right)+\cos(dx+c)-3\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*2^(1/2)*10^(1/2)*(EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+
c),I*5^(1/2))-2*EllipticPi(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5,I*5^(
1/2)))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*(-3-2*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(2*cos(d*x+c)^2+cos(d*x+c)-3)/co
s(d*x+c)^(1/2)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{-2\cos(c+d\,x)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2\cos(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(-2*cos(c + d*x) - 3), x)
```

3.668 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

```
[Out] -4/3*csc(d*x+c)*EllipticPi(1/5*(2+3*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1
/2),5/3,5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1-sec(d*x+c))^(1/2)
*(1-sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2810, 2809}

$$-\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[5/3, Arc
Sin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], 5]*Sqrt[-1 - Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(3*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{4\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-}}{3d}$$

Mathematica [A] time = 0.77, size = 194, normalized size = 1.96

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}\right.}}{3d\sqrt{-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*(3*EllipticF[ArcSin[Sq
rt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4] - 5*EllipticPi[-2/3, Ar
cSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4])*Sin[(c + d*x)/2
]^4)/(3*d*Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]])
```

fricas [F] time = 1.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

maple [A] time = 0.17, size = 159, normalized size = 1.61

$$-\frac{\sqrt{2}\,\sqrt{10}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{5\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{5\sin(dx+c)},-5,\sqrt{5}\right)\right)\sqrt{-\cos(dx+c)}\,\left(\sin^2(dx\right.}{5d\sqrt{2+3\cos(dx+c)}\,(-1+\cos(dx+c))\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*2^(1/2)*10^(1/2)*(EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5
^(1/2))-2*EllipticPi(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),-5,5^(1/2)))/(2
+3*cos(d*x+c))^(1/2)*(-cos(d*x+c))^(1/2)*sin(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*
x+c)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\,\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\cos(c+d\,x)}}{\sqrt{3\,\cos(c+d\,x)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) + 2)^(1/2),x)
```

```
[Out] int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3*cos(c + d*x) + 2), x)
```

3.669 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$-\frac{4\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}\,d}$$

```
[Out] -4/15*csc(d*x+c)*EllipticPi((-2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/3,1/
5*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+sec(d*x+c))^(1/2)*(1+se
c(d*x+c))^(1/2)/d*5^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2810, 2809}

$$-\frac{4\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[1/3, Arc
Sin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d
*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+}}{3\sqrt{5}\,d}$$

Mathematica [A] time = 0.22, size = 142, normalized size = 1.46

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{3\cos(c+dx)-2}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{1}{5}\right)-2\Pi\left(-\frac{1}{5};\sin^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c\right.\right.\right.\right.}{\sqrt{5}\,d\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(-2 + 3*Co
s[c + d*x])/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Sqrt[5]*Tan[(c + d*x)/2]]
, 1/5] - 2*EllipticPi[-1/5, ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5]))/(Sqrt[
5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

maple [A] time = 0.19, size = 142, normalized size = 1.46

$$-\frac{2\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{5}\right)\right)\left(\sin^{2}(dx+c)\right)\sqrt{-\cos(dx+c)}\ \sqrt{\frac{\cos(dx+}{1+\cos(dx}}}{d\sqrt{-2+3\cos(dx+c)}\ (-1+\cos(dx+c))\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,5^(1/2)))*sin(d*x+c)^2*(-cos(d*x+c))^(1/2)/(-2+3*cos(d*
x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\,\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\cos(c+d\,x)}}{\sqrt{3\,\cos(c+d\,x)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) - 2)^(1/2),x)
```

```
[Out] int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3*cos(c + d*x) - 2), x)
```

3.670 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\, dx$

Optimal. Leaf size=77

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}\,d}$$

```
[Out] -4/15*cot(d*x+c)*EllipticPi((2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/3,
1/5*5^(1/2))*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2809}

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos[
c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]
*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\, dx = -\frac{4\cot(c+dx)\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}{3\sqrt{5}\,d}$$

Mathematica [A] time = 0.51, size = 147, normalized size = 1.91

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{-\frac{(2-3\cos(c+dx))^2}{(\cos(c+dx)+1)^2}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|5\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{d\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}\sqrt{\frac{2-3\cos(c+dx)}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[-((2 - 3*Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Tan[(c + d*x)/2]],
5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], 5]))/(d*Sqrt[2 - 3*Cos[c +
d*x]]*Sqrt[-Cos[c + d*x]]*Sqrt[(2 - 3*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}}{3\cos(dx+c)-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)/(3*cos(d*x + c) - 2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

maple [B] time = 0.17, size = 154, normalized size = 2.00

$$\frac{2\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{5}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\left(\sin^2(dx+c\right.}{d\left(3\left(\cos^2(dx+c)\right)-5\cos(dx+c)+2\right)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,5^(1/2)))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*(-cos(d*x+c))^(1/2)*(2-3*cos(d*x+
c))^(1/2)/(3*cos(d*x+c)^2-5*cos(d*x+c)+2)/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{2-3\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(c + d*x))^(1/2)/(2 - 3*cos(c + d*x))^(1/2),x)
```

```
[Out] int((-cos(c + d*x))^(1/2)/(2 - 3*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos{\left(c+dx\right)}}}{\sqrt{2-3\cos{\left(c+dx\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2 - 3*cos(c + d*x)), x)
```

3.671 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

```
[Out] -4/3*cot(d*x+c)*EllipticPi(1/5*(-2-3*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c)
)^(1/2),5/3,5^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2809}

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[-2 - 3*Cos[c + d*x]]/(Sqrt[5]*
Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(
3*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = -\frac{4\cot(c+dx)\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\,\sqrt{1-\sec(c+dx)}}{3d}$$

Mathematica [A] time = 0.51, size = 156, normalized size = 1.97

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{-\frac{(3\cos(c+dx)+2)^2}{(\cos(c+dx)+1)^2}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{1}{5}\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right.\right.\right.\right.}{\sqrt{5}\,d\sqrt{-3\cos(c+dx)-2}\sqrt{-\cos(c+dx)}\sqrt{\frac{-3\cos(c+dx)-2}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-((2 + 3*C
os[c + d*x])^2/(1 + Cos[c + d*x])^2)]*(EllipticF[ArcSin[Tan[(c + d*x)/2]],
1/5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], 1/5]))/(Sqrt[5]*d*Sqrt[-2
 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]*Sqrt[(-2 - 3*Cos[c + d*x])/(1 + Cos[
c + d*x])])
```

fricas [F] time = 1.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos\left(dx+c\right)}\sqrt{-3\,\cos\left(dx+c\right)-2}}{3\,\cos\left(dx+c\right)+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)/(3*cos(d*x + c) + 2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-3\,\cos\left(dx+c\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

maple [B] time = 0.18, size = 164, normalized size = 2.08

$$\frac{\sqrt{10}\,\sqrt{2}\,\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\frac{\sqrt{5}}{5}\right)\right)\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{-2-3}}{5d\left(3\left(\cos^2\left(dx+c\right)\right)-\cos\left(dx+c\right)-2\right)\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*10^(1/2)*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))-2
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,1/5*5^(1/2)))*((2+3*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2-3*cos(d*x+c))^(1
/2)*(-cos(d*x+c))^(1/2)*sin(d*x+c)^2/(3*cos(d*x+c)^2-cos(d*x+c)-2)/cos(d*x+
c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos\left(dx + c\right)}}{\sqrt{-3 \cos\left(dx + c\right) - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{-3\,\cos\left(c+d\,x\right)-2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(c + d*x))^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2),x)
```

```
[Out] int((-cos(c + d*x))^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos{\left(c + dx \right)}}}{\sqrt{-3 \cos{\left(c + dx \right)} - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(-3*cos(c + d*x) - 2), x)
```

3.672 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{3\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] -3*csc(d*x+c)*EllipticPi(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2
),5/2,I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(1-sec(d*x+c))^(1/2)*
(1+sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2810, 2808}

$$-\frac{3\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

```
[Out] (-3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[5/2, Arc
Sin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[1 - Se
c[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-}}{d}$$

Mathematica [A] time = 0.50, size = 117, normalized size = 1.23

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)+3}\,\sec^2\left(\frac{1}{2}(c+dx)\right)\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-\frac{1}{5}\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\right.\right.\right.\right.}{\sqrt{5}\,d\sqrt{(3\cos(c+dx)+\cos(2(c+dx))+1)\sec^4\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*(EllipticF[ArcSin[Tan[(c +
 d*x)/2]], -1/5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], -1/5])*Sec[(c
 + d*x)/2]^2)/(Sqrt[5]*d*Sqrt[(1 + 3*Cos[c + d*x] + Cos[2*(c + d*x)])*Sec[(
c + d*x)/2]^4])
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)

maple [A] time = 0.18, size = 168, normalized size = 1.77

$$\frac{i\sqrt{2}\,\sqrt{10}\,\left(\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)}, i\sqrt{5}\right) - 2\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)}, 5, i\sqrt{5}\right)\right)\sqrt{-\cos(dx+c)}\,\left(\sin^2(dx\right.}{5d\sqrt{3+2\cos(dx+c)}\,\left(-1+\cos(dx+c)\right)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x)

[Out] 1/5*I/d*2^(1/2)*10^(1/2)*(EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))-2*EllipticPi(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5,I*5^(1/2)))/(3+2*cos(d*x+c))^(1/2)*(-cos(d*x+c))^(1/2)*sin(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)*5^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\,\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\cos(c+d\,x)}}{\sqrt{2\,\cos(c+d\,x)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) + 3)^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) + 3)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(d*x+c))**(1/2)/(3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2*cos(c + d*x) + 3), x)

3.673 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{3\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

```
[Out] 3/5*csc(d*x+c)*EllipticPi((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),-1/2,1/5*
I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(1-sec(d*x+c))^(1/2)*(1+sec
(d*x+c))^(1/2)/d*5^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2810, 2808}

$$\frac{3\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] (3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[-1/2, Arc
Sin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*
x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-s}}{\sqrt{5}\,d}$$

Mathematica [A] time = 0.22, size = 119, normalized size = 1.23

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{3-2\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-5\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{d\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[(3 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], -5] - 2
*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], -5]))/(d*Sqrt[3 - 2*Cos[c + d*x]]
*Sqrt[-Cos[c + d*x]])
```

fricas [F] time = 1.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}}{2\cos(dx+c)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)/(2*cos(d*x + c) - 3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)

maple [B] time = 0.17, size = 180, normalized size = 1.86

$$\frac{i\sqrt{2}\,\left(2\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{1}{5},\frac{i\sqrt{5}}{5}\right)-\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\right)\sqrt{3-2\cos\left(dx+c\right)}\,\sqrt{-\cos\left(dx\right.}}{5d\left(2\left(\cos^{2}\left(dx+c\right)\right)-5\cos\left(dx+c\right)+3\right)\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x)

[Out] 1/5*I/d*2^(1/2)*(2*EllipticPi(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5,1/5*
I*5^(1/2))-EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2)))*(
3-2*cos(d*x+c))^(1/2)*(-cos(d*x+c))^(1/2)*sin(d*x+c)^2*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(2*cos(d*x+c)^2-5
*cos(d*x+c)+3)/cos(d*x+c)*5^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{3-2\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(c + d*x))^(1/2)/(3 - 2*cos(c + d*x))^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(3 - 2*cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos\left(c+dx\right)}}{\sqrt{3-2\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(d*x+c))**(1/2)/(3-2*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3 - 2*cos(c + d*x)), x)

3.674 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=77

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

```
[Out] 3/5*cot(d*x+c)*EllipticPi((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),-1/2,
1/5*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2808}

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] (3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos
[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*
d)
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \frac{3\cot(c+dx)\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}{\sqrt{5}\,d}$$

Mathematica [C] time = 0.16, size = 140, normalized size = 1.82

$$\frac{2i\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{2\cos(c+dx)-3}\left(F\left(i\sinh^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-\frac{1}{5}\right)-2\Pi\left(\frac{1}{5};i\sinh^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx\right.\right.\right.}{\sqrt{5}d\sqrt{-\cos(c+dx)}\sqrt{\frac{3-2\cos(c+dx)}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-3 + 2*Cos[c + d*x]]*(Ell
ipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5] - 2*EllipticPi[1/5, I*Arc
Sinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5]))/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt
[(3 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

fricas [F] time = 2.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

maple [B] time = 0.17, size = 152, normalized size = 1.97

$$-\frac{\sqrt{2}\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},i\sqrt{5}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,i\sqrt{5}\right)\right)\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-c}}{d\sqrt{-3+2\cos(dx+c)}\,(-1+\cos(dx+c))\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))-2*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,I*5^(1/2)))*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-3+2*cos(d*x+c))^(1/2)*(-cos(
d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\cos(c+d\,x)}}{\sqrt{2\cos(c+d\,x)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) - 3)^(1/2),x)
```

```
[Out] int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2*cos(c + d*x) - 3), x)
```

3.675 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] -3*cot(d*x+c)*EllipticPi(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^
(1/2),5/2,I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2808}

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] (-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*
Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+d}}{d}$$

Mathematica [A] time = 0.31, size = 115, normalized size = 1.53

$$\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(2\cos(c+dx)+3)\sec^4\left(\frac{1}{2}(c+dx)\right)}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{1}{5}\right)-2\Pi\left(-1;\right.\right.}{\sqrt{5}d\sqrt{-2\cos(c+dx)-3}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]^2*(EllipticF[ArcSin[Tan[(c + d*x)/2]], -1/5] - 2*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], -1/5])*Sqrt[Cos[c + d*x]*(3 + 2*Cos[c +
d*x])*Sec[(c + d*x)/2]^4])/(Sqrt[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c
 + d*x]])
```

fricas [F] time = 3.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)-3}}{2\cos(dx+c)+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)/(2*cos(d*x + c) + 3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

maple [B] time = 0.18, size = 164, normalized size = 2.19

$$\frac{\sqrt{10}\sqrt{2}\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \frac{i\sqrt{5}}{5}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, -1, \frac{i\sqrt{5}}{5}\right)\right)\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-3-2}}{5d\left(2\left(\cos^2(dx+c)\right)+\cos(dx+c)-3\right)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*10^(1/2)*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))
-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,1/5*I*5^(1/2)))*((3+2*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3-2*cos(d*x+c)
)^(1/2)*(-cos(d*x+c))^(1/2)*sin(d*x+c)^2/(2*cos(d*x+c)^2+cos(d*x+c)-3)/cos(
d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-2\,\cos\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\cos\left(c+d\,x\right)}}{\sqrt{-2\,\cos\left(c+d\,x\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(c + d*x))^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2),x)
```

```
[Out] int((-cos(c + d*x))^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\cos\left(c+dx\right)}}{\sqrt{-2\cos\left(c+dx\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(-2*cos(c + d*x) - 3), x)
```

3.676 $\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+dx)}}-\frac{b\sin(c+dx)\cos^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+}}$$

```
[Out] -b*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*
x+c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)+a*AppellF1(1/2,1/6,1
,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sin(d*x
+c)/(a^2-b^2)/d/cos(d*x+c)^(1/3)
```

Rubi [A] time = 0.20, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+dx)}}-\frac{b\sin(c+dx)\cos^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(2/3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^
2 - b^2))]*Cos[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)
^(1/3))) + (a*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x
]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[
c + d*x]^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
```

```
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx = a\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx - b\int \frac{\cos^{\frac{5}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx$$

$$= -\frac{\left(b\cos^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{a^2-b^2+b^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}} + \frac{\left(a\sqrt[6]{\cos^2(c+dx)}\right)\text{Subst}}{d}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{\frac{2}{3}}(c+dx)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[3]{\cos^2(c+dx)}} + \frac{aF_1\left(\frac{1}{2};\frac{1}{6},1;\right.}{}$$

Mathematica [B] time = 21.70, size = 4614, normalized size = 26.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(2/3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*
AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b
^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*
x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1
/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9
*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/
```

2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))
/(d*Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(5/6)*(-b^2 +
a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((a*AppellF1[1/2
, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[S
ec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF
1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*
Tan[c + d*x]^2) + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan
[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/
6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 -
 b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(
a^2 - b^2))])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^2
 - b^2)*(Sec[c + d*x]^2)^(1/6)*Tan[c + d*x]^2*((a*AppellF1[1/2, 1/3, 1, 3/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2]
)/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1,
 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2
) + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(
a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -
((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1
[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*
Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2)^2 - (15*(a^2 - b^2)*Tan[c + d
x]^2((a*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2
)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1,
3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*Appel
lF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
 + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d
*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[
1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (
6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -(
(a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^(5/
6)*(-b^2 + a^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((a*AppellF1[
1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqr
t[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -
Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/
2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^
2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Tan[c + d*x]^2) + (a*Sqrt[Sec[c + d*x]^2]*((-2*a^2*AppellF1
[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Se

c[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*((-2*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (5*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) - (a*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2]*(-4*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] + 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) - 2*Tan[c + d*x]^2*(3*a^2*((-12*a^2*AppellF1[5/2, 1/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (8*AppellF1[5/2, 7/3, 1, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2 - (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*(2*(6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (5*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(6*a^2*((-12*a^2*AppellF1[5/2, 5/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - AppellF1[5/2, 11/6, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x]) + 5*(a^2 - b^2)*((-6*a^2*AppellF1[5/2, 11/6, 2, 7/2, -Tan[c

```
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])
/(5*(a^2 - b^2)) - (11*AppellF1[5/2, 17/6, 1, 7/2, -Tan[c + d*x]^2, -((a^2*
Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(-9*(a^2 -
 b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan
[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/((Se
c[c + d*x]^2)^(5/6)*(-b^2 + a^2*Sec[c + d*x]^2))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{2}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(2/3)/(b*cos(d*x + c) + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{2}{3}}\left(dx+c\right)}{a+b\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{2}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(2/3)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + d\,x)^{2/3}}{a + b\,\cos(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(2/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.677 $\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos(c+dx)}\,F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\mathrm{si}\right.}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+}}$$

```
[Out] -b*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*
x+c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)+a*AppellF1(1/2,1/3,1
,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sin(d*x
+c)/(a^2-b^2)/d/cos(d*x+c)^(2/3)
```

Rubi [A] time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos(c+dx)}\,F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\mathrm{si}\right.}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(1/3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^
2 - b^2))]*Cos[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)
^(1/6))) + (a*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x
]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[
c + d*x]^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
```

```
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = a\int \frac{\sqrt[3]{\cos(c+dx)}}{a^2-b^2\cos^2(c+dx)}\,dx - b\int \frac{\cos^{\frac{4}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx$$

$$= -\frac{\left(b\sqrt[3]{\cos(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{a^2-b^2+b^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}} + \frac{\left(a\sqrt[3]{\cos^2(c+dx)}\right)\text{Subst}}{}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[6]{\cos^2(c+dx)}} + \frac{aF_1\left(\frac{1}{2};\frac{1}{3},1;\right.}{}$$

Mathematica [B] time = 21.33, size = 4613, normalized size = 26.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(1/3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*
AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b
^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*
x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1
/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9
*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -
((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5
```

/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2))
)/(d*Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(2/3)*(-b^2 +
 a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*((a*AppellF1[1/
2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[
Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*Appell
F1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])
*Tan[c + d*x]^2) + (b*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Ta
n[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2,
 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(a^
2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^
2 - b^2)*(Sec[c + d*x]^2)^(1/3)*Tan[c + d*x]^2*((a*AppellF1[1/2, 1/6, 1, 3/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^
2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]
^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6,
1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]
^2) + (b*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*Appe
llF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2)^2 - (12*(a^2 - b^2)*Tan[c
+ d*x]^2*((a*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6,
1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*App
ellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2)
)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
+ d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 2/3, 1, 3/2, -T
an[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*Appell
F1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
+ 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^
2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)
^(2/3)*(-b^2 + a^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((a*Appel
lF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1
[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] +
(-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (a*Sqrt[Sec[c + d*x]^2]*((-2*a^2*Appe
llF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))

]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (AppellF1[3/2, 7/6, 1, 5/2
, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[
c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -(
(a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[
3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Ta
n[c + d*x]^2) + (b*((-2*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^
2)) - (4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1[1/
2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(
3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) - (a*AppellF1[1/2, 1/6,
1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c +
d*x]^2]*(2*(-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c +
 d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x]
+ 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2))
 - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(-6*a^2*((-12*a^
2*AppellF1[5/2, 1/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (AppellF1[5/2, 7/6,
2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^
2*Tan[c + d*x])/5) + (-a^2 + b^2)*((-6*a^2*AppellF1[5/2, 7/6, 2, 7/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x
])/(5*(a^2 - b^2)) - (7*AppellF1[5/2, 13/6, 1, 7/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 -
 b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Ta
n[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2 - (b*Ap
pellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
))]*(4*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*
x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] - 9*
(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (
4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + 2*Tan[c + d*x]^2*(3*a^2*((-12*a^2
*AppellF1[5/2, 2/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (4*AppellF1[5/2, 5/3,
 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]
^2*Tan[c + d*x])/5) + 2*(a^2 - b^2)*((-6*a^2*AppellF1[5/2, 5/3, 2, 7/2, -Ta

```
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d
*x])/(5*(a^2 - b^2)) - 2*AppellF1[5/2, 8/3, 1, 7/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])))))/(-9*(a^2 - b
^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
 - b^2))] + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/((Sec
[c + d*x]^2)^(2/3)*(-b^2 + a^2*Sec[c + d*x]^2))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(1/3)/(b*cos(d*x + c) + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{1}{3}}\left(dx+c\right)}{a+b\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(1/3)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{1/3}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.678 $\int \frac{1}{\sqrt[3]{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=176

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{2/3}F_1\left(\frac{1}{2};\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{4}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\text{si}\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+}}$$

[Out] -b*AppellF1(1/2,1/6,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/3)+a*AppellF1(1/2,2/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(2/3)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(4/3)

Rubi [A] time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{2/3}F_1\left(\frac{1}{2};\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{4}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\text{si}\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+}}$$

Antiderivative was successfully verified.

[In] Int[1/(Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])),x]

[Out] -((b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(1/3))) + (a*AppellF1[1/2, 2/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(4/3))

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx = a\int \frac{1}{\sqrt[3]{\cos(c+dx)}\,\left(a^2-b^2\cos^2(c+dx)\right)}\,dx - b\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a^2-b^2\cos^2(c+d\ldots}$$

$$= -\frac{\left(b\sqrt[6]{\cos^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x^2}\left(a^2-b^2+b^2x^2\right)}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos(c+dx)}} + \ldots$$

$$= -\frac{bF_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[3]{\cos(c+dx)}}\ldots$$

Mathematica [B] time = 21.34, size = 4605, normalized size = 26.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)
*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1
[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(
-9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d
*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2))
)/(d*Cos[c + d*x]^(4/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(1/3)*(-b^2 +
```

a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(2/3)*((a*AppellF1[1/
2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt
[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*Appe
llF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*
Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -
Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/
2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^
2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^
2 - b^2)*(Sec[c + d*x]^2)^(2/3)*Tan[c + d*x]^2*((a*AppellF1[1/2, -1/6, 1, 3
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]
^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d
*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6
, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*
x]^2) + (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^
2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*Appe
llF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2)^2 - (6*(a^2 - b^2)*Tan[c +
d*x]^2*((a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6,
1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*Ap
pellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^
2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
+ d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/3, 1, 3/2, -
Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*Appel
lF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
+ 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^
(1/3)*(-b^2 + a^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((a*Appell
F1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF
1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
+ (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*
x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (a*Sqrt[Sec[c + d*x]^2]*((-2*a^2*App
ellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) + (AppellF1[3/2, 5/6, 1, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Ta

```
n[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*Appell
F1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])
*Tan[c + d*x]^2) + (b*((-2*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 -
 b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1
[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] +
2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -(
(a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) - (a*AppellF1[1/2, -1/6
, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c
+ d*x]^2]*(2*(-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Ta
n[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c
 + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x
] + 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^
2)) + (AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(
a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(-6*a^2*((-12
*a^2*AppellF1[5/2, -1/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) + (AppellF1[5/2, 5
/6, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d
*x]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 5/6, 2, 7/2, -T
an[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c +
d*x])/(5*(a^2 - b^2)) - AppellF1[5/2, 11/6, 1, 7/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x]))))/(9*(a^2 - b^
2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2 - (b*Appe
llF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
]*(4*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2
 - b^2)*((-2*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (2*Ap
pellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + 2*Tan[c + d*x]^2*(3*a^2*((-12*a^2*App
ellF1[5/2, 1/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2)
)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (2*AppellF1[5/2, 4/3, 2,
7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*T
an[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(
5*(a^2 - b^2)) - (8*AppellF1[5/2, 7/3, 1, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[
```

```
c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(-9*(a^2 - b^2
)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))] + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/((Sec[c +
 d*x]^2)^(1/3)*(-b^2 + a^2*Sec[c + d*x]^2))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos(dx+c)^{\frac{1}{3}}(a+b\cos(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x)
[Out] int(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos (c+d x)^{1/3} (a+b \cos (c+d x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/3)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

3.679 $\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=176

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{5/6}F_1\left(\frac{1}{2};\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{5}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\mathrm{si}\right.}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+}$$

```
[Out] -b*AppellF1(1/2,1/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*
x+c)^2)^(1/3)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(2/3)+a*AppellF1(1/2,5/6,1,
3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(5/6)*sin(d*x+
c)/(a^2-b^2)/d/cos(d*x+c)^(5/3)
```

Rubi [A] time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{5/6}F_1\left(\frac{1}{2};\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{5}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\mathrm{si}\right.}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])),x]
```

```
[Out] -((b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2
 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^
(2/3))) + (a*AppellF1[1/2, 5/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]
^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(5/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c
 + d*x]^(5/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
```

```
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx = a\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\left(a^2-b^2\cos^2(c+dx)\right)}\,dx - b\int \frac{\sqrt[3]{\cos(c+dx)}}{a^2-b^2\cos^2(c+dx)}$$

$$= -\frac{\left(b\sqrt[3]{\cos^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^2}\left(a^2-b^2+b^2x^2\right)}\,dx, x, \sin(c+dx)\right)}{d\cos^{\frac{2}{3}}(c+dx)} + \ldots$$

$$= -\frac{bF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\cos^{\frac{2}{3}}(c+dx)}$$

Mathematica [B] time = 21.20, size = 4608, normalized size = 26.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)
*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[
c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*Appell
F1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])
/(-9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2,
```

-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2))
)/(d*Cos[c + d*x]^(5/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(1/6)*(-b^2 +
a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(5/6)*((a*AppellF1[1/
2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt
[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*Ap
pellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^
2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/
2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^
2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^
2 - b^2)*(Sec[c + d*x]^2)^(5/6)*Tan[c + d*x]^2*((a*AppellF1[1/2, -1/3, 1, 3
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]
^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2
/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c +
d*x]^2) + (b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*Appe
llF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2)^2 - (3*(a^2 - b^2)*Tan[c +
d*x]^2*((a*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3,
1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*A
ppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b
^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/6, 1, 3/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*App
ellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2)
)] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^
(1/6)*(-b^2 + a^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((a*Appell
F1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*Appell
F1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
+ (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (a*Sqrt[Sec[c + d*x]^2]*((-2*a^2*A

ppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) + (2*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*((-2*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) - (a*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2]*(-4*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] + 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) + (2*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) - 2*Tan[c + d*x]^2*(3*a^2*((-12*a^2*AppellF1[5/2, -1/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) + (2*AppellF1[5/2, 2/3, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5) + (-a^2 + b^2)*((-6*a^2*AppellF1[5/2, 2/3, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (4*AppellF1[5/2, 5/3, 1, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2 - (b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*(2*(6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(6*a^2*((-12*a^2*AppellF1[5/2, 1/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (AppellF1[5/2, 7/6, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c

```
+ d*x]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 7/6, 2, 7/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c
 + d*x])/(5*(a^2 - b^2)) - (7*AppellF1[5/2, 13/6, 1, 7/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(-9
*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/
((Sec[c + d*x]^2)^(1/6)*(-b^2 + a^2*Sec[c + d*x]^2))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos(dx+c)^{\frac{2}{3}}\,(a+b\cos(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{2/3}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(2/3)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

3.680 $\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 33.56, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

fricas [A] time = 5.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{7}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{7}{3}}(dx+c)}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
[Out] int(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{7}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(c+d\,x)^{7/3}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/3)/(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(7/3)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/3)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.681 $\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 80.14, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

fricas [A] time = 5.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{5}{3}}\left(dx+c\right)}{\sqrt{a+b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/3}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/3)/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/3)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.682 $\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 19.41, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

fricas [A] time = 3.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{4}{3}}\left(dx+c\right)}{\sqrt{a+b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
[Out] int(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{4}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\cos\left(c+d\,x\right)}^{4/3}}{\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(4/3)/(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(4/3)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(4/3)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**(4/3)/sqrt(a + b*cos(c + d*x)), x)
```

3.683 $\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]],x]
[Out] Defer[Int][Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 9.18, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]],x]
[Out] Integrate[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

fricas [A] time = 3.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{2}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx + c\right)^{\frac{2}{3}}}{\sqrt{b\cos\left(dx + c\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{2}{3}}\left(dx + c\right)}{\sqrt{a + b\cos\left(dx + c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
[Out] int(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx + c\right)^{\frac{2}{3}}}{\sqrt{b\cos\left(dx + c\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\cos\left(c + d\,x\right)}^{2/3}}{\sqrt{a + b\,\cos\left(c + d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{2}{3}}\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(2/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(2/3)/sqrt(a + b*cos(c + d*x)), x)
```

3.684 $\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

fricas [A] time = 1.97, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{1}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{1}{3}}(dx+c)}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
[Out] int(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{1}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(c+d\,x)^{1/3}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(1/3)/sqrt(a + b*cos(c + d*x)), x)
```

3.685 $\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]),x]
[Out] Defer[Int][1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]),x]
[Out] Integrate[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

fricas [A] time = 3.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{2}{3}}}{b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^2 + a*
cos(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos\left(dx+c\right)^{\frac{1}{3}}\sqrt{a+b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{1/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cos(c + dx)}\,\sqrt[3]{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(1/3)), x)
```

3.686 $\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]),x]
[Out] Defer[Int][1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]),x]
[Out] Integrate[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

fricas [A] time = 1.11, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{1}{3}}}{b\cos(dx+c)^{2}+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)/(b*cos(d*x + c)^2 + a*
cos(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos(dx+c)^{\frac{2}{3}}\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{2/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cos\left(c + dx\right)}\,\cos^{\frac{2}{3}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(2/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(2/3)), x)
```

3.687 $\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 82.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Integrate[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

fricas [A] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{2}{3}}}{b\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^3 + a*
cos(d*x + c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(4/3)), x)
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos\left(dx+c\right)^{\frac{4}{3}}\sqrt{a+b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(4/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{4/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(4/3)*(a + b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(4/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cos\left(c + dx\right)}\cos^{\frac{4}{3}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(4/3)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(4/3)), x)
```

3.688 $\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]),x]
[Out] Defer[Int][1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 29.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]),x]
[Out] Integrate[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

fricas [A] time = 1.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{1}{3}}}{b\cos(dx+c)^{3}+a\cos(dx+c)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)/(b*cos(d*x + c)^3 + a*
cos(d*x + c)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/3)), x)
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos(dx+c)^{\frac{5}{3}}\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{5/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/3)*(a + b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(5/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{5}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/3)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(5/3)), x)
```

3.689 $\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]),x]
[Out] Defer[Int][1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 86.23, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]),x]
[Out] Integrate[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

fricas [A] time = 1.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{2}{3}}}{b\cos(dx+c)^4+a\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^4 + a*
cos(d*x + c)^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/3)), x)
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos\left(dx+c\right)^{\frac{7}{3}}\,\sqrt{a+b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{7/3}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/3)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.690 $\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=151

$$\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{6A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\text{ s}}{}$$

```
[Out] 2/3*B*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*A*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/5
*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3768, 3771, 2641, 2639}

$$\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{6A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\text{ s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*B*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))\,dx$$

$$= A\int \sec^{\frac{7}{2}}(c+dx)\,dx + B\int \sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}(3A)\int$$

$$= \frac{6A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2A\sec^{\frac{5}{2}}}{}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6A\sqrt{\sec(c+dx)}\,\sin}{5d}$$

$$= -\frac{6A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2B\sqrt{\cos(c+dx)}\,F}{}$$

Mathematica [A] time = 0.35, size = 97, normalized size = 0.64

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(21A\sin(c+dx)+9A\sin(3(c+dx))-36A\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10B\sin(2(c+dx))+20\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-36*A*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 2
0*B*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*A*Sin[c + d*x] + 10*B
*Sin[2*(c + d*x)] + 9*A*Sin[3*(c + d*x)]))/(30*d)
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

maple [B] time = 2.18, size = 502, normalized size = 3.32

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2A\left(12\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*A/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2),x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.691 $\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{}}{}$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*B*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3768, 3771, 2639, 2641}

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))\,dx$$
$$= A\int \sec^{\frac{5}{2}}(c+dx)\,dx + B\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}A\int \sqrt{}$$
$$= \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(A\sqrt{c}\right.$$
$$= -\frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2A\sqrt{\cos(c+dx)}\,F}{}$$

Mathematica [A] time = 0.24, size = 85, normalized size = 0.69

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(2\sin(c+dx)(A+3B\cos(c+dx))+2A\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6B\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-6*B*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
A*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(A + 3*B*Cos[c + d*x])*S
in[c + d*x]))/(3*d)
```

fricas [F] time = 1.48, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos\left(dx+c\right)+A)\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(B\cos\left(dx+c\right)+A)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 1.91, size = 397, normalized size = 3.23

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
```

```
1/2))+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.692 $\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\big(}{d}$$

```
[Out] 2*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3771, 2641, 3768, 2639}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\big(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))\,dx$$
$$= A\int \sec^{\frac{3}{2}}(c+dx)\,dx + B\int \sqrt{\sec(c+dx)}\,dx$$
$$= \frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \Big(B\sqrt{\cos(c+d$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2A\sqrt{\sec(c+dx)}\,\sin}{d}$$
$$= -\frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\cos(c+dx)}\,F}{}$$

Mathematica [A] time = 0.11, size = 71, normalized size = 0.73

$$\frac{2\sqrt{\sec(c+dx)}\left(A\sin(c+dx)-A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/d
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.74, size = 148, normalized size = 1.53

$$-\frac{2\left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos (c+d\,x))\,{\left(\frac{1}{\cos (c+d\,x)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.693 $\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*A*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3238, 3787, 3771, 2639, 2641}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx &= \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\\ &= A\int\sqrt{\sec(c+dx)}\,dx+B\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx\\ &= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx+\left(B\sqrt{\cos(c+dx\right.\\ &= \frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\right.}{}\end{aligned}$$

Mathematica [A] time = 0.07, size = 52, normalized size = 0.69

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2])*Sqrt[Sec[c + d*x]])/d
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)

maple [A] time = 0.77, size = 152, normalized size = 2.03

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\sqrt{\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.694 $\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{2A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/3*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2641, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx &= \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + B\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}B\int \sqrt{\sec(c+dx)}\,dx + \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\mathrm{co}
\\
&= \frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(B\sqrt{\cos(c+\right.
\\
&= \frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 76, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(6A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(\sin(2(c+dx))+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + B*(
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.74, size = 229, normalized size = 2.27

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.695 $\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{5d}$$

```
[Out] 2/5*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*A*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5
*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*A*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2639, 2641}

$$\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*A*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= A\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + B\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}A\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}(3B)\int \frac{1}{\sqrt{\sec(c+}$$

$$= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c-}$$

$$= \frac{6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [A] time = 0.32, size = 88, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(5A+3B\cos(c+dx))+10A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+18B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.\right.}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]

[Out] (Sqrt[Sec[c + d*x]]*(18*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 3*B*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)

maple [A] time = 0.86, size = 262, normalized size = 2.06

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.696 $\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10B\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10B\sqrt{\cos}}{}$$

```
[Out] 2/7*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*A*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10/
21*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2641, 2639}

$$\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10B\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10B\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*A*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*B*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= A\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + B\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3A)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}(5B)\int \frac{1}{\sec^{\frac{3}{2}}(c$$

$$= \frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10B\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5B)\int \sqrt{\sec(c+dx)}\,d$$

$$= \frac{6A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{6A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Mathematica [A] time = 0.56, size = 99, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|2}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
00*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*
x] + 15*B*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

fricas [F] time = 1.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)
```

maple [A] time = 0.74, size = 290, normalized size = 1.92

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A
-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.697 $\int (a+b\cos(c+dx))^2 \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=200

$$\frac{2\left(5a^2+7b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+d}{7}$$

```
[Out] 2/21*(5*a^2+7*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a*b*sec(d*x+c)^(5/2)*s
in(d*x+c)/d+2/7*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a*b*sin(d*x+c)*sec(d
*x+c)^(1/2)/d-12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*
(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(5a^2+7b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+d}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2),x]
```

```
[Out] (-12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (12*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (
2*(5*a^2 + 7*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a*b*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7
*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sec^{\frac{5}{2}}(c+dx)(b+a\sec(c+dx))^2\,dx$$
$$= (2ab)\int \sec^{\frac{7}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\left(b^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(6ab)\int$$
$$= \frac{12ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2\left(5a^2+7b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}$$
$$= \frac{12ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2\left(5a^2+7b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}$$
$$= -\frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(5a^2+7b^2\right)\sqrt{co}}{}$$

Mathematica [A] time = 0.87, size = 139, normalized size = 0.70

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(20\left(5a^2+7b^2\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\left(5\left(5a^2+7b^2\right)\cos(2(c+dx))+55a^2+\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2]
+ 20*(5*a^2 + 7*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(55*a
^2 + 35*b^2 + 273*a*b*Cos[c + d*x] + 5*(5*a^2 + 7*b^2)*Cos[2*(c + d*x)] + 6
3*a*b*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(9/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2), x)
```

maple [B] time = 2.78, size = 689, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/5*a*b/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*a^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1
/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2} (a+b\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.698 $\int (a+b\cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx)\, dx$

Optimal. Leaf size=175

$$\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+}{}$$

```
[Out] 4/3*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+2/5*(3*a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(3*a^2+5*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*(3*a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d) + (4*a*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2\,dx$$
$$= (2ab)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \int \sec^{\frac{3}{2}}(c+dx)\left(b^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(2ab)$$
$$= \frac{2\left(3a^2+5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{3d}$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2\left(3a^2+5b^2\right)\sqrt{\sec(}}{5}$$
$$= -\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos}}{}$$

Mathematica [A] time = 1.31, size = 126, normalized size = 0.72

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12\left(3a^2+5b^2\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\left(3\left(3a^2+5b^2\right)\cos(2(c+dx))+15\left(\right.\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a
^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(3*a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(30*d)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(7/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

maple [B] time = 2.47, size = 660, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*a^2/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*b^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1)+4*a*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.699 $\int (a+b\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+}{d}$$

```
[Out] 2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(a^2+3*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (4*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2\,dx$$
$$= (2ab)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\sec(c+dx)}\,\left(b^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{4ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - (2ab)\int$$
$$= \frac{4ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \left(2ab\sqrt{\mathrm{co}\right.$$
$$= -\frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(a^2+3b^2\right)\sqrt{\cos(}}{}$$

Mathematica [A] time = 0.34, size = 93, normalized size = 0.69

$$\frac{2\sec^{\frac{3}{2}}(c+dx)\left(\left(a^2+3b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6ab\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(c+dx)(a+6b\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
+ (a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + a*(a + 6*b*C
os[c + d*x])*Sin[c + d*x]))/(3*d)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

maple [B] time = 1.94, size = 514, normalized size = 3.81

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+6*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*
d*x+1/2*c)^2+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2-24*a*b*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*b^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*a^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2+12*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.700 $\int (a+b\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=108

$$-\frac{2(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

```
[Out] 2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3788, 3771, 2641, 4046, 2639}

$$-\frac{2(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx &= \int \frac{(b+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx\\
&= (2ab)\int \sqrt{\sec(c+dx)}\,dx+\int \frac{b^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\\
&= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\left(-a^2+b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\Big(2ab\sqrt{}\\
&= \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^2\sqrt{\sec(c+dx)}\,\sin}{d}\\
&= -\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c}}{}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 83, normalized size = 0.77

$$
\frac{2\sqrt{\sec(c+dx)}\,\left(a\left(a\sin(c+dx)+2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)-\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]) + a*(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*Sin[c + d
*x])))/d
```

fricas [F] time = 1.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.91, size = 202, normalized size = 1.87

$$-\frac{2\left(2ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*(2*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-2*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2} (a+b\,\cos(c+d\,x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.701 $\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=112

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2}{3d\sqrt{}}$$

```
[Out] 2/3*b^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3788, 3771, 2639, 4045, 2641}

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2}{3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx &= \int \frac{(b+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= (2ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(-3a^2-b^2\right)\int \sqrt{\sec(c+dx)}\,dx + \left(2ab\sqrt{\cos(c+a\right. \\
&= \frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3} \\
&= \frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(3a^2+b^2\right)\sqrt{\cos(c}}{}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 87, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+b^2\sin(2(c+d\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b^2*Sin[2*(c
 + d*x)]))/(3*d)
```

fricas [F] time = 1.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.81, size = 283, normalized size = 2.53

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*b^2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x)), x)
```

3.702 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\right.}{3d}$$

```
[Out] 2/5*b^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a*b*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2/5*(5*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= (2ab)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(2ab)\int \sqrt{\sec(c+dx)}\,dx - \frac{1}{5}\left(-5a^2-3b^2\right.$$
$$= \frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos}}$$
$$= \frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [A] time = 0.46, size = 100, normalized size = 0.71

$$\frac{\sqrt{\sec(c+dx)}\left(6\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(2(c+dx))(10a+3b\cos(c+dx))+20ab\sqrt{\cos}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(10*a +
3*b*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

fricas [F] time = 2.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/sqrt(sec(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.78, size = 321, normalized size = 2.28

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(40ab+24b^2\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(40*a*b+24*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-20*a*b-6*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10
*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\cos(c+d\,x))^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cos(c + dx))^2}{\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
[Out] Integral((a + b*cos(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

3.703 $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{2(7a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{12ab\sqrt{\mathrm{co}}}{}$$

```
[Out] 2/7*b^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+2/21*(7*a^2+5*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a*b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{2(7a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{12ab\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(21*d) + (2*b^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*a^2 + 5*b^2)*Sin[c + d*x
])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= (2ab)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(6ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{7}\left(-7a^2-5b^2\right)$$

$$= \frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} - \frac{1}{21}\left(-7a^2-5b^2\right)\int$$

$$= \frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+d}{5d\sec^{\frac{3}{2}}(c+d}$$

$$= \frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{21d}$$

Mathematica [A] time = 0.68, size = 120, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(70a^2+84ab\cos(c+dx)+15b^2\cos(2(c+dx))+65b^2\right)+20\left(7a^2+5b^2\right)\sqrt{\cos(c+\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 20*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (70*a^2
+ 65*b^2 + 84*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])
)/(210*d)
```

fricas [F] time = 2.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/sec(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.90, size = 362, normalized size = 2.07

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-336ab-360b^2\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-336*a*b-360*b^2)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*a^2+336*a*b+280*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-70*a^2-84*a*b-80*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+35*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\cos(c+d\,x))^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
[Out] Integral((a + b*cos(c + d*x))**2/sec(c + d*x)**(3/2), x)
```

3.704 $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{2(9a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{20ab\,\mathrm{si}}{21d\sqrt{\mathrm{se}}}$$

```
[Out] 2/9*b^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)+4/7*a*b*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+2/45*(9*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+20/21*a*b*sin(d*x+c)/d/se
c(d*x+c)^(1/2)+2/15*(9*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2(9a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{20ab\,\mathrm{si}}{21d\sqrt{\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(9*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (2*b^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (4
*a*b*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(9*a^2 + 7*b^2)*Sin[c + d*
x])/(45*d*Sec[c + d*x]^(3/2)) + (20*a*b*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= (2ab)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}(10ab)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx - \frac{1}{9}\left(-9a^2-7b^2\right.$$

$$= \frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(9a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{20ab\sin(c+dx}{21d\sqrt{\sec(c+dx}}$$

$$= \frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(9a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{20ab\sin(c+dx}{21d\sqrt{\sec(c+dx}}$$

$$= \frac{2\left(9a^2+7b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{20ab\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

Mathematica [A] time = 1.11, size = 135, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(7\left(36a^2+43b^2\right)\cos(c+dx)+5b(36a\cos(2(c+dx))+156a+7b\cos(3(c+dx)))\right.\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(9*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2] + 1200*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36
*a^2 + 43*b^2)*Cos[c + d*x] + 5*b*(156*a + 36*a*Cos[2*(c + d*x)] + 7*b*Cos[
3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

fricas [F] time = 2.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2}{\sec\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/sec(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

maple [A] time = 1.18, size = 398, normalized size = 1.99

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(1440ab+2240b^2\right)\left(\sin^8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1440*a*b+2240*b^2)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-504*a^2-2160*a*b-2072*b^2)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(504*a^2+1680*a*b+952*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-126*a^2-480*a*b-168*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+150*a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\, \cos (c + d\, x))^2}{\left(\frac{1}{\cos(c+d\, x)}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
[Out] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.705 $\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=234

$$\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2b\left(9a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx}}{2}$$

```
[Out] 2/21*a*(5*a^2+21*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+32/35*a^2*b*sec(d*x+c)^
(5/2)*sin(d*x+c)/d+2/7*a^2*sec(d*x+c)^(5/2)*(b+a*sec(d*x+c))*sin(d*x+c)/d+2
/5*b*(9*a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*b*(9*a^2+5*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(5*a^2+21*b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3842, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2b\left(9a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

```
[Out] (-2*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*(9*a^2 + 5*b^2)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (32*a^2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a^2
*Sec[c + d*x]^(5/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^3\,dx$$
$$= \frac{2a^2\sec^{\frac{5}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}b\right.$$
$$= \frac{2a^2\sec^{\frac{5}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}b\right.$$
$$= \frac{2a(5a^2+21b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{32a^2b\sec^{\frac{5}{2}}(c+dx)\sin(c+d}{35d}$$
$$= \frac{2b(9a^2+5b^2)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a(5a^2+21b^2)\sec^{\frac{3}{2}}(c+dx}{21d}$$
$$= \frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2b(9a^2}{}$$
$$= -\frac{2b(9a^2+5b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(5a^2}{}$$

Mathematica [A] time = 1.20, size = 191, normalized size = 0.82

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(30a^3\sin(c+dx)+50a^3\sin(c+dx)\cos^2(c+dx)+10a(5a^2+21b^2)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-42*b*(9*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(c
+ d*x)/2, 2] + 10*a*(5*a^2 + 21*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)
/2, 2] + 30*a^3*Sin[c + d*x] + 50*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 210*a*b
^2*Cos[c + d*x]^2*Sin[c + d*x] + 378*a^2*b*Cos[c + d*x]^3*Sin[c + d*x] + 21
0*b^3*Cos[c + d*x]^3*Sin[c + d*x] + 63*a^2*b*Sin[2*(c + d*x)]))/(105*d)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

maple [B] time = 3.34, size = 847, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-6/5*a^2*b/(8*s
in(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1
/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)+2*b^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1)+6*b^2*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
)))+2*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2
+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2} (a+b\,\cos(c+d\,x))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.706 $\int (a+b\cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=189

$$\frac{6a\left(a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6a\left(a^2+5b^2\right)}{}$$

```
[Out] 8/5*a^2*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(3/2)*(b+a*sec(d
*x+c))*sin(d*x+c)/d+6/5*a*(a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a*(
a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*b*(a^2+b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3842, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{6a\left(a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6a\left(a^2+5b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (6*a*(a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(5*d) + (8*a^2*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Sec[
c + d*x]^(3/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^3\,dx$$

$$= \frac{2a^2\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+dx)}\,\Big($$

$$= \frac{2a^2\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+dx)}\,\Big($$

$$= \frac{6a\left(a^2+5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{8a^2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{5d}$$

$$= \frac{6a\left(a^2+5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{8a^2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{5d}$$

$$= -\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\left(a^2+\right.}{}$$

Mathematica [A] time = 1.67, size = 134, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(5b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)\left(3\left(a^2+5b^2\right)\right.}{}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*a*(a^2 + 5*b^2)*EllipticE[(c +
 d*x)/2, 2] + 5*b*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (a*(5*(a^2 + 3*b^
2) + 10*a*b*Cos[c + d*x] + 3*(a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/
(2*Cos[c + d*x]^(5/2))))/(5*d)
```

fricas [F] time = 2.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

maple [B] time = 2.57, size = 738, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*a
^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1
)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*b^2*a*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2
)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2} (a+b\,\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.707 $\int (a+b\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{d}$$

```
[Out] 16/3*a^2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*a^2*(b+a*sec(d*x+c))*sin(d*x+c
)*sec(d*x+c)^(1/2)/d-2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3842, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a^2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
3*d) + (2*a^2*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
```

```
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx=\int \frac{(b+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$

$$=\frac{2a^2\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{-\frac{1}{2}b\left(a^2-3b^2\right)}{}$$

$$=\frac{2a^2\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{-\frac{1}{2}b\left(a^2-3b^2\right)}{\sqrt{\sec}}$$

$$=\frac{16a^2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{2a^2\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))\,\mathrm{s}}{3d}$$

$$=\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{16a^2b\sqrt{\sec}}{}$$

$$=-\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\left(a^2+\right.}{}$$

Mathematica [A] time = 0.51, size = 106, normalized size = 0.66

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(6b\left(b^2-3a^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(2\left(a^2+9b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2a\sin(\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(6*b*(-3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d
*x)/2, 2] + a*(2*(a^2 + 9*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]
 + 2*a*(a + 9*b*Cos[c + d*x])*Sin[c + d*x])))/(3*d)
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

maple [B] time = 2.23, size = 631, normalized size = 3.94

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+18*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b^2*sin(1
/2*d*x+1/2*c)^2+18*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-6*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2-36*a^2*b*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4-a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*b^2*a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+2*
a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*a^2*b*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2} (a+b\,\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.708 $\int (a+b\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=166

$$\frac{2a\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2-3b^2\right.}{}$$

```
[Out] 2/3*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*a*(3*a^2-b^2)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3841, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2-3b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*d) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
 d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{4ab^2+\frac{1}{2}b\left(9a^2+b^2\right)\sec(c+$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{4ab^2+\frac{1}{2}a\left(3a^2-b^2\right)\sec^2(c}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2a\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2b^2(b+a\sec(c+dx))\sin(c}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\left(3a^2-\right.}{}$$

$$= -\frac{2a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\left(9a^2\right.}{}$$

Mathematica [A] time = 0.63, size = 108, normalized size = 0.65

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(c+dx)\left(3a^3+b^3\cos(c+dx)\right)+2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-6a\left(a^2-3b^2\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ 2*(3*a^3 + b^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

maple [A] time = 0.96, size = 303, normalized size = 1.83

$$-\frac{2\left(4b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+9a^2b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*a^2*b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b^2-6*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}\,{\left(a+b\,\cos(c+d\,x)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.709 $\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{2a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{5d}$$

```
[Out] 2/5*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a*b^2*sin(d*x+c)
/d/sec(d*x+c)^(1/2)+6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3841, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d + (8*a*b^2*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
 + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{6ab^2+\frac{3}{2}b\left(5a^2+b^2\right)\sec(c+}{\sec^{\frac{3}{2}}}$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{6ab^2+\frac{1}{2}a\left(5a^2+b^2\right)\sec^2(c+}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{8ab^2\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \left(a\left(a^2+b^2\right)\right)\int$$

$$= \frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8ab^2\sin(c}{5d\sqrt{\sec(c}}$$

$$= \frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\left(a^2+b\right.}{}$$

Mathematica [A] time = 0.47, size = 106, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(10a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+b^2\sin\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
b^2*(5*a + b*Cos[c + d*x])*Sin[2*(c + d*x)]))/(5*d)
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

maple [A] time = 1.09, size = 376, normalized size = 2.41

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-8b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20b^2a+8b^3\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x)
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*b^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*a*b^2+8*b^3)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-10*a*b^2-2*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5
*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+5*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{\cos(c+d\,x)}}\ (a+b\ \cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(c + dx))^3 \sqrt{\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(1/2),x)
[Out] Integral((a + b*cos(c + d*x))**3*sqrt(sec(c + d*x)), x)
```

3.710 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=199

$$\frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5a^2+9b^2)\sqrt{\cos}}{}$$

```
[Out] 32/35*a*b^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*b^2*(b+a*sec(d*x+c))*sin(d*x+
c)/d/sec(d*x+c)^(5/2)+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2
/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*b*(2
1*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3841, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5a^2+9b^2)\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (32*a*b^2*Sin[c + d*x])/(35*d*Sec[c
 + d*x]^(3/2)) + (2*b*(21*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8ab^2+\frac{1}{2}b\left(21a^2+5b^2\right)\sec(c+dx)+\frac{1}{2}a}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8ab^2+\frac{1}{2}a\left(7a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx +$$

$$= \frac{32ab^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a\sec(c+dx))\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{32ab^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a\sec(c+dx))\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\left(21a^2+5b^2\right)\sqrt{\text{co}}}{}$$

Mathematica [A] time = 0.96, size = 132, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(b\sin(2(c+dx))\left(210a^2+126ab\cos(c+dx)+15b^2\cos(2(c+dx))+65b^2\right)+20b\left(21a^2+5b^2\right)\sqrt{}\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 20*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2] + b*(210*a^2 + 65*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)
])*Sin[2*(c + d*x)]))/(210*d)
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.93, size = 421, normalized size = 2.12

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-504b^2a-360b^3\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-504*a*b^2-360*b^3)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(420*a^2*b+504*a*b^2+280*b^3)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-210*a^2*b-126*a*b^2-80*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+105*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\cos\left(c+d\,x\right))^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cos\left(c+dx\right))^3}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
[Out] Integral((a + b*cos(c + d*x))**3/sqrt(sec(c + d*x)), x)
```

3.711 $\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{2b(27a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7a^2+15b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7a^2+15b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+d\right.}{21d}$$

```
[Out] 40/63*a*b^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*b*(27*a^2+7*b^2)*sin(d*x+c)/
d/sec(d*x+c)^(3/2)+2/9*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2
/21*a*(7*a^2+15*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*b*(27*a^2+7*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(7*a^2+15*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.28, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3238, 3841, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2b(27a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7a^2+15b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7a^2+15b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+d\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*b*(27*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (2*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (40*a*b^2*Sin[c + d*x])/(63*d*Sec
[c + d*x]^(5/2)) + (2*b*(27*a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(
3/2)) + (2*a*(7*a^2 + 15*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*
b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{10ab^2+\frac{1}{2}b\left(27a^2+7b^2\right)\sec(c+dx)+\frac{1}{2}a}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{10ab^2+\frac{1}{2}a\left(9a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx +$$

$$= \frac{40ab^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b^2(b+a\sec(c+dx))\sin(c+dx}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{40ab^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(7a^2+15b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + 2b$$

$$= \frac{2b\left(27a^2+7b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{40ab^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} +$$

$$= \frac{2b\left(27a^2+7b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2a\left(7a^2+15b^2\right)\sqrt{co}}{}$$

Mathematica [A] time = 1.24, size = 159, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(120a\left(7a^2+15b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+168b\left(27a^2+7b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right.\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*b*(27*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 120*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] + (7*b*(108*a^2 + 43*b^2)*Cos[c + d*x] + 5*(84*a^3 + 234*a*b^2 + 5
4*a*b^2*Cos[2*(c + d*x)] + 7*b^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(126
0*d)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.02, size = 470, normalized size = 2.01

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-1120b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2160b^2a+2240b^3\right)\left(\mathrm{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2160*a*b^2+2240*b^3)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*a^2*b-3240*a*b^2-2072*b^3)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^3+1512*a^2*b+2520*a*b^2+952*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^3-378*a^2*b-720*a*b^2-168*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*b^2
*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
[Out] Integral((a + b*cos(c + d*x))**3/sec(c + d*x)**(3/2), x)
```

3.712 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=188

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}}{a^2d}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+2
*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2/3*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

Rubi [A] time = 0.55, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3851, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a*d) + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^2*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx$$

$$= \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{b}{2}+\frac{1}{2}a\sec(c+dx)-\frac{3}{2}b\sec^2(c+dx)\right)}{b+a\sec(c+dx)}\,dx}{3a}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{4\int \frac{\frac{3b^2}{4}+ab\sec(c+dx)+\frac{1}{4}(a^2+ \ldots}{\sqrt{\sec(c+dx)}\,(b+a\,\mathrm{s}\ldots}}{3a^2}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{4\int \frac{\frac{3b^3}{4}+\frac{1}{4}ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^2b^2} \ldots$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a} + \ldots$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} - \frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \ldots$$

$$= \frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}\ldots}}{3ad} \ldots$$

Mathematica [A] time = 3.02, size = 165, normalized size = 0.88

$$-\frac{\cot(c+dx)\left(-2\left(a^2+3ab+3b^2\right)\sqrt{-\tan^2(c+dx)}\,F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-a^2\sec^{\frac{5}{2}}(c+dx)+a^2\cos(2(c\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -1/3*(Cot[c + d*x]*(-(a^2*Sec[c + d*x]^(5/2)) + a^2*Cos[2*(c + d*x)]*Sec[c
+ d*x]^(5/2) + 6*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c
+ d*x]^2] - 2*(a^2 + 3*a*b + 3*b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[-Tan[c + d*x]^2] + 6*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

maple [A] time = 2.22, size = 452, normalized size = 2.40

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{a^2\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{2b\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*b^3/a^2/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-2/a^2*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{b\cos (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+b\,\cos (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.713 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E}{ad}$$

```
[Out] 2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a+
b)/d
```

Rubi [A] time = 0.21, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3850, 3768, 3771, 2639, 3849, 2805}

$$-\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(a*(a + b)*d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3850

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[(a*
d)/b, Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx$$

$$= \frac{\int \sec^{\frac{3}{2}}(c+dx)\,dx}{a} - \frac{b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{a}$$

$$= \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c}}}{a}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a+b)d} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} - $$

$$= -\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}$$

Mathematica [A] time = 5.16, size = 83, normalized size = 0.71

$$\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+b\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+aE}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(a*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSin[Sqrt[Se
c[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

maple [B] time = 1.05, size = 354, normalized size = 3.03

$$-\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/a/(a-b)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x)),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x)), x)
```

3.714 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a+b)/d
```

Rubi [A] time = 0.13, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3238, 3849, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/((a + b)*d)
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
```

```
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a+b)d}$$

Mathematica [A] time = 0.34, size = 63, normalized size = 1.29

$$\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a
/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.83, size = 150, normalized size = 3.06

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x)), x)
```

3.715 $\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d-2*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a+b)/d
```

Rubi [A] time = 0.19, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3848, 2803, 2641, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d) -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(b*(a + b)*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx &= \int \frac{\sqrt{\sec(c+dx)}}{b+a\sec(c+dx)}\,dx \\
&= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx \\
&= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{} \\
&= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b}\right.}{b(}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 47, normalized size = 0.51

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Ta
n[c + d*x]^2])/(b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.90, size = 188, normalized size = 2.02

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{(a-b)\,b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2)))/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\;\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
[Out] Integral(1/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.716 $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sqrt{\cos(c+}}{}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d-2*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

Rubi [A] time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3852, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d) -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2
*d) + (2*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx$$
$$= \frac{\int \frac{b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{b^2} + \frac{a^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{b^2}$$
$$= -\frac{a\int \sqrt{\sec(c+dx)}\,dx}{b^2} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} + \frac{\left(a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$
$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d} - \frac{\left(a\sqrt{\cos(c+dx\right.}{}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{b}$$

Mathematica [A] time = 6.52, size = 176, normalized size = 1.30

$$\frac{\cot(c+dx)\left(-2a\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+b\sec^{\frac{7}{2}}(c+dx)-b\sec^{\frac{3}{2}}(c+dx)+b\cos(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cot[c + d*x]*(-(b*Sec[c + d*x]^(3/2)) - b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3
/2) + b*Sec[c + d*x]^(7/2) + b*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*b*El
lipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*a*EllipticPi[-
(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^2*d)
```

fricas [F] time = 80.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 1.20, size = 227, normalized size = 1.68

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\ldots}\right.\right.}{b^2\left(a-b\right)\sqrt{-2\left(\sin^4\left(\frac{d\ldots}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x)
```

3.717 $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$-\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-$$

```
[Out] 2/3*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/b^2/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/b^3/d-2*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(
a+b)/d
```

Rubi [A] time = 0.39, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3853, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-2$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2
*d) + (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*b^3*d) - (2*a^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a + b)*d) + (2*Sin[c + d*x])/(3
*b*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
```

```
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}\,dx$$

$$= \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx}{3b}$$

$$= \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3ab}{2}-\left(-\frac{3a^2}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3b^3} - \frac{a^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{b^3}$$

$$= \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b^2} + \frac{\left(3a^2+b^2\right)\int \sqrt{\sec(c+dx)}\,dx}{3b^3} -$$

$$= -\frac{2a^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d} + \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+a}$$

$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2\left(3a^2+b^2\right)\sqrt{\cos(c}{}$$

Mathematica [A] time = 6.79, size = 196, normalized size = 1.14

$$-\frac{\cot(c+dx)\left(-12a^2\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+6ab\sec^{\frac{3}{2}}(c+dx)-6ab\cos(2(c+dx))\,s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -1/6*(Cot[c + d*x]*(-(b^2*Sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) -
6*a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + b^2*Cos[3*(c + d*x)]*Sec[c + d*
x]^(3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d
*x]^2] + 4*(3*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
```

```
c + d*x]^2] - 12*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[-Tan[c + d*x]^2]))/(b^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

maple [B] time = 1.03, size = 516, normalized size = 3.00

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\left(4b^2a-4b^3\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2b^2a+2b^3\right)\left(\sin^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*a*b^2-4*b^
3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a*b^2+2*b^3)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*a^2*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*a^3*(sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.718 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx$

Optimal. Leaf size=341

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}+\frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,I}{3a^2d\left(a^2-b^2\right)}$$

```
[Out] 1/3*(2*a^2-5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*sec(d*x+c
)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-b*(4*a^2-5*b^2)*sin(d*x+c
)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b^2*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(a+b)^2/d
```

Rubi [A] time = 0.97, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^3d\left(a^2-b^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d) + (b^2*(7*a^2 -
 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) - (b*(4*a^2 - 5*b^2)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3b^2}{2}-ab\sec(c+dx)+\frac{1}{2}\left(2a^2-5b^2\right)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{2\int \frac{\sqrt{\sec(c+}}{}}{}$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \ldots$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \ldots$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \ldots$$

$$= \frac{b^2\left(7a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a-b)(a+b)^2d} - \frac{b\left(4a^2-5b^2\right)\sqrt{\mathrm{se}}}{a^3\left(a\right.}$$

$$= \frac{b\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+d}}{3a}$$

Mathematica [A] time = 6.80, size = 655, normalized size = 1.92

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{b^3\sin(c+dx)}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{2\tan(c+dx)}{3a^2}-\frac{b\left(4a^2-5b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)}\right)}{d} + \frac{2\left(40ab^3-28a^3b\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\,(}{b\left(1-\cos^2(c+dx)\right)(a+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*(-4*a^4 - 44*a^2*b^2 + 45*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Se
c[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b
```

```
+ a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)) + (2*(-28*a^3*b + 40*a*b^3)*Cos[c + d*x]^2*Elli
pticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2
)) + ((-12*a^2*b^2 + 15*b^4)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b
+ 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^
2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a
+ b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]
^2)))/(12*a^3*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((b*(4*a^2 - 5*b^
2)*Sin[c + d*x])/(a^3*(a^2 - b^2))) - (b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(
a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*a^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

maple [B] time = 3.86, size = 1008, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*b^3/a^3/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
```

```
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-4/a^3*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))+2*b^2/a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2
)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.719 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=277

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{ad\left(a^2-b^2\right)}$$

```
[Out] b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))+(2*a^2-3*b^2
)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-(2*a^2-3*b^2)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-b*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)/(a+b)^2/d
```

Rubi [A] time = 0.71, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{ad\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -(((2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^2*(a^2 - b^2)*d)) + (b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - (b*(5*a^2 - 3*b^2)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
(a - b)*(a + b)^2*d) + ((2*a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*
(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
```

```
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{b^2}{2}-ab\sec(c+dx)+\frac{1}{2}\left(2a^2-3b^2\right)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{2\int \frac{-\frac{1}{4}b(}{}}{}$$

$$= \frac{\left(2a^2-3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{2\int \frac{-\frac{1}{4}b^2}{}}{}$$

$$= \frac{\left(2a^2-3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{b\int \sqrt{\mathrm{se}}}{2a(}$$

$$= -\frac{b\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} + \frac{\left(2a^2-3b^2\right)\sqrt{\mathrm{s}}}{a^2(}$$

$$= -\frac{\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} + \frac{b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{a\left(a^2\right.}$$

Mathematica [A] time = 4.63, size = 351, normalized size = 1.27

$$\frac{\frac{2a\sin(c+dx)\left(2a\left(a^2-b^2\right)\sec(c+dx)+2a^2b-3b^3\right)}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))} + \frac{\cot(c+dx)\left(-2a^3\sec^{\frac{3}{2}}(c+dx)+2a^3\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+2a\left(2a^2-3b^2\right)\sqrt{-\tan^2(c+dx)}\,E\left(\sin^{-1}\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*a*(2*a^2*b - 3*b^3 + 2*a*(a^2 - b^2)*Sec[c + d*x])*Sin[c + d*x])/((a^2
- b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-2*a^3*Sec
[c + d*x]^(3/2) + 3*a*b^2*Sec[c + d*x]^(3/2) + 2*a^3*Cos[2*(c + d*x)]*Sec[c
 + d*x]^(3/2) - 3*a*b^2*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*a*(2*a^2 -
3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*
(2*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
```

```
1]*Sqrt[-Tan[c + d*x]^2] + 10*a^2*b*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*b^3*EllipticPi[-(a/b), ArcSin[Sqrt[Se
c[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

maple [B] time = 2.56, size = 874, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/a^2*b^2/(-2*a
*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))+2/a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c
)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/a*b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^
2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x))**2, x)
```

3.720 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{ad\left(a^2-b^2\right)}$$

```
[Out] b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-b*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a/(a-b)/(a+b)^2/d
```

Rubi [A] time = 0.44, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3238, 3845, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{ad\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a
^2 - b^2)*d)) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/((a^2 - b^2)*d) + ((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*(a + b)^2*d) + (b^
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{b^2}{2}-ab\sec(c+dx)+\frac{1}{2}\left(2a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{b^3}{2}-\frac{1}{2}ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{ab^2\left(a^2-b^2\right)} + \frac{\left(3a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,d}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2\left(a^2-b^2\right)} - \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a\left(a^2-b^2\right)} + \frac{\left(\left(3a^2-\right.\right.}{}$$

$$= \frac{\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} + \frac{b^2\sqrt{\sec(c+dx)}\,\sin}{a\left(a^2-b^2\right)d(b+a\,\mathrm{se}}$$

$$= -\frac{b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} - \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 6.67, size = 584, normalized size = 2.69

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{b\sin(c+dx)}{a\left(a^2-b^2\right)}+\frac{b\sin(c+dx)}{\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d} + \frac{\frac{2\left(3b^2-4a^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\,(a\sec(c+dx)+b)\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+d}\right.\right.\right.}{a\left(1-\cos^2(c+dx)\right)(a+b\cos(c+dx))}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((b*Sin[c + d*x])/(a*(a^2 - b^2)) + (b*Sin[c + d*x])/((
-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d + ((2*(-4*a^2 + 3*b^2)*Cos[c + d*x]^2
*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqr
t[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c
+ d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*a*Cos[c + d*x]^2
*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sq
rt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((a + b*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)) + (Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]
^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b),
 ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - C
os[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*a*(-a + b)*(a +
 b)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx + c\right)}}{\left(b\cos\left(dx + c\right) + a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

maple [B] time = 1.68, size = 612, normalized size = 2.82

$$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2b^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a\left(a^{2}-b^{2}\right)\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\cdots\right.\right.}}{(a+b)a\sqrt{-2\left(\sin\cdots\right.}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^2/a/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b/a/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b/a/(a^2-b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-6*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a/(a^2-b^2)/(-2*a*b
+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2, x)
```

3.721 $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d\left(a^2-b^2\right)}$$

```
[Out] -b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))+(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/b/(a^2-b^2)/d-(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/(a-b)/b/(a+b)^2/d
```

Rubi [A] time = 0.42, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3844, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a^2 - b
^2)*d) + (a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b*(a^2 - b^2)*d) - ((a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b*(a + b)^2*d) - (b*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b+a\sec(c+dx))^2}\,dx$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{-\frac{b}{2}-a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx}{a^2-b^2}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{-\frac{b^2}{2}-\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{b^2\left(a^2-b^2\right)} - \frac{\left(a^2+b^2\right)\int \frac{}{b}}{2b\left(a^2\right.}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2\left(a^2-b^2\right)} + \frac{a\int \sqrt{\sec(c+dx)}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} - \frac{b\sqrt{\mathrm{se}}}{\left(a^2-b\right.}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{\left(a^2-b^2\right)d} + \frac{a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{b\left(a^2-\right.}$$

Mathematica [B] time = 6.67, size = 574, normalized size = 2.76

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\sin(c+dx)}{a^2-b^2}\right)}{d} + \frac{\sin(c+dx)\cos(2(c+dx))(a\sec(c+dx)+b)\left(-4a^2\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\,\Pi\left(-\frac{a}{b};\sin^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]/(a^2 - b^2)) + (a*Sin[c + d*x])/((a^2 -
 b^2)*(a + b*Cos[c + d*x]))))/d + ((-2*b*Cos[c + d*x]^2*(EllipticF[ArcSin[S
```

```
qrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1
])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Co
s[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*a*Cos[c + d*x]^2*EllipticPi[-(a/b),
ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^
2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (Cos[2*(c
+ d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ellipti
cE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*Elliptic
Pi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2])*Sin[c + d*x])/(a*b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*S
qrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a + b)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

maple [B] time = 2.31, size = 713, normalized size = 3.43

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{2a}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/(-2*a*b+2*b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))-2/b*a*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/
(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a
*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x))), x)
```

3.722 $\int \frac{1}{(a+b\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=223

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{bd\left(a^2-\right.}$$

```
[Out] a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))-a*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(a^2-2*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

Rubi [A] time = 0.41, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3843, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{bd\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a
^2 - b^2)*d)) + ((a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - (a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*
b^2*(a + b)^2*d) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b +
a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{a}{2}-b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx}{a^2-b^2}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\left(a\left(3-\frac{a^2}{b^2}\right)\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{2\left(a^2-b^2\right)} + \frac{\int \frac{-\frac{ab}{2}-}{}}{}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2b\left(a^2-b^2\right)} + \frac{\left(a^2-2b^2\right)\int \sqrt{\sec(}}{2b^2\left(a^2-b\right.}$$

$$= -\frac{a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} + \frac{a\sqrt{}}{\left(a^2-\right.}$$

$$= -\frac{a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d} + \frac{\left(a^2-2b^2\right)\sqrt{\cos(c+}}{}$$

Mathematica [A] time = 5.64, size = 250, normalized size = 1.12

$$\frac{\cos(2(c+dx))\csc(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(-\left(a^2-3b^2\right)\sqrt{-\tan^2(c+dx)}\,\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))\Pi\left(-\frac{a}{b};\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[2*(c + d*x)]*Csc[c + d*x]*Sec[c + d*x]^(3/2)*(-(b*(-a + b)*(a + b*Cos[
c + d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
```

```
[-Tan[c + d*x]^2]) - (a^2 - 3*b^2)*(a + b*Cos[c + d*x])*EllipticPi[-(a/b),
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] +
a*b*(a*Tan[c + d*x]^2 - (a + b*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*b^2*(a + b
)*d*(b + a*Sec[c + d*x])*(-2 + Sec[c + d*x]^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

maple [B] time = 1.97, size = 794, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8/b*a
/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/b^2*a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*
a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
[Out] Integral(1/((a + b*cos(c + d*x))**2*sec(c + d*x)**(3/2)), x)
```

3.723 $\int \frac{1}{(a+b\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)} dx$

Optimal. Leaf size=245

$$-\frac{a^2 \sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2 d\left(a^2-b^2\right)}-\frac{a\left(3a^2-4b^2\right)\sqrt{\cos(}}{}$$

```
[Out] -a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))+(3*a^2-2*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(3*a^2-4*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+a^2*(3*a^
2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2
*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^3/
(a+b)^2/d
```

Rubi [A] time = 0.47, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3847, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2 \sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3 d\left(a^2-b^2\right)}+\frac{\left(3a^2-2b^2\right)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(b^2*(a^2 - b^2)*d) - (a*(3*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) + (a^2*(3*a^2 - 5
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - (a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2}\,dx$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{3a^2}{2}-b^2+ab\sec(c+dx)-\frac{1}{2}a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{b\left(\frac{3a^2}{2}-b^2\right)-\left(-ab^2+a\left(\frac{3a^2}{2}-b^2\right)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}}{b^3\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\left(a\left(3a^2-4b^2\right)\right)\int\sqrt{\sec(c+dx)}\,dx}{2b^3\left(a^2-b^2\right)}$$

$$= \frac{a^2\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} - \frac{a}{b(}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} - \frac{a\left(3a^2-4\right.}{}$$

Mathematica [A] time = 6.43, size = 319, normalized size = 1.30

$$\frac{\frac{4a^2\sin(c+dx)}{b\left(b^2-a^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))} - \frac{2\cot(c+dx)\left(6a^3\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+2b\left(-3a^2+ab+2b^2\right)\sqrt{-\tan^2(c+dx)}\,F\left(\sin^{-1}\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((4*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x
]]) - (2*Cot[c + d*x]*(-3*a^2*b*Sec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(3/
2) + 3*a^2*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)]*S
ec[c + d*x]^(3/2) + 2*b*(3*a^2 - 2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*(-3*a^2 + a*b + 2*b^2)*EllipticF[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^3*EllipticPi[-(a/b), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a*b^2*EllipticPi[
-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*b^
3*(a + b)))/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos\left(dx+c\right)+a\right)^{2} \sec\left(dx+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

maple [B] time = 2.40, size = 815, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-12*a^2/b^2/(-2*a*b+2*b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
```

```
^(1/2))-2/b^3*a^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^
2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.724 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=455

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{4a^4d\left(a^2-b^2\right)^2}$$

```
[Out] 1/12*(8*a^4-61*a^2*b^2+35*b^4)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/(a^2-b^2)^2/
d+1/2*b^2*sec(d*x+c)^(7/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*
b^2*(13*a^2-7*b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*sec(d
*x+c))-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^4/(a^
2-b^2)^2/d+1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/12*(8*a^4-61*a^2*b^2+35*b^4)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1/4*b^2*(63*a^4-86*a^2*b
^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a-b
)^2/(a+b)^3/d
```

Rubi [A] time = 1.45, antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3238, 3845, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-61a^2b^2+8a^4+35b^4\right)\sin(c+dx)}{12a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35
*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*
a^3*(a^2 - b^2)^2*d) + (b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a
- b)^2*(a + b)^3*d) - (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]^(
7/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (b^2*(13*a^
```

```
2 - 7*b^2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*S
ec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
```

```
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5b^2}{2}-2ab\sec(c+dx)+\frac{1}{2}\left(4a^2-7b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b^2\left(13a^2-7b^2\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} + \int$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))}$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sec}{12a^3\left(a^2-b\right.}$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sec}{12a^3\left(a^2-b\right.}$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sec}{12a^3\left(a^2-b\right.}$$

$$= \frac{b^2\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4(a-b)^2(a+b)^3 d} - \frac{b\left(2\right.}{}$$

$$= \frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61\right.}{}$$

Mathematica [A] time = 6.83, size = 747, normalized size = 1.64

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{2\tan(c+dx)}{3a^3}-\frac{b^3\sin(c+dx)}{2a^2\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2}-\frac{3\left(5a^2b^3\sin(c+dx)-3b^5\sin(c+dx)\right)}{4a^3\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d} + \frac{2(1}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*(16*a^6 + 328*a^4*b^2 - 641*a^2*b^4 + 315*b^6)*Cos[c + d*x]^2*(Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a
*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(160*a^5*b - 512*a^3*b^3 +
 280*a*b^5)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -
1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Co
s[c + d*x])*(1 - Cos[c + d*x]^2)) + ((72*a^4*b^2 - 195*a^2*b^4 + 105*b^6)*C
os[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b
*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sq
rt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*
EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c +
 d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^4*(a - b)^2*(a + b
)^2*d) + (Sqrt[Sec[c + d*x]]*(-1/4*(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sin[c
+ d*x])/(a^4*(a^2 - b^2)^2) - (b^3*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*(a + b*
Cos[c + d*x])^2) - (3*(5*a^2*b^3*Sin[c + d*x] - 3*b^5*Sin[c + d*x]))/(4*a^3
*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*a^3)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

maple [B] time = 5.95, size = 2128, normalized size = 4.68

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2/a^2*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-12*b^3/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-6/a^4*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4*b^2/a^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^

```
2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a
^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

3.725 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=388

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{b\left(11a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{4a^2d\left(a^2-b^2\right.}$$

```
[Out] 1/2*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*b^
2*(11*a^2-5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*sec(d*x
+c))+1/4*(8*a^4-29*a^2*b^2+15*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2
)^2/d-1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/a^3/(a^2-b^2)^2/d+1/4*b*(11*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*b*(35*a^4-38*a^2*b^2+15*b^4)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a
+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)^2/(a+b)^3/d
```

Rubi [A] time = 1.03, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3238, 3845, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-29a^2b^2+8a^4+15b^4\right)\sin(c+dx}{4a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) + (b*(11*a^2 - 5*b^2)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2
)^2*d) - (b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3
*d) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^
3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*(b + a*Sec[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
```

```
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3b^2}{2}-2ab\sec(c+dx)+\frac{1}{2}\left(4a^2-5b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}\,d}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} + \frac{\int \frac{\sqrt{s}}{}}{}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}$$

$$= -\frac{b\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3 d} + \frac{\left(8a^4\right.}{}$$

$$= -\frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3\left(a^2-b^2\right)^2 d} + \frac{b\left(11a^2-5b^2\right.}{}$$

Mathematica [A] time = 6.90, size = 723, normalized size = 1.86

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{11a^2b^2\sin(c+dx)-5b^4\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2}\right)}{d} - \frac{2\left(16a^5-80a^3b^2+40ab^4\right)\sin}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/16*((2*(56*a^4*b - 95*a^2*b^3 + 45*b^5)*Cos[c + d*x]^2*(EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]],
```

```
-1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*
Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^5 - 80*a^3*b^2 + 40*a*b^4)*C
os[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) + ((8*a^4*b - 29*a^2*b^3 + 15*b^5)*Cos[2*(c + d*x)]*(b
 + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2
*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b),
 ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[
c + d*x]]*(2 - Sec[c + d*x]^2)))/(a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c
+ d*x]]*(((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2)
 + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (11*a^2*b^
2*Sin[c + d*x] - 5*b^4*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*
x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

maple [B] time = 4.54, size = 1992, normalized size = 5.13

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*b*(-1/2*b^
2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2
/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
))+4*b^2/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a^3*(-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/a^2*b*(-b^2/a/(a^2-b^2)*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(
a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
```

```
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(
-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.726 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=321

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2}$$

```
[Out] 1/2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+3/4*b^
2*(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(b+a*sec(d*x+c)
)-3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b
^2)^2/d-1/4*(7*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2
-b^2)^2/d+3/4*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a^2/(a-b)^2/(a+b)^3/d
```

Rubi [A] time = 0.77, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3845, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-3*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a^2 - b^2)^2*d) + (3*(5*a^4
- 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*(a + b)^3*d) + (b^2*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (3*b^2*(3*
a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*S
ec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{b^2}{2}-2ab\sec(c+dx)+\frac{1}{2}\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2d(b+a\sec(c+dx))} + \frac{\int}{}$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2d(b+a\sec(c+dx))} + \frac{\int}{}$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2d(b+a\sec(c+dx))} - \frac{(3b}{}$$

$$= \frac{3\left(5a^4-2a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3d} + \frac{b^2\sec}{2a\left(a^2-b\right.}$$

$$= -\frac{3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2d} - \frac{\left(7a^2-b^2\right)\sqrt{\cos(c+}}{}$$

Mathematica [B] time = 6.77, size = 694, normalized size = 2.16

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3b\left(3a^2-b^2\right)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2} - \frac{b\sin(c+dx)}{2\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{b^3\sin(c+dx)-7a^2b\sin(c+dx)}{4a\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d} + \frac{2\left(8ab^3-32a^3b\right)\sin(c+dx)\cos^2(c+dx)}{b(1-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*(16*a^4 - 19*a^2*b^2 + 9*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b +
 a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d
*x])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*b + 8*a*b^3)*Cos[c + d*x]^2*Ellipt
icPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 -
```

```
Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
 + ((-9*a^2*b^2 + 3*b^4)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*
a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*El
lipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*
Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))
)/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((3*b*(3*a^2 - b^2)*
Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*(a +
b*Cos[c + d*x])^2) + (-7*a^2*b*Sin[c + d*x] + b^3*Sin[c + d*x])/(4*a*(a^2 -
 b^2)^2*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

maple [B] time = 2.15, size = 1176, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-b^2/a/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/2*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
```

```
*d*x+1/2*c)^2*b+a-b)-7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a+b)/(a^2-b^2)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/
4/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*b^2-9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*b^3/a^2/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-3/4*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/2*a^2/(a^2-b^2)^2
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2))-3/2/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec (c + dx)}}{(a + b \cos (c + dx))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**3, x)
```

3.727 $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=317

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2}$$

```
[Out] 1/2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2-1/4*b*
(7*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(b+a*sec(d*x+c))+1/
4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d+
3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d-
1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a/(a-b)^2/b/(a+b)^3/d
```

Rubi [A] time = 0.75, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3845, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(4*a*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b*(a^2 - b^2)^2*d) - ((3*a^4 + 10*a^2*
b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*(a - b)^2*b*(a + b)^3*d) + (b^2*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) - (b*(7*a^2 - b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{b^2}{2}-2ab\sec(c+dx)+\frac{1}{2}\left(4a^2-b^2\right)\sec^2(c+d}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+}{4a\left(a^2-b^2\right)^2 d(b+a\sec(c+dx}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+}{4a\left(a^2-b^2\right)^2 d(b+a\sec(c+dx}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+}{4a\left(a^2-b^2\right)^2 d(b+a\sec(c+dx}$$

$$= -\frac{\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b(a+b)^3d}$$

$$= \frac{\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)}{}$$

Mathematica [A] time = 5.98, size = 395, normalized size = 1.25

$$\frac{2\cot(c+dx)\left(6a^4\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+5a^3b\sec^{\frac{3}{2}}(c+dx)-5a^3b\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-2ab\left(5a^2+b^2\right)\sqrt{-\tan^2(c+dx)}\,E\left(\sin^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((-4*b*(7*a^3 - a*b^2 + b*(5*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a*(a^2
 - b^2)^2*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (2*Cot[c + d*x]*(5*a
^3*b*Sec[c + d*x]^(3/2) + a*b^3*Sec[c + d*x]^(3/2) - 5*a^3*b*Cos[2*(c + d*x
)]*Sec[c + d*x]^(3/2) - a*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - 2*a*b*(
5*a^2 + b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2
```

```
] + 2*b*(5*a^3 - 7*a^2*b + a*b^2 + b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^4*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 20*a^2*b^2*EllipticPi[-(a/b), ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b^4*EllipticPi[-(a/b), Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*(a - b)^2*b*(a +
 b)^2))/(16*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

maple [B] time = 3.83, size = 1736, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b*a*(-1/2*b^
2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
```

```
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2
/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
))+2/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/
a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)^3\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
[Out] Integral(1/((a + b*cos(c + d*x))**3*sqrt(sec(c + d*x))), x)
```

3.728 $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=302

$$\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2\left(a\sec(c+dx)+b\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}+\frac{a\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F}{4b^2d\left(a^2-b^2\right)^2}$$

```
[Out] -1/2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+3/4*(a^2+
b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-1/4*(a^2+5*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d+1/4*a*(a^
2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d-1/4
*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/(a-b)^2/b^2/(a+b)^3/d
```

Rubi [A] time = 0.70, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3844, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2\left(a\sec(c+dx)+b\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a\sec(c+dx)+b\right)^2}+\frac{a\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F}{4b^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(4*b*(a^2 - b^2)^2*d) + (a*(a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) - ((a^4 - 10*a
^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^2*(a + b)^3*d) - (b*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
```

```
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{b}{2}-2a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+}{4\left(a^2-b^2\right)^2 d(b+a\sec(c+dx}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+}{4\left(a^2-b^2\right)^2 d(b+a\sec(c+dx}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+}{4\left(a^2-b^2\right)^2 d(b+a\sec(c+dx}$$

$$= -\frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{4(a-b)^2b^2(a+b)^3d}$$

$$= -\frac{\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b\left(a^2-b^2\right)^2 d} + \frac{a\left(a^2-7b\right.}{}$$

Mathematica [B] time = 6.75, size = 665, normalized size = 2.20

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(a^2+5b^2\right)\sin(c+dx)}{4b\left(b^2-a^2\right)^2}+\frac{a^2\sin(c+dx)}{2b\left(b^2-a^2\right)(a+b\cos(c+dx))^2}+\frac{a^3\sin(c+dx)-7ab^2\sin(c+dx)}{4b\left(b^2-a^2\right)^2(a+b\cos(c+dx))}\right)}{d} - \frac{2\left(-5a^2-b^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-s}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -1/16*((2*(-5*a^2 - b^2)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c
+ d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 -
 Cos[c + d*x]^2)) + (48*a*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec
[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
```

```
)/((a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((a^2 + 5*b^2)*Cos[2*(c + d
*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
 + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[
-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sq
rt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)^2*(a + b)^2*d) + (Sqrt[Sec
[c + d*x]]*(((a^2 + 5*b^2)*Sin[c + d*x])/(4*b*(-a^2 + b^2)^2) + (a^2*Sin[c
+ d*x])/(2*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (a^3*Sin[c + d*x] - 7*a
*b^2*Sin[c + d*x])/(4*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

maple [B] time = 3.51, size = 1836, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*a^2*(-1/2
*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2
-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)
```

```
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-
b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/
a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*
a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*
b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2)))-4/b/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-4/b^2*a*(-b^2/a/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2
*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.729 $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=319

$$\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+a}}{4b^2d\left(a^2-b^2\right)^2}$$

```
[Out] 1/2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*a*(a^2
-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-3/4*a*
(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d+
1/4*(3*a^4-5*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^
3/(a^2-b^2)^2/d-3/4*a*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)^3/d
```

Rubi [A] time = 0.69, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3843, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(-5a^2b^2+3a^4+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{}}{4b^3d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (-3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4 - 5*a^2*b^2 + 8*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2
*d) - (3*a*(a^4 - 2*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^3*(a + b)^3*d) +
(a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2
) + (a*(a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*
(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
```

```
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2(a^2-b^2)\,d(b+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{a}{2}-2b\sec(c+dx)+\frac{3}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2(a^2-b^2)\,d(b+a\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+d}{4b(a^2-b^2)^2\,d(b+a\sec(c+dx))}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2(a^2-b^2)\,d(b+a\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+d}{4b(a^2-b^2)^2\,d(b+a\sec(c+dx))}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2(a^2-b^2)\,d(b+a\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\,\sin(c+d}{4b(a^2-b^2)^2\,d(b+a\sec(c+dx))}$$

$$= -\frac{3a(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{4(a-b)^2b^3(a+b)^3d}$$

$$= -\frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2\,d} + \frac{(3a^4-5}{}$$

Mathematica [A] time = 5.66, size = 280, normalized size = 0.88

$$\frac{\frac{2ab^2\sin(c+dx)\left(a^3+3b(a^2-3b^2)\cos(c+dx)-7ab^2\right)}{(a^2-b^2)^2\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^2} + \frac{\cot(c+dx)\left(-6ab(a^2-3b^2)\sin^2(c+dx)\sec^{\frac{3}{2}}(c+dx)+6ab(a^2-3b^2)\sqrt{-\tan^2(c+dx)}\,E\left(\sin^{-1}\left(\sqrt{\sec(c+d}\right.\right.\right.}{}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((2*a*b^2*(a^3 - 7*a*b^2 + 3*b*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/((
a^2 - b^2)^2*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-6
*a*b*(a^2 - 3*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 + 6*a*b*(a^2 - 3*b^2)*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b*(3*a^
3 - a^2*b - 9*a*b^2 + 7*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
```

```
[-Tan[c + d*x]^2] + 6*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[-(a/b), ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*b
^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

maple [B] time = 3.46, size = 1914, normalized size = 6.00

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/b^3
*a^3*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2
)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(
a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/
(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/
(a-b),2^(1/2)))+12/b^2*a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+6/b^3*a^2*(-b^2/a/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a
^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.730 $\int \sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=369

$$-\frac{2(a-b)\sqrt{a+b}\,(9a+2b)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/15*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/5*sec(d*x+c
)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*a^2-2*b^2)*csc(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)-2/15*(a-b)*(9*a+2*b)*c
sc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2796, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(9
*a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*S
qrt[Sec[c + d*x]]) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(15*a*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(5*d)
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
```

```
1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} + \frac{2\sqrt{a+b\cos(c+dx)}\,\text{s}}{}$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} + \frac{2\sqrt{a+b\cos(c+dx)}\,\text{s}}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.08, size = 353, normalized size = 0.96

$$2\left(\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))\left(\left(9a^2-2b^2\right)\sin(c+dx)+a\tan(c+dx)(3a\sec(c+dx)+b)\right)+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

[Out] (2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (9*a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2] + (a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((9*a^2 - 2*b^2)*Sin[c + d*x] + a*(b + 3*a*Sec[c + d*x])*Tan[c + d*x])))/(15*a^2*d*Sqrt[a + b*Cos[c + d*x]])

fricas [F] time = 1.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

maple [B] time = 0.25, size = 1563, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x)

[Out] -2/15/d*(cos(d*x+c)^2*a*b^2-9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*a^3+2*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+7*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(

```
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*cos(d*x+c)^
2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2-2*cos(d*x+c)^4*b^3+9*cos(d*x+c)^3*a^3-4*cos(d*x+c)*a^2*b+9*cos(d*x+
c)^4*a^2*b+cos(d*x+c)^4*a*b^2-5*cos(d*x+c)^3*a^2*b-2*cos(d*x+c)^3*a*b^2+2*c
os(d*x+c)^3*b^3-6*cos(d*x+c)^2*a^3-9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3+2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^3+9*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)
/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.731 $\int \sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=311

$$\frac{2b(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{2\sin(}{}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(a-b)*b*csc(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/3*(a-b)*csc(d*x+c)*E
llipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2796, 2998, 2816, 2994}

$$\frac{2b(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{2\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a
 + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec\right.$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left((a-b)\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|-}{3a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 10.99, size = 301, normalized size = 0.97

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2b\sin(c+dx)}{3a}+\frac{2}{3}\tan(c+dx)\right)}{d} - \frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(b\cos(c+dx)\,\mathrm{ta}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a + b)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Sq
rt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*
b*Sin[c + d*x])/(3*a) + (2*Tan[c + d*x])/3))/d
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 0.29, size = 888, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*a^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2
+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*a^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+cos(d*x+c)^3*a
*b+cos(d*x+c)^3*b^2+cos(d*x+c)^2*a^2+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-2*a*
b*cos(d*x+c)-a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/si
n(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.732 $\int \sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=269

$$\frac{2(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2(a-$$

```
[Out] 2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*(a-b)*cs
c(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4222, 2795, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2(a-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx +$$

$$= \frac{2(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 6.05, size = 215, normalized size = 0.80

$$\frac{2\left(\sin(c+dx)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))+\frac{-\tan\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))-\frac{(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\right.}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}}}{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}}\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (-(((a + b)*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) - (a + b*Cos[c + d*x]
)*Tan[(c + d*x)/2])/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 0.25, size = 797, normalized size = 2.96

$$-\frac{2\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a+\text{EllipticF}\left(\frac{-1+}{\text{si}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c
)^2*b+a*cos(d*x+c)-b*cos(d*x+c)-a)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos
(d*x+c))^(1/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.733 $\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)}\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c}}{\sqrt{a+b\,\mathrm{co}}}\right.\right.}{d\sqrt{a+b}}$$

```
[Out] -2*(a+b*cos(d*x+c))*csc(d*x+c)*EllipticPi((a+b)^(1/2)*cos(d*x+c)^(1/2)/(a+b
*cos(d*x+c))^(1/2),b/(a+b),((-a+b)/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-cos
(d*x+c))/(a+b*cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c))/(a+b*cos(d*x+c)))^(1/2)*
sec(d*x+c)^(1/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4222, 2811}

$$-\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)}\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c}}{\sqrt{a+b\,\mathrm{co}}}\right.\right.}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sqrt[(a*(1 - Cos[c + d*x]))/(a + b*Cos[c + d*x])]*Sq
rt[(a*(1 + Cos[c + d*x]))/(a + b*Cos[c + d*x])]*(a + b*Cos[c + d*x])*Csc[c
+ d*x]*EllipticPi[b/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[Cos[c + d*x]])/Sqrt[a
 + b*Cos[c + d*x]]], -((a - b)/(a + b))]*Sqrt[Sec[c + d*x]])/(Sqrt[a + b]*d
)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= -\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(1+\cos(c+dx))}{a+b\cos(c+dx)}}\,(a+b\cos(c+dx))\csc}{\sqrt{a+b\,c}}$$

Mathematica [A] time = 1.36, size = 146, normalized size = 0.94

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a+b\cos(c+dx)}\left((a-b)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2b\Gamma\right.}{d(a+b)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] + 2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]])/((a + b)*d
*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(dx+c)+a}\,\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [A] time = 0.24, size = 199, normalized size = 1.28

$$\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)a-\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)b+2b\,\mathrm{Ell}\right.}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*b*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/(a+b*cos(d*x+c))^(1/2)*(1
/cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos(dx+c)+a}\,\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.734 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=431

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{d}+\frac{\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\Big(s}{d\sqrt{\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(a-b)*csc(d*x+c)*Ellip
ticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+csc(d*x+c)*EllipticF((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*co
s(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/d/sec(d*x+c)^(1/2)-a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec
(d*x+c)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2821, 3054, 2809, 12, 2801, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{d}+\frac{\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\Big(s}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (a*Sqrt[a + b]*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]
]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 3054

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= -\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.85, size = 565, normalized size = 1.31

$$-\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(2a\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\sin^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-(a*Tan[(c + d*x)/2]) - b*Tan[(c +
d*x)/2] + 2*b*Tan[(c + d*x)/2]^3 + a*Tan[(c + d*x)/2]^5 - b*Tan[(c + d*x)/2
]^5 - 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] - 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*EllipticE[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(
```

```
c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
a + b)] + 2*a*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x
)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*(1 + Tan[(c + d*x)/2]^2)^(3/2)
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)]))
```

fricas [F] time = 2.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [B] time = 0.34, size = 803, normalized size = 1.86

$$-\frac{\left(-2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)\,a+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+2*(cos(d*x+c)/(1+cos(d*x+c)
```

```
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+
2*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)+cos(d*x+c)^3*b+a*cos(d*x+c)^2-cos(d*x+c)^2*b-a*cos(d*x+c))*(1/cos(d
*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.735 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=498

$$\frac{\sqrt{a+b}\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*a*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/4*(a-b)*csc(d*x+c)*EllipticE((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*(a+2*b)*csc(d*x+c)*EllipticF((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*(a^2-4*b^2)*csc(d*x+c)*EllipticPi((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2821, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(
a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
```

```
(2*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(4*b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{ab}{2}+b^2\cos(c+dx)+\frac{1}{2}a}{\sqrt{\cos(c+dx)}\sqrt{a+b}}}{2b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4bd} +$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4bd} +$$

$$= \frac{\sqrt{a+b}\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{4bd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 18.16, size = 1113, normalized size = 2.23

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(2(c+dx))}{4d} + \frac{a^2\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right) - ab\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right) + 2ab\sqrt{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (-(a
^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]) - a*b*Sqrt[(a - b)/(a + b)]*Tan[
(c + d*x)/2] + 2*a*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + a^2*Sqrt[(a
 - b)/(a + b)]*Tan[(c + d*x)/2]^5 - a*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]^5 - (2*I)*a^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b
)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*b^2
```

```
*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/
2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*EllipticPi[(a +
b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
 - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*b^2*EllipticPi[(a
+ b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/
(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*a*(a - b)*EllipticE[
I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a^2 + a*b - 2*b^2)*Elli
pticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(4*b*Sqrt[(a - b)/(a + b)
]*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[
(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 59.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [B] time = 0.27, size = 1241, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+2*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*cos(d*x+c)^4*b^2-2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+8*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x
+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*s
in(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b*sin(d*x+c)+3*cos(d*x+c)^3*a*b+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*
b-2*cos(d*x+c)^2*b^2-a^2*cos(d*x+c)-2*a*b*cos(d*x+c))*cos(d*x+c)*(1/cos(d*x
+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\cos\left(c+d\,x\right)}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cos\left(c+dx\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.736 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=427

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\sqrt{\cos(c+dx)}}{}$$

```
[Out] 2/105*(25*a^2+3*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d
+16/35*b*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/7*a*sec(d*x
+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+4/105*(a-b)*b*(41*a^2-3*b^2)*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/105*(a-b)*(25*
a^2-57*a*b-6*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2
)
```

Rubi [A] time = 1.05, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2799, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b
]*(25*a^2 - 57*a*b - 6*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(105*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[
c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (16*b*Sqrt[a + b*Cos
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*Sqrt[a + b*Cos[c
+ d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{16b\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2a\sqrt{a+b\cos(c+}}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105ad} + \frac{16b\sqrt{a}}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105ad} + \frac{16b\sqrt{a}}{}$$

$$= \frac{4(a-b)b\sqrt{a+b}\left(41a^2-3b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+}}\right.\right.}{105a^3d\sqrt{\sec(c+dx}}$$

Mathematica [A] time = 13.88, size = 441, normalized size = 1.03

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{4b(3b^2-41a^2)\sin(c+dx)}{105a^2}+\frac{2\sec(c+dx)(25a^2\sin(c+dx)+3b^2\sin(c+dx))}{105a}+\frac{2}{7}a\tan(c+dx)\sec^2\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-41*a^3 - 41*a^2*b + 3*a*b^2
 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-41*a^2 + 3*b^2)*Cos[
c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^
2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*(-41*a^2 + 3*b^2)*Sin[c + d*x])/(105*a^2)
 + (2*Sec[c + d*x]*(25*a^2*Sin[c + d*x] + 3*b^2*Sin[c + d*x]))/(105*a) + (1
6*b*Sec[c + d*x]*Tan[c + d*x])/35 + (2*a*Sec[c + d*x]^2*Tan[c + d*x])/7))/d
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

maple [B] time = 0.29, size = 1835, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(25*cos(d*x+c)^5*a^3*b+25*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4-15*a^4+82*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*
a^3*b+51*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^4*sin(d*x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-82*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-82
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^4*sin(d*x+c)*a^2*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+82*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b+51*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d
*x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-82*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-82*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^
2*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^3*sin(d*x+c)*a*b^3+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4+25*cos(d*x+c)^3*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+6*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4+82*cos(d*x+c)^5*a^2*b^2+3*cos(d*x+c)^5*a*b^3+82*cos(d*x+c)^4*a^3*
b-55*cos(d*x+c)^4*a^2*b^2-6*cos(d*x+c)^4*a*b^3-68*cos(d*x+c)^3*a^3*b+3*cos(
d*x+c)^3*a*b^3-27*cos(d*x+c)^2*a^2*b^2-39*cos(d*x+c)*a^3*b+25*cos(d*x+c)^4*
a^4-10*cos(d*x+c)^2*a^4-6*cos(d*x+c)^5*b^4+6*cos(d*x+c)^4*b^4)*cos(d*x+c)/(
```

```
a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.737 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=365

$$\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\dots\right.}{5a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 4/5*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*a*sec(d*x+c)
^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*(a-b)*(3*a^2+b^2)*csc(d*x+c)
*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2/5*(a-b)*(3*a-b)*csc(d*x+
c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2799, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\dots\right.}{5a^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(5*a^2*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*(3*a - b)*Sqrt[a
+ b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a*d*Sqrt[Sec[
c + d*x]]) + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(5*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*
d)
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
```

```
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{4b\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\sqrt{a+b\cos(c+}}{}$$

$$= \frac{4b\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\sqrt{a+b\cos(c+}}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{5a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 11.87, size = 345, normalized size = 0.95

$$\frac{2\left(\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))\left(\left(3a^2+b^2\right)\sin(c+dx)+a\tan(c+dx)(a\sec(c+dx)+2b)\right)+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

[Out] (2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(3*a^2 + 4*a*b + b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2] + (a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((3*a^2 + b^2)*Sin[c + d*x] + a*(2*b + a*Sec[c + d*x])*Tan[c + d*x])))/(5*a*d*Sqrt[a + b*Cos[c + d*x]])

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)

maple [B] time = 0.26, size = 1547, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x)

[Out] -2/5/d*(3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+4*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co

```
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3-3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b-cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-cos(d*x+c)^3*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+3*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3+4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*co
s(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+3*cos(d*x+c)^4*a^2*b+2
*cos(d*x+c)^4*a*b^2+cos(d*x+c)^4*b^3+3*cos(d*x+c)^3*a^3+cos(d*x+c)^3*a*b^2-
cos(d*x+c)^3*b^3-2*cos(d*x+c)^2*a^3-3*cos(d*x+c)^2*a*b^2-3*cos(d*x+c)*a^2*b
-a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.738 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=317

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{2(a-3b)(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+8/3*(a-b)*b*csc(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/3*(a-3*b)*(a-b)*csc(
d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2799, 2998, 2816, 2994}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{2(a-3b)(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (8*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*(a - 3*b)*(a - b)*Sqrt[a + b]*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (
2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
```

```
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left((a-3b)(a-b)\right.$$

$$= \frac{8(a-b)b\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right|}{3ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 7.22, size = 291, normalized size = 0.92

$$\frac{\sqrt{\sec(c+dx)}\left(4\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\left(a^2+4ab+3b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right|\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*Cos[(c + d*x)/2]^2*(-4*b*(a + b)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a^2 + 4*a*b + 3*b^
2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
 + b*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c
+ d*x))/2])) + 2*(a + b*Cos[c + d*x])*(a + 4*b*Cos[c + d*x])*Tan[c + d*x]))
/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 0.32, size = 1085, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3/d*(4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)*a*b+4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-4*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c
)*a*b-3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*b^2+4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-4
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-cos(d*x+c)^3*a*b-4*cos(d*x+c)^3*b^2-cos(d*
x+c)^2*a^2-4*cos(d*x+c)^2*a*b+4*cos(d*x+c)^2*b^2+5*a*b*cos(d*x+c)+a^2)*cos(
d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2} (a+b\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.739 $\int (a + b\cos(c + dx))^{3/2} \sec^{\frac{3}{2}}(c + dx)\, dx$

Optimal. Leaf size=397

$$-\frac{2(a-2b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}+\frac{2(a\cdots}{\cdots}$$

```
[Out] 2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-2*(a-2*b)*cs
c(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-2*b*csc(d*x+c)*Ellipti
cPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-
b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2798, 2809, 2998, 2816, 2994}

$$-\frac{2(a-2b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}+\frac{2(a\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(d*Sqrt[Sec[c + d*x]]) - (2*(a - 2*b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec
[c + d*x]])
```

Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]]
```

```
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a+2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{d\sqrt{\sec(c+dx)}}$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.49, size = 639, normalized size = 1.61

$$\frac{2\left(-\left(a^2+2ab-b^2\right)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\sin^{-1}\left(\tan\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*(a^2*
Tan[(c + d*x)/2] + a*b*Tan[(c + d*x)/2] - 2*a*b*Tan[(c + d*x)/2]^3 - a^2*Ta
n[(c + d*x)/2]^5 + a*b*Tan[(c + d*x)/2]^5 - 2*b^2*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*
```

```
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a^2 + 2*a*b - b^2)*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[
(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan
[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 48.68, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 0.28, size = 1191, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x)
[Out] -2/d*(2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*b^2+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-c
os(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
```

```
))^(1/2))*a^2-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+cos(d*x+c)^2*a
*b+a^2*cos(d*x+c)-a*b*cos(d*x+c)-a^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/
cos(d*x+c))^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.740 $\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=435

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}+\frac{\sqrt{a+b}\,(2a+b)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c}{a}}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(a-b)*b*csc(d*x+c)*E
llipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+(2*a+b)*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-3*a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/
2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2821, 3053, 2809, 2998, 2816, 2994}

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}+\frac{\sqrt{a+b}\,(2a+b)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c}{a}}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -(((a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*a + b)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b]
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[
Sec[c + d*x]]) + (b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/d
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(\sqrt{\cos(c+dx)}\sqrt{\text{se}\right.$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(\sqrt{\cos(c+dx)}\sqrt{\text{se}\right.$$

$$= -\frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 11.67, size = 322, normalized size = 0.74

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(b\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))+4a(a-2b)\sqrt{\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a - 2*b)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 12*a*b*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
 + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 53.10, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

maple [B] time = 0.33, size = 1005, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(2*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-4*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
```

```
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b+EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b+2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)
+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*
x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
a*b*sin(d*x+c)+cos(d*x+c)^3*b^2+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-a*b*cos(d
*x+c))/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.741 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=493

$$-\frac{\sqrt{a+b}\,(3a^2+4b^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{4bd\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+3/4*a*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-5/4*(a-b)*csc(d*x+c)*EllipticE((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(
a-b))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(5*a+2*b)*csc(d*x+c)*EllipticF((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2
)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/d/sec(d*x+c)^(1/2)-1/4*(3*a^2+4*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+
b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.27, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4222, 2821, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\,(3a^2+4b^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{4bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-5*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a + 2*b)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(
3*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(4*b*d*Sqrt[Sec[c + d*x]]) + (3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
```

```
 + d*x]]*Sin[c + d*x])/(4*d) + ((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
```

```
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\,dx$$

$$= \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots}{\cdots}$$

$$= -\frac{a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+a\cdots}}{2d}$$

$$= \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+a\cdots}}{2d}$$

$$= \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+a\cdots}}{2d}$$

$$= -\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\cdots\right.}{4bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{5(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\cdots}{\cdots}}}{4d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.80, size = 845, normalized size = 1.71

$$\frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(2(c+dx))}{4d} - \frac{-5a^2\tan^5\left(\frac{1}{2}(c+dx)\right)+5ab\tan^5\left(\frac{1}{2}(c+dx)\right)-10ab\tan^3\left(\frac{1}{2}(c\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) - (5
*a^2*Tan[(c + d*x)/2] + 5*a*b*Tan[(c + d*x)/2] - 10*a*b*Tan[(c + d*x)/2]^3
- 5*a^2*Tan[(c + d*x)/2]^5 + 5*a*b*Tan[(c + d*x)/2]^5 + 6*a^2*EllipticPi[-1
, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2
*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 6*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*T
an[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x
)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*EllipticPi[-1, ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] +
5*a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(4*a^2 - a*b + 2*b^2)*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(a + b)])/(4*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x
)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 2.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

maple [B] time = 0.28, size = 1423, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(-8*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+5*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2+5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+2*cos(d*x+c)^4
*b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^2*sin(d*x+c)+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*sin(d*x+c)+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+7*cos(d*x+c)^3*a*b+5
*cos(d*x+c)^2*a^2-5*cos(d*x+c)^2*a*b-2*cos(d*x+c)^2*b^2-5*a^2*cos(d*x+c)-2*
a*b*cos(d*x+c))*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\ \cos(c + d\,x))^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cos(c + dx))^{\frac{3}{2}}}{\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
[Out] Integral((a + b*cos(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

3.742 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=568

$$\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24bd}-\frac{(a-b)\sqrt{a+b}\,(3a^2+16b^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)}{24abd}$$

```
[Out] 1/3*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/4*a*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(3*a^2+16*b^2)*sin(d*x+c)*(a+
b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/24*(a-b)*(3*a^2+16*b^2)*csc(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+1/24*(a+2*b)*(3*a+8*b)*c
sc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/8*a*(a^2-12*b^2)*
csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.36, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4222, 2821, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24bd}-\frac{(a-b)\sqrt{a+b}\,(3a^2+16b^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)}{24abd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a + 2*b)*(3
*a + 8*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqr
t[Sec[c + d*x]]) + (a*Sqrt[a + b]*(a^2 - 12*b^2)*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
```

```
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (a*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + ((a + b*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((3*a^2 + 16*b^2
)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$= \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cdots}}{3b}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c\cdots}\right.}{\cdots}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(3a^2+16\cdots\right.}{\cdots}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(3a^2+16\cdots\right.}{\cdots}$$

$$= \frac{a\sqrt{a+b}\left(a^2-12b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\cdots\right.}{8b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(3a^2+16b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\cdots\right.}{24abd\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.62, size = 961, normalized size = 1.69

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{12}b\sin(c+dx)+\frac{7}{24}a\sin(2(c+dx))+\frac{1}{12}b\sin(3(c+dx))\right)}{d} + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*Sin[c + d*x])/12 + (7*a*Si
n[2*(c + d*x)])/24 + (b*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[(c + d*x)
/2]^2)^(-1)]*(3*a^3*Tan[(c + d*x)/2] + 3*a^2*b*Tan[(c + d*x)/2] + 16*a*b^2*
Tan[(c + d*x)/2] + 16*b^3*Tan[(c + d*x)/2] - 6*a^2*b*Tan[(c + d*x)/2]^3 - 3
2*b^3*Tan[(c + d*x)/2]^3 - 3*a^3*Tan[(c + d*x)/2]^5 + 3*a^2*b*Tan[(c + d*x)
/2]^5 - 16*a*b^2*Tan[(c + d*x)/2]^5 + 16*b^3*Tan[(c + d*x)/2]^5 - 6*a^3*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)] + 72*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(a + b)] - 6*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a*b^2*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + (3*a^3 + 3*a^2*b + 16*a*b^2 + 16*b^3)*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
 d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 2*a*(7*a - 26*b)*b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*b*d*(1 + Tan[(c +
 d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 2.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.31, size = 1691, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*(-6*cos(d*x+c)^2*a*b^2-6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-6*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-
3*a^3*cos(d*x+c)-16*cos(d*x+c)*a*b^2-3*cos(d*x+c)^2*a^2*b+8*cos(d*x+c)^5*b^
3-16*cos(d*x+c)^2*b^3-14*cos(d*x+c)*a^2*b+22*cos(d*x+c)^4*a*b^2+17*cos(d*x+
c)^3*a^2*b+8*cos(d*x+c)^3*b^3+3*cos(d*x+c)^2*a^3+3*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^3+72*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b*sin(d*x+c)+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-52*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+72*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
a*b^2+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b+16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+14*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-52*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
```

```
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2
)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.743 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=494

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad}$$

```
[Out] 2/315*b*(163*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/
a/d+2/315*(49*a^2+75*b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2
)/d+38/63*a*b*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/9*a^2*
sec(d*x+c)^(9/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/315*(a-b)*(147*a^4+2
79*a^2*b^2-10*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/
2)-2/315*(a-b)*(147*a^3-114*a^2*b+165*a*b^2+10*b^3)*csc(d*x+c)*EllipticF((a
+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+
b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.52, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a -
 b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b + 165*a*b^2 + 10*b^3)*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*b
*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(315*a*d) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5
/2)*Sin[c + d*x])/(315*d) + (38*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(
7/2)*Sin[c + d*x])/(63*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(9
/2)*Sin[c + d*x])/(9*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{38ab\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2a^2\sqrt{a+b\cos(c}}{}$$

$$= \frac{2\left(49a^2+75b^2\right)\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d} + \frac{38ab}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315ad} + \frac{2\left(49\right.}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315ad} + \frac{2\left(49\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4+279a^2b^2-10b^4\right)\sqrt{\cos(c+dx)}\ \csc(c+dx)E}{315a^3d\sqrt{\sec(c}}$$

Mathematica [A] time = 16.29, size = 521, normalized size = 1.05

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)\left(163a^2b\sin(c+dx)+5b^3\sin(c+dx)\right)}{315a} + \frac{2}{315}\sec^2(c+dx)\left(49a^2\sin(c+dx)+75b^2\right.\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(147*a^5 + 147*a^4*b + 279*a^3
*b^2 + 279*a^2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(147*a^4 + 261*a^3*b + 279*a^2*
b^2 + 155*a*b^3 - 10*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
```

```
/2]], (-a + b)/(a + b)] - (147*a^4 + 279*a^2*b^2 - 10*b^4)*Cos[c + d*x]*(a
+ b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^2*d*Sqrt[a +
 b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]*((2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sin[c + d*x])/(315*a^2)
 + (2*Sec[c + d*x]^2*(49*a^2*Sin[c + d*x] + 75*b^2*Sin[c + d*x]))/315 + (2*
Sec[c + d*x]*(163*a^2*b*Sin[c + d*x] + 5*b^3*Sin[c + d*x]))/(315*a) + (38*a
*b*Sec[c + d*x]^2*Tan[c + d*x])/63 + (2*a^2*Sec[c + d*x]^3*Tan[c + d*x])/9)
)/d
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^{2} \cos\left(dx + c\right)^{2} + 2\, ab \cos\left(dx + c\right) + a^{2}\right) \sqrt{b \cos\left(dx + c\right) + a} \sec\left(dx + c\right)^{\frac{11}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c)^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos\left(dx + c\right) + a)^{\frac{5}{2}} \sec\left(dx + c\right)^{\frac{11}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/2), x)
```

maple [B] time = 0.37, size = 2512, normalized size = 5.09

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(-80*cos(d*x+c)^3*a^2*b^3+147*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+65*cos(d*x+c)^5*a^4
*b+279*cos(d*x+c)^5*a^3*b^2-199*cos(d*x+c)^5*a^2*b^3-10*cos(d*x+c)^5*a*b^4-
272*cos(d*x+c)^4*a^3*b^2+5*cos(d*x+c)^4*a*b^4-82*cos(d*x+c)^3*a^4*b-170*cos
(d*x+c)^2*a^3*b^2-130*cos(d*x+c)*a^4*b+147*cos(d*x+c)^6*a^4*b+163*cos(d*x+c
)^6*a^3*b^2+279*cos(d*x+c)^6*a^2*b^3+5*cos(d*x+c)^6*a*b^4-35*a^5-10*cos(d*x
+c)^6*b^5+147*cos(d*x+c)^5*a^5+10*cos(d*x+c)^5*b^5-98*cos(d*x+c)^4*a^5-14*c
```

os(d*x+c)^2*a^5-10*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-279*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+261*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+155*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-10*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-147*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-279*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+10*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+261*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+155*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-147*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+10*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+147*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-147*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b

```
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^5+10*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5)*cos(d*x+c)/(a+b*cos(d*x+c
))^(1/2)*(1/cos(d*x+c))^(11/2)/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{11/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(11/2),x)
[Out] Timed out
```

3.744 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=427

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}+\frac{2(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\sqrt{\cos(c+dx)}\,\csc(\ldots)}{2\ldots}$$

```
[Out] 2/21*(5*a^2+9*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+6/7
*a*b*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/7*a^2*sec(d*x+c
)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/21*(a-b)*b*(29*a^2+3*b^2)*csc
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/21*(a-b)*(5*a^2-2
4*a*b+3*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}+\frac{2(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\sqrt{\cos(c+dx)}\,\csc(\ldots)}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(21*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]
*(5*a^2 - 24*a*b + 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(21*a*d*Sqrt[Sec[c + d*x]]) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a*b*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
 + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{6ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\sqrt{a+b\cos(c+}}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{6ab\sqrt{a+}}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{6ab\sqrt{a+}}{}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(29a^2+3b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{21a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.83, size = 443, normalized size = 1.04

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2b(29a^2+3b^2)\sin(c+dx)}{21a}+\frac{2}{21}\sec(c+dx)\left(5a^2\sin(c+dx)+9b^2\sin(c+dx)\right)+\frac{2}{7}a^2\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*b*(29*a^3 + 29*a^2*b + 3*a*b^2
 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + 2*a*(5*a^3 + 29*a^2*b + 27*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(29*a^2 + 3*b^2)*Cos[
c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(21*a*d
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]*((2*b*(29*a^2 + 3*b^2)*Sin[c + d*x])/(21*a) + (2*Se
c[c + d*x]*(5*a^2*Sin[c + d*x] + 9*b^2*Sin[c + d*x]))/21 + (6*a*b*Sec[c + d
*x]*Tan[c + d*x])/7 + (2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/7))/d
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2), x)
```

maple [B] time = 0.28, size = 1835, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x)

[Out] -2/21/d*(5*cos(d*x+c)^5*a^3*b+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^4-3*a^4+29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+27*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^2*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b+27*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*b^4+5*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+29*cos(d*x+c)^5*a^2*b^2+9*cos(d*x+c)^5*a*b^3+29*cos(d*x+c)^4*a^3*b-11*cos(d*x+c)^4*a^2*b^2+3*cos(d*x+c)^4*a*b^3-22*cos(d*x+c)^3*a^3*b-12*cos(d*x+c)^3*a*b^3-18*cos(d*x+c)^2*a^2*b^2-12*cos(d*x+c)*a^3*b+5*cos(d*x+c)^4*a^4-2*cos(d*x+c)^2*a^4+3*cos(d*x+c)^5*b^4-3*cos(d*x+c)^4*b^4)*cos(d*x+c)/(a+b*co

```
s(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.745 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=378

$$-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{15ad\sqrt{\sec(c+dx)}}$$

```
[Out] 22/15*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*a^2*sec(
d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*a^2+23*b^2)*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2/15*(a-b)*(9*a^2-
8*a*b+15*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2792, 3055, 2998, 2816, 2994}

$$-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{15ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(9*
a^2 - 8*a*b + 15*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(15*a*d*Sqrt[Sec[c + d*x]]) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(15*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
```

```
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{22ab\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2a^2\sqrt{a+b\cos(c+}}{}$$

$$= \frac{22ab\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2a^2\sqrt{a+b\cos(c+}}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2+23b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{15ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 15.17, size = 376, normalized size = 0.99

$$\frac{2\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))\left(\left(9a^2+23b^2\right)\sin(c+dx)+a\tan(c+dx)(3a\sec(c+dx)+11b)\right)+\frac{2\left(\left(9a^2+23b^2\right)\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] ((2*(2*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d
*x])/((a + b)*(1 + Sec[c + d*x]))] - 2*(9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^
3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c +
d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + (9*a^
2 + 23*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]))/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-
1 + Tan[(c + d*x)/2]^2)) + 2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((9*a^
2 + 23*b^2)*Sin[c + d*x] + a*(11*b + 3*a*Sec[c + d*x])*Tan[c + d*x]))/(15*d
*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2), x)
```

maple [B] time = 0.29, size = 1758, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-34*cos(d*x+c)^2*a*b^2-9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*a^3-23*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
```

```
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2+17*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b+23*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-23*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^2+23*cos(d*x+c)^4*b^3+9*cos(d*x+c)^3*a^3-14*cos(d*x+c)*a^2*b+
9*cos(d*x+c)^4*a^2*b+11*cos(d*x+c)^4*a*b^2+5*cos(d*x+c)^3*a^2*b+23*cos(d*x+
c)^3*a*b^2-23*cos(d*x+c)^3*b^3-6*cos(d*x+c)^2*a^3+15*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-9*cos
(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b+17*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+23*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-23*cos(d*x+c)^3
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^3+9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3-9*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-23*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*cos(d*x+c)
^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3+15*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c)
)^(7/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.746 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=452

$$\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+14/3*(a-b)*b*c
sc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)+2/3*(a^2-7*a*b+9*b^2)
*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-2*b^2*csc(d*x+c)*El
lipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2792, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (14*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(a^2 - 7*a*b + 9*b^2)*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d*Sqrt[Sec[c + d*x]]) -
(2*b^2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx=\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$=\frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(2\sqrt{\cos(c+dx)}\right.$$

$$=\frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(2\sqrt{\cos(c+dx)}\right.$$

$$=-\frac{2b^2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{d\sqrt{\sec(c+dx)}}$$

$$=\frac{14(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 12.00, size = 399, normalized size = 0.88

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2}{3}a^2\tan(c+dx)+\frac{14}{3}ab\sin(c+dx)\right)}{d}-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-4\left(a^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] -1/3*(Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(28*a*b*(a + b)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(a^3 + 7*a^2*
b + 9*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)] - 24*b^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)] + 14*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(d*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((14*a*b*Sin[c + d*x])/3 + (2*a^2*Tan
[c + d*x])/3))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 0.35, size = 1493, normalized size = 3.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3+7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)*b^3+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-7*cos(d*x+c)^2*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2
+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^3+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*b^3-7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+cos(d*x+c)^3*a^2*b+7
*cos(d*x+c)^3*a*b^2+cos(d*x+c)^2*a^3+7*cos(d*x+c)^2*a^2*b-7*cos(d*x+c)^2*a*
b^2-8*cos(d*x+c)*a^2*b-a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c)
)^(5/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}} \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2} (a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.747 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=505

$$-\frac{(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}\,(2a^2-6ab-b^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(}{}}}{d\sqrt{\mathrm{se}}}$$

```
[Out] 2*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(2*a^2-b^2)*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+(a-b)*(2*a^2-b^2)*csc(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-(2*a^2-6*a*b-b^2)*csc(d*x+
c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-5*a*b*csc(d*x+c)*EllipticPi(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^
(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2792, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}\,(2a^2-6ab-b^2)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(}{}}}{d\sqrt{\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]
]) - (5*a*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/
b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \left(2\sqrt{\cos(c+dx)}\,\sqrt{\right.$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{\left(2a^2-b^2\right)\sqrt{a+b}}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{\left(2a^2-b^2\right)\sqrt{a+b}}{}$$

$$= -\frac{5ab\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{d\sqrt{\sec(c+dx)}}$$

$$= \frac{(a-b)\sqrt{a+b}\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 16.68, size = 736, normalized size = 1.46

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d} + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(2a^3\tan^5\left(\frac{1}{2}(c+dx)\right) - 2a^3\tan\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-2*a^3*Tan[(c + d*x)/2] - 2*a^2*b*Tan[(c +
d*x)/2] + a*b^2*Tan[(c + d*x)/2] + b^3*Tan[(c + d*x)/2] + 4*a^2*b*Tan[(c +
d*x)/2]^3 - 2*b^3*Tan[(c + d*x)/2]^3 + 2*a^3*Tan[(c + d*x)/2]^5 - 2*a^2*b*T
an[(c + d*x)/2]^5 - a*b^2*Tan[(c + d*x)/2]^5 + b^3*Tan[(c + d*x)/2]^5 + 10*
a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] + 10*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
```

```
 + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*a^3 + 2*a^2*b - a*b^2
- b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*(a^2 + 3*a*b - 3*b^2)*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)]))/(d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 67.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 0.22, size = 1631, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+cos(d*x+c)*sin(
```

```
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+
10*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-6*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*s
in(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*b^3*sin(d*x+c)+10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-6*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+cos(d*x+c)^3
*b^3+2*cos(d*x+c)^2*a^2*b+cos(d*x+c)^2*a*b^2-cos(d*x+c)^2*b^3+2*a^3*cos(d*x
+c)-2*cos(d*x+c)*a^2*b-cos(d*x+c)*a*b^2-2*a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^
(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}\,{\left(a+b\,\cos(c+d\,x)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.748 $\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=503

$$\frac{\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{4d\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+9/4*a*b*sin(d*
x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-9/4*(a-b)*b*csc(d*x+c)*Ellip
ticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(8*a^2+9*a*b+2*b^2)*csc(d*x+c)*E
llipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/4*(15*a^2+4*b^2)*csc(d*x+c)*El
lipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2793, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-9*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(4*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a^2 + 9*a*b + 2*b^2)*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) -
(Sqrt[a + b]*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a
 + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{9ab\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+}}{4d}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{9ab\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+}}{4d}$$

$$= -\frac{\sqrt{a+b}\left(15a^2+4b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{4d\sqrt{\sec(c+dx)}}$$

$$= -\frac{9(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{4d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 22.32, size = 3679, normalized size = 7.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) +
(((3*a^2*b)/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + b^3/(2*Sqrt[a +
 b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^3*Sqrt[Sec[c + d*x]])/Sqrt[a + b*
Cos[c + d*x]] + (11*a*b^2*Sqrt[Sec[c + d*x]])/(8*Sqrt[a + b*Cos[c + d*x]])
+ (9*a*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(8*Sqrt[a + b*Cos[c + d*x]]
))*(-18*a*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)] - 4*(4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*b*(15*a^2 + 4*b^
2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] - 9*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c +
```

d*x)/2]))/(4*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-1/4*(Sqrt[Sec[(c +
d*x)/2]^2]*Tan[(c + d*x)/2]*(-18*a*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(4*a^3 - 12*a^2*b + a*b^2 -
2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] - 4*b*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] - 9*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)^2) + (b*Sin[c + d*x]*(-
18*a*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - 4*(4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*b*(15*a^2 + 4*b^2)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
- 9*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(8*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) - (Tan[(c + d*x)/2]*(-18
*a*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] - 4*(4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*b*(15*a^2 + 4*b^2)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] -
9*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(8*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + ((-9*a*b*Cos[c + d*x]*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - (9*a*b*(a + b)*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c +
d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (2*(4*a^
3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c +
d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))
/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (2*b*(15*a^2 + 4*b^2)*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])
^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
] - (9*a*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c

```
 + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
)^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (2*(4*a^3
- 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos
[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (
2*b*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticPi[-1, A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c
 + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + 9*a
*b^2*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + 9*a*b*
(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - 9*a
*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2
- (2*(4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]
^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a
 + b)]) - (2*b*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqr
t[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 - ((-a + b)*Tan[(
c + d*x)/2]^2)/(a + b)]) - (9*a*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x
)/2]^2])/(4*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) - ((-18*a*b*(a + b)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(
4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] - 4*b*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 9*a*b*Cos[c + d
*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d
*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan
[c + d*x]))/(8*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*(Cos[(c +
d*x)/2]^2*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2))))
```

fricas [F] time = 59.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

maple [B] time = 0.24, size = 1631, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x)
[Out] -1/4/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(30*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b
+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*b^3+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-24*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*cos(d*x+c)*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^2-4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*b^3+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)^4*b^
3+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^
2*b*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*b^3*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3*sin(d*x+c)-24*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
```

```
,(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+9*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+9*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+11*co
s(d*x+c)^3*a*b^2+9*cos(d*x+c)^2*a^2*b-9*cos(d*x+c)^2*a*b^2-2*cos(d*x+c)^2*b
^3-9*cos(d*x+c)*a^2*b-2*cos(d*x+c)*a*b^2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.749 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=566

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\sqrt{a+b}\,(33a^2+26ab+16b^2)\sqrt{\cos(c+dx)}\,\csc(c+\cdots}{\cdots}$$

```
[Out] 1/3*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+13/12*a*b*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(33*a^2+16*b^2)*sin(d
*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/24*(a-b)*(33*a^2+16*b^2)*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+1/24*(33*a^2+26*a*
b+16*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-5/8*a*(a^2
+4*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.45, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4222, 2793, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\sqrt{a+b}\,(33a^2+26ab+16b^2)\sqrt{\cos(c+dx)}\,\csc(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(33*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(33*a^2 + 26*
a*b + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*S
qrt[Sec[c + d*x]]) - (5*a*Sqrt[a + b]*(a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
```

```
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) + (b^
2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (13*a*b
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Sec[c + d*x]]) + ((33*a^
2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*d
)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+d}}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{12d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{12d\sqrt{\sec(c+dx)}} + \frac{\left(33a^2\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{12d\sqrt{\sec(c+dx)}} + \frac{\left(33a^2\right.}{}$$

$$= -\frac{5a\sqrt{a+b}\,\left(a^2+4b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{8bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(33a^2+16b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|\right.}{24ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.70, size = 970, normalized size = 1.71

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{12}\sin(c+dx)b^2+\frac{1}{12}\sin(3(c+dx))b^2+\frac{13}{24}a\sin(2(c+dx))b\right)}{d}+\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+d\right.}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sin[c + d*x])/12 + (13*a
*b*Sin[2*(c + d*x)])/24 + (b^2*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[(c
 + d*x)/2]^2)^(-1)]*(33*a^3*Tan[(c + d*x)/2] + 33*a^2*b*Tan[(c + d*x)/2] +
16*a*b^2*Tan[(c + d*x)/2] + 16*b^3*Tan[(c + d*x)/2] - 66*a^2*b*Tan[(c + d*x
)/2]^3 - 32*b^3*Tan[(c + d*x)/2]^3 - 33*a^3*Tan[(c + d*x)/2]^5 + 33*a^2*b*T
an[(c + d*x)/2]^5 - 16*a*b^2*Tan[(c + d*x)/2]^5 + 16*b^3*Tan[(c + d*x)/2]^5
 + 30*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + 120*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^3*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 120*a
*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] + (33*a^3 + 33*a^2*b + 16*a*b^2 + 16*b^3)*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 2*a*(24*a^2 - 13*a*b + 38*b^2)*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)]))/(24*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 1.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/sqrt(sec(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.30, size = 1868, normalized size = 3.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d*(18*cos(d*x+c)^2*a*b^2+48*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-30*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-30*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d
*x+c)-33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*sin(d*x+c)-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^3*sin(d*x+c)+33*a^3*cos(d*x+c)+16*cos(d*x+c)*a*b^2+33*cos(d*x+c)^2*a
^2*b-8*cos(d*x+c)^5*b^3+16*cos(d*x+c)^2*b^3+26*cos(d*x+c)*a^2*b-34*cos(d*x+
c)^4*a*b^2-59*cos(d*x+c)^3*a^2*b-8*cos(d*x+c)^3*b^3-33*cos(d*x+c)^2*a^3-33*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3-16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-120*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-33*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-16*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-26*(cos(d*
```

```
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+7
6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin
(d*x+c)+48*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^3*sin(d*x+c)-120*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-33*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-16*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^2-26*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b+76*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)*(1/cos(d*x+c))^(1/2)/sin(d*x+
c)/(a+b*cos(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\cos(c+d\,x))^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.750 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=638

$$\frac{\left(59a^2+36b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{96d\sqrt{\sec(c+dx)}}+\frac{a\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{192bd}-$$

```
[Out] 1/4*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+17/24*a*b*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/96*(59*a^2+36*b^2)*sin(d
*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/192*a*(15*a^2+284*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/192*(a-b)*(15*a^2+28
4*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/192*(15*a
^3+118*a^2*b+284*a*b^2+72*b^3)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(
1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(
d*x+c)^(1/2)+1/64*(5*a^4-120*a^2*b^2-48*b^4)*csc(d*x+c)*EllipticPi((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.88, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4222, 2793, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(59a^2+36b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{96d\sqrt{\sec(c+dx)}}+\frac{a\left(15a^2+284b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{192bd}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(15*a^2 + 284*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(15*a^3 + 1
18*a^2*b + 284*a*b^2 + 72*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a^4 - 120*a^2*b^2
```

```
- 48*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(64*b^2*d*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(4*d*Sec[c + d*x]^(5/2)) + (17*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(24*d*Sec[c + d*x]^(3/2)) + ((59*a^2 + 36*b^2)*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(96*d*Sqrt[Sec[c + d*x]]) + (a*(15*a^2 + 284*b^2)*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
```

```
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
```

```
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+d}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(59a^2\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(59a^2\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(59a^2\right.}{}$$

$$= \frac{\sqrt{a+b}\left(5a^4-120a^2b^2-48b^4\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{64b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(15a^2+284b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{192bd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 16.61, size = 1642, normalized size = 2.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((17*a*b*Sin[c + d*x])/96 + ((
59*a^2 + 48*b^2)*Sin[2*(c + d*x)])/192 + (17*a*b*Sin[3*(c + d*x)])/96 + (b^
2*Sin[4*(c + d*x)])/32))/d + (-15*a^4*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
] - 15*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - 284*a^2*b^2*Sqrt[(a -
 b)/(a + b)]*Tan[(c + d*x)/2] - 284*a*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*
x)/2] + 30*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 568*a*b^3*Sqrt[
(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 15*a^4*Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]^5 - 15*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + 284*a^2*b^
2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - 284*a*b^3*Sqrt[(a - b)/(a + b)
]*Tan[(c + d*x)/2]^5 - (30*I)*a^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqr
t[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] + (720*I)*a^2*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (288
*I)*b^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c
+ d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*a^4*Elliptic
Pi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
 + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (720*I)*a^2*b^2*
EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (288*I)*
b^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*
x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*a*
(15*a^3 - 15*a^2*b + 284*a*b^2 - 284*b^3)*EllipticE[I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(a + b)] + (2*I)*(15*a^4 + 59*a^3*b - 38*a^2*b^2 + 36*a*b^3 - 72
*b^4)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b
)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(192*b*Sqrt[(a
- b)/(a + b)]*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^
2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 2.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/sec(d*x + c)^(3/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.44, size = 2327, normalized size = 3.65

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d*(-30*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+48*cos(d*x+c)^6*b^4-72*cos(d*x+c)*a*b^3-15
*cos(d*x+c)^2*a^3*b-284*cos(d*x+c)^2*a*b^3-284*cos(d*x+c)*a^2*b^2-72*cos(d*
x+c)^2*b^4+184*cos(d*x+c)^5*a*b^3+254*cos(d*x+c)^4*a^2*b^2+133*cos(d*x+c)^3
*a^3*b+172*cos(d*x+c)^3*a*b^3+30*cos(d*x+c)^2*a^2*b^2-118*cos(d*x+c)*a^3*b+
284*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^2+284*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^3+118*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b-644*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+15*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
```

```
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+15*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+284*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^3+118*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-644*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+72*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^3+15*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b+284*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-30*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+288*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)-144
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4*sin(d*
x+c)+15*cos(d*x+c)^2*a^4+24*cos(d*x+c)^4*b^4+288*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-144*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^4+720*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a^2*b^2*sin(d*x+c)+72*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-15*a^4*cos(d*x+c)+15*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+720*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a^2*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^
(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\cos(c + d\,x))^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.751 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=314

$$-\frac{4b(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{}}{}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d-4/3*(a-b)*b*csc(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/3*(a+2*b)*csc(d*x+
c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2802, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{4b(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(a + 2*b)*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
```

```
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{3a}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\left(2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{3a}$$

$$= -\frac{4(a-b)b\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{3a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 14.04, size = 322, normalized size = 1.03

$$\frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}\left(\frac{2\tan(c+dx)}{3a}-\frac{4b\sin(c+dx)}{3a^2}\right)}{d}+\frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(b\cos(c+dx)\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a - 2*b)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c +
 d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]*((-4*b*Sin[c + d*x])/(3*a^2) + (2*Tan[c + d*x])/(3*a)
))/d
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.33, size = 891, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*a^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+2*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*b^2+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+co
s(d*x+c)^3*a*b-2*cos(d*x+c)^3*b^2+cos(d*x+c)^2*a^2-2*cos(d*x+c)^2*a*b+2*cos
(d*x+c)^2*b^2+a*b*cos(d*x+c)-a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(
d*x+c))^(1/2)/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.752 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=264

$$\frac{2(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+}}{}$$

[Out] 2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)

Rubi [A] time = 0.31, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4222, 2801, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\left(\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx\right) + \left(\sqrt{\cos\right.$$

$$= \frac{2(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 12.59, size = 296, normalized size = 1.12

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{ad} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,\left(\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\mathrm{se}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2*Sqr
t[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*d*Sqrt[a + b*Cos[c + d*x]
]*Sqrt[Sec[(c + d*x)/2]^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.24, size = 620, normalized size = 2.35

$$-\frac{2\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*b+a*cos(d*x
+c)-b*cos(d*x+c)-a)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/
sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)
```

3.753 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=129

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4222, 2816}

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*S
qrt[Sec[c + d*x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.87, size = 103, normalized size = 0.80

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 0.29, size = 125, normalized size = 0.97

$$\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{1}{\cos(dx+c)}}\,\left(\sin^2(dx+c)\right)}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))/(a+b*co
s(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

3.754 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

```
[Out] -2*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4222, 2809}

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{bd\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 1.79, size = 146, normalized size = 1.07

$$-\frac{2\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c\right.\right.\right.\right.}{d\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[1 +
 Sec[c + d*x]])/(d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 0.22, size = 143, normalized size = 1.05

$$\frac{2\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, \sqrt{-\frac{a-b}{a+b}}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, -1, \sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}}{d\sqrt{a+b\cos\left(dx+c\right)}\,\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)/(1/cos(d*x+c))^(1/2)/(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.755 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=474

$$\frac{a\sqrt{a+b}\ \sqrt{\cos(c+dx)}\ \csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\ \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\ \Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\ \sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{a\sin(c}{bd\sqrt{}}$$

```
[Out] sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*sin(d*x+c)*sec(d*x+c
)^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*co
s(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/a/b/d/sec(d*x+c)^(1/2)+csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)
^(1/2)+a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2
)
```

Rubi [A] time = 0.80, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4222, 2820, 2809, 3003, 2993, 12, 2801, 2816, 2994}

$$\frac{a\sqrt{a+b}\ \sqrt{\cos(c+dx)}\ \csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\ \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\ \Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\ \sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{a\sin(c}{bd\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c
+ d*x]]) + Sin[c + d*x]/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (
a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2820

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> -Dist[(a*d)/(2*b), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a
 + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[(Sqrt[d*Sin[e + f*x]]*(a + 2
*b*Sin[e + f*x]))/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
```

```
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{\left(a\sqrt{\cos(c\right.}{}$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{abd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 12.10, size = 507, normalized size = 1.07

$$\frac{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)+1}\left(2a\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\tan\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2*Sqrt[1 + Sec[c +
d*x]]*((2*I)*(a - b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
 - b))] - (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
b))] + (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
```

```
pticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
-((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a
+ b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*b*Sqrt[(a
 - b)/(a + b)]*d*((1 + Cos[c + d*x])^(-1))^(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 2.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 0.28, size = 630, normalized size = 1.33

$$-\frac{\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
```

```
),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*a*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)+cos(d*x+c)^3
*b+a*cos(d*x+c)^2-cos(d*x+c)^2*b-a*cos(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3
/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\,\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x)
```

3.756 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=505

$$-\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\ \csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\ \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\ \Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\ \sqrt{\cos(c+dx)}}\right)\middle|-\frac{a}{a}\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)-3/4*a*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/d+3/4*(a-b)*csc(d*x+c)*Ellipti
cE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2)
)*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)-1/4*(3*a-2*b)*csc(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)-1/4*(3*a^2+4*b^2)*csc(d*x+c)*Ellipti
cPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-
b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2793, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\ \csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\ \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\ \Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\ \sqrt{\cos(c+dx)}}\right)\middle|-\frac{a}{a}\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (3*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(4*b^2*d*Sqrt[Sec[c + d*x]]) - ((3*a - 2*b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a
 + b]*(3*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
 ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(4*b^3*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c
```

```
 + d*x])/(2*b*d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(4*b^2*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{a}{2}+b\,\mathrm{c}}{\sqrt{\cos}}}{2b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{s}}{4b^2d}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{s}}{4b^2d}$$

$$= -\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

$$= \frac{3(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a}{a}\right.}{4b^2d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 18.92, size = 1153, normalized size = 2.28

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4bd} - \frac{\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(-3a^2\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) - (S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x
)/2]^2)]*(3*a^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] + 3*a*b*Sqrt[(a - b)
/(a + b)]*Tan[(c + d*x)/2] - 6*a*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3
 - 3*a^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*a*b*Sqrt[(a - b)/(a +
 b)]*Tan[(c + d*x)/2]^5 + (6*I)*a^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[S
```

```
qrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + (8*I)*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a +
b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*a^
2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*
b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*
x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (3*I
)*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -(
(a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*(3
*a^2 - a*b + 2*b^2)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2
]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/
(4*b^2*Sqrt[(a - b)/(a + b)]*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c +
 d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 +
 Tan[(c + d*x)/2]^2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

maple [B] time = 0.28, size = 1248, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*(-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*b^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b-4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*cos(d*x+c)^4*b^2-3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+
6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*s
in(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*b^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^2*sin(d*x+c)-cos(d*x+c)^3*a*b-3*cos(d*x+c)^2*a^2+3*cos(d
*x+c)^2*a*b-2*cos(d*x+c)^2*b^2+3*a^2*cos(d*x+c)-2*a*b*cos(d*x+c))*cos(d*x+c
)^2*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2)),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.757 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=397

$$\frac{2(a+2b)(a+4b)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}+\frac{2b^2}{ad\left(a\right.}$$

```
[Out] 2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*
(a^2-4*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2
)/d-2/3*b*(5*a^2-8*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)/sec(d*x+c
)^(1/2)+2/3*(a+2*b)*(a+4*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)/sec(d
*x+c)^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2802, 3055, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}-\frac{2b\left(5a^2-8b^2\right)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*(5*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(a + 2*b)*(a + 4*b)*Sqrt[Cos
[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec
[c + d*x]]) + (2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}\left(a^2-4b^2\right)-\frac{1}{2}ab\,\mathrm{c}}{\cos^{\frac{5}{2}}(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin}{3a^2\left(a^2-b^2\right)d}$$

$$= -\frac{2b\left(5a^2-8b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{s}}{}}}{3a^4\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 14.44, size = 440, normalized size = 1.11

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2b^3\sin(c+dx)}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{2\tan(c+dx)}{3a^2}-\frac{2b\left(5a^2-8b^2\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)}\right)}{d} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\mathrm{s}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-5*a^3 - 5*a^2*b + 8*a*b^2
+ 8*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] - 2*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-5*a^2 + 8*b^2)*Cos[c + d
*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2
- b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*(5*a^2 - 8*b^2)*Sin[c + d*x])/(3*a^3
*(a^2 - b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x]))
 + (2*Tan[c + d*x])/(3*a^2)))/d
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.20, size = 1789, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

[Out] -2/3/d*(-8*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-a^4-5*cos(d*x+c)^3*a^2*b^2-4*cos(d*x+c)*a*b^3-5*cos(d*x+c)^2*a^3*b+8*cos(d*x+c)^2*a*b^3+8*cos(d*x+c)^3*b^4-8*cos(d*x+c)^2*b^4+a^2*b^2+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+cos(d*x+c)^3*a^3*b-4*cos(d*x+c)^3*a*b^3+4*cos(d*x+c)^2*a^2*b^2+4*cos(d*x+c)*a^3*b-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+5*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-5*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+cos(d*x+c)^2*a^4-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b)/(a-b)/a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.758 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=325

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(a+2b)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{a^2d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

```
[Out] 2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(a
^2-2*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*(a+2
*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2802, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{a^3d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3
*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b^
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
```

```
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}\left(a^2-2b^2\right)-\frac{1}{2}ab\,c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left((a-b)(a+2b)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{a\left(a^2-b^2\right)}$$

$$= \frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{a^3\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 12.13, size = 369, normalized size = 1.14

$$\frac{2\left(\sin(c+dx)\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\left(\left(a^2-2b^2\right)(a+b\cos(c+dx))+ab^2\right)-\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*((a*b^2 + (a^2 - 2*b^2)*(a + b*Cos[c + d*x]))*Sqrt[Sec[(c + d*x)/2]^2]*S
qrt[Sec[c + d*x]]*Sin[c + d*x] - Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(
a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] - 2*a*(a^2 - a*b - 2*b^2)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a^2 - 2*b^2)*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])))/(a^2*(a^
2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)

maple [B] time = 0.27, size = 1457, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)

[Out] 2/d*(cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*(cos(d

```
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c
)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin
(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^2*sin(d*x+c)-cos(d*x+c)^2*a^2*b-cos(d*x+c)^2*a*b^2+2*cos(d*x+c)^2*b^3-a^
3*cos(d*x+c)+cos(d*x+c)*a^2*b+2*cos(d*x+c)*a*b^2-2*cos(d*x+c)*b^3+a^3-b^2*a
)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^2/(a-
b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}{\left(c+dx\right)}}{\left(a+b\cos{\left(c+dx\right)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(3/2), x)
```

3.759 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(}}\right.\right.}{a^2d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

```
[Out] -2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*b*csc
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2*csc(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2800, 2998, 2816, 2994}

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c}}{\sqrt{a+b}\,\sqrt{\cos(}}\right.\right.}{a^2d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]
*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2800

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[(2*b*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(b + a
*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{b+a\cos(c+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{a^2-b^2}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left((a-b)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c+}}}{a^2-b^2}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(}{}}}{a^2\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 8.46, size = 237, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left(b(b-a)\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)+2a(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\sec(c+dx)+1}}\,\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1}}}{ad\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-2*b*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + 2*a*(a + b)*Cos[(c + d*x)/2]
^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + b*(-a
 + b)*Cos[c + d*x]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]])
```

fricas [F] time = 2.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sqrt{\sec(dx+c)}}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 0.31, size = 832, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(-EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b+EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*a*b-cos(d*x
+c)^2*b^2-a*b*cos(d*x+c)+cos(d*x+c)*b^2)/sin(d*x+c)/(a+b)/(a-b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**(3/2), x)
```

3.760 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=306

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{ad\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*csc(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*co
s(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4222, 2794, 2795, 2816, 2994}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{ad\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d*
Sqrt[Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+}}{\mathrm{co}}}{a^2-b^2}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\mathrm{co}}}{a^2-b^2}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{a\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 4.02, size = 235, normalized size = 0.77

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left((a-b)\sin(c+dx)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}-(a+b\cos(c+dx))F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{d\left(a^2-b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\sqrt{\frac{a}{(a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*((a + b*Cos[c + d*x])*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)] - (a + b*Cos[c + d*x])*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + (a - b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sin[c + d*x]))/((a^2 -
b^2)*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)

maple [B] time = 0.29, size = 811, normalized size = 2.65

$$2\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\;\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)\,a+\mathrm{EllipticF}\left(\frac{-1+\mathrm{co}}{\mathrm{sin}(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)

[Out] 2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-a*cos(d*x+c)^2+cos(d*x+c)^2*b+a*cos(d*x+c)-b*cos(d*x+c))*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/(a-b)/(a+b)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cos(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)
```

3.761 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=447

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-}\right.}{b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] -2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*c
sc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*csc(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*csc(d*x+c)*EllipticPi((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(
a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4222, 2797, 2809, 2794, 2795, 2816, 2994}

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-}\right.}{b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d*S
qrt[Sec[c + d*x]]) - (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c + d*x]])
- (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]])
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2797

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(3/2), x_Symbol] :> Dist[d/b, Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin
[e + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Sin[e + f*x]]/(a + b*Sin[e +
f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{}}}{b\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 18.05, size = 1175, normalized size = 2.63

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)a^2}{b\left(b^2-a^2\right)(a+b\cos(c+dx))}+\frac{2\sin(c+dx)a}{b\left(a^2-b^2\right)}\right)}{d} - \frac{2\left(a^2\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)-ab\sqrt{\frac{a-b}{a+b}}\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(b*(a^2 -
b^2)) + (2*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - (2
*(-(a^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]) - a*b*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2] + 2*a*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + a^2*Sq
rt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - a*b*Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]^5 - (2*I)*a^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I
)*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*EllipticPi[
(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a +
b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*EllipticP
i[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
+ b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*a*(a - b)*Ellip
ticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(2*a^2 - a*b - b^2)*Ell
ipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b)
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b*Sqrt[(a - b)/(a + b)
]*(a^2 - b^2)*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^
2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 61.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\left(b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
 + a^2)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

maple [B] time = 0.37, size = 1214, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
[Out] 2/d*(-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*a^2+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+cos(d*
```

```
x+c)^2*a^2-cos(d*x+c)^2*a*b-a^2*cos(d*x+c)+a*b*cos(d*x+c))*cos(d*x+c)*(1/co
s(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/(a+b)/(a-b)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.762 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=525

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{\left(3a^2\right.}{}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(3*
a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d
-(3*a^2-b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+(
3*a+b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+3*a*csc(d
*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/
b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2792, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{\left(3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -(((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b
^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])) + ((3*a + b)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (3*a
*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt
```

```
[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a+b\,\mathrm{c}}}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a+b\,\mathrm{c}}}{}$$

$$= \frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(3a^2-b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{ab^2\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 15.19, size = 1025, normalized size = 1.95

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\sin(c+dx)a^3}{b^2(b^2-a^2)(a+b\cos(c+dx))}-\frac{2\sin(c+dx)a^2}{b^2(a^2-b^2)}\right)}{d} - \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(\right.}{\tan^2\left(\frac{1}{2}(c+dx)\right)+}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sin[c + d*x])/(b^2*(a
^2 - b^2)) - (2*a^3*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))
/d - (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a^3*Tan[(c + d*x)/2
] - 3*a^2*b*Tan[(c + d*x)/2] + a*b^2*Tan[(c + d*x)/2] + b^3*Tan[(c + d*x)/2
] + 6*a^2*b*Tan[(c + d*x)/2]^3 - 2*b^3*Tan[(c + d*x)/2]^3 + 3*a^3*Tan[(c +
```

```
d*x)/2]^5 - 3*a^2*b*Tan[(c + d*x)/2]^5 - a*b^2*Tan[(c + d*x)/2]^5 + b^3*Tan
[(c + d*x)/2]^5 + 6*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
 + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
b)] - 6*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (3*a^3 + 3*a^2*b - a*b^2 - b^3)*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + 2*a*b*(a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b^2*(-
a^2 + b^2)*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*
(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 40.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
 + a^2)*sec(d*x + c)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

maple [B] time = 0.25, size = 1675, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)

[Out] -1/d*(-cos(d*x+c)^2*a*b^2-6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-3*a^3*cos(d*x+c)+cos(d*x+c)*a*b^2-3*cos(d*x+c)^2*a^2*b+cos(d*x+c)^2*b^3+2*cos(d*x+c)*a^2*b+cos(d*x+c)^3*a^2*b-cos(d*x+c)^3*b^3+3*cos(d*x+c)^2*a^3+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)*cos(d*x+c)^2*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/(a+b)/(a-b)/b^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2)),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.763 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=513

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{\cos(c+a}}{}$$

```
[Out] 2/3*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4/
3*b^2*(5*a^2-3*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(
d*x+c))^(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*c
os(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*csc(d*x+c)
*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/a^5/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(a^4+9*a^3*b+16*
a^2*b^2-12*a*b^3-16*b^4)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d/se
c(d*x+c)^(1/2)
```

Rubi [A] time = 1.30, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2802, 3055, 2998, 2816, 2994}

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-13a^2b^2+a^4+8b^4\right)\sin(c+dx}{3a^3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(3*a^5*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a^4 +
 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) +
(2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^(3/2)) + (4*b^2*(5*a^2 - 3*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^
```

```
2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^2 - b^
2)^2*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{3}{2}\left(a^2-2b^2\right)-}{\cos^{\frac{5}{2}}(}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} +$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} +$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} +$$

$$= -\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle| -\frac{a\cdots}{a\cdots}\right.}{3a^5(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.55, size = 546, normalized size = 1.06

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\tan(c+dx)}{3a^3} - \frac{2b^3\sin(c+dx)}{3a^2\left(a^2-b^2\right)(a+b\cos(c+dx))^2} - \frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)^2} - \frac{2\left(11a^2b^3\sin(c+dx)\cdots\right.}{3a^3\left(a^2-b^2\right)^2(a+\cdots}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2
- 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] + a*(a^5 - 8*a^4*b + 7*a^3*b^2 + 28*a^2*b^3 -
4*a*b^4 - 16*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
```

```
a + b)/(a + b)] + 2*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]*((-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(3*a^4*(
a^2 - b^2)^2) - (2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x]
)^2) - (2*(11*a^2*b^3*Sin[c + d*x] - 7*b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2
)^2*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*a^3)))/d
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.27, size = 4197, normalized size = 8.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^3*sin(d*x+c)*b^7+cos(d*x+c)^4*a^5*b^2-16*cos(d*x+c)^2*a^2*b^5+24
*cos(d*x+c)^2*a*b^6+6*cos(d*x+c)*a^6*b-12*cos(d*x+c)*a^4*b^3+6*cos(d*x+c)*a
^2*b^5-8*cos(d*x+c)^4*a^4*b^3-13*cos(d*x+c)^4*a^3*b^4+28*cos(d*x+c)^4*a^2*b
^5+8*cos(d*x+c)^4*a*b^6+2*cos(d*x+c)^3*a^6*b-16*cos(d*x+c)^3*a^5*b^2-8*cos(
```

d*x+c)^3*a^4*b^3+56*cos(d*x+c)^3*a^3*b^4-18*cos(d*x+c)^3*a^2*b^5-32*cos(d*x+c)^3*a*b^6-8*cos(d*x+c)^2*a^6*b+13*cos(d*x+c)^2*a^5*b^2+28*cos(d*x+c)^2*a^4*b^3-42*cos(d*x+c)^2*a^3*b^4+cos(d*x+c)^2*a^7-16*cos(d*x+c)^4*b^7+16*cos(d*x+c)^3*b^7-a^7+2*a^5*b^2-a^3*b^4+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^7+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^7+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^7+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^6*b-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5*b^2+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4*b^3+28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b^4-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^5-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^6+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4*b^3-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b^4-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^5+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^6-7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*b-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b^2+35*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3+24*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^4-20*cos(d*x+c)^2*sin(d*x+

```
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5-
16*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^6+8*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^6*b+16*cos(d*x+c)^2*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*b^2-20*cos(d
*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a^4*b^3-56*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^4-12*cos(d*x+c)^2*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^5+32*cos(d*x+
c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*a*b^6-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*a^6*b+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b^2+28*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3-4*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3*b^4-16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5+8*cos(d*x+c)*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^6*b+8*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^
5*b^2-28*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*a^4*b^3-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^4+16*cos(d*x+c)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^5+16*
cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*a*b^6)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(3/2)/si
n(d*x+c)/(a-b)^2/(a+b)^2/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.764 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=438

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{\cos(c+d}}{}$$

```
[Out] 2/3*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+8/
3*b^2*(2*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*
x+c))^(1/2)+2/3*(3*a^4-15*a^2*b^2+8*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/
2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/
(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(3*a^3+9*a^2*b-6*a*b^2-8*b^3)*csc(d*x+c)
*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2802, 3055, 2998, 2816, 2994}

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(9a^2b+3a^3-6ab^2-8b^3\right)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*a^3 +
 9*a^2*b - 6*a*b^2 - 8*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (
8*b^2*(2*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d
*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}\left(3a^2-4b^2\right)-\frac{3}{2}\cdots}{\cos^{\frac{3}{2}}(c+\cdots}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \cdots$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \cdots$$

$$= \frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\cdots}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.27, size = 525, normalized size = 1.20

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2b^2\sin(c+dx)}{3a(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{8\left(2a^2b^2\sin(c+dx)-b^4\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)^2}\right)}{d}.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4 - 15*a^2*b^2 + 8*b^
4)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b
^2)*(a + b*Cos[c + d*x])^2) + (8*(2*a^2*b^2*Sin[c + d*x] - b^4*Sin[c + d*x]
))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]*(-2*(3*a^5 + 3*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 8*a*b^4
+ 8*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + 2*a*(3*a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 + 8*b^4)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4 - 15*a
^2*b^2 + 8*b^4)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c +
d*x)/2]^2])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)

maple [B] time = 0.29, size = 3701, normalized size = 8.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)

[Out] 2/3/d*(15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)-8*cos(d*x+c)^3*b^6+8*cos(d*x+c)^2*b^6-3*cos(d*x+c)*a^6-6*b^2*a^4+3*b^4*a^2+3*a^6-3*cos(d*x+c)^3*a^4*b^2+15*cos(d*x+c)^3*a^2*b^4+4*cos(d*x+c)^3*a*b^5-6*cos(d*x+c)^2*a^5*b+30*cos(d*x+c)^2*a^3*b^3-10*cos(d*x+c)^2*a^2*b^4-16*cos(d*x+c)^2*a*b^5+15*cos(d*x+c)*a^4*b^2-22*cos(d*x+c)*a^3*b^3-8*cos(d*x+c)*a^2*b^4+12*cos(d*x+c)*a*b^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)-8*cos(d*x+c)^3*a^3*b^3-6*cos(d*x+c)^2*a^4*b^2+6*cos(d*x+c)*a^5*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b-15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2-15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^6-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(

(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^6+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^4+16*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*a*b^5+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*sin(d*x+c)*a^5*b+21*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b^2+13*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3-10*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a^2*b^4-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^5+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b-12*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b^
2-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^3*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5*b+6*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b^2+15*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*si
n(d*x+c)*a^3*b^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^4-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^5+3*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)
*a^5*b+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)*a^4*b^2-15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^3-15*(cos(d*x+c)/(1+cos

```
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b
^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^2*sin(d*x+c)*a*b^5)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(
3/2)/sin(d*x+c)/(a-b)^2/(a+b)^2/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.765 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=421

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2-3ab-2b^2\right)}{}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-4/
3*b*(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*cos(d*x+c)
)^(1/2)+4/3*b*(3*a^2-b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d/s
ec(d*x+c)^(1/2)+2/3*(3*a^2-3*a*b-2*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(
a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2802, 2993, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2-3ab-2b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(3*a^2 - 3*a*b - 2*b^
2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(a - b)*(a
+ b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (4*b*(3*a^2 - b^2)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.}{3a\left(a\right.}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{4b\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\ \mathrm{si}}{3a\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{4b\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\ \mathrm{si}}{3a\left(a^2-b^2\right)^2d\sqrt{a+b\cos(c}}$$

$$= \frac{4b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\ \csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\ \sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{a}}}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 14.45, size = 471, normalized size = 1.12

$$\frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}\left(\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2}-\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{2\left(5a^2b\sin(c+dx)-b^3\sin(c+dx)\right)}{3a\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d}+\frac{4\sqrt{\cos}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*b*(3*a^2 - b^2)*Sin[c + d*
x])/(3*a^2*(a^2 - b^2)^2) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[c
```

```
+ d*x])^2) - (2*(5*a^2*b*Sin[c + d*x] - b^3*Sin[c + d*x]))/(3*a*(a^2 - b^2)
^2*(a + b*Cos[c + d*x]))))/d + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*
b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)] + a*(3*a^3 + 6*a^2*b + a*b^2 - 2*b^3)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-
3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2]))/(3*(a^3 - a*b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/
2]^2])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\sec\left(dx+c\right)}}{b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 0.31, size = 2745, normalized size = 6.52

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(3/2)*(6*cos(d*x+c)^2*a^4*b-6*c
os(d*x+c)^3*a^2*b^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^4*sin(d*x+c)+2*cos(d*x+c)^3*b^5-3*(cos(d*x+c)/(1+cos(d*x+c))
```

```
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5-2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*b^5+4*cos(d*x+c)^2*a^2*b^3+7*cos(d*x+c)*a^3*b^2-12*cos(d*x+c)^2*a^3*b^2-6
*cos(d*x+c)*a^4*b-3*cos(d*x+c)*a*b^4-2*cos(d*x+c)^2*b^5+5*cos(d*x+c)^3*a^3*
b^2-cos(d*x+c)^3*a*b^4+4*cos(d*x+c)^2*a*b^4+2*cos(d*x+c)*a^2*b^3-3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)-6*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x
+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2
*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b^3*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^4*b*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^4+12*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)*a^3*b^2-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^4+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4*b+4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^3-
9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)*a^4*b-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b^2-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-3*cos(d*x+c)^2*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^4*b-6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b^2-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
```

```
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+2*cos(d*x+c)^2*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+6*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^3*b^2+6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^
2*b^3)/sin(d*x+c)/a^2/(a+b)^2/(a-b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.766 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=399

$$\frac{2\left(3a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+}}{}$$

```
[Out] -2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2/3*(
3*a^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
-2/3*(3*a^2+b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+
b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)
^(1/2)+2/3*(3*a-b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)
^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2796, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(3*a - b)*Sqrt[Cos[c +
 d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt
[Sec[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)*Sqrt[Sec[c + d*x]]) + (2*(3*a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
```

```
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(3a^2+b^2\right)\sqrt{s}}{3\left(a^2-b^2\right)^2 a}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(3a^2+b^2\right)\sqrt{s}}{3\left(a^2-b^2\right)^2 a}$$

$$= -\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(c+d}}$$

Mathematica [A] time = 13.51, size = 455, normalized size = 1.14

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3a\left(a^2-b^2\right)^2}+\frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{4\left(a^2\sin(c+dx)+b^2\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d} - \frac{2\sqrt{\cos^2}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(3*a^2 + b^2)*Sin[c + d*x
])/(3*a*(a^2 - b^2)^2) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[c + d
*x])^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^2*(a + b
*Cos[c + d*x]))))/d - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a^3 +
```

```
3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] + 2*a*(3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^2 + b^2)*Cos[c + d*x
]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*(a^2 - b^
2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\left(b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

maple [B] time = 0.27, size = 2419, normalized size = 6.06

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*b^4-3*cos(d*x+c)^3*a^2*b^2-6*cos(d*x+c)^2*a^3*b-2*cos(
d*x+c)^2*a*b^3-cos(d*x+c)*a^2*b^2-cos(d*x+c)^3*b^4+cos(d*x+c)^2*b^4+2*cos(d
*x+c)^3*a^3*b+2*cos(d*x+c)^3*a*b^3+4*cos(d*x+c)^2*a^2*b^2+4*cos(d*x+c)*a^3*
b+sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
```

)*a^2*b^2+sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^3-4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*cos(d*x+c)*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3
-7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^3*b-5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+3*cos(d*x+c)^2*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*b+3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^2+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2*b^2-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+4*cos(d*x+c)*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3
*cos(d*x+c)^2*a^4-3*a^4*cos(d*x+c)+3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-3*cos(d*x+c)*

```
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^4)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(3/2)/a/(a+b)^2/(a-b)
^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.767 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=382

$$-\frac{8ab\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}+\frac{2(a-3b)\sqrt{\cos(c+dx)}\ \mathrm{cs}}{}$$

```
[Out] 2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-8/3*a*
b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+8/3*b*cs
c(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(a-3*b)*cs
c(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4222, 2799, 2993, 2998, 2816, 2994}

$$-\frac{8ab\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}+\frac{2(a-3b)\sqrt{\cos(c+dx)}\ \mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (8*b*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a
+ b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a - 3*b)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]])
 + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[
c + d*x]]) - (8*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqr
t[a + b*Cos[c + d*x]])
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
```

```
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{8ab\sqrt{\sec(c+d}}{3\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{8ab\sqrt{\sec(c+d}}{3\left(a^2-b^2\right)^2 d\sqrt{a}}$$

$$= \frac{8b\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{3a(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 8.41, size = 359, normalized size = 0.94

$$-\frac{2\sqrt{\sec(c+dx)}\left(a^2\left(a^2-b^2\right)\sin(c+dx)-a\left(a^2-5b^2\right)\sin(c+dx)(a+b\cos(c+dx))+2b\cos^2\left(\frac{1}{2}(c+dx)\right)(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[Sec[c + d*x]]*(a^2*(a^2 - b^2)*Sin[c + d*x] - a*(a^2 - 5*b^2)*(a +
 b*Cos[c + d*x])*Sin[c + d*x] - 4*b^2*(a + b*Cos[c + d*x])^2*Sin[c + d*x] +
```

```
2*b*Cos[(c + d*x)/2]^2*(a + b*Cos[c + d*x])*(4*b*(a + b)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (a^2 + 4*a*b + 3*
b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] - b*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(
c + d*x))/2]))))/(3*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\left(b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.34, size = 1790, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*(8*cos(d*x+c)^2*a*b^2+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-a^3*cos(d*x+c)-3*cos(d*x+c)*a*b
^2-4*cos(d*x+c)^2*a^2*b-4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*cos(d*x+c)^2*b^3+cos(d*x+c)^
3*a^3+4*cos(d*x+c)*a^2*b-5*cos(d*x+c)^3*a*b^2+4*cos(d*x+c)^3*b^3+3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
```

```
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*
x+c)*b^3+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*b^3+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*cos(d*x+c)^2*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b^2*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-4*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^2+5*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+7*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2)*cos(d*x+c)
*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(3/2)/(a+b)^2/(a-b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)),x)
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
[Out] Timed out
```

3.768 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=557

$$-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+ab-6b^2\right.}{}$$

```
[Out] -2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-2
/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*cos
(d*x+c))^(1/2)+2/3*(3*a^2-7*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3
/2)/d/sec(d*x+c)^(1/2)-2/3*(3*a^2+a*b-6*b^2)*csc(d*x+c)*EllipticF((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/
b^2/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/
2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.21, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4222, 2792, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+ab-6b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (2*(3*a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*a^2 + a*b - 6*b^2)
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^2*(a +
b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
```

```
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*a^2*
Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*
x]]) - (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2
- b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{b^3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(3a^2-7b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{3(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 14.45, size = 1716, normalized size = 3.08

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(3*a^2 - 7*b^2)*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2) - (2*a^3*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)*(a +
 b*Cos[c + d*x])^2) - (8*(a^4*Sin[c + d*x] - 2*a^2*b^2*Sin[c + d*x]))/(3*b^
2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x])))))/d + (2*(3*a^4*Sqrt[(a - b)/(a + b)
]*Tan[(c + d*x)/2] + 3*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - 7*a^2
*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - 7*a*b^3*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2] - 6*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 14*a
*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 - 3*a^4*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2]^5 + 3*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + 7*
```

```
a^2*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - 7*a*b^3*Sqrt[(a - b)/(a
+ b)]*Tan[(c + d*x)/2]^5 + (6*I)*a^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[
Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
a + b)] - (12*I)*a^2*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)
/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6
*I)*b^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c
+ d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*a^4*EllipticP
i[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
+ b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (12*I)*a^2*b^2*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*b^4*
EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a
^3 - 3*a^2*b - 7*a*b^2 + 7*b^3)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*T
an[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] - I*(6*a^4 - 2*a^3*b - 13*a^2*b^2 + 6*a*b^3 + 3*b^4)*EllipticF[I*
ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*b^2*Sqrt[(a - b)/(a + b)]*(a^
2 - b^2)^2*d*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(
a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^
2)]*(-1 + Tan[(c + d*x)/2]^4))
```

fricas [F] time = 45.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\left(b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)

maple [B] time = 0.28, size = 3920, normalized size = 7.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)

[Out] 2/3/d*(-6*cos(d*x+c)^2*a^4*b-8*cos(d*x+c)^3*a^2*b^3+14*cos(d*x+c)^2*a^2*b^3
+7*cos(d*x+c)*a^3*b^2+4*cos(d*x+c)^3*a^4*b-4*cos(d*x+c)^2*a^3*b^2+2*cos(d*x
+c)*a^4*b+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^5*sin(d*x+c)-6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*sin(d*x+c)-3*a^5*cos(d*x+c)-3*cos(d*x+c)^3*a^3*b^2+7*cos(d*x
+c)^3*a*b^4-7*cos(d*x+c)^2*a*b^4-6*cos(d*x+c)*a^2*b^3+3*cos(d*x+c)^2*a^5-2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d
*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
^2*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b^3*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)-6*cos(d*x+c)^2*sin(d*x+
c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5+
3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^5+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-6*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5-6*cos(d*x+c)*sin(d
*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b

^5+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+12*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^4-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b^2-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^4+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4*b-14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b^2-6*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b+12*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3-6*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b+12*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^2+12*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3-6*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d

```
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+3*cos(d*x+c)^2*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
^2-7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^3-7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+7*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^3)*cos(d*
x+c)^2*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(3/2)/b^2/(a+b)^2/(
a-b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)),x)
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.769 $\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=330

$$-\frac{4ab\left(a^2(m+3)+b^2(m+2)\right)\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(c+dx)\right)}{d(m+2)(m+3)\sqrt{\sin^2(c+dx)}}+\frac{b^2\left(a^2(5m+22)+b^2\right.}{}$$

```
[Out] b^2*(b^2*(3+m)+a^2*(22+5*m))*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(4+m)+2*a*
b^3*(5+m)*cos(d*x+c)^(2+m)*sin(d*x+c)/d/(3+m)/(4+m)+b^2*cos(d*x+c)^(1+m)*(a
+b*cos(d*x+c))^2*sin(d*x+c)/d/(4+m)-(b^4*(m^2+4*m+3)+6*a^2*b^2*(m^2+5*m+4)+
a^4*(m^2+6*m+8))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],co
s(d*x+c)^2)*sin(d*x+c)/d/(4+m)/(m^2+3*m+2)/(sin(d*x+c)^2)^(1/2)-4*a*b*(b^2*
(2+m)+a^2*(3+m))*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(d*
x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2793, 3033, 3023, 2748, 2643}

$$-\frac{\left(6a^2b^2\left(m^2+5m+4\right)+a^4\left(m^2+6m+8\right)+b^4\left(m^2+4m+3\right)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\text{c}\right.}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (b^2*(b^2*(3 + m) + a^2*(22 + 5*m))*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(
2 + m)*(4 + m)) + (2*a*b^3*(5 + m)*Cos[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(3
 + m)*(4 + m)) + (b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(d*(4 + m)) - ((b^4*(3 + 4*m + m^2) + 6*a^2*b^2*(4 + 5*m + m^2) + a^4*
(8 + 6*m + m^2))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c
 + d*x]^2]) - (4*a*b*(b^2*(2 + m) + a^2*(3 + m))*Cos[c + d*x]^(2 + m)*Hyper
geometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2
 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx = \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{d(4+m)} + \frac{\int \cos^m(c+dx)(a$$

$$= \frac{2ab^3(5+m)\cos^{2+m}(c+dx)\sin(c+dx)}{d(3+m)(4+m)} + \frac{b^2\cos^{1+m}(c+dx)(a+b\cos}{d(4+}$$

$$= \frac{b^2\left(b^2(3+m)+a^2(22+5m)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(4+m)} + \frac{2ab^3(5+}{}$$

$$= \frac{b^2\left(b^2(3+m)+a^2(22+5m)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(4+m)} + \frac{2ab^3(5+}{}$$

$$= \frac{b^2\left(b^2(3+m)+a^2(22+5m)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(4+m)} + \frac{2ab^3(5+}{}$$

Mathematica [A] time = 1.78, size = 242, normalized size = 0.73

$$\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(b\cos(c+dx)\left(b\cos(c+dx)\left(b\cos(c+dx)\left(-\frac{4a\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(c+dx\right.}{m+4}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((a^4*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[c + d*x]^2])/(1 + m)) + b*Cos[c + d*x]*((-4*a^3*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m) + b*Cos[c + d*x
]*((-6*a^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])/(3
 + m) + b*Cos[c + d*x]*((-4*a*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2,
Cos[c + d*x]^2])/(4 + m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + m)/2
, (7 + m)/2, Cos[c + d*x]^2])/(5 + m)))))*Sqrt[Sin[c + d*x]^2])/d
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4\cos(dx+c)^4 + 4\,ab^3\cos(dx+c)^3 + 6\,a^2b^2\cos(dx+c)^2 + 4\,a^3b\cos(dx+c) + a^4\right)\cos(dx+c)^m,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + 4*a*b^3*cos(d*x + c)^3 + 6*a^2*b^2*cos(d*x +
 c)^2 + 4*a^3*b*cos(d*x + c) + a^4)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))(a+b\cos(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x)
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^m\,(a+b\,\cos(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^4,x)
[Out] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**4,x)
[Out] Timed out
```

3.770 $\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=250

$$-\frac{a\left(a^2(m+2)+3b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{b\left(3a^2(m+3)+b^2(m\right.}{}$$

```
[Out] a*b^2*(7+2*m)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(3+m)+b^2*cos(d*x+c)^(1+m
)*(a+b*cos(d*x+c))*sin(d*x+c)/d/(3+m)-a*(3*b^2*(1+m)+a^2*(2+m))*cos(d*x+c)^
(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1+
m)/(2+m)/(sin(d*x+c)^2)^(1/2)-b*(b^2*(2+m)+3*a^2*(3+m))*cos(d*x+c)^(2+m)*hy
pergeom([1/2, 1+1/2*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(si
n(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2793, 3023, 2748, 2643}

$$-\frac{a\left(a^2(m+2)+3b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{b\left(3a^2(m+3)+b^2(m\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (a*b^2*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (
b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) - (
a*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2,
 (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqr
t[Sin[c + d*x]^2]) - (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[c + d*x]^(2 + m)*
Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/
(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{d(3+m)} + \frac{\int \cos^m(c+dx)\left(a\left(b\right.\right.}{}$$

$$= \frac{ab^2(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(}{d(3+m)}$$

$$= \frac{ab^2(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(}{d(3+m)}$$

$$= \frac{ab^2(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(}{d(3+m)}$$

Mathematica [A] time = 0.91, size = 197, normalized size = 0.79

$$\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(b\cos(c+dx)\left(b\cos(c+dx)\left(-\frac{3a\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(c+dx)\right)}{m+3} - \frac{b\cos(c+dx)\,{}_2F_1}{}\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((a^3*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[c + d*x]^2])/(1 + m)) + b*Cos[c + d*x]*((-3*a^2*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m) + b*Cos[c + d*x
]*((-3*a*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])/(3 +
 m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c +
d*x]^2])/(4 + m))))*Sqrt[Sin[c + d*x]^2])/d
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)+a^3\right)\cos\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

maple [F] time = 1.30, size = 0, normalized size = 0.00

$$\int\left(\cos^m\left(dx+c\right)\right)\left(a+b\cos\left(dx+c\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^m\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.771 $\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=179

$$-\frac{\left(a^2(m+2)+b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2ab\sin(c+dx)\cos^{m+2}(c}{d(m-}$$

```
[Out] b^2*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)-(b^2*(1+m)+a^2*(2+m))*cos(d*x+c)^(1
+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)
/(2+m)/(sin(d*x+c)^2)^(1/2)-2*a*b*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m]
,[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2789, 2643, 3014}

$$-\frac{\left(a^2(m+2)+b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2ab\sin(c+dx)\cos^{m+2}(c}{d(m-}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (b^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - ((b^2*(1 + m) + a^2*(
2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*
a*b*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^{1+m}(c+dx)\,dx + \int \cos^m(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$
$$= \frac{b^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} - \frac{2ab\cos^{2+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\mathrm{co}\right.}{d(2+m)\sqrt{\sin^2(c+}}$$
$$= \frac{b^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} - \frac{\left(a^2+\frac{b^2(1+m)}{2+m}\right)\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{}\right.}{d(1+m)\sqrt{\sin}}$$

Mathematica [A] time = 0.36, size = 168, normalized size = 0.94

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(a^2\left(m^2+5m+6\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)+b(m+1)\cos(c\right.}{d(m+1)(m+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(a^2*(6 + 5*m + m^2)*Hypergeometric2F1
[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2] + b*(1 + m)*Cos[c + d*x]*(2*a*(
3 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2] + b*(2
+ m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]
^2]))*Sqrt[Sin[c + d*x]^2])/(d*(1 + m)*(2 + m)*(3 + m)))
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\cos\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))\,(a+b\cos(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x)
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^m\,(a+b\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^2\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**2,x)
[Out] Integral((a + b*cos(c + d*x))**2*cos(c + d*x)**m, x)
```

3.772 $\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=131

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\bigg(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -a*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*si
n(d*x+c)/d/(1+m)/(sin(d*x+c)^2)^(1/2)-b*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+
1/2*m],[2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2748, 2643}

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\bigg(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (b*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^m(c+dx)\,dx + b\int \cos^{1+m}(c+dx)\,dx$$
$$= -\frac{a\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)\sqrt{\sin^2(c+dx)}} - \frac{b\cos^2}{}$$

Mathematica [A] time = 0.17, size = 112, normalized size = 0.85

$$-\frac{\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\cos^{m+1}(c+dx)\left(a(m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)+b(m+1)\cos(c+dx)\,{}_2F\right.}{d(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(a*(2 + m)*Hypergeometric2F1[1/2, (1 +
 m)/2, (3 + m)/2, Cos[c + d*x]^2] + b*(1 + m)*Cos[c + d*x]*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 +
 m)*(2 + m)))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)+a)\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))(a+b\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^m\,\left(a+b\,\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a + b*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^m*(a + b*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((a + b*cos(c + d*x))*cos(c + d*x)**m, x)
```

3.773 $\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=190

$$\frac{a\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\cos^m(c+$$

```
[Out] a*AppellF1(1/2,1/2-1/2*m,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*co
s(d*x+c)^(-1+m)*(cos(d*x+c)^2)^(1/2-1/2*m)*sin(d*x+c)/(a^2-b^2)/d-b*AppellF
1(1/2,-1/2*m,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*x+c)^m*s
in(d*x+c)/(a^2-b^2)/d/((cos(d*x+c)^2)^(1/2*m))
```

Rubi [A] time = 0.23, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\cos^m(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/
(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)*Sin[c + d*
x])/((a^2 - b^2)*d) - (b*AppellF1[1/2, -m/2, 1, 3/2, Sin[c + d*x]^2, -((b^2
*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^m*Sin[c + d*x])/((a^2 - b^2)*d*
(Cos[c + d*x]^2)^(m/2))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx = a\int \frac{\cos^m(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx - b\int \frac{\cos^{1+m}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx$$

$$= \frac{\left(a\cos^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}(c+dx)\cos^2(c+dx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1}{2}(-1+m)}}{a^2-b^2+b^2x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{(b}{}$$

$$= \frac{aF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{-1+m}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\sin(c+d}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 24.62, size = 6703, normalized size = 35.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^m/(a + b*Cos[c + d*x]),x]
[Out] Result too large to show
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^m}{b\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] integral(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^m}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(dx+c)}{a+b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a+b*cos(d*x+c)),x)
[Out] int(cos(d*x+c)^m/(a+b*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^m}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^m}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(a + b*cos(c + d*x)),x)
[Out] int(cos(c + d*x)^m/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.774 $\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=294

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)\cos^2(c+dx)^{\frac{1}{2}(-m-1)}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)^2}+\frac{a^2\sin(c+$$

```
[Out] b^2*AppellF1(1/2,-1/2-1/2*m,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))
*cos(d*x+c)^(1+m)*(cos(d*x+c)^2)^(-1/2-1/2*m)*sin(d*x+c)/(a^2-b^2)^2/d+a^2*
AppellF1(1/2,1/2-1/2*m,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(
d*x+c)^(-1+m)*(cos(d*x+c)^2)^(1/2-1/2*m)*sin(d*x+c)/(a^2-b^2)^2/d-2*a*b*App
ellF1(1/2,-1/2*m,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*x+c)
^m*sin(d*x+c)/(a^2-b^2)^2/d/((cos(d*x+c)^2)^(1/2*m))
```

Rubi [A] time = 0.35, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2824, 3189, 429}

$$\frac{b^2\sin(c+dx)\cos^{m+1}(c+dx)\cos^2(c+dx)^{\frac{1}{2}(-m-1)}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)^2}+\frac{a^2\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^
2)/(a^2 - b^2))]*Cos[c + d*x]^(1 + m)*(Cos[c + d*x]^2)^((-1 - m)/2)*Sin[c +
 d*x])/((a^2 - b^2)^2*d) + (a^2*AppellF1[1/2, (1 - m)/2, 2, 3/2, Sin[c + d*
x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d
*x]^2)^((1 - m)/2)*Sin[c + d*x])/((a^2 - b^2)^2*d) - (2*a*b*AppellF1[1/2, -
m/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d
*x]^m*Sin[c + d*x])/((a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \left(\frac{a^2\cos^m(c+dx)}{\left(a^2-b^2\cos^2(c+dx)\right)^2} - \frac{2ab\cos^{1+m}(c+dx)}{\left(a^2-b^2\cos^2(c+dx)\right)^2} + \frac{b^2\cos^{2+m}(c+dx)}{\left(-a^2+b^2\cos^2(c+dx)\right)^2}\right) d$$

$$= a^2\int \frac{\cos^m(c+dx)}{\left(a^2-b^2\cos^2(c+dx)\right)^2}\,dx - (2ab)\int \frac{\cos^{1+m}(c+dx)}{\left(a^2-b^2\cos^2(c+dx)\right)^2}\,dx + b^2\int \frac{c}{\left(-a^2\right.}$$

$$= \frac{\left(b^2\cos^{2\left(\frac{1}{2}+\frac{m}{2}\right)}(c+dx)\cos^2(c+dx)^{-\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1+m}{2}}}{\left(-a^2+b^2-b^2x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{b^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),2;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{1+m}(c+dx)\cos^2(c+dx)^{\frac{1}{2}(-1}}{\left(a^2-b^2\right)^2 d}$$

Mathematica [B] time = 26.46, size = 7214, normalized size = 24.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^m/(a + b*Cos[c + d*x])^2,x]
[Out] Result too large to show
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^m}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^m}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(dx+c)}{(a+b\cos(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^m}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^m}{(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^m/(a + b*cos(c + d*x))^2, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.775 $\int (a+b\cos(c+dx))^3 \sec^m(c+dx)\,dx$

Optimal. Leaf size=282

$$-\frac{b\left(3a^2(3-m)+b^2(2-m)\right)\sin(c+dx)\sec^{m-4}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(c+dx)\right)}{d(2-m)(4-m)\sqrt{\sin^2(c+dx)}}-\frac{a\left(a^2(2-m)+3b^2(1-\right.}{}$$

```
[Out] -a^2*b*(1-2*m)*sec(d*x+c)^(-2+m)*sin(d*x+c)/d/(m^2-3*m+2)-a^2*sec(d*x+c)^(-
2+m)*(b+a*sec(d*x+c))*sin(d*x+c)/d/(1-m)-b*(b^2*(2-m)+3*a^2*(3-m))*hypergeo
m([1/2, 2-1/2*m],[3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-4+m)*sin(d*x+c)/d/(m^
2-6*m+8)/(sin(d*x+c)^2)^(1/2)-a*(3*b^2*(1-m)+a^2*(2-m))*hypergeom([1/2, 3/2
-1/2*m],[5/2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-3+m)*sin(d*x+c)/d/(m^2-4*m+3
)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3842, 4047, 3772, 2643, 4046}

$$-\frac{b\left(3a^2(3-m)+b^2(2-m)\right)\sin(c+dx)\sec^{m-4}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(c+dx)\right)}{d(2-m)(4-m)\sqrt{\sin^2(c+dx)}}-\frac{a\left(a^2(2-m)+3b^2(1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^m,x]
```

```
[Out] -((a^2*b*(1 - 2*m)*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(d*(1 - m)*(2 - m)))
 - (a^2*Sec[c + d*x]^(-2 + m)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(d*(1 - m)
) - (b*(b^2*(2 - m) + 3*a^2*(3 - m))*Hypergeometric2F1[1/2, (4 - m)/2, (6 -
 m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-4 + m)*Sin[c + d*x])/(d*(2 - m)*(4 -
m)*Sqrt[Sin[c + d*x]^2]) - (a*(3*b^2*(1 - m) + a^2*(2 - m))*Hypergeometric2
F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-3 + m)*Sin[c +
 d*x])/(d*(1 - m)*(3 - m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
```

```
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx = \int \sec^{-3+m}(c+dx)(b+a\sec(c+dx))^3\,dx$$
$$= -\frac{a^2\sec^{-2+m}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{d(1-m)} + \frac{\int \sec^{-3+m}(c+d}{}$$
$$= -\frac{a^2\sec^{-2+m}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{d(1-m)} + \Bigg(a\Bigg(3b^2+\frac{a^2(2-}{1-}$$
$$= -\frac{a^2b(1-2m)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(1-m)(2-m)} - \frac{a^2\sec^{-2+m}(c+dx)(b+a}{d(1-}$$
$$= -\frac{a^2b(1-2m)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(1-m)(2-m)} - \frac{a^2\sec^{-2+m}(c+dx)(b+a}{d(1-}$$
$$= -\frac{a^2b(1-2m)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(1-m)(2-m)} - \frac{a^2\sec^{-2+m}(c+dx)(b+a}{d(1-}$$

Mathematica [A] time = 0.88, size = 222, normalized size = 0.79

$$\frac{\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)\sec^{m-4}(c+dx)\left(\frac{1}{2}a(m-3)\sec^3(c+dx)\left(2a(m-2)\left(a(m-1)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\sec^2(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^m,x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]^(-4 + m)*(b^3*m*(2 - 3*m + m^2)*Hypergeometric2F
1[1/2, (-3 + m)/2, (-1 + m)/2, Sec[c + d*x]^2] + (a*(-3 + m)*(6*b^2*(-1 + m
)*m*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[c + d*x]^2]
+ 2*a*(-2 + m)*(3*b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 +
m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, S
ec[c + d*x]^2]))*Sec[c + d*x]^3)/2)*Sqrt[-Tan[c + d*x]^2])/(d*(-3 + m)*(-2
+ m)*(-1 + m)*m)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="fricas")
```

[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^m, x)

maple [F] time = 1.51, size = 0, normalized size = 0.00

$$\int (a+b\cos(dx+c))^3 (\sec^m(dx+c)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x)

[Out] int((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^m (a+b\,\cos(c+d\,x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^3 \sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**m,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**m, x)
```

3.776 $\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx$

Optimal. Leaf size=200

$$-\frac{\left(a^2(2-m)+b^2(1-m)\right)\sin(c+dx)\sec^{m-3}(c+dx)\,_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(c+dx)\right)}{d(1-m)(3-m)\sqrt{\sin^2(c+dx)}}-\frac{a^2\sin(c+dx)\sec^{m-1}(c+d}{d(1-m)}$$

```
[Out] -a^2*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(1-m)-(b^2*(1-m)+a^2*(2-m))*hypergeom([
1/2, 3/2-1/2*m],[5/2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-3+m)*sin(d*x+c)/d/(m
^2-4*m+3)/(sin(d*x+c)^2)^(1/2)-2*a*b*hypergeom([1/2, 1-1/2*m],[2-1/2*m],cos
(d*x+c)^2)*sec(d*x+c)^(-2+m)*sin(d*x+c)/d/(2-m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3238, 3788, 3772, 2643, 4046}

$$-\frac{\left(a^2(2-m)+b^2(1-m)\right)\sin(c+dx)\sec^{m-3}(c+dx)\,_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(c+dx)\right)}{d(1-m)(3-m)\sqrt{\sin^2(c+dx)}}-\frac{a^2\sin(c+dx)\sec^{m-1}(c+d}{d(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^m,x]
```

```
[Out] -((a^2*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m))) - ((b^2*(1 - m) + a
^2*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Se
c[c + d*x]^(-3 + m)*Sin[c + d*x])/(d*(1 - m)*(3 - m)*Sqrt[Sin[c + d*x]^2])
- (2*a*b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx = \int \sec^{-2+m}(c+dx)(b+a\sec(c+dx))^2\,dx$$
$$= (2ab)\int \sec^{-1+m}(c+dx)\,dx + \int \sec^{-2+m}(c+dx)\left(b^2+a^2\sec^2(c+dx\right.$$
$$= -\frac{a^2\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m)} + \left(b^2+\frac{a^2(2-m)}{1-m}\right)\int \sec^{-2+m}(c+d$$
$$= -\frac{a^2\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m)} - \frac{2ab\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2}}$$
$$= -\frac{a^2\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m)} - \frac{\left(b^2+\frac{a^2(2-m)}{1-m}\right){}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos\right.}{d(3-m)}$$

Mathematica [A] time = 0.34, size = 159, normalized size = 0.80

$$\frac{\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)\sec^{m-3}(c+dx)\left(a(m-2)\sec^2(c+dx)\left(a(m-1)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\sec^2(c+dx)\right)+2b\right.\right.}{d(m-2)(m-1)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^m,x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]^(-3 + m)*(b^2*(-1 + m)*m*Hypergeometric2F1[1/2,
(-2 + m)/2, m/2, Sec[c + d*x]^2] + a*(-2 + m)*(2*b*m*Cos[c + d*x]*Hypergeom
etric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeom
etric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2])*Sec[c + d*x]^2)*Sqrt[-Tan[c
+ d*x]^2])/(d*(-2 + m)*(-1 + m)*m)
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

maple [F] time = 1.30, size = 0, normalized size = 0.00

$$\int\left(a+b\cos\left(dx+c\right)\right)^2\left(\sec^m\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x)
```

```
[Out] int((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^m (a+b\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))^2 \sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**m,x)
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**m, x)
```

3.777 $\int (a + b\cos(c + dx))\sec^m(c + dx)\,dx$

Optimal. Leaf size=143

$$-\frac{a\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d(1-m)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2\right.}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b*hypergeom([1/2, 1-1/2*m],[2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-2+m)*sin(d
*x+c)/d/(2-m)/(sin(d*x+c)^2)^(1/2)-a*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m]
,cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(1-m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3238, 3787, 3772, 2643}

$$-\frac{a\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d(1-m)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2\right.}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^m,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c + d
*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])) - (a*Hypergeom
etric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*S
in[c + d*x])/(d*(1 - m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx &= \int \sec^{-1+m}(c+dx)(b+a\sec(c+dx))\,dx\\ &= a\int \sec^m(c+dx)\,dx + b\int \sec^{-1+m}(c+dx)\,dx\\ &= (a\cos^m(c+dx)\sec^m(c+dx))\int \cos^{-m}(c+dx)\,dx + (b\cos^m(c+dx)\operatorname{sec}\\ &= -\frac{b\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(2-m)\sqrt{\sin^2(c+dx)}} - \frac{a\,{}_2F_1}{}\end{aligned}$$

Mathematica [A] time = 0.20, size = 107, normalized size = 0.75

$$\frac{\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)\sec^{m-1}(c+dx)\left(a(m-1)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\sec^2(c+dx)\right)+bm\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m-}{2}\right.\right.}{d(m-1)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^m,x]
```

```
[Out] (Csc[c + d*x]*(b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/
2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[
c + d*x]^2])*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + m)*m)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((b\cos(dx+c)+a)\sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

maple [F] time = 1.10, size = 0, normalized size = 0.00

$$\int (a+b\cos(dx+c))\left(\sec^m(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^m,x)
```

```
[Out] int((a+b*cos(d*x+c))*sec(d*x+c)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c)+a)\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(c+d\,x)}\right)^m\,(a+b\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**m,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**m, x)
```

3.778 $\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx$

Optimal. Leaf size=26

$$-2\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\,\sqrt{a-\cos(x)}}\right)$$

```
[Out] -2*arctan(sin(x)/(1-cos(x))^(1/2)/(a-cos(x))^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2775, 204}

$$-2\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\,\sqrt{a-\cos(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[x]]/Sqrt[a - Cos[x]],x]
```

```
[Out] -2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{1-\cos(x)}\,\sqrt{a-\cos(x)}}\right)\\ &= -2\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\,\sqrt{a-\cos(x)}}\right)\end{aligned}$$

Mathematica [C] time = 0.08, size = 47, normalized size = 1.81

$$i\sqrt{2-2\cos(x)}\,\csc\left(\frac{x}{2}\right)\log\left(\sqrt{a-\cos(x)}+i\sqrt{2}\,\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]]/Sqrt[a - Cos[x]],x]
[Out] I*Sqrt[2 - 2*Cos[x]]*Csc[x/2]*Log[I*Sqrt[2]*Cos[x/2] + Sqrt[a - Cos[x]]]
```

fricas [A] time = 1.85, size = 30, normalized size = 1.15

$$\arctan\left(\frac{(a-2\,\cos(x)-1)\sqrt{-\cos(x)+1}}{2\,\sqrt{a-\cos(x)}\,\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="fricas")
[Out] arctan(1/2*(a - 2*cos(x) - 1)*sqrt(-cos(x) + 1)/(sqrt(a - cos(x))*sin(x)))
```

giac [B] time = 0.53, size = 71, normalized size = 2.73

$$4\,\arctan\left(-\frac{1}{4}\,\sqrt{2}\left(\sqrt{a-1}\,\tan\left(\frac{1}{4}\,x\right)^{2}-\sqrt{a\tan\left(\frac{1}{4}\,x\right)^{4}-\tan\left(\frac{1}{4}\,x\right)^{4}+2\,a\tan\left(\frac{1}{4}\,x\right)^{2}+6\,\tan\left(\frac{1}{4}\,x\right)^{2}+a-1}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="giac")
[Out] 4*arctan(-1/4*sqrt(2)*(sqrt(a - 1)*tan(1/4*x)^2 - sqrt(a*tan(1/4*x)^4 - tan
(1/4*x)^4 + 2*a*tan(1/4*x)^2 + 6*tan(1/4*x)^2 + a - 1) + sqrt(a - 1)))*sgn(
sin(1/2*x))
```

maple [B] time = 0.19, size = 67, normalized size = 2.58

$$-\frac{(2-2\cos(x))^{\frac{3}{2}}\,\sqrt{a-\cos(x)}\,\arctan\left(\frac{\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}\,\sqrt{2}}{2}\right)}{\sin(x)\,(-1+\cos(x))\,\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x)
```

```
[Out] -(2-2*cos(x))^(3/2)*(a-cos(x))^(1/2)*arctan(1/2*(-2*(-a+cos(x))/(cos(x)+1))
^(1/2)*2^(1/2))/sin(x)/(-1+cos(x))/(-2*(-a+cos(x))/(cos(x)+1))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(x))^(1/2)/(a - cos(x))^(1/2),x)
```

```
[Out] int((1 - cos(x))^(1/2)/(a - cos(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\cos{\left(x \right)}}}{\sqrt{a-\cos{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))**(1/2)/(a-cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cos(x))/sqrt(a - cos(x)), x)
```

3.779 $\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}}$$

```
[Out] -2*arctan(sin(x)/(1-cos(x))^(1/2)/(a-cos(x))^(1/2))*((1-cos(x))/(a-cos(x)))
^(1/2)*(a-cos(x))^(1/2)/(1-cos(x))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4400, 2775, 204}

$$-\frac{2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - Cos[x])/(a - Cos[x])],x]
```

```
[Out] (-2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]*Sqrt[(1 - Cos[x])/(a
 - Cos[x])]*Sqrt[a - Cos[x]])/Sqrt[1 - Cos[x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
```

```
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx = \frac{\left(\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\right)\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx}{\sqrt{1-\cos(x)}}$$

$$= \frac{\left(2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}}{\sqrt{1-\cos(x)}}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 0.98

$$-\sqrt{2}\csc\left(\frac{x}{2}\right)\sqrt{\frac{\cos(x)-1}{\cos(x)-a}}\sqrt{\cos(x)-a}\log\left(\sqrt{\cos(x)-a}+\sqrt{2}\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - Cos[x])/(a - Cos[x])],x]
```

```
[Out] -(Sqrt[2]*Sqrt[(-1 + Cos[x])/(-a + Cos[x])]*Sqrt[-a + Cos[x]]*Csc[x/2]*Log[
Sqrt[2]*Cos[x/2] + Sqrt[-a + Cos[x]]])
```

fricas [A] time = 1.47, size = 32, normalized size = 0.49

$$-\arctan\left(-\frac{(a-2\cos(x)-1)\sqrt{-\frac{\cos(x)-1}{a-\cos(x)}}}{2\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*(a - 2*cos(x) - 1)*sqrt(-(cos(x) - 1)/(a - cos(x)))/sin(x))
```

giac [A] time = 0.86, size = 46, normalized size = 0.71

$$2 \arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)^{2}+a-1}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)\right)\operatorname{sgn}\left(a-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*x)^2 + tan(1/2*x)^2 + a - 1))*sgn(tan(1
/2*x)^3 + tan(1/2*x))*sgn(a - cos(x))
```

maple [A] time = 0.14, size = 67, normalized size = 1.03

$$-\frac{\sqrt{2}\,\sqrt{\frac{-1+\cos(x)}{-a+\cos(x)}}\,\sin(x)\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}\,\arctan\left(\frac{\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}\,\sqrt{2}}{2}\right)}{-1+\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-cos(x))/(a-cos(x)))^(1/2),x)
```

```
[Out] -2^(1/2)*((-1+cos(x))/(-a+cos(x)))^(1/2)*sin(x)*(-2*(-a+cos(x))/(cos(x)+1))
^(1/2)*arctan(1/2*(-2*(-a+cos(x))/(cos(x)+1))^(1/2)*2^(1/2))/(-1+cos(x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-\frac{\cos(x)-1}{a-\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(cos(x) - 1)/(a - cos(x)))^(1/2),x)
[Out] int((-(cos(x) - 1)/(a - cos(x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-cos(x))/(a-cos(x)))**(1/2),x)
[Out] Integral(sqrt((1 - cos(x))/(a - cos(x))), x)
```

3.780 $\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx$

Optimal. Leaf size=37

$$\frac{aB\sin(c+dx)}{2d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] 1/2*a*B*sin(d*x+c)/d+1/2*a*B*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2734}

$$\frac{aB\sin(c+dx)}{2d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(-B/2 + B*Cos[c + d*x]),x]
```

```
[Out] (a*B*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right)\,dx=\frac{aB\sin(c+dx)}{2d}+\frac{aB\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.06, size = 29, normalized size = 0.78

$$\frac{aB(2\sin(c+dx)+\sin(2(c+dx))+2c)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(-1/2*B + B*Cos[c + d*x]),x]
```

```
[Out] (a*B*(2*c + 2*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 0.74, size = 24, normalized size = 0.65

$$\frac{(Ba\cos(dx+c)+Ba)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*a*cos(d*x + c) + B*a)*sin(d*x + c)/d
```

giac [A] time = 0.50, size = 30, normalized size = 0.81

$$\frac{Ba\sin(2\,dx+2\,c)}{4\,d}+\frac{Ba\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*B*a*sin(2*d*x + 2*c)/d + 1/2*B*a*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 51, normalized size = 1.38

$$\frac{2aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aB\sin(dx+c)-aB(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x)
```

```
[Out] 1/2/d*(2*a*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*B*sin(d*x+c)-a*B*(
d*x+c))
```

maxima [A] time = 0.32, size = 45, normalized size = 1.22

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba-2\,(dx+c)Ba+2\,Ba\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 2*(d*x + c)*B*a + 2*B*a*sin(d*x
 + c))/d
```

mupad [B] time = 0.63, size = 25, normalized size = 0.68

$$\frac{B\,a\,\left(2\,\sin(c+d\,x)+\sin(2\,c+2\,d\,x)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B/2 - B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (B*a*(2*sin(c + d*x) + sin(2*c + 2*d*x)))/(4*d)
```

sympy [A] time = 0.28, size = 87, normalized size = 2.35

$$\begin{cases} \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} - \frac{Bax}{2} + \frac{Ba\sin(c+dx)\cos(c+dx)}{2d} + \frac{Ba\sin(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(B\cos(c) - \frac{B}{2}\right)(a\cos(c) + a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*a*x*sin(c + d*x)**2/2 + B*a*x*cos(c + d*x)**2/2 - B*a*x/2 + B*
a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/(2*d), Ne(d, 0)), (x*(
B*cos(c) - B/2)*(a*cos(c) + a), True))
```

3.781 $\int (a+a\cos(c+dx))^4 \left(-\frac{4B}{5}+B\cos(c+dx)\right) dx$

Optimal. Leaf size=26

$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

[Out] 1/5*B*(a+a*cos(d*x+c))^4*sin(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2749}

$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^4}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^4*((-4*B)/5 + B*Cos[c + d*x]),x]

[Out] (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\left(-\frac{4B}{5}+B\cos(c+dx)\right)dx=\frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d}$$

Mathematica [A] time = 0.21, size = 31, normalized size = 1.19

$$\frac{a^4B\sin^9(c+dx)\csc^8\left(\frac{1}{2}(c+dx)\right)}{80d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^4*((-4*B)/5 + B*Cos[c + d*x]),x]

[Out] (a^4*B*Csc[(c + d*x)/2]^8*Sin[c + d*x]^9)/(80*d)

fricas [B] time = 0.82, size = 70, normalized size = 2.69

$$\frac{\left(Ba^4 \cos\left(dx+c\right)^4 + 4\,Ba^4 \cos\left(dx+c\right)^3 + 6\,Ba^4 \cos\left(dx+c\right)^2 + 4\,Ba^4 \cos\left(dx+c\right) + Ba^4\right) \sin\left(dx+c\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5*(B*a^4*cos(d*x + c)^4 + 4*B*a^4*cos(d*x + c)^3 + 6*B*a^4*cos(d*x + c)^2
 + 4*B*a^4*cos(d*x + c) + B*a^4)*sin(d*x + c)/d
```

giac [B] time = 0.40, size = 88, normalized size = 3.38

$$\frac{Ba^4 \sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{Ba^4 \sin\left(4\,dx+4\,c\right)}{10\,d} + \frac{27\,Ba^4 \sin\left(3\,dx+3\,c\right)}{80\,d} + \frac{3\,Ba^4 \sin\left(2\,dx+2\,c\right)}{5\,d} + \frac{21\,Ba^4 \sin\left(dx+c\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/80*B*a^4*sin(5*d*x + 5*c)/d + 1/10*B*a^4*sin(4*d*x + 4*c)/d + 27/80*B*a^4
*sin(3*d*x + 3*c)/d + 3/5*B*a^4*sin(2*d*x + 2*c)/d + 21/40*B*a^4*sin(d*x +
c)/d
```

maple [B] time = 0.06, size = 150, normalized size = 5.77

$$\frac{a^4B\left(\frac{8}{3}+\cos^4\left(dx+c\right)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin\left(dx+c\right)+16a^4B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{14a^4B\left(2+\mathrm{co}\right.}{}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x)
```

```
[Out] 1/5/d*(a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+16*a^4*B*(1/4*(
cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+14/3*a^4*B*(2+cos(d*
x+c)^2)*sin(d*x+c)-4*a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-11*a^4
*B*sin(d*x+c)-4*a^4*B*(d*x+c))
```

maxima [B] time = 0.34, size = 144, normalized size = 5.54

$$\frac{2\left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)Ba^4 - 28\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)Ba^4 + 3\left(12\,dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(2*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 28
*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4
- 24*(d*x + c)*B*a^4 - 66*B*a^4*sin(d*x + c))/d
```

mupad [B] time = 0.68, size = 29, normalized size = 1.12

$$\frac{32\,B\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((4*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (32*B*a^4*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2))/(5*d)
```

sympy [A] time = 2.46, size = 333, normalized size = 12.81

$$\begin{cases} \frac{6Ba^4x\sin^4(c+dx)}{5} + \frac{12Ba^4x\sin^2(c+dx)\cos^2(c+dx)}{5} - \frac{2Ba^4x\sin^2(c+dx)}{5} + \frac{6Ba^4x\cos^4(c+dx)}{5} - \frac{2Ba^4x\cos^2(c+dx)}{5} - \frac{4Ba^4x}{5} + \frac{8Ba^4\sin^5(c+\end{matrix}}{15d} \\ x\left(B\cos(c) - \frac{4B}{5}\right)(a\cos(c)+a)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(-4/5*B+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((6*B*a**4*x*sin(c + d*x)**4/5 + 12*B*a**4*x*sin(c + d*x)**2*cos(c
 + d*x)**2/5 - 2*B*a**4*x*sin(c + d*x)**2/5 + 6*B*a**4*x*cos(c + d*x)**4/5
- 2*B*a**4*x*cos(c + d*x)**2/5 - 4*B*a**4*x/5 + 8*B*a**4*sin(c + d*x)**5/(1
5*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 6*B*a**4*sin(c + d*
x)**3*cos(c + d*x)/(5*d) + 28*B*a**4*sin(c + d*x)**3/(15*d) + B*a**4*sin(c
+ d*x)*cos(c + d*x)**4/d + 2*B*a**4*sin(c + d*x)*cos(c + d*x)**3/d + 14*B*a
**4*sin(c + d*x)*cos(c + d*x)**2/(5*d) - 2*B*a**4*sin(c + d*x)*cos(c + d*x)
/(5*d) - 11*B*a**4*sin(c + d*x)/(5*d), Ne(d, 0)), (x*(B*cos(c) - 4*B/5)*(a*
cos(c) + a)**4, True))
```

3.782 $\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n} + B\cos(c+dx)\right) dx$

Optimal. Leaf size=28

$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^n}{d(n+1)}$$

```
[Out] B*(a+a*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2749}

$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^n*(-((B*n)/(1 + n)) + B*Cos[c + d*x]),x]
```

```
[Out] (B*(a + a*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n))
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n}+B\cos(c+dx)\right) dx = \frac{B(a+a\cos(c+dx))^n\sin(c+dx)}{d(1+n)}$$

Mathematica [A] time = 0.20, size = 28, normalized size = 1.00

$$\frac{B\sin(c+dx)(a(\cos(c+dx)+1))^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^n*(-((B*n)/(1 + n)) + B*Cos[c + d*x]),x]
```

```
[Out] (B*(a*(1 + Cos[c + d*x]))^n*Sin[c + d*x])/(d*(1 + n))
```

fricas [A] time = 1.44, size = 27, normalized size = 0.96

$$\frac{(a\cos(dx+c)+a)^n B\sin(dx+c)}{dn+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] (a*cos(d*x + c) + a)^n*B*sin(d*x + c)/(d*n + d)
```

giac [B] time = 55.25, size = 1370, normalized size = 48.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(B*(sqrt(-tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^4 + 2*tan(d*x + c)^4*tan(1
/2*d*x + 1/2*c)^2 - tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^4 + 3*tan(d*x + c)^
4 + 6*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 7*tan(d*x + c)^2 + 4*tan(1/2*
d*x + 1/2*c)^2 + 4)*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x
 + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))^n*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x
+ 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*
d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)
*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/
2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*
x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/
2*c) - B*(sqrt(-tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^4 + 2*tan(d*x + c)^4*ta
n(1/2*d*x + 1/2*c)^2 - tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^4 + 3*tan(d*x +
c)^4 + 6*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 7*tan(d*x + c)^2 + 4*tan(1
/2*d*x + 1/2*c)^2 + 4)*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(
d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))^n*tan(1/2*d*x + 1/2*c))/(d*n*tan(
-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/
2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))
- 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/
2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x
+ 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x
 + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + d*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x
+ 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*
d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)
```

```
*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/
2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*
x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/
2*c)^2 + d*n*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn
(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1
/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1
/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) -
 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)
*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*
n*sgn(tan(1/2*d*x + 1/2*c)))^2 + d*n*tan(1/2*d*x + 1/2*c)^2 + d*tan(-1/4*pi
*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) +
 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*p
i*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x +
 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c
)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*
c)))^2 + d*tan(1/2*d*x + 1/2*c)^2 + d*n + d)
```

maple [B] time = 0.37, size = 74, normalized size = 2.64

$$\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{e}^{n\ln\left(a+\frac{a\left(1-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{d\left(1+n\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x)
```

```
[Out] 2*B/d/(1+n)*tan(1/2*d*x+1/2*c)*exp(n*ln(a+a*(1-tan(1/2*d*x+1/2*c)^2)/(1+tan
(1/2*d*x+1/2*c)^2)))/(1+tan(1/2*d*x+1/2*c)^2)
```

maxima [B] time = 0.63, size = 143, normalized size = 5.11

$$-\frac{\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)^n Ba^n\sin\left(-(dx+c)(n+1)+2\,n\arctan\left(\sin\left(dx+c\right),\cos\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -1/2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*B*a^n*sin(-(
d*x + c)*(n + 1) + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1)) - (cos(d*x
+ c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*B*a^n*sin(-(d*x + c)*(n - 1
) + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1)))/(2^n*d*(n + 1))
```

mupad [B] time = 0.89, size = 28, normalized size = 1.00

$$\frac{B\,\sin\left(c+d\,x\right)\,\left(a\,\left(\cos\left(c+d\,x\right)+1\right)\right)^n}{d\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(c + d*x) - (B*n)/(n + 1))*(a + a*cos(c + d*x))^n,x)
[Out] (B*sin(c + d*x)*(a*(cos(c + d*x) + 1))^n)/(d*(n + 1))
```

sympy [A] time = 4.99, size = 114, normalized size = 4.07

$$\begin{cases} \frac{2B\left(a-\frac{a\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{a}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^n\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{dn\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dn+d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+d} & \text{for } d\neq 0 \\ x\left(a\cos\left(c\right)+a\right)^n\left(-\frac{Bn}{n+1}+B\cos\left(c\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**n*(-B*n/(1+n)+B*cos(d*x+c)),x)
[Out] Piecewise((2*B*(a - a*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + a/(ta
n(c/2 + d*x/2)**2 + 1))**n*tan(c/2 + d*x/2)/(d*n*tan(c/2 + d*x/2)**2 + d*n
+ d*tan(c/2 + d*x/2)**2 + d), Ne(d, 0)), (x*(a*cos(c) + a)**n*(-B*n/(n + 1)
 + B*cos(c)), True))
```

3.783 $\int \frac{-\frac{3B}{2}+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{B\sin(c+dx)}{2d(a\cos(c+dx)+a)^3}$$

```
[Out] -1/2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2749}

$$-\frac{B\sin(c+dx)}{2d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3*B)/2 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(B*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^3)
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int \frac{-\frac{3B}{2}+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{B\sin(c+dx)}{2d(a+a\cos(c+dx))^3}$$

Mathematica [A] time = 0.13, size = 27, normalized size = 1.04

$$-\frac{B\sin(c+dx)}{2a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3*B)/2 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/2*(B*Sin[c + d*x])/(a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [B] time = 0.75, size = 56, normalized size = 2.15

$$-\frac{B\sin(dx+c)}{2\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*B*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.36, size = 47, normalized size = 1.81

$$-\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(B*tan(1/2*d*x + 1/2*c)^5 + 2*B*tan(1/2*d*x + 1/2*c)^3 + B*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

maple [A] time = 0.07, size = 48, normalized size = 1.85

$$\frac{B\left(-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/8/d*B/a^3*(-tan(1/2*d*x+1/2*c)^5-2*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c
))
```

maxima [B] time = 0.34, size = 115, normalized size = 4.42

$$-\frac{\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}-\frac{2\,B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/40*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 2*B*(5*sin(d*x +
c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

mupad [B] time = 0.64, size = 33, normalized size = 1.27

$$-\frac{B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((3*B)/2 - B*cos(c + d*x))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] -(B*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 1)^2)/(8*a^3*d)
```

sympy [A] time = 2.37, size = 80, normalized size = 3.08

$$\begin{cases} -\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^3d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} - \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^3d} & \text{for}\ d \neq 0 \\ \frac{x\left(B\cos(c)-\frac{3B}{2}\right)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-B*tan(c/2 + d*x/2)**5/(8*a**3*d) - B*tan(c/2 + d*x/2)**3/(4*a**
3*d) - B*tan(c/2 + d*x/2)/(8*a**3*d), Ne(d, 0)), (x*(B*cos(c) - 3*B/2)/(a*c
os(c) + a)**3, True))
```

3.784 $\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)dx$

Optimal. Leaf size=28

$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

```
[Out] 2/5*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2749}

$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*((-3*B)/5 + B*Cos[c + d*x]),x]
```

```
[Out] (2*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)dx=\frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

Mathematica [A] time = 0.12, size = 45, normalized size = 1.61

$$\frac{8aB\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*((-3*B)/5 + B*Cos[c + d*x]),x]
```

```
[Out] (8*a*B*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Cos[c + d*x])]*Sin[(c + d*x)/2])/(5*d
)
```

fricas [A] time = 1.03, size = 36, normalized size = 1.29

$$\frac{2\left(Ba\cos\left(dx+c\right)+Ba\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 2/5*(B*a*cos(d*x + c) + B*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/d
```

giac [B] time = 0.45, size = 86, normalized size = 3.07

$$\frac{1}{10}\,\sqrt{2}\left(\frac{Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}{d}+\frac{2\,Ba\mathrm{sgn}\left(\mathrm{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2)*(B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 3*B*a*
sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 2*B*a*sgn(cos(1/2*d*x +
1/2*c))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.30, size = 48, normalized size = 1.71

$$\frac{8\left(\cos^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)B\sqrt{2}}{5\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x)
```

```
[Out] 8/5*cos(1/2*d*x+1/2*c)^5*a^2*sin(1/2*d*x+1/2*c)*B*2^(1/2)/(a*cos(1/2*d*x+1/
2*c)^2)^(1/2)/d
```

maxima [B] time = 0.54, size = 92, normalized size = 3.29

$$\frac{\left(\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}\,-2\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\right.}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/10*((sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) +
20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) - 2*(sqrt(2)*a*sin(3/2*d*x + 3
/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\left(\frac{3\,B}{5}-B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((3*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(-((3*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{B\left(\int\left(-3a\sqrt{a\cos\left(c+dx\right)+a}\right)dx+\int 2a\sqrt{a\cos\left(c+dx\right)+a}\cos\left(c+dx\right)dx+\int 5a\sqrt{a\cos\left(c+dx\right)+a}\cos^{2}\left(c+\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(-3/5*B+B*cos(d*x+c)),x)
```

```
[Out] B*(Integral(-3*a*sqrt(a*cos(c + d*x) + a), x) + Integral(2*a*sqrt(a*cos(c +
 d*x) + a)*cos(c + d*x), x) + Integral(5*a*sqrt(a*cos(c + d*x) + a)*cos(c +
 d*x)**2, x))/5
```

3.785 $\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=26

$$\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {21, 2646}

$$\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(B + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx &= \frac{B\int \sqrt{a+a\cos(c+dx)}\,dx}{a}\\ &= \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 33, normalized size = 1.27

$$\frac{2B\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
[Out] (2*B*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/(a*d)
```

fricas [A] time = 0.97, size = 36, normalized size = 1.38

$$\frac{2\sqrt{a\cos(dx+c)+a}\,B\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.96, size = 35, normalized size = 1.35

$$\frac{2\sqrt{2}\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*B*tan(1/2*d*x + 1/2*c)/(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

maple [A] time = 0.23, size = 43, normalized size = 1.65

$$\frac{2B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

[Out] 2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 0.74, size = 37, normalized size = 1.42

$$\frac{2\,B\,\sin\left(c+d\,x\right)\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}}{a\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/2),x)

[Out] (2*B*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(a*d*(cos(c + d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\left(\int\frac{\cos\left(c+dx\right)}{\sqrt{a\cos\left(c+dx\right)+a}}\,dx+\int\frac{1}{\sqrt{a\cos\left(c+dx\right)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)

[Out] B*(Integral(cos(c + d*x)/sqrt(a*cos(c + d*x) + a), x) + Integral(1/sqrt(a*cos(c + d*x) + a), x))

3.786 $\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2B\sin(c+dx)}{3d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] -2/3*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2749}

$$-\frac{2B\sin(c+dx)}{3d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((-5*B)/3 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*B*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^(5/2))
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{2B\sin(c+dx)}{3d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [A] time = 0.08, size = 28, normalized size = 1.00

$$-\frac{2B\sin(c+dx)}{3d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-5*B)/3 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*B*Sin[c + d*x])/(3*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [B] time = 0.96, size = 68, normalized size = 2.43

$$-\frac{2\sqrt{a\cos(dx+c)+a}\,B\sin(dx+c)}{3\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas
")
```

```
[Out] -2/3*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [B] time = 1.83, size = 59, normalized size = 2.11

$$-\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{\sqrt{2}\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}+\frac{\sqrt{2}\,B}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*B*tan(1/2*d*x + 1/2*c)^2/a
^3 + sqrt(2)*B/a^3)*tan(1/2*d*x + 1/2*c)/d
```

maple [A] time = 0.20, size = 48, normalized size = 1.71

$$-\frac{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)B\sqrt{2}}{6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^2\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/6/cos(1/2*d*x+1/2*c)^3/a^2*sin(1/2*d*x+1/2*c)*B*2^(1/2)/(a*cos(1/2*d*x+1
/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima
")
```

```
[Out] Timed out
```

mupad [B] time = 5.33, size = 85, normalized size = 3.04

$$\frac{8\,B\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,\sqrt{a+a\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((5*B)/3 - B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] (8*B*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2))^(1/2)*(exp(c*1i + d*x*1i)*1i - 1i))/(3*a^3*d*(exp(c*1i + d*x*1i) + 1)^
5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.787 $\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt[6]{2}\,(5A+2B)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{5d(\cos(c+dx)+1)^{7/6}}+\frac{3B\sin(c+dx)(a\cos(c+dx}{5d}$$

```
[Out] 3/5*B*(a+a*cos(d*x+c))^(2/3)*sin(d*x+c)/d+2/5*2^(1/6)*(5*A+2*B)*(a+a*cos(d*
x+c))^(2/3)*hypergeom([-1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)/d/(1
+cos(d*x+c))^(7/6)
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2652, 2651}

$$\frac{2\sqrt[6]{2}\,(5A+2B)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{5d(\cos(c+dx)+1)^{7/6}}+\frac{3B\sin(c+dx)(a\cos(c+dx}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*B*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) + (2*2^(1/6)*(5*A + 2*B
)*(a + a*Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c +
 d*x])/2]*Sin[c + d*x])/(5*d*(1 + Cos[c + d*x])^(7/6))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{3B(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{1}{5}(5A+2B)\int (a+a\cos$$

$$= \frac{3B(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{\left((5A+2B)(a+a\cos(c\right.}{5(1+}$$

$$= \frac{3B(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{2\sqrt[6]{2}\,(5A+2B)(a+a\cos}{}$$

Mathematica [A] time = 0.66, size = 164, normalized size = 1.58

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{2/3}\left(3\,2^{5/6}\sin(c+dx)(5A+2B\cos(c+dx)+4B)\sqrt[6]{1-\cos\left(dx-2\tan^{-1}\left(\cot\right.\right.}}{20\,2^{5/6}d\sqrt[6]{1-\cos\left(dx-2\tan^{-1}\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*(3*2^(5/6)*(5*A + 4*B + 2*
B*Cos[c + d*x])*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] - 2*
(5*A + 2*B)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/2]]
]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]]))/(20*2^(5/6)*d*(1 - Cos[d*x - 2*ArcTan[
Cot[c/2]]])^(1/6))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int (a + a\cos(dx + c))^{\frac{2}{3}} (A + B\cos(dx + c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
[Out] int((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx + c) + A)(a\cos(dx + c) + a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\,\cos(c + d\,x))\,(a + a\,\cos(c + d\,x))^{2/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(2/3),x)
[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\cos(c + dx) + 1))^{\frac{2}{3}} (A + B\cos(c + dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
[Out] Integral((a*(cos(c + d*x) + 1))**(2/3)*(A + B*cos(c + d*x)), x)
```

3.788 $\int \sqrt[3]{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{(4A+B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}\,d(\cos(c+dx)+1)^{5/6}}+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

```
[Out] 3/4*B*(a+a*cos(d*x+c))^(1/3)*sin(d*x+c)/d+1/4*(4*A+B)*(a+a*cos(d*x+c))^(1/3
)*hypergeom([1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/d/(1+co
s(d*x+c))^(5/6)
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2652, 2651}

$$\frac{(4A+B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}\,d(\cos(c+dx)+1)^{5/6}}+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*B*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) + ((4*A + B)*(a + a*Cos
[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin
[c + d*x])/(2*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sqrt[3]{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{3B\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}(4A+B)\int \sqrt[3]{a+a\cos(\ldots}$$

$$= \frac{3B\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{\left((4A+B)\sqrt[3]{a+a\cos(c+\ldots}\right.}{4\sqrt[3]{1+\text{co}\ldots}}$$

$$= \frac{3B\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{(4A+B)\sqrt[3]{a+a\cos(c+d\ldots}}{2\sqrt[6]{\ldots}}$$

Mathematica [C] time = 3.53, size = 213, normalized size = 2.09

$$\frac{3\sqrt[3]{a(\cos(c+dx)+1)}\left(\frac{2(4A+B)\csc\left(\frac{c}{4}\right)\sec\left(\frac{c}{4}\right)\sqrt[3]{i\sin(c)e^{idx}+\cos(c)e^{idx}+1}\left(2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-e^{idx}(\cos(c)+i\sin(c))\right)+e^{idx}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-e^{idx}(\cos(c\ldots\right.\right.}{i\sin\left(\frac{c}{2}\right)\left(-1+e^{idx}\right)+\cos\left(\frac{c}{2}\right)\left(1+e^{idx}\right)}\ldots}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*(a*(1 + Cos[c + d*x]))^(1/3)*(-8*(4*A + B)*Cot[c/2] + 8*B*Cos[d*x]*Sin[c
] + (2*(4*A + B)*Csc[c/4]*(2*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^(I*d*x)*
(Cos[c] + I*Sin[c]))] + E^(I*d*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -(E^(I*d
*x)*(Cos[c] + I*Sin[c]))])*Sec[c/4]*(1 + E^(I*d*x)*Cos[c] + I*E^(I*d*x)*Sin
[c])^(1/3))/((1 + E^(I*d*x))*Cos[c/2] + I*(-1 + E^(I*d*x))*Sin[c/2]) + 8*B*
Cos[c]*Sin[d*x]))/(32*d)
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int (a+a\cos(dx+c))^{\frac{1}{3}}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
[Out] int((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/3),x)
[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + B*cos(c + d*x)), x)
```

3.789 $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{(2A-B)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\,\sqrt[3]{a\cos(c+dx)+a}}+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

```
[Out] 3/2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+1/2*(2*A-B)*hypergeom([1/2, 5/6],
[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*co
s(d*x+c))^(1/3)
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2652, 2651}

$$\frac{(2A-B)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\,\sqrt[3]{a\cos(c+dx)+a}}+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*B*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(1/3)) + ((2*A - B)*Hypergeome
tric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(5/6)*d*(1 +
Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{3B\sin(c+dx)}{2d\sqrt[3]{a+a\cos(c+dx)}}+\frac{1}{2}(2A-B)\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$$

$$= \frac{3B\sin(c+dx)}{2d\sqrt[3]{a+a\cos(c+dx)}}+\frac{\left((2A-B)\sqrt[3]{1+\cos(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\cos(c+dx)}}\,dx}{2\sqrt[3]{a+a\cos(c+dx)}}$$

$$= \frac{3B\sin(c+dx)}{2d\sqrt[3]{a+a\cos(c+dx)}}+\frac{(2A-B)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\,\sqrt[3]{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.27, size = 133, normalized size = 1.32

$$\frac{3\,2^{5/6}B\sin(c+dx)\sqrt[6]{1-\cos\left(dx-2\tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right)}-2(2A-B)\sin\left(dx-2\tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\cos^2\left(\right.\right.}{4d\sqrt[3]{a(\cos(c+dx)+1)}\,\sqrt[6]{\sin^2\left(\frac{dx}{2}-\tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*2^(5/6)*B*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] - 2*(2*
A - B)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*
Sin[d*x - 2*ArcTan[Cot[c/2]]])/(4*d*(a*(1 + Cos[c + d*x]))^(1/3)*(Sin[(d*x)
/2 - ArcTan[Cot[c/2]]]^2)^(1/6))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(dx+c)}{(a+a\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x)
[Out] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{(a+a\,\cos(c+d\,x))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/3),x)
[Out] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(1/3), x)
```

3.790 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=105

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{2^{5/6}(A-2B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{ad(\cos(c+dx)+1)^{5/6}}$$

```
[Out] 3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)-2^(5/6)*(A-2*B)*(a+a*cos(d*x+c)
)^(1/3)*hypergeom([1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)/a/d/(1+co
s(d*x+c))^(5/6)
```

Rubi [A] time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2750, 2652, 2651}

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{2^{5/6}(A-2B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{ad(\cos(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*(A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) - (2^(5/6)*(A - 2*B
)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c +
d*x])/2]*Sin[c + d*x])/(a*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx &= \frac{3(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} - \frac{(A-2B)\int \sqrt[3]{a+a\cos(c+dx)}\,dx}{a}\\ &= \frac{3(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} - \frac{\left((A-2B)\sqrt[3]{a+a\cos(c+dx)}\right)\int \sqrt[3]{1+\cos(c+dx)}\,dx}{a\sqrt[3]{1+\cos(c+dx)}}\\ &= \frac{3(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} - \frac{2^{5/6}(A-2B)\sqrt[3]{a+a\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c\right.}{ad(1+\cos(c+dx))^{5/6}}\end{aligned}$$

Mathematica [C] time = 1.42, size = 197, normalized size = 1.88

$$\frac{3\cos\left(\frac{1}{2}(c+dx)\right)\left(-4\csc\left(\frac{c}{2}\right)\left((3B-2A)\cos\left(\frac{dx}{2}\right)+B\cos\left(c+\frac{dx}{2}\right)\right)-(A-2B)\csc\left(\frac{c}{4}\right)\sec\left(\frac{c}{4}\right)e^{-\frac{1}{2}idx}\sqrt[3]{i\sin(c)e^{idx}}\right.}{4d(a(\cos(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*Cos[(c + d*x)/2]*(-4*((-2*A + 3*B)*Cos[(d*x)/2] + B*Cos[c + (d*x)/2])*Cs
c[c/2] - ((A - 2*B)*Csc[c/4]*(2*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^(I*d*
x)*(Cos[c] + I*Sin[c]))] + E^(I*d*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -(E^(
I*d*x)*(Cos[c] + I*Sin[c]))])*Sec[c/4]*(1 + E^(I*d*x)*Cos[c] + I*E^(I*d*x)*
Sin[c])^(1/3))/E^((I/2)*d*x)))/(4*d*(a*(1 + Cos[c + d*x]))^(2/3))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(dx+c\right)}{\left(a+a\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x)
[Out] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(2/3),x)
[Out] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(2/3), x)
```

3.791 $\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

```
[Out] B*x/b-2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a
+b)^(1/2)/a/b/d
```

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2735, 2659, 205}

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*x)/b - (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{Bx}{b}-\frac{\left(aB-\frac{b^2B}{a}\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}$$
$$= \frac{Bx}{b}-\frac{\left(2\left(a-\frac{b^2}{a}\right)B\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$
$$= \frac{Bx}{b}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

Mathematica [**A**] time = 0.13, size = 64, normalized size = 1.02

$$\frac{B\left(2\sqrt{b^2-a^2}\,\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)+a(c+dx)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(a*(c + d*x) + 2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[-a^2 + b^2]]))/(a*b*d)
```

fricas [**A**] time = 3.91, size = 194, normalized size = 3.08

$$\left[\frac{2\,Badx+\sqrt{-a^2+b^2}\,B\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\,abd},\ \frac{Badx-\sqrt{a^2-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*d*x + sqrt(-a^2 + b^2)*B*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/(a*b*d), (B*a
*d*x - sqrt(a^2 - b^2)*B*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(
d*x + c))))/(a*b*d)]
```

giac [B] time = 0.74, size = 281, normalized size = 4.46

$$-\frac{\frac{\left(\sqrt{a^2-b^2}\,B|a-b||a||b|+\left(2\,a^2+ab\right)\sqrt{a^2-b^2}\,B|a-b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{\frac{a^2+\sqrt{a^4-\left(a^2+ab\right)\left(a^2-ab\right)}}{a^2-ab}}}\right)\right)}{(a-b)a^2b^2+\left(a^3-a^2b\right)|a||b|}+\frac{\left(2\,Ba^3-Ba^2b-Bab^2-Ba|a||b|+Bb|a||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}\right.\right.}{a^2b^2-a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((sqrt(a^2 - b^2)*B*abs(a - b)*abs(a)*abs(b) + (2*a^2 + a*b)*sqrt(a^2 - b^
2)*B*abs(a - b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1
/2*c)/sqrt((a^2 + sqrt(a^4 - (a^2 + a*b)*(a^2 - a*b)))/(a^2 - a*b))))/((a -
 b)*a^2*b^2 + (a^3 - a^2*b)*abs(a)*abs(b)) + (2*B*a^3 - B*a^2*b - B*a*b^2 -
 B*a*abs(a)*abs(b) + B*b*abs(a)*abs(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) +
 arctan(tan(1/2*d*x + 1/2*c)/sqrt((a^2 - sqrt(a^4 - (a^2 + a*b)*(a^2 - a*b)
))/(a^2 - a*b))))/(a^2*b^2 - a^2*abs(a)*abs(b)))/d
```

maple [B] time = 0.09, size = 117, normalized size = 1.86

$$-\frac{2Ba\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{db\sqrt{(a-b)(a+b)}}+\frac{2Bb\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{da\sqrt{(a-b)(a+b)}}+\frac{2B\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*B*a/b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)
)^(1/2))+2/d*B/a*b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-
b)*(a+b))^(1/2))+2/d*B/b*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.94, size = 93, normalized size = 1.48

$$\frac{2\,B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{2\,B\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a+b)}\right)\,\sqrt{b^2-a^2}}{a\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(c + d*x) + (B*b)/a)/(a + b*cos(c + d*x)),x)
```

```
[Out] (2*B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*B*atanh((sin(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a + b)))*(b^2 - a^2)^
(1/2))/(a*b*d)
```

sympy [A] time = 32.92, size = 235, normalized size = 3.73

$$\begin{cases} \text{NaN} \\ \frac{x\left(B\cos(c)+\frac{Bb}{a}\right)}{a+b\cos(c)} \\ \frac{Bx}{b} \\ \frac{B\sin(c+dx)}{ad} \\ \frac{Bx}{b} \\ \frac{Bx}{b}-\frac{B\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{B\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{B\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{B\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\mathrm{t}\right.}{ad\sqrt{-\frac{a}{a-b}-\frac{}{a}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x*(B*cos(c) + B*b/a)/(a +
 b*cos(c)), Eq(d, 0)), (B*x/b, Eq(a, b)), (B*sin(c + d*x)/(a*d), Eq(b, 0)),
 (B*x/b, Eq(a, -b)), (B*x/b - B*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2
 + d*x/2))/(b*d*sqrt(-a/(a - b) - b/(a - b))) + B*log(sqrt(-a/(a - b) - b/(
a - b)) + tan(c/2 + d*x/2))/(b*d*sqrt(-a/(a - b) - b/(a - b))) - B*log(-sqr
t(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a -
 b))) + B*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a
/(a - b) - b/(a - b))), True))
```

3.792 $\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=22

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+b)}$$

```
[Out] sin(d*x+c)/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2754, 8}

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])/(b + a*Cos[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx &= \frac{\sin(c+dx)}{d(b+a\cos(c+dx))}+\frac{\int 0\,dx}{a^2-b^2}\\ &= \frac{\sin(c+dx)}{d(b+a\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.11, size = 22, normalized size = 1.00

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])/(b + a*Cos[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))
```

fricas [A] time = 0.82, size = 22, normalized size = 1.00

$$\frac{\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(a*d*cos(d*x + c) + b*d)
```

giac [B] time = 1.09, size = 50, normalized size = 2.27

$$-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a-b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)
^2 - a - b)*d)
```

maple [B] time = 0.06, size = 51, normalized size = 2.32

$$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x)
```

```
[Out] -2/d*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [**B**] time = 0.85, size = 37, normalized size = 1.68

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\left(b-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/(b + a*cos(c + d*x))^2,x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(d*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.793 $\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{10\tan^{-1}\left(\frac{\sin(c+dx)}{-\cos(c+dx)+\sqrt{3}+2}\right)}{\sqrt{3}\,d}+\frac{5x}{\sqrt{3}}-x$$

```
[Out] -x+5/3*x*3^(1/2)+10/3*arctan(sin(d*x+c)/(2-cos(d*x+c)+3^(1/2)))/d*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2735, 2657}

$$\frac{10\tan^{-1}\left(\frac{\sin(c+dx)}{-\cos(c+dx)+\sqrt{3}+2}\right)}{\sqrt{3}\,d}+\frac{5x}{\sqrt{3}}-x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Cos[c + d*x])/(2 - Cos[c + d*x]),x]
```

```
[Out] -x + (5*x)/Sqrt[3] + (10*ArcTan[Sin[c + d*x]/(2 + Sqrt[3] - Cos[c + d*x])])
/(Sqrt[3]*d)
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx = -x+5\int \frac{1}{2-\cos(c+dx)}\,dx$$
$$= -x+\frac{5x}{\sqrt{3}}+\frac{10\tan^{-1}\left(\frac{\sin(c+dx)}{2+\sqrt{3}-\cos(c+dx)}\right)}{\sqrt{3}\,d}$$

Mathematica [A] time = 0.06, size = 31, normalized size = 0.66

$$\frac{10\tan^{-1}\left(\sqrt{3}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{3}\,d}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + Cos[c + d*x])/(2 - Cos[c + d*x]),x]
```

```
[Out] -x + (10*ArcTan[Sqrt[3]*Tan[(c + d*x)/2]])/(Sqrt[3]*d)
```

fricas [A] time = 1.88, size = 43, normalized size = 0.91

$$-\frac{3\,dx+5\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(dx+c)-\sqrt{3}}{3\sin(dx+c)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*d*x + 5*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(d*x + c) - sqrt(3))/sin(d
*x + c)))/d
```

giac [A] time = 0.33, size = 72, normalized size = 1.53

$$-\frac{3\,dx-5\sqrt{3}\left(dx+c+2\arctan\left(-\frac{\sqrt{3}\sin(dx+c)-3\sin(dx+c)}{\sqrt{3}\cos(dx+c)+\sqrt{3}-3\cos(dx+c)+3}\right)\right)+3\,c}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*d*x - 5*sqrt(3)*(d*x + c + 2*arctan(-(sqrt(3)*sin(d*x + c) - 3*sin(
d*x + c))/(sqrt(3)*cos(d*x + c) + sqrt(3) - 3*cos(d*x + c) + 3))) + 3*c)/d
```

maple [A] time = 0.10, size = 39, normalized size = 0.83

$$\frac{10\sqrt{3}\ \arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{3}\right)}{3d}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+cos(d*x+c))/(2-cos(d*x+c)),x)
```

```
[Out] 10/3/d*3^(1/2)*arctan(tan(1/2*d*x+1/2*c)*3^(1/2))-2/d*arctan(tan(1/2*d*x+1/
2*c))
```

maxima [A] time = 0.43, size = 52, normalized size = 1.11

$$\frac{2\left(5\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\sin(dx+c)}{\cos(dx+c)+1}\right)-3\ \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(5*sqrt(3)*arctan(sqrt(3)*sin(d*x + c)/(cos(d*x + c) + 1)) - 3*arctan(s
in(d*x + c)/(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.67, size = 74, normalized size = 1.57

$$\frac{\left(\frac{\pi-\frac{5\,\pi\,\sqrt{3}}{3}}{d}-\frac{\pi+\frac{5\,\pi\,\sqrt{3}}{3}}{d}\right)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{\pi}-\frac{d\,x-\frac{10\,\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(c + d*x) + 3)/(cos(c + d*x) - 2),x)
```

```
[Out] (((pi - (5*3^(1/2)*pi)/3)/d - (pi + (5*3^(1/2)*pi)/3)/d)*(atan(tan(c/2 + (d
*x)/2)) - (d*x)/2))/pi - (d*x - (10*3^(1/2)*atan(3^(1/2)*tan(c/2 + (d*x)/2)
))/3)/d
```

sympy [A] time = 2.42, size = 56, normalized size = 1.19

$$\begin{cases} -x+\frac{10\sqrt{3}\left(\mathrm{atan}\left(\sqrt{3}\ \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\pi\left\lfloor\frac{\frac{c}{2}+\frac{dx}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3d} & \text{for}\ d\neq 0\\ \frac{x(\cos(c)+3)}{2-\cos(c)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x)
```

```
[Out] Piecewise((-x + 10*sqrt(3)*(atan(sqrt(3)*tan(c/2 + d*x/2)) + pi*floor((c/2
+ d*x/2 - pi/2)/pi))/(3*d), Ne(d, 0)), (x*(cos(c) + 3)/(2 - cos(c)), True))
```

3.794 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(
a+b))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {21, 2655, 2653}

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= B\int \sqrt{a+b\cos(c+dx)}\,dx\\ &= \frac{\left(B\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 58, normalized size = 1.00

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\,B,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.98, size = 171, normalized size = 2.95

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(a-b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b
+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [B] time = 1.03, size = 56, normalized size = 0.97

$$\frac{2\,B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,(a+b)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*B*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b)*((a + b*cos(c + d*x))/
(a + b))^(1/2))/(d*(a + b*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \sqrt{a + b\cos{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
[Out] B*Integral(sqrt(a + b*cos(c + d*x)), x)
```

3.795 $\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=229

$$\frac{\sqrt{2}\,(Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{\sqrt{2}\,B(a+b)\sin(}{}$$

```
[Out] (a+b)*B*AppellF1(1/2,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c)
)*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(2
/3)/(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c
))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/
((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}\,(Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{\sqrt{2}\,B(a+b)\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(
1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*S
qrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b
 - a*B)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{B\int (a+b\cos(c+dx))^{5/3}\,dx}{b} + \frac{(Ab-aB)\int (a+b\cos(c+dx))}{b}$$

$$= -\frac{(B\sin(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{5/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}} - \frac{((A}{}$$

$$= \frac{\left((-a-b)B(a+b\cos(c+dx))^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\right.}{\sqrt{1-x}}\right.}{bd\sqrt{1-\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}\,\left(-\frac{a+b\cos}{-a}\right.}$$

$$= \frac{\sqrt{2}\,(a+b)BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a}{bd\sqrt{1+\cos(c+dx)}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)}$$

Mathematica [A] time = 2.11, size = 259, normalized size = 1.13

$$3(a+b\cos(c+dx))^{2/3}\left(5B\left(a^2-b^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(}{a+}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*(a + b*Cos[c + d*x])^(2/3)*(5*(a^2 - b^2)*B*AppellF1[2/3, 1/2, 1/2, 5/3,
 (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1
+ Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c +
d*x] - (5*A*b + 2*a*B)*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a
 - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b)
)]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])*Csc[c + d*x]
+ 5*b^2*B*Sin[c + d*x]))/(25*b^2*d)
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int(a+b\cos(dx+c))^{\frac{2}{3}}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(2/3),x)
[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(2/3), x)
```

3.796 $\int \sqrt[3]{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=229

$$\frac{\sqrt{2}\,(Ab-aB)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}\,B(a+b)\sin(c\,\cdots}{\cdots}$$

```
[Out] (a+b)*B*AppellF1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c)
)*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1
/3)/(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*x+c
))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/
((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}\,(Ab-aB)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}\,B(a+b)\sin(c\,\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(
1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*S
qrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b
 - a*B)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{B\int(a+b\cos(c+dx))^{4/3}\,dx}{b}+\frac{(Ab-aB)\int\sqrt[3]{a+b\cos(c+dx)}$$

$$= -\frac{(B\sin(c+dx))\,\text{Subst}\left(\int\frac{(a+bx)^{4/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}-\frac{((Ab$$

$$= \frac{\left((-a-b)B\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)\right)\text{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right.}{\sqrt{1-x}\,\sqrt{1+}}\right.}{bd\sqrt{1-\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}\,\sqrt[3]{-\frac{a+b\cos}{-a}}}$$

$$= \frac{\sqrt{2}\,(a+b)BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a}}{bd\sqrt{1+\cos(c+dx)}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.92, size = 253, normalized size = 1.10

$$-\frac{3\csc(c+dx)\sqrt[3]{a+b\cos(c+dx)}\left(4B\left(b^2-a^2\right)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos}{a+}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*B*AppellF1[1/3,
 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]
*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a
- b))] + (4*A*b + a*B)*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a
 - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b)
)]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*B*Sin
[c + d*x]^2))/(16*b^2*d)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (a+b\cos(dx+c))^{\frac{1}{3}}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

[Out] int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/3),x)

[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\sqrt[3]{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)

[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(1/3), x)

3.797 $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{2}\,(Ab-aB)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\,\sqrt[3]{a+b\cos(c+dx)}}+\frac{\sqrt{2}\,B\sin(c+dx)(a+b\cos$$

```
[Out] B*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b
*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1
+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b
),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)/b/d
/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}\,(Ab-aB)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\,\sqrt[3]{a+b\cos(c+dx)}}+\frac{\sqrt{2}\,B\sin(c+dx)(a+b\cos$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[
c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 +
Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b - a*B)*
AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(
a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Co
s[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{B\int(a+b\cos(c+dx))^{2/3}\,dx}{b}+\frac{(Ab-aB)\int\frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx}{b}$$

$$= -\frac{(B\sin(c+dx))\,\mathrm{Subst}\left(\int\frac{(a+bx)^{2/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}-\frac{((Ab-aB)\sin(c+dx))\,\mathrm{S}}{bd\sqrt{1-\mathrm{c}}}$$

$$= -\frac{\left(B(a+b\cos(c+dx))^{2/3}\sin(c+dx)\right)\mathrm{Subst}\left(\int\frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}}-$$

$$= \frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+}{bd\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [A] time = 0.46, size = 189, normalized size = 0.84

$$-\frac{3\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}(a+b\cos(c+dx))^{2/3}\left(5(Ab-aB)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\right.}{10b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-
a + b)]*(a + b*Cos[c + d*x])^(2/3)*(5*(A*b - a*B)*AppellF1[2/3, 1/2, 1/2, 5
/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)] + 2*B*Appel
lF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/
(a + b)]*(a + b*Cos[c + d*x]))*Csc[c + d*x])/(10*b^2*d)
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int\frac{A+B\cos(dx+c)}{(a+b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\sqrt[3]{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a + b*cos(c + d*x))**(1/3), x)
```

3.798 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{2}\,(Ab-aB)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\,(a+b\cos(c+dx))^{2/3}}+\frac{\sqrt{2}\,B\sin(c+dx)\sqrt[3]{a+}}{}$$

```
[Out] B*AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b
*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1
+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b
),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)*2^(1/2)/b/d
/(a+b*cos(d*x+c))^(2/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}\,(Ab-aB)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\,(a+b\cos(c+dx))^{2/3}}+\frac{\sqrt{2}\,B\sin(c+dx)\sqrt[3]{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[
c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 +
Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b - a*B)*
AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(
a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Co
s[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{B\int\sqrt[3]{a+b\cos(c+dx)}\,dx}{b} + \frac{(Ab-aB)\int\frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx}{b}$$

$$= -\frac{(B\sin(c+dx))\,\text{Subst}\left(\int\frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}} - \frac{((Ab-aB)\sin(c+dx)}{bd\sqrt{1\cdots}}$$

$$= -\frac{\left(B\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}\,\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}} - \cdots$$

$$= \frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+\cdots}{bd\sqrt{1+\cos(c+dx)}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.48, size = 188, normalized size = 0.83

$$-\frac{3\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\sqrt[3]{a+b\cos(c+dx)}\left(4(Ab-aB)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\right.}{4b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-
a + b)]*(a + b*Cos[c + d*x])^(1/3)*(4*(A*b - a*B)*AppellF1[1/3, 1/2, 1/2, 4
/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)] + B*AppellF
1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a
 + b)]*(a + b*Cos[c + d*x]))*Csc[c + d*x])/(4*b^2*d)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int\frac{A+B\cos(dx+c)}{(a+b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x)
```

[Out] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{(a+b\,\cos(c+d\,x))^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(2/3),x)

[Out] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(2/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(2/3),x)

[Out] Integral((A + B*cos(c + d*x))/(a + b*cos(c + d*x))**(2/3), x)

3.799 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=168

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{10B\sin}{}$$

```
[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*
x+c)/b^2/d+10/21*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/
21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2
)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{10B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (10*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos
[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*(b*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c
+ d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^3}$$
$$= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2B(b\cos(c+dx))^{5/2}\,\mathrm{s}}{7b^2d}$$
$$= \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2A(b\cos(c+dx))^{3/2}\,\mathrm{si}}{5bd}$$
$$= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx}}{21d}$$
$$= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10bB\sqrt{\cos(c+dx)}}{21d\sqrt{b\cos}}$$

Mathematica [A] time = 0.58, size = 100, normalized size = 0.60

$$\frac{\sqrt{b\cos(c+dx)}\left(2\sin(c+dx)\sqrt{\cos(c+dx)}\,(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252AE\left(\frac{1}{2}(c+dx)\middle|2\right.\right.}{210d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[(c
 + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2
*(c + d*x)])*Sin[c + d*x]))/(210*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(B\cos(dx+c)^3 + A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

maple [A] time = 0.88, size = 299, normalized size = 1.78

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{d}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(240*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-
42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,\sqrt{b\,\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.800 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=139

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2Ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6BE\left(\frac{1}{2}(c\right.}{}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/3*A*b*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*B*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2Ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6BE\left(\frac{1}{2}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*(b*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx}{b}$$

$$= \frac{A\int (b\cos(c+dx))^{3/2}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^2}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\,\mathrm{sin}}{5bd}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\,\mathrm{sin}}{5bd}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab\sqrt{\cos(c+dx)}\,F}{3d\sqrt{b\cos(c}}$$

Mathematica [A] time = 0.31, size = 91, normalized size = 0.65

$$\frac{2(b\cos(c+dx))^{3/2}\left(\sin(c+dx)\sqrt{\cos(c+dx)}\,(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c
 + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/
(15*b*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 2.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

maple [A] time = 0.89, size = 271, normalized size = 1.95

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-24*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c
)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c
)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\sqrt{b\,\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.801 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=108

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)
*(b*cos(d*x+c))^(1/2)/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
 + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
 d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx &= A\int \sqrt{b\cos(c+dx)}\,dx+\frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b}\\ &= \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}(bB)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx+\frac{\left(A\sqrt{\right.}}{}\\ &= \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}\\ &= \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 0.69

$$\frac{2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c +
d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 2.18, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 0.98, size = 238, normalized size = 2.20

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2 \right.}\cdots}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-4*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x)), x)
```

3.802 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{2Ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2Ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
 + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d
*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= (Ab)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx + B\int \sqrt{b\cos(c+dx)}\,dx$$

$$= \frac{\left(Ab\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)}{\sqrt{\cos(c}}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab\sqrt{\cos(c+dx)}\,F}{d\sqrt{b\cos(c}}$$

Mathematica [A] time = 0.09, size = 55, normalized size = 0.69

$$\frac{2b\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
[Out] (2*b*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x
)/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

maple [A] time = 0.94, size = 161, normalized size = 2.01

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(c+dx\right)}\,\left(A+B\cos\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

3.803 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=105

$$\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d
/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2
)
```

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c +
d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + (bB)\int \frac{1}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - A\int \sqrt{b\cos(c+dx)}\,dx + \frac{\left(bB\sqrt{\cos(\right.}{}$$

$$= \frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} -$$

$$= -\frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx}}{d\sqrt{b\cos}}$$

Mathematica [A] time = 0.19, size = 73, normalized size = 0.70

$$\frac{2\sqrt{b\cos(c+dx)}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + 
d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.34, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

maple [A] time = 0.99, size = 213, normalized size = 2.03

$$-\frac{2b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -2*b*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/
2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.804 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=136

$$\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2/3*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b*B*sin(d*x+c)/d/(b*cos(d*x+c
))^(1/2)+2/3*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*B*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b*B*Sin
[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx)}$$

$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{1}{3}(Ab)\int \frac{}{\sqrt{b}}$$

$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(Ab\sqrt{\cos(c+}\right.}{3\sqrt{b}}$$

$$= -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab\sqrt{\cos(c+dx}}{3d\sqrt{b\cos}}$$

Mathematica [A] time = 0.26, size = 85, normalized size = 0.62

$$\frac{2b\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*Sq
rt[b*Cos[c + d*x]])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

maple [B] time = 1.09, size = 453, normalized size = 3.33

$$-\frac{2\left(12B\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(s\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

[Out] -2/3*(12*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*b/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\cos(c+d\,x)}\,\left(A+B\cos(c+d\,x)\right)}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)

[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)

[Out] Timed out

3.805 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sqrt{\cos(}}{3d}$$

```
[Out] 2/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^2*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+6/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2
)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sqrt{\cos(}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*B
*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b*Sin[c + d*x])/(5*d*Sqr
t[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx = b^4\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \left(Ab^4\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \left(b^3B\right)\int \frac{1}{(b\cos(c+dx$$
$$= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{5}\left(3Ab^2\right.$$
$$= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6Ab\sin}{5d\sqrt{b\cos}}$$
$$= \frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx}{5d(b\cos(c+dx))^{5}}$$
$$= -\frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+d}}{3d\sqrt{b\,c}}$$

Mathematica [A] time = 0.49, size = 107, normalized size = 0.63

$$\frac{2\sec^2(c+dx)\sqrt{b\cos(c+dx)}\left(\frac{9}{2}A\sin(2(c+dx)) + 3A\tan(c+dx) - 9A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 5B\sin(\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(-9*A*Cos[c + d*x]^(3/2)*EllipticE[(
c + d*x)/2, 2] + 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*B*Sin
[c + d*x] + (9*A*Sin[2*(c + d*x)])/2 + 3*A*Tan[c + d*x]))/(15*d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

maple [B] time = 2.37, size = 575, normalized size = 3.40

$$2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*
c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+s
in(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
[Out] Timed out
```

3.806 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=169

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{10b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2B}{}$$

```
[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*x+
c)/b/d+10/21*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21
*b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1
/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{10b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2B}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (10*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (10*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*
(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Si
n[c + d*x])/(7*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^2}$$
$$= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2B(b\cos(c+dx))^{5/2}}{7bd}$$
$$= \frac{10bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2A(b\cos(c+dx))^{3/2}}{5d}$$
$$= \frac{6Ab\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10bB\sqrt{b\cos(c+}}{21}$$
$$= \frac{6Ab\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10b^2B\sqrt{\cos(c+}}{21d\sqrt{b\cos}}$$

Mathematica [A] time = 0.30, size = 103, normalized size = 0.61

$$\frac{(b\cos(c+dx))^{5/2}\left(2\sin(c+dx)\sqrt{\cos(c+dx)}\,(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252AE\left(\frac{1}{2}(c+dx)\right.\right.}{210bd\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[
(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos
[2*(c + d*x)])*Sin[c + d*x]))/(210*b*d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^3 + Ab\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

maple [A] time = 0.79, size = 301, normalized size = 1.78

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(240*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\ \left(b\ \cos(c+d\,x)\right)^{3/2}\ \left(A+B\ \cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.807 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bBE\left(\right.}{}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*b*B
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bBE\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*C
os[c + d*x]]) + (2*A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = A\int (b\cos(c+dx))^{3/2}\,dx + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b}$$
$$= \frac{2Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$
$$= \frac{2Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$
$$= \frac{6bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.63

$$\frac{2(b\cos(c+dx))^{3/2}\left(\sin(c+dx)\sqrt{\cos(c+dx)}\,(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c
 + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/
(15*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)
```

maple [A] time = 0.87, size = 273, normalized size = 1.95

$$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^{2}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20A+24B\right)\left(\sin^{4}\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-24*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (b \cos (c + d x))^{3/2} (A + B \cos (c + d x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
[Out] int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.808 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[Out] 2/3*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/d+2*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos
[c + d*x]]) + (2*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$$

$$= (Ab)\int \sqrt{b\cos(c+dx)}\,dx + B\int (b\cos(c+dx))^{3/2}\,dx$$

$$= \frac{2bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\left(b^2B\right)\int \frac{1}{\sqrt{b\cos(c+\cdots}}$$

$$= \frac{2Ab\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{b\cos(c+d\cdots}}{3d\cdots}$$

$$= \frac{2Ab\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+d\cdots}}{3d\sqrt{b\cos\cdots}}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.68

$$\frac{2b\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c
+ d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

maple [A] time = 0.90, size = 240, normalized size = 2.14

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-4*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

3.809 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=83

$$\frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*b*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c
 + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left(Ab^2\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx + (bB)\int \sqrt{b\cos(c+dx)}\,d$$

$$= \frac{\left(Ab^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+d\right.}{\sqrt{c}}$$

$$= \frac{2bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+d}}{d\sqrt{b\,c}}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.69

$$\frac{2b^2\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d
*x)/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 2.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

maple [A] time = 0.85, size = 163, normalized size = 1.96

$$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^{2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{3/2}\ (A+B\cos(c+dx))}{\cos(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.810 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)-2*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (-2*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + \left(b^2B\right)\int \frac{1}{\sqrt{b\cos(c+d}}$$

$$= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - (Ab)\int \sqrt{b\cos(c+dx)}\,dx + \frac{\left(b^2B\right.}{}$$

$$= \frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

$$= -\frac{2Ab\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+}}{d\sqrt{b\,c}}$$

Mathematica [A] time = 0.15, size = 73, normalized size = 0.66

$$\frac{2(b\cos(c+dx))^{3/2}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c
+ d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 3.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

maple [A] time = 1.02, size = 215, normalized size = 1.95

$$-\frac{2b^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -2*b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{3/2}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.811 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2/3*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b^2*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(1/2)+2/3*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*b
*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (-2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^2
*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = b^4\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \left(Ab^4\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + \left(b^3B\right)\int \frac{1}{(b\cos(c+dx$$

$$= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{1}{3}\left(Ab^2\right)\int$$

$$= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(Ab^2\sqrt{\cos(}\right.}{3\sqrt{}}$$

$$= -\frac{2bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+}}{3d\sqrt{b}}$$

Mathematica [A] time = 0.16, size = 87, normalized size = 0.62

$$\frac{2b^2\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

maple [B] time = 0.97, size = 455, normalized size = 3.23

$$-\frac{2\left(12B\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\mathrm{s}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -2/3*(12*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*sin
(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))*b^2/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{3/2}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.812 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=174

$$\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sqrt{\text{co}}}{3}$$

```
[Out] 2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^3*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+6/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*b*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c)
)^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sqrt{\text{co}}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (-6*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b
^3*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b^2*Sin[c + d*x])/(5
*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = b^5\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \left(Ab^5\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \left(b^4B\right)\int \frac{1}{(b\cos(c+}$$
$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{5}\left(3Ab\right.$$
$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6Ab^2 s}{5d\sqrt{b\,c}}$$
$$= \frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+}{5d(b\cos(c+dx}$$
$$= -\frac{6Ab\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c}}{3d\sqrt{}}$$

Mathematica [A] time = 0.29, size = 107, normalized size = 0.61

$$\frac{2\sec^3(c+dx)(b\cos(c+dx))^{3/2}\left(\frac{9}{2}A\sin(2(c+dx))+3A\tan(c+dx)-9A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sin\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(-9*A*Cos[c + d*x]^(3/2)*EllipticE
[(c + d*x)/2, 2] + 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*B*S
in[c + d*x] + (9*A*Sin[2*(c + d*x)])/2 + 3*A*Tan[c + d*x]))/(15*d)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

maple [B] time = 2.37, size = 576, normalized size = 3.31

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(36A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4*b
+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx))}{\cos(c+dx)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.813 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=171

$$\frac{6Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{10b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+$$

```
[Out] 2/5*A*b*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*
x+c)/d+10/21*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21
*b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c)
)^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{6Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{10b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (10*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (10*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*A*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*B*(b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = A\int (b\cos(c+dx))^{5/2}\,dx+\frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b}$$

$$=\frac{2Ab(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+d}{7d}$$

$$=\frac{10b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2Ab(b\cos(c+dx))^{3/2}\sin(c+}{5d}$$

$$=\frac{6Ab^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{10b^2B\sqrt{b\cos(c+dx)}\,\sin(}{21d}$$

$$=\frac{6Ab^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{10b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{21d\sqrt{b\cos(c+dx}}$$

Mathematica [A] time = 0.06, size = 100, normalized size = 0.58

$$\frac{(b\cos(c+dx))^{5/2}\left(2\sin(c+dx)\sqrt{\cos(c+dx)}\,(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252AE\left(\frac{1}{2}(c+dx\right.\right.}{210d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[
(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos
[2*(c + d*x)])*Sin[c + d*x]))/(210*d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 2.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos\left(dx+c\right)^3+Ab^2\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)
```

maple [A] time = 0.90, size = 301, normalized size = 1.76

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-168A-360B\right)\left(\sin^6\left(\frac{d}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(240*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
```

```
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)),x)
[Out] int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.814 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{2Ab^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{6b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b}{}$$

```
[Out] 2/5*b*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*A*b^3*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5
*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2Ab^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{6b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*
B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$

$$= (Ab)\int (b\cos(c+dx))^{3/2}\,dx + B\int (b\cos(c+dx))^{5/2}\,dx$$

$$= \frac{2Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2bB(b\cos(c+dx))}{5d}$$

$$= \frac{2Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2bB(b\cos(c+dx))}{5d}$$

$$= \frac{6b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sqrt{\cos(c+}}{3d\sqrt{b}}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 0.61

$$\frac{2b(b\cos(c+dx))^{3/2}\left(\sin(c+dx)\sqrt{\cos(c+dx)}\,(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*b*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[
(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x])
)/(15*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb^2\cos\left(dx+c\right)^3+Ab^2\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)}\,\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

maple [A] time = 0.96, size = 273, normalized size = 1.88

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-24*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{5/2}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

3.815 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[Out] 2/3*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*C
os[c + d*x]]) + (2*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$$

$$= \left(Ab^2\right)\int \sqrt{b\cos(c+dx)}\,dx + (bB)\int (b\cos(c+dx))^3$$

$$= \frac{2b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\left(b^3B\right)\int \frac{}{\sqrt{b\cos}}$$

$$= \frac{2Ab^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{b\cos(}}{}$$

$$= \frac{2Ab^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c}}{3d\sqrt{}}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.67

$$\frac{2b^2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x]]
)
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

maple [A] time = 0.88, size = 240, normalized size = 2.07

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\right.}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-4*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.816 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{2Ab^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*A*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*b^2*B*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2Ab^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos
[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left(Ab^3\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx + \left(b^2B\right)\int \sqrt{b\cos(c+d$$

$$= \frac{\left(Ab^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c\right.}{\sqrt{}}$$

$$= \frac{2b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sqrt{\cos(c}}{d\sqrt{b}}$$

Mathematica [A] time = 0.10, size = 54, normalized size = 0.64

$$\frac{2(b\cos(c+dx))^{5/2}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d
*x)/2, 2]))/(d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 2.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

maple [A] time = 0.81, size = 163, normalized size = 1.92

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\Big(A\,\text{EllipticF}\Big(\cos\Big(\frac{dx}{2}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\Big(2\Big(\cos^2\Big(\frac{dx}{2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx))}{\cos(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.817 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)-2*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (-2*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = b^4\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$= \left(Ab^4\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + \left(b^3B\right)\int \frac{1}{\sqrt{b\cos(c+}$$

$$= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \left(Ab^2\right)\int \sqrt{b\cos(c+dx)}\,dx + \frac{\left(b^3\right.}{}$$

$$= \frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+d}{d\sqrt{b\cos(c+dx}}$$

$$= -\frac{2Ab^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(}}{d\sqrt{}}$$

Mathematica [A] time = 0.21, size = 73, normalized size = 0.65

$$\frac{2(b\cos(c+dx))^{5/2}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c
+ d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

maple [A] time = 0.99, size = 215, normalized size = 1.92

$$-\frac{2b^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{}{}\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -2*b^3*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.818 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2/3*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b^3*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(1/2)+2/3*A*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*b
^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (-2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b
^3*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = b^5\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \left(Ab^5\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + \left(b^4B\right)\int \frac{1}{(b\cos(c+}$$

$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{1}{3}\left(Ab^3\right)\int$$

$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(Ab^3\sqrt{\cos}\right.}{3}$$

$$= -\frac{2b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sqrt{\cos(}}{3d}$$

Mathematica [A] time = 0.21, size = 87, normalized size = 0.61

$$\frac{2b^3\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos\left(dx+c\right)^3+Ab^2\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

maple [B] time = 1.14, size = 455, normalized size = 3.18

$$-\frac{2\left(12B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\mathrm{si}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] -2/3*(12*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*sin
(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))*b^3/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{5/2}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.819 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=176

$$\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\sqrt{\cos}}{3}$$

```
[Out] 2/5*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^4*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+6/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^3*B*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*b^2*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\sqrt{\cos}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (-6*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2
*b^4*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b^3*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = b^6\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \left(Ab^6\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \left(b^5B\right)\int \frac{1}{(b\cos(c+dx$$
$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{5}\left(3Ab^4\right.$$
$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6Ab^3\sin}{5d\sqrt{b\cos}}$$
$$= \frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+d}{5d(b\cos(c+dx)}$$
$$= -\frac{6Ab^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c}}{3d\sqrt{}}$$

Mathematica [A] time = 0.29, size = 102, normalized size = 0.58

$$-\frac{2b^4\left(-\frac{9}{2}A\sin(2(c+dx)) - 3A\tan(c+dx) + 9A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) - 5B\sin(c+dx) - 5B\cos^{\frac{3}{2}}(c+dx}{15d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (-2*b^4*(9*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 5*B*Cos[c + d*x
]^(3/2)*EllipticF[(c + d*x)/2, 2] - 5*B*Sin[c + d*x] - (9*A*Sin[2*(c + d*x)
])/2 - 3*A*Tan[c + d*x]))/(15*d*(b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

maple [B] time = 2.44, size = 578, normalized size = 3.28

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(36A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4
*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{5/2}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^6,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.820 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=173

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{10B\sin}{}$$

```
[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^3/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/
21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1
/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{10B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*
x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos
[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*A*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Si
n[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b^3}$$

$$= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b^3} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^4}$$

$$= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + $$

$$= \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + $$

$$= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd}$$

$$= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{21d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.45, size = 101, normalized size = 0.58

$$\frac{\sin(2(c+dx))(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos}}{210d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c +
 d*x)])*Sin[2*(c + d*x)])/(210*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^3 + A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 0.96, size = 298, normalized size = 1.72

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42
*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)}{\sqrt{b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.821 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=144

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6BE\left(\frac{1}{2}(c+\right.}{5ba}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+6/5*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6BE\left(\frac{1}{2}(c+\right.}{5ba}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d) + (2*B*(b*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx}{b^2}$$

$$= \frac{A\int (b\cos(c+dx))^{3/2}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^3}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{1}{3}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{(}{}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.21, size = 88, normalized size = 0.61

$$\frac{2\sqrt{\cos(c+dx)}\left(\sin(c+dx)\sqrt{\cos(c+dx)}\,(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d
*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*
d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 0.96, size = 270, normalized size = 1.88

$$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^
4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^
2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,\left(A+B\,\cos(c+d\,x)\right)}{\sqrt{b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.822 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/b/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^
(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]
]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
 d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx}{b}$$

$$= \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^2}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{1}{3}B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx + \frac{\left(A\sqrt{b\cos\right.}{}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} +$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.69

$$\frac{2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c +
d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*b*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 1.01, size = 237, normalized size = 2.10

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\cdots\right.\right.}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d
*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

mupad [B] time = 0.28, size = 94, normalized size = 0.83

$$\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2A\sqrt{\cos(c+dx)}\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*B*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2)
*ellipticE(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*B*cos(c + d*x
)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.823 $\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*co
s(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2748, 2642, 2641, 2640, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]
]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d
*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx+\frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b}\\ &= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}}\\ &= \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.66

$$\frac{2\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 0.90, size = 160, normalized size = 1.95

$$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

mupad [B] time = 0.34, size = 48, normalized size = 0.59

$$\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\left(A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d\,\sqrt{b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*cos(c + d*x)^(1/2)*(A*ellipticF(c/2 + (d*x)/2, 2) + B*ellipticE(c/2 + (d
*x)/2, 2)))/(d*(b*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{\left(c + dx \right)}}{\sqrt{b\cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(b*cos(c + d*x)), x)
```

3.824 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*
cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c +
d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx \\
&= (Ab)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{}{\sqrt{\cos(}}}{\sqrt{b\cos(c+dx)}} \\
&= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{b\cos(c+}\right.}{b\sqrt{}} \\
&= -\frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 73, normalized size = 0.69

$$\frac{2\left(A\sin(c+dx)-A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b*cos(d*x
+ c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 1.16, size = 212, normalized size = 2.00

$$-\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\ \left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x)

[Out] -2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(b*cos(c + d*x)), x)

3.825 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] 2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/d/(b*cos(d*x+c))^(
1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*B*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c +
 d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx+(bB)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{1}{3}A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c}}}{3\sqrt{b\cos(c+dx)}} \\
&= -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{3d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 84, normalized size = 0.62

$$\frac{2\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*Sqrt
[b*Cos[c + d*x]])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b*cos(d*
x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

maple [B] time = 2.31, size = 405, normalized size = 3.00

$$2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x
+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4
*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^2\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

3.826 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=168

$$\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6A\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+}}{3d\sqrt{b}}$$

```
[Out] 2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(3/2)+6/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/
cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6A\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+}}{3d\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*B*S
in[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*d*Sqrt[b*
Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$

$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{5}(3Ab)\int \frac{1}{(b\cos(c+dx)}$$

$$= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6A\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{(3A}{}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2bB\sin(c}{3d(b\cos(c+}$$

$$= -\frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.26, size = 101, normalized size = 0.60

$$\frac{2\left(9A\sin(c+dx) - 9A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + 3A\tan(c+dx)\sec(c+dx) + 5B\tan(c+dx) + 5B\sqrt{\cos(c\right.}{15d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*S
ec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^3}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b*cos(d*
x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

maple [B] time = 2.60, size = 578, normalized size = 3.44

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4*b
+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,\sqrt{b\,\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(b*cos(c + d*x)), x)
```

3.827 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{10B\sin}{}$$

```
[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^4/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+1
0/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c)
)^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{10B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b
*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*
A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d) + (2*B*(b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b^4}$$

$$= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b^4} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^5}$$

$$= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} +$$

$$= \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} +$$

$$= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d}$$

$$= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{21bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.17, size = 104, normalized size = 0.59

$$\frac{\sin(2(c+dx))(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos}}{210bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c +
 d*x)])*Sin[2*(c + d*x)])/(210*b*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)}}{b^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

maple [A] time = 1.08, size = 301, normalized size = 1.71

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(240*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-
42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c + d\,x)^4\ (A + B\ \cos (c + d\,x))}{(b\ \cos (c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.828 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6BE\left(\frac{1}{2}(c+\right.}{5b^2}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+6/5
*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6BE\left(\frac{1}{2}(c+\right.}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*C
os[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*B*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx}{b^3}$$

$$= \frac{A\int (b\cos(c+dx))^{3/2}\,dx}{b^3} + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^4}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{A}{-}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{(}{-}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.15, size = 88, normalized size = 0.60

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\left(\sin(c+dx)\sqrt{\cos(c+dx)}\left(5A+3B\cos(c+dx)\right)+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d
*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*
d*(b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 1.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

maple [A] time = 0.99, size = 273, normalized size = 1.86

$$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-24*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c
)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c
)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)}{\left(b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.829 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)
*(b*cos(d*x+c))^(1/2)/b^2/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos
[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx}{b^2} \\
&= \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^3} \\
&= \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} + \frac{\left(A\sqrt{b\cos(c+\right.}{b^2\sqrt{}} \\
&= \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \\
&= \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3bd\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.65

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*
x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*(b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

maple [A] time = 0.94, size = 240, normalized size = 2.07

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{}{}\right.\right.}}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-4*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,\left(A+B\,\cos(c+d\,x)\right)}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.830 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*
cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c
 + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b} \\
&= \frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b} + \frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b^2} \\
&= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}} \\
&= \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.67

$$
\frac{2\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{bd\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2]))/(b*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

maple [A] time = 0.85, size = 163, normalized size = 1.92

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/b/(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(b\,\cos(c+d\,x)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.831 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}} dx$

Optimal. Leaf size=112

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d
/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^
(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2636, 2640, 2639, 2642, 2641}

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= A\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b}\\ &= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b^2}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos}}{b^2\sqrt{\cos(c+dx)}}\\ &= -\frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin}{bd\sqrt{b\cos}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 0.68

$$\frac{2\left(A\sin(c+dx)-A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)
```

maple [A] time = 1.00, size = 215, normalized size = 1.92

$$-\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\ \left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/
2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\ \cos\left(c + d\,x\right)}{\left(b\ \cos\left(c + d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(3/2),x)
[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.832 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b/d/(b*cos(d*x+c))^(
1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-2*B*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c
 + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= (Ab)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + B\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} - \frac{B\int \cdots}{\cdots} \\
&= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c\cdots}}}{3b\sqrt{b\cos(c+dx)}} \cdots \\
&= -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{3bd\sqrt{b\cos(c+dx)}} \cdots
\end{aligned}$$

Mathematica [A] time = 0.08, size = 87, normalized size = 0.62

$$\frac{2\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*b*d*Sq
rt[b*Cos[c + d*x]])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b^2*cos(d*
x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

maple [B] time = 1.06, size = 455, normalized size = 3.25

$$-\frac{2\left(12B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\mathrm{si}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(12*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*sin
(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/b/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

3.833 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{6A\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(}}{3b}$$

```
[Out] 2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/d/(b*cos(d*x+c))
^(3/2)+6/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/
d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{6A\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*S
in[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b*d*Sqrt[
b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + (bB)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{5}(3A)\int \frac{1}{(b\cos(c+dx))^{3}}$$
$$= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6A\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} - \frac{(3A}{}$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c-}{3d(b\cos(c+}$$
$$= -\frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.11, size = 104, normalized size = 0.61

$$\frac{2\left(9A\sin(c+dx) - 9A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + 3A\tan(c+dx)\sec(c+dx) + 5B\tan(c+dx) + 5B\sqrt{\cos(c}\right.}{15bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*S
ec[c + d*x]*Tan[c + d*x]))/(15*b*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)^2}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b^2*cos(
d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

maple [B] time = 2.73, size = 578, normalized size = 3.38

$$2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(36A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\sqrt{\frac{1}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4
*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \cos{\left(c + d x \right)}\right) \sec^{2}{\left(c + d x \right)}}{\left(b \cos{\left(c + d x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(b*cos(c + d*x))**(3/2), x)
```

3.834 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10B\sin}{}$$

```
[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^5/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
+10/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+
c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt
[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (
2*A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d) + (2*B*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b^5}$$
$$= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b^5} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^6}$$
$$= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} +$$
$$= \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} +$$
$$= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d}$$
$$= \frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{21b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.10, size = 104, normalized size = 0.59

$$\frac{\sin(2(c+dx))(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos}}{210b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c +
 d*x)])*Sin[2*(c + d*x)])/(210*b^2*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^3 + A\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)}}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

maple [A] time = 0.93, size = 301, normalized size = 1.71

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(240*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^5\,\left(A+B\,\cos(c+d\,x)\right)}{\left(b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.835 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6BE\left(\frac{1}{2}(c+\right.}{5b^3}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+6
/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6BE\left(\frac{1}{2}(c+\right.}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d) + (2*B*
(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx}{b^4}$$

$$= \frac{A\int (b\cos(c+dx))^{3/2}\,dx}{b^4} + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^5}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{A}{-}$$

$$= \frac{2A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{(}{-}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.09, size = 91, normalized size = 0.62

$$\frac{2\sqrt{\cos(c+dx)}\left(\sin(c+dx)\sqrt{\cos(c+dx)}\,(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d
*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*
b^2*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{b^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{4}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

maple [A] time = 0.92, size = 273, normalized size = 1.86

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)

[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4\,\left(A+B\,\cos(c+d\,x)\right)}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)

[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

3.836 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/b^3/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos
(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*
x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*C
os[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx}{b^3}\\
&= \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b^3} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^4}\\
&= \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2} + \frac{\left(A\sqrt{b\cos(c+d\cdots\right.}{b^3\sqrt{\cdots}}\\
&= \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \cdots\\
&= \frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.\cdots}{3b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.67

$$\frac{2\sqrt{\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*
x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*b^2*d*Sqrt[b*Cos[c + d*x]]
)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

maple [A] time = 1.05, size = 240, normalized size = 2.07

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\right.\right.}}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-4*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(b\,\cos(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.837 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(
b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^2} \\
&= \frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b^2} + \frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b^3} \\
&= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}} \\
&= \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.67

$$\frac{2\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2]))/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

maple [A] time = 1.00, size = 163, normalized size = 1.92

$$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/b^2/(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.838 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b
^2/d/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x
+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{b}$$

$$= \frac{A\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{b} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}$$

$$= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b^3} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cdots}{b^2\sqrt{b\cos(c+d\cdots}}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{b\cos(c\cdots}\right.}{b^3\cdots}$$

$$= -\frac{2A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 0.68

$$\frac{2\left(A\sin(c+dx)-A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

maple [A] time = 1.11, size = 215, normalized size = 1.92

$$-\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\ \left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(b\,\cos(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.839 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b^2/d/(b*cos(d*x+c
))^(1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-2*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d
*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (2*B*S
in[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= A\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx+\frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{b}\\ &= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}-\frac{B\int \sqrt{b\cos(c+}\\ &= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}-\\ &= -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin}{3bd(b\cos}\end{aligned}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.61

$$\frac{2\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*b^2*d*
Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)
```

maple [B] time = 1.06, size = 455, normalized size = 3.18

$$-\frac{2\left(12B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\mathrm{si}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(12*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*sin
(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
```

```
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3*B*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/b^2/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.840 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{6A\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b/d/(b*cos(d*x+c))
^(3/2)+6/5*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/
b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{6A\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*S
in[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b^2*d*S
qrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= (Ab)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + B\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{(3A)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b} +$$
$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{6A\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{(3}{}$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+}{3bd(b\cos(c+}$$
$$= -\frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.08, size = 104, normalized size = 0.60

$$\frac{2\left(9A\sin(c+dx) - 9A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + 3A\tan(c+dx)\sec(c+dx) + 5B\tan(c+dx) + 5B\sqrt{\cos(c\right.}{15b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*S
ec[c + d*x]*Tan[c + d*x]))/(15*b^2*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sec(dx+c)}{b^3\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b^3*cos(d*
x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

maple [B] time = 2.53, size = 578, normalized size = 3.34

$$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4
*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.841 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{6A\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{3b^3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b^2/d/(b*cos(d*x
+c))^(3/2)+6/5*A*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/b^3/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1
/2)/b^4/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{6A\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (2*B
*Sin[c + d*x])/(3*b^2*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b^3
*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = A\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx}{b}$$

$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{(3A)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2} + \frac{B\int \frac{1}{\sqrt{b\cos(}}}{3b^{}}$$

$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{6A\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{(3A)\int \sqrt{b\cos}}{5b}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} +$$

$$= -\frac{6A\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(}{5bd(b\cos(}$$

Mathematica [A] time = 0.08, size = 104, normalized size = 0.59

$$\frac{2\left(9A\sin(c+dx)-9A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+3A\tan(c+dx)\sec(c+dx)+5B\tan(c+dx)+5B\sqrt{\cos}\right.}{15b^3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*S
ec[c + d*x]*Tan[c + d*x]))/(15*b^3*d*Sqrt[b*Cos[c + d*x]])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^4\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^4*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)
```

maple [B] time = 2.51, size = 578, normalized size = 3.28

$$2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(36A\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4
*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(7/2),x)
[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.842 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=172

$$-\frac{A\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx}{4d}$$

```
[Out] 1/4*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*B*x*(b*cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)+A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)-1/3*A*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+3/8*B*sin(
d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2633, 2635, 8}

$$-\frac{A\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x
]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]
]*Sin[c + d*x])/(4*d) - (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[C
os[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int \cos^3(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+d\right.}{\sqrt{\text{co}}}$$

$$= \frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{\left(3B\sqrt{b\cos\right.}{}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3B\sqrt{\cos(c+dx)}\,\sqrt{b\cos}}{8d}$$

$$= \frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3}{}$$

Mathematica [A] time = 0.17, size = 81, normalized size = 0.47

$$\frac{\sqrt{b\cos(c+dx)}\,(72A\sin(c+dx)+8A\sin(3(c+dx))+24B\sin(2(c+dx))+3B\sin(4(c+dx))+36Bc+36Bdx)}{96d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

[Out] (Sqrt[b*Cos[c + d*x]]*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Sqrt[Cos[c + d*x]])

fricas [A] time = 0.84, size = 252, normalized size = 1.47

$$\left[\frac{9\,B\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(6\,B\cos\right.}{48\,d\cos\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/48*(9*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*cos(d*x + c)^3
 + 8*A*cos(d*x + c)^2 + 9*B*cos(d*x + c) + 16*A)*sqrt(b*cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*sqrt(b)*arctan(sqrt
(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) +
(6*B*cos(d*x + c)^3 + 8*A*cos(d*x + c)^2 + 9*B*cos(d*x + c) + 16*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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n((c+d*x)/2)^2+9*sqrt(b)*B*d*x+(-30*sqrt(b))*B*tan((c+d*x)/2)^7+18*sqrt(b)*
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)/2))/(24*d*tan((c+d*x)/2)^8+96*d*tan((c+d*x)/2)^6+144*d*tan((c+d*x)/2)^4+9
6*d*tan((c+d*x)/2)^2+24*d)

maple [A] time = 0.32, size = 91, normalized size = 0.53

$$\frac{\sqrt{b\cos\left(dx+c\right)}\ \left(6B\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)+8A\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)+9B\cos\left(dx+c\right)\sin\left(dx+c\right)}{24d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(1/2)*(6*B*cos(d*x+c)^3*sin(d*x+c)+8*A*cos(d*x+c)^2*s
in(d*x+c)+9*B*cos(d*x+c)*sin(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)
^(1/2)
```

maxima [A] time = 0.71, size = 93, normalized size = 0.54

$$\frac{3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)B\sqrt{b}\,+8\,A\sqrt{b}\left(\sin\left(3\,dx\right.\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 8*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

mupad [B] time = 2.30, size = 105, normalized size = 0.61

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,B\,\sin\left(c+d\,x\right)+80\,A\,\sin\left(2\,c+2\,d\,x\right)+8\,A\,\sin\left(4\,c+4\,d\,x\right)+27\,B\,\sin\left(3\right.\right.}{96\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + d*x) + 80*A*sin(2*
c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5*c + 5
*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.843 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=136

$$\frac{Ax\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}-\frac{B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b}}{d\sqrt{\cos(c}}$$

```
[Out] 1/2*A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)-1/3*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)+1/2*A*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ax\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}-\frac{B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b}}{d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(2*d) - (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*
Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+\right.}{\sqrt{c}}$$
$$= \frac{A\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\left(A\sqrt{b\,\mathrm{co}\right.}{2\sqrt{}}$$
$$= \frac{Ax\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \cdots$$

Mathematica [A] time = 0.13, size = 69, normalized size = 0.51

$$\frac{\sqrt{b\cos(c+dx)}\,(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c +
d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

fricas [A] time = 1.32, size = 230, normalized size = 1.69

$$\left[\frac{3\,A\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(2\,B\cos\right.}{12\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*cos(d*x + c)^2
 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^2 + 3
*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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tan((c+d*x)/2)+12*sqrt(b)*B*tan((c+d*x)/2)^5+8*sqrt(b)*B*tan((c+d*x)/2)^3+1
2*sqrt(b)*B*tan((c+d*x)/2))/(6*d*tan((c+d*x)/2)^6+18*d*tan((c+d*x)/2)^4+18*
d*tan((c+d*x)/2)^2+6*d)
```

maple [A] time = 0.26, size = 74, normalized size = 0.54

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(2B\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)+3A\cos\left(dx+c\right)\sin\left(dx+c\right)+3A\left(dx+c\right)+4B\sin\left(dx+c\right)\right)}{6d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(
d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [A] time = 0.73, size = 68, normalized size = 0.50

$$\frac{3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)A\sqrt{b}\,+B\sqrt{b}\left(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + B*sqrt(b)*(sin(3*d*x +
 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

mupad [B] time = 1.25, size = 92, normalized size = 0.68

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,A\,\sin\left(c+d\,x\right)+3\,A\,\sin\left(3\,c+3\,d\,x\right)+10\,B\,\sin\left(2\,c+2\,d\,x\right)+B\,\sin\left(4\,c+4\,d\,x\right)+12\,A\,d\,x\,\cos\left(c+d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c
+ 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*
x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.844 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=98

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] 1/2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*sin(d*x+c)*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)+1/2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1
/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} +$$

Mathematica [A] time = 0.12, size = 57, normalized size = 0.58

$$\frac{\sqrt{b\cos(c+dx)}\,(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]
)))/(4*d*Sqrt[Cos[c + d*x]])
```

fricas [A] time = 0.68, size = 204, normalized size = 2.08

$$\left[\frac{B\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)+2\,(B\cos(dx}{4\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c
))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*cos(d*x + c) + 2*A)
*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/
2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)
^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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)/2)^2+sqrt(b)*B*d*x+(-2*sqrt(b))*B*tan((c+d*x)/2)^3+2*sqrt(b)*B*tan((c+d*x
)/2))/(2*d*tan((c+d*x)/2)^4+4*d*tan((c+d*x)/2)^2+2*d)
```

maple [A] time = 0.21, size = 55, normalized size = 0.56

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(B\cos\left(dx+c\right)\sin\left(dx+c\right)+2A\sin\left(dx+c\right)+B\left(dx+c\right)\right)}{2d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c
))/cos(d*x+c)^(1/2)
```

maxima [A] time = 0.65, size = 40, normalized size = 0.41

$$\frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))B\sqrt{b}\,+4\,A\sqrt{b}\,\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*sqrt(b) + 4*A*sqrt(b)*sin(d*x + c))
/d
```

mupad [B] time = 0.82, size = 79, normalized size = 0.81

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(B\,\sin\left(c+d\,x\right)+4\,A\,\sin\left(2\,c+2\,d\,x\right)+B\,\sin\left(3\,c+3\,d\,x\right)+4\,B\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c +
2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*d*(cos(2*c + 2*d*x)
 + 1))
```

sympy [A] time = 40.28, size = 99, normalized size = 1.01

$$\begin{cases} \frac{A\sqrt{b}\sin{(c+dx)}}{d} + \frac{B\sqrt{b}\,x\sin^2{(c+dx)}}{2} + \frac{B\sqrt{b}\,x\cos^2{(c+dx)}}{2} + \frac{B\sqrt{b}\sin{(c+dx)}\cos{(c+dx)}}{2d} & \text{for } d \neq 0 \\ x\sqrt{b\cos{(c)}}\,(A + B\cos{(c)})\sqrt{\cos{(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*sqrt(b)*sin(c + d*x)/d + B*sqrt(b)*x*sin(c + d*x)**2/2 + B*sqr
t(b)*x*cos(c + d*x)**2/2 + B*sqrt(b)*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*sqrt(b*cos(c))*(A + B*cos(c))*sqrt(cos(c)), True))
```

3.845 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=59

$$\frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (B*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.71

$$\frac{\sqrt{b\cos(c+dx)}\,(A(c+dx)+B\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]]
)
```

fricas [A] time = 0.94, size = 181, normalized size = 3.07

$$\left[\frac{A\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)+2\,\sqrt{b\cos(d}}{2\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c
))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))*B
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*sqrt(b)*arctan(sqrt(
b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + s
qrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.18, size = 39, normalized size = 0.66

$$\frac{\sqrt{b\cos(dx+c)}\ (A(dx+c)+B\sin(dx+c))}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [A] time = 0.60, size = 40, normalized size = 0.68

$$\frac{2A\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+B\sqrt{b}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*sqrt(b)*sin(d*x +
c))/d
```

mupad [B] time = 0.29, size = 35, normalized size = 0.59

$$\frac{\sqrt{b\cos(c+dx)}\ (B\sin(c+dx)+A\,d\,x)}{d\sqrt{\cos(c+dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2))
```

sympy [A] time = 12.09, size = 46, normalized size = 0.78

$$\begin{cases} A\sqrt{b}x+\frac{B\sqrt{b}\sin(c+dx)}{d} & \text{for } d\neq 0\\ \frac{x\sqrt{b\cos(c)}(A+B\cos(c))}{\sqrt{\cos(c)}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((A*sqrt(b)*x + B*sqrt(b)*sin(c + d*x)/d, Ne(d, 0)), (x*sqrt(b*cos
(c))*(A + B*cos(c))/sqrt(cos(c)), True))
```

3.846 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{A\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*arctanh(sin(d*x+c))*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{A\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.67

$$\frac{\sqrt{b\cos(c+dx)}\,\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*
x]])
```

fricas [A] time = 1.86, size = 210, normalized size = 3.50

$$\left[-\frac{2\,A\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) - B\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^2 - 2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right.}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) - B*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*sqrt(b)*arc
tan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A*sqr
t(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.16, size = 54, normalized size = 0.90

$$-\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\left(dx+c\right)\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(b*cos(d*x+c))^(1/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c
))/cos(d*x+c)^(1/2)
```

maxima [A] time = 0.59, size = 92, normalized size = 1.53

$$\frac{A\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right.\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*B*sqrt(b)*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos\left(c+dx\right)}\,\left(A+B\cos\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.847 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+B*arctanh(sin(d*x+c))*
(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 8, 3770}

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sec(c$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{\left(A\sqrt{b\cos(c+dx)}\right)\text{Subst}(}{d\sqrt{\cos(c}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+d}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.74

$$\frac{\sqrt{b\cos(c+dx)}\,\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x
]))/(d*Cos[c + d*x]^(3/2))
```

fricas [A] time = 0.96, size = 205, normalized size = 3.01

$$\left[\frac{B\sqrt{b}\,\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos(dx+c)}\,A\sqrt{\cos(d}{2\,d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

maple [A] time = 0.17, size = 59, normalized size = 0.87

$$\frac{\left(-2B\cos(dx+c)\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*c
os(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
```

maxima [A] time = 0.65, size = 120, normalized size = 1.76

$$\frac{B\sqrt{b}\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(B*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*A*sqrt(b)*s
```

```
in(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\cos(c+dx)}\ (A+B\cos(c+dx))}{\cos(c+dx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

3.848 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{A\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+B*sin(d*x+c)*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*A*arctanh(sin(d*x+c))*(b*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {17, 2748, 3768, 3770, 3767, 8}

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{A\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (B*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\,\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sec^2(\\
&= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,d}{2\sqrt{\cos(c+dx)}}\\
&= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+}{2d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 65, normalized size = 0.61

$$\frac{\sqrt{b\cos(c+dx)}\,\left(\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Co
s[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 1.02, size = 225, normalized size = 2.10

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\right.}{4d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))
*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*cos(d*
x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

maple [A] time = 0.23, size = 120, normalized size = 1.12

$$\frac{\left(-A\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A\left(\cos^{2}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+2B\cos\left(dx+c\right)\sin\left(\right.\right.}{2d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+2*B*cos(d*x+c)*sin(d*x+c)+A*
sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

maxima [B] time = 0.71, size = 716, normalized size = 6.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] -1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c)
+ 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1) - 8*B*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

3.849 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{A\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos}}$$

```
[Out] 1/2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+A*sin(d*x+c)*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*A*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/
d/cos(d*x+c)^(7/2)+1/2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 3768, 3770}

$$\frac{A\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\,\int (A+B\cos(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sec^3(c}{\sqrt{\cos(c+dx)}} \\
&= \frac{B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}} \\
&= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\,\sin(c+dx}{2d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [A] time = 0.32, size = 76, normalized size = 0.52

$$
\frac{\sqrt{b\cos(c+dx)}\,\left(2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{6d\cos^{\frac{5}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c
 + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(5/2)
)
```

fricas [A] time = 1.07, size = 253, normalized size = 1.74

$$\left[\frac{3\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,A\cos\left(dx+c\right)^{2}+3\,B\right.}{12\,d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)
*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*
cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

maple [A] time = 0.24, size = 139, normalized size = 0.96

$$\frac{\left(-3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+4A\left(\cos^{2}\left(dx+c\right.\right.\right.}{6d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] 1/6/d*(-3*B*cos(d*x+c)^3*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*B*cos
(d*x+c)^3*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*A*cos(d*x+c)^2*sin(d*x
+c)+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+
c)^(7/2)
```

maxima [B] time = 0.72, size = 957, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*
d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*si
n(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c
)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c
) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4
*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)
+ 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(
2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

3.850 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=177

$$-\frac{Ab\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\cos^{\frac{5}{2}}(c+ }{4d}$$

```
[Out] 1/4*b*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b*B*x*(b*cos
(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)-1/3*A*b*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+3/
8*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2633, 2635, 8}

$$-\frac{Ab\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\cos^{\frac{5}{2}}(c+ }{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*b*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c +
 d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*b*B*Sqrt[Cos[c + d*x]]*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) - (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(
3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)(A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos\right.}{}$$

$$= \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{\left(3bB\sqrt{\right.}{}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3bB\sqrt{\cos(c+dx)}\sqrt{}}{}$$

$$= \frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.16, size = 81, normalized size = 0.46

$$\frac{(b\cos(c+dx))^{3/2}(72A\sin(c+dx)+8A\sin(3(c+dx))+24B\sin(2(c+dx))+3B\sin(4(c+dx))+36Bc+36Bd}{96d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2
*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Cos[c + d
*x]^(3/2))
```

fricas [A] time = 1.13, size = 261, normalized size = 1.47

$$\left[\frac{9\,B\sqrt{-b}\,b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(6\,Bb\cos\right.}{48\,d\cos\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/48*(9*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*b*cos(d*x +
c)^3 + 8*A*b*cos(d*x + c)^2 + 9*B*b*cos(d*x + c) + 16*A*b)*sqrt(b*cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*b^(3/2)*a
rctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d
*x + c) + (6*B*b*cos(d*x + c)^3 + 8*A*b*cos(d*x + c)^2 + 9*B*b*cos(d*x + c)
 + 16*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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9*sqrt(b)*B*d*x*tan((c+d*x)/2)^8+36*sqrt(b)*B*d*x*tan((c+d*x)/2)^6+54*sqrt(
b)*B*d*x*tan((c+d*x)/2)^4+36*sqrt(b)*B*d*x*tan((c+d*x)/2)^2+9*sqrt(b)*B*d*x
+(-30*sqrt(b))*B*tan((c+d*x)/2)^7+18*sqrt(b)*B*tan((c+d*x)/2)^5+(-18*sqrt(b
))*B*tan((c+d*x)/2)^3+30*sqrt(b)*B*tan((c+d*x)/2))/(24*d*tan((c+d*x)/2)^8+9
6*d*tan((c+d*x)/2)^6+144*d*tan((c+d*x)/2)^4+96*d*tan((c+d*x)/2)^2+24*d)
```

maple [A] time = 0.26, size = 91, normalized size = 0.51

$$\frac{(b\cos(dx+c))^{\frac{3}{2}}\left(6B\left(\cos^3(dx+c)\right)\sin(dx+c)+8A\left(\cos^2(dx+c)\right)\sin(dx+c)+9B\cos(dx+c)\sin(dx+c)}{24d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(3/2)*(6*B*cos(d*x+c)^3*sin(d*x+c)+8*A*cos(d*x+c)^2*s
in(d*x+c)+9*B*cos(d*x+c)*sin(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)
^(3/2)
```

maxima [A] time = 0.75, size = 100, normalized size = 0.56

$$\frac{8\left(b\sin(3\,dx+3\,c)+9\,b\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)A\sqrt{b}+3\left(12\,(dx+c)b+b\sin(4\,dx}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/96*(8*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))))*A*sqrt(b) + 3*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b))/d
```

mupad [B] time = 1.91, size = 106, normalized size = 0.60

$$\frac{b\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,B\,\sin\left(c+d\,x\right)+80\,A\,\sin\left(2\,c+2\,d\,x\right)+8\,A\,\sin\left(4\,c+4\,d\,x\right)+27\,B\,\sin\left(3\right.}{96\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + d*x) + 80*A*sin(
2*c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5*c +
 5*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.851 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{Abx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}-\frac{bB\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+ }{d\sqrt{}}$$

```
[Out] 1/2*A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)+1/2*A*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Abx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}-\frac{bB\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+ }{d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[Cos[c + d*x]]*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) - (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^
3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(}\right.}{}$$

$$= \frac{Ab\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{\left(Ab\sqrt{}\right.}{}$$

$$= \frac{Abx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.14, size = 69, normalized size = 0.49

$$\frac{(b\cos(c+dx))^{3/2}(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c
+ d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(3/2))
```

fricas [A] time = 1.02, size = 237, normalized size = 1.69

$$\left[\frac{3 A\sqrt{-b}\, b\cos\left(dx+c\right)\log\left(2 b\cos\left(dx+c\right)^{2}-2 \sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2 \left(2 Bb\cos\right.}{12 d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*A*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*b*cos(d*x +
c)^2 + 3*A*b*cos(d*x + c) + 4*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*A*b^(3/2)*arctan(sqrt(b*cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*b*cos(d*x
+ c)^2 + 3*A*b*cos(d*x + c) + 4*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
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[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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tan((c+d*x)/2)^2+3*sqrt(b)*A*d*x+(-6*sqrt(b))*A*tan((c+d*x)/2)^5+6*sqrt(b)*
A*tan((c+d*x)/2)+12*sqrt(b)*B*tan((c+d*x)/2)^5+8*sqrt(b)*B*tan((c+d*x)/2)^3
+12*sqrt(b)*B*tan((c+d*x)/2))/(6*d*tan((c+d*x)/2)^6+18*d*tan((c+d*x)/2)^4+1
8*d*tan((c+d*x)/2)^2+6*d)
```

maple [A] time = 0.23, size = 74, normalized size = 0.53

$$\frac{(b\cos(dx+c))^{\frac{3}{2}}\left(2B\sin(dx+c)\left(\cos^2(dx+c)\right)+3A\cos(dx+c)\sin(dx+c)+3A(dx+c)+4B\sin(dx+c)\right)}{6d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(3/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(
d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(3/2)
```

maxima [A] time = 1.07, size = 74, normalized size = 0.53

$$\frac{3\,(2\,(dx+c)b+b\sin(2\,dx+2\,c))A\sqrt{b}+\left(b\sin(3\,dx+3\,c)+9\,b\sin\left(\frac{1}{3}\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c$$
$$12\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + (b*sin(3*d*x + 3*c
) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b))/d
```

mupad [B] time = 1.23, size = 93, normalized size = 0.66

$$\frac{b\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(3\,A\,\sin(c+d\,x)+3\,A\,\sin(3\,c+3\,d\,x)+10\,B\,\sin(2\,c+2\,d\,x)+B\,\sin(4\,c+}{12\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*
c + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c +
d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.852 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] 1/2*b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*sin(d*x+c)*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+
c))^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos(c}{}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.57

$$\frac{b\sqrt{b\cos(c+dx)}\,(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x
)])))/(4*d*Sqrt[Cos[c + d*x]])
```

fricas [A] time = 1.99, size = 209, normalized size = 2.07

$$\left[\frac{B\sqrt{-b}\,b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)+2\,(Bb\cos(d}{4\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*b*cos(d*x + c) +
2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
)), 1/2*(B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2)))*cos(d*x + c) + (B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)), x
)
```

maple [A] time = 0.19, size = 55, normalized size = 0.54

$$\frac{(b\cos(dx+c))^{\frac{3}{2}}\,(B\cos(dx+c)\sin(dx+c)+2A\sin(dx+c)+B(dx+c))}{2d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(3/2)*(B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c
))/cos(d*x+c)^(3/2)
```

maxima [A] time = 1.01, size = 43, normalized size = 0.43

$$\frac{4\,Ab^{\frac{3}{2}}\sin(dx+c)+(2\,(dx+c)b+b\sin(2\,dx+2\,c))B\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(4*A*b^(3/2)*sin(d*x + c) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*B*sqrt
(b))/d
```

mupad [B] time = 0.52, size = 50, normalized size = 0.50

$$\frac{b\,\sqrt{b\,\cos(c+d\,x)}\,\left(4\,A\,\sin(c+d\,x)+B\,\sin(2\,c+2\,d\,x)+2\,B\,d\,x\right)}{4\,d\,\sqrt{\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(4*A*sin(c + d*x) + B*sin(2*c + 2*d*x) + 2*B*d*x)
)/(4*d*cos(c + d*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.853 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*B*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.07, size = 42, normalized size = 0.69

$$\frac{(b\cos(c+dx))^{3/2}(A(c+dx)+B\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/
2))
```

fricas [A] time = 0.82, size = 184, normalized size = 3.02

$$\left[\frac{A\sqrt{-b}\,b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)+2\,\sqrt{b\cos(d}}{2\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(A*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))
*B*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*b^(3/2)*arctan(s
qrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
 + sqrt(b*cos(d*x + c))*B*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x
)
```

maple [A] time = 0.18, size = 39, normalized size = 0.64

$$\frac{(b\cos(dx+c))^{\frac{3}{2}}\,(A(dx+c)+B\sin(dx+c))}{d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(3/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(3/2)
```

maxima [A] time = 1.14, size = 40, normalized size = 0.66

$$\frac{2\,Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+Bb^{\frac{3}{2}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*b^(3/2)*sin(d*x +
c))/d
```

mupad [B] time = 0.85, size = 36, normalized size = 0.59

$$\frac{b\,\sqrt{b\,\cos(c+d\,x)}\,\left(B\,\sin(c+d\,x)+A\,d\,x\right)}{d\,\sqrt{\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.854 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{Ab\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*arctanh(sin(d*x+c))*(b*cos(
d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{Ab\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*ArcTanh[Sin[c + d*x]
]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.65

$$\frac{(b\cos(c+dx))^{3/2}\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*(b*Cos[c + d*x])^(3/2))/(d*Cos[c + d*x]^
(3/2))
```

fricas [A] time = 1.10, size = 212, normalized size = 3.42

$$\left[-\frac{2\,A\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) - B\sqrt{-b}\,b\log\left(2\,b\cos\left(dx+c\right)^2 - 2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+\right.}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*
sqrt(cos(d*x + c)))) - B*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*b^(3/2)
*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A
*b^(3/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d
*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2), x
)
```

maple [A] time = 0.15, size = 54, normalized size = 0.87

$$-\frac{(b\cos(dx+c))^{\frac{3}{2}}\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)\right)}{d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/d*(b*cos(d*x+c))^(3/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c
))/cos(d*x+c)^(3/2)
```

maxima [A] time = 1.25, size = 95, normalized size = 1.53

$$\frac{4\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(b\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-b\log\left(\cos(dx+c)^2+s\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*B*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b*log(cos(d*x +
 c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d
*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,\cos(c+d\,x))^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.855 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b*B*arctanh(sin(d*x+
c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 8, 3770}

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (
A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \text{se}}{\sqrt{\cos(c+dx)}}$$

$$= \frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\text{Sub}}{d\sqrt{\cos}}$$

$$= \frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.71

$$\frac{(b\cos(c+dx))^{3/2}\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d
*x]))/(d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 1.22, size = 208, normalized size = 2.97

$$\left[\frac{Bb^{\frac{3}{2}}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}\,Ab\sqrt{\cos(dx}{2\,d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*b^(3/2)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2), -(B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*b*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2), x
)
```

maple [A] time = 0.16, size = 59, normalized size = 0.84

$$\frac{\left(-2B\cos(dx+c)\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)(b\cos(dx+c))^{\frac{3}{2}}}{d\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*c
os(d*x+c))^(3/2)/cos(d*x+c)^(5/2)
```

maxima [A] time = 1.13, size = 123, normalized size = 1.76

$$\frac{\left(b\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-b\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*((b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(c
os(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b) + 4*A*b^(3/
2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{3/2}\ (A+B\cos(c+dx))}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.856 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b*B*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*A*b*arctanh(sin(d*x+c))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {17, 2748, 3768, 3770, 3767, 8}

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*B*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \text{se}}{\sqrt{\cos(c+dx}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 0.59

$$\frac{(b\cos(c+dx))^{3/2}\left(\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*
Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))
```

fricas [A] time = 1.09, size = 232, normalized size = 2.11

$$\left[\frac{Ab^{\frac{3}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2Bb\cos(dx+c)+Ab\right)\sqrt{b}}{4d\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x
 + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*
b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2), x
)
```

maple [A] time = 0.17, size = 120, normalized size = 1.09

$$\frac{\left(-A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A\left(\cos^2(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+2B\cos(dx+c)\sin\right.}{2d\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+2*B*cos(d*x+c)*sin(d*x+c)+A*
sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2)
```

maxima [B] time = 1.27, size = 747, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*B*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - (4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c
) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2
 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*
cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*si
n(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x +
2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*
c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x
 + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*co
s(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx))}{\cos(c+dx)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.857 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{Ab\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+d}{2d\sqrt{}}$$

```
[Out] 1/2*b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+A*b*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*A*b*sin(d*x+c)^3*(b*cos(d*x+c))^
(1/2)/d/cos(d*x+c)^(7/2)+1/2*b*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d
/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 3768, 3770}

$$\frac{Ab\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+d}{2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*b*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \text{s}}{\sqrt{\cos(c+d}}$$
$$= \frac{bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \sec(c+d}{2\sqrt{\cos(c+dx)}}$$
$$= \frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\,\sin(c}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.52

$$\frac{b\sqrt{b\cos(c+dx)}\left(2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2)
,x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin
[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(5/
2))
```

fricas [A] time = 1.04, size = 260, normalized size = 1.74

$$\left[\frac{3\,Bb^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,Ab\cos\left(dx+c\right)^{2}+3\,Bb\right.}{12\,d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] [1/12*(3*B*b^(3/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*B*sq
rt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x
+ c))))*cos(d*x + c)^4 - (4*A*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2),
x)
```

maple [A] time = 0.20, size = 139, normalized size = 0.93

$$\frac{\left(-3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+4A\left(\cos^{2}\left(dx+c\right)\right)\right.}{6d\cos\left(dx+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] 1/6/d*(-3*B*cos(d*x+c)^3*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*B*cos
(d*x+c)^3*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*A*cos(d*x+c)^2*sin(d*x
```

```
+c)+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+
c)^(9/2)
```

maxima [B] time = 1.13, size = 992, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] -1/12*(16*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*
d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 +
6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)
^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*
d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(4*(b*sin(4*d*x + 4*c) + 2*b*sin
(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*
sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin
(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2
*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c
) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x
+ 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c)
+ 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*
cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos
(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{3/2}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2),x)
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
[Out] Timed out
```

3.858 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=187

$$-\frac{Ab^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\cos}{}$$

```
[Out] 1/4*b^2*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b^2*B*x*(b
*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
/cos(d*x+c)^(1/2)-1/3*A*b^2*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)+3/8*b^2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2633, 2635, 8}

$$-\frac{Ab^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*b^2*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*b^2*B*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*B*Cos[c + d*x]^(5/2)*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c +
 d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)(A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\operatorname{co}\right.}{}$$
$$= \frac{b^2B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{\left(3b^2B\right.}{}$$
$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3b^2B\sqrt{\cos(c+dx)}}{}$$
$$= \frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.20, size = 81, normalized size = 0.43

$$\frac{(b\cos(c+dx))^{5/2}(72A\sin(c+dx)+8A\sin(3(c+dx))+24B\sin(2(c+dx))+3B\sin(4(c+dx))+36Bc+36Bdx}{96d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2
*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Cos[c + d
*x]^(5/2))
```

fricas [A] time = 1.03, size = 279, normalized size = 1.49

$$\left[\frac{9\,B\sqrt{-b}\,b^{2}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(6\,Bb^{2}\right.}{48\,d\cos\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/48*(9*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*b^2*cos(d*
x + c)^3 + 8*A*b^2*cos(d*x + c)^2 + 9*B*b^2*cos(d*x + c) + 16*A*b^2)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B
*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/
2)))*cos(d*x + c) + (6*B*b^2*cos(d*x + c)^3 + 8*A*b^2*cos(d*x + c)^2 + 9*B*
b^2*cos(d*x + c) + 16*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
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tan((c+d*x)/2)+9*b^2*sqrt(b)*B*d*x*tan((c+d*x)/2)^8+36*b^2*sqrt(b)*B*d*x*ta
n((c+d*x)/2)^6+54*b^2*sqrt(b)*B*d*x*tan((c+d*x)/2)^4+36*b^2*sqrt(b)*B*d*x*t
an((c+d*x)/2)^2+9*b^2*sqrt(b)*B*d*x+(-30*b^2*sqrt(b))*B*tan((c+d*x)/2)^7+18
*b^2*sqrt(b)*B*tan((c+d*x)/2)^5+(-18*b^2*sqrt(b))*B*tan((c+d*x)/2)^3+30*b^2
*sqrt(b)*B*tan((c+d*x)/2))/(24*d*tan((c+d*x)/2)^8+96*d*tan((c+d*x)/2)^6+144
*d*tan((c+d*x)/2)^4+96*d*tan((c+d*x)/2)^2+24*d)
```

maple [A] time = 0.26, size = 91, normalized size = 0.49

$$\frac{(b\cos(dx+c))^{\frac{5}{2}}\left(6B\left(\cos^3(dx+c)\right)\sin(dx+c)+8A\left(\cos^2(dx+c)\right)\sin(dx+c)+9B\cos(dx+c)\sin(dx+c)+\right.}{24d\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(5/2)*(6*B*cos(d*x+c)^3*sin(d*x+c)+8*A*cos(d*x+c)^2*s
in(d*x+c)+9*B*cos(d*x+c)*sin(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)
^(5/2)
```

maxima [A] time = 0.66, size = 110, normalized size = 0.59

$$\frac{8\left(b^2\sin(3\,dx+3\,c)+9\,b^2\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)A\sqrt{b}\,+3\left(12\,(dx+c)b^2+b^2\sin(4\,d\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/96*(8*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))))*A*sqrt(b) + 3*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8
*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b))/d
```

mupad [B] time = 2.19, size = 108, normalized size = 0.58

$$\frac{b^2\,\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(24\,B\,\sin(c+d\,x)+80\,A\,\sin(2\,c+2\,d\,x)+8\,A\,\sin(4\,c+4\,d\,x)+27\,B\,\sin(3\right.}{96\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + d*x) + 80*A*si
n(2*c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5*c
 + 5*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.859 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=148

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}-\frac{b^2B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(}{}$$

```
[Out] 1/2*A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)+1/2*A*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1
/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}-\frac{b^2B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) - (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c
 + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int}{\sqrt{\cos(c+d}}$$
$$= \frac{Ab^2\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{\left(Ab^2\sqrt{b\cos(c+d}\right.}{2\sqrt{\cos(c+}}$$
$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{\text{co}}}{}$$

Mathematica [A] time = 0.16, size = 69, normalized size = 0.47

$$\frac{(b\cos(c+dx))^{5/2}(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c
+ d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 1.01, size = 251, normalized size = 1.70

$$\left[\frac{3\,A\sqrt{-b}\,b^2\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(2\,Bb\right.}{12\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*A*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*b^2*cos(d*
x + c)^2 + 3*A*b^2*cos(d*x + c) + 4*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*A*b^(5/2)*arctan(sqrt(b*cos(
d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*b^
2*cos(d*x + c)^2 + 3*A*b^2*cos(d*x + c) + 4*B*b^2)*sqrt(b*cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)), x
)
```

maple [A] time = 0.22, size = 74, normalized size = 0.50

$$\frac{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(2B\sin\left(dx+c\right)\left(\cos^2\left(dx+c\right)\right)+3A\cos\left(dx+c\right)\sin\left(dx+c\right)+3A\left(dx+c\right)+4B\sin\left(dx+c\right)\right)}{6d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(5/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(
d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(5/2)
```

maxima [A] time = 0.66, size = 82, normalized size = 0.55

$$\frac{3\left(2\,(dx+c)b^2+b^2\sin\left(2\,dx+2\,c\right)\right)A\sqrt{b}+\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+\right.\right.\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + (b^2*sin(3*d*x
 + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqr
t(b))/d
```

mupad [B] time = 0.71, size = 64, normalized size = 0.43

$$\frac{b^2\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(9\,B\,\sin\left(c+d\,x\right)+3\,A\,\sin\left(2\,c+2\,d\,x\right)+B\,\sin\left(3\,c+3\,d\,x\right)+6\,A\,d\,x\right)}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(9*B*sin(c + d*x) + 3*A*sin(2*c + 2*d*x) + B*si
n(3*c + 3*d*x) + 6*A*d*x))/(12*d*cos(c + d*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.860 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] 1/2*b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*sin(d*x+c)*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b^2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*co
s(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{\cos}}{}$$

Mathematica [A] time = 0.15, size = 57, normalized size = 0.53

$$\frac{(b\cos(c+dx))^{5/2}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x
)])))/(4*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 2.47, size = 219, normalized size = 2.05

$$\left[\frac{B\sqrt{-b}\,b^2\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\left(Bb^2\cos\right.}{4\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*b^2*cos(d*x + c
) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)), 1/2*(B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
cos(d*x + c)^(3/2)))*cos(d*x + c) + (B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2), x
)
```

maple [A] time = 0.17, size = 55, normalized size = 0.51

$$\frac{(b\cos(dx+c))^{\frac{5}{2}}\,(B\cos(dx+c)\sin(dx+c)+2A\sin(dx+c)+B(dx+c))}{2d\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(5/2)*(B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c
))/cos(d*x+c)^(5/2)
```

maxima [A] time = 0.65, size = 47, normalized size = 0.44

$$\frac{4\,Ab^{\frac{5}{2}}\sin(dx+c)+\left(2\,(dx+c)b^2+b^2\sin(2\,dx+2\,c)\right)B\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(4*A*b^(5/2)*sin(d*x + c) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*B*
sqrt(b))/d
```

mupad [B] time = 1.05, size = 52, normalized size = 0.49

$$\frac{b^2\,\sqrt{b\,\cos(c+d\,x)}\,\left(4\,A\,\sin(c+d\,x)+B\,\sin(2\,c+2\,d\,x)+2\,B\,d\,x\right)}{4\,d\,\sqrt{\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(4*A*sin(c + d*x) + B*sin(2*c + 2*d*x) + 2*B*d*
x))/(4*d*cos(c + d*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.861 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*B*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.08, size = 42, normalized size = 0.65

$$\frac{(b\cos(c+dx))^{5/2}(A(c+dx)+B\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Cos[c + d*x]^(5/
2))
```

fricas [A] time = 1.97, size = 190, normalized size = 2.92

$$\left[\frac{A\sqrt{-b}\,b^2\cos(dx+c)\log\left(2\,b\cos(dx+c)^2 - 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\,\sin(dx+c) - b\right) + 2\sqrt{b\cos(}}{2\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(A*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c
))*B*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*b^(5/2)*arct
an(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
+ c) + sqrt(b*cos(d*x + c))*B*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2), x
)
```

maple [A] time = 0.14, size = 39, normalized size = 0.60

$$\frac{(b\cos(dx+c))^{\frac{5}{2}}(A(dx+c)+B\sin(dx+c))}{d\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(5/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(5/2)
```

maxima [A] time = 0.58, size = 40, normalized size = 0.62

$$\frac{2\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+Bb^{\frac{5}{2}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*b^(5/2)*sin(d*x +
c))/d
```

mupad [B] time = 1.04, size = 38, normalized size = 0.58

$$\frac{b^2\sqrt{b\,\cos(c+d\,x)}\,\left(B\,\sin(c+d\,x)+A\,d\,x\right)}{d\,\sqrt{\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.862 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{Ab^2\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*arctanh(sin(d*x+c))*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{Ab^2\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*ArcTanh[Sin[c +
d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 0.61

$$\frac{(b\cos(c+dx))^{5/2}\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*(b*Cos[c + d*x])^(5/2))/(d*Cos[c + d*x]^
(5/2))
```

fricas [A] time = 1.69, size = 216, normalized size = 3.27

$$\left[-\frac{2\,A\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) - B\sqrt{-b}\,b^2\log\left(2\,b\cos\left(dx+c\right)^2 - 2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx\right.}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c)))) - B*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*b^(
5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))
 + A*b^(5/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2), x
)
```

maple [A] time = 0.19, size = 54, normalized size = 0.82

$$-\frac{(b\cos(dx+c))^{\frac{5}{2}}\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)\right)}{d\cos(dx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/d*(b*cos(d*x+c))^(5/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c
))/cos(d*x+c)^(5/2)
```

maxima [A] time = 0.59, size = 99, normalized size = 1.50

$$\frac{4Bb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-b^2\log\left(\cos(dx+c)^2+\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*B*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b^2*log(cos(d*x
 + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + s
in(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx))}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.863 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b^2*B*arctanh(sin(
d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 8, 3770}

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) +
 (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \mathrm{s}}{\sqrt{\cos(c+d}}$$
$$= \frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\mathrm{Su}}{d\sqrt{\mathrm{co}}}$$
$$= \frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.09, size = 50, normalized size = 0.68

$$\frac{(b\cos(c+dx))^{5/2}\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d
*x]))/(d*Cos[c + d*x]^(7/2))
```

fricas [A] time = 1.43, size = 214, normalized size = 2.89

$$\left[\frac{Bb^{\frac{5}{2}}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}\,Ab^2\sqrt{\cos(dx}}{2\,d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*b^(5/2)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos
(d*x + c)^2), -(B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x
 + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*b^2*s
qrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2), x
)
```

maple [A] time = 0.16, size = 59, normalized size = 0.80

$$\frac{\left(-2B\cos(dx+c)\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)(b\cos(dx+c))^{\frac{5}{2}}}{d\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*c
os(d*x+c))^(5/2)/cos(d*x+c)^(7/2)
```

maxima [A] time = 0.65, size = 127, normalized size = 1.72

$$\frac{\frac{4Ab^{\frac{5}{2}}\sin(2dx+2c)}{\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1}+\left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-b^2\log\left(\mathrm{co}\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*A*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*si
n(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c)
+ 1))*B*sqrt(b))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx))}{\cos(c+dx)^{9/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.864 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab^2\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b^2*B*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*A*b^2*arctanh(sin(d*x+c))*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {17, 2748, 3768, 3770, 3767, 8}

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab^2\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]])
 + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^
2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int s}{\sqrt{\cos(c+d}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] time = 0.15, size = 65, normalized size = 0.56

$$\frac{(b\cos(c+dx))^{5/2}\left(\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2)
,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*
Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))
```

fricas [A] time = 0.94, size = 242, normalized size = 2.09

$$\left[\frac{Ab^{\frac{5}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2Bb^2\cos(dx+c)+Ab^2\right)}{4d\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] [1/4*(A*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*b^2*arctan(sqrt(b*c
os(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 -
 (2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2),
x)
```

maple [A] time = 0.18, size = 121, normalized size = 1.04

$$-\frac{\left(A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-A\left(\cos^2(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-2B\cos(dx+c)\sin}{2d\cos(dx+c)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-2*B*cos(d*x+c)*sin(d*x+c)-A*
sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2)
```

maxima [B] time = 0.70, size = 803, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] 1/4*(8*B*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - (4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*
c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x
 + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(
4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x
+ 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*
cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4
*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 +
 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*
cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c
)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.865 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{Ab^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c}{2}$$

```
[Out] 1/2*b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+A*b^2*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*A*b^2*sin(d*x+c)^3*(b*cos(d*
x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*b^2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 3768, 3770}

$$\frac{Ab^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x]
```

```
[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]])
 + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*
b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*b^2*Sqrt
[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int}{\sqrt{\cos(c+}}\\ &= \frac{b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int\sec(c+}{2\sqrt{\cos(c+dx)}}\\ &= \frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\sin}{2d\cos^{\frac{5}{2}}(c+dx}\end{aligned}$$

Mathematica [A] time = 0.19, size = 76, normalized size = 0.48

$$\frac{(b\cos(c+dx))^{5/2}\left(2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{6d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2)
,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin
[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(9/
2))
```

fricas [A] time = 1.03, size = 274, normalized size = 1.75

$$\left[\frac{3\,Bb^{\frac{5}{2}}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,Ab^{2}\cos\left(dx+c\right)^{2}+3\,Bb\right.}{12\,d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algori
thm="fricas")
```

```
[Out] [1/12*(3*B*b^(5/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(
b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(
3*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(
cos(d*x + c))))*cos(d*x + c)^4 - (4*A*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x
+ c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algori
thm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2),
x)
```

maple [A] time = 0.21, size = 139, normalized size = 0.89

$$\frac{\left(-3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+4A\left(\cos^{2}\left(dx+c\right)\right)\right.}{6d\cos\left(dx+c\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x)
```

```
[Out] 1/6/d*(-3*B*cos(d*x+c)^3*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*B*cos
(d*x+c)^3*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*A*cos(d*x+c)^2*sin(d*x
```

```
+c)+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+
c)^(11/2)
```

maxima [B] time = 0.70, size = 1060, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algori
thm="maxima")
```

```
[Out] -1/12*(16*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)
*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*
b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
 + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(4*(b^2*sin(4*d*x +
4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*co
s(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b
^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
(b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2
 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b
^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*
c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x
 + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^
2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
+ 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx))}{\cos(c+dx)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(13/2),x)
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(13/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
[Out] Timed out
```

3.866 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=136

$$\frac{Ax\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*A*x*cos(d*x+c)
^(1/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^
(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ax\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sq
rt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+a}\right.}{}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c}{2d\sqrt{b\cos(c+dx}}$$

Mathematica [A] time = 0.11, size = 69, normalized size = 0.51

$$\frac{\sqrt{\cos(c+dx)}\,(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*
x)] + B*Sin[3*(c + d*x)]))/(12*d*Sqrt[b*Cos[c + d*x]])
```

fricas [A] time = 2.00, size = 236, normalized size = 1.74

$$\left[-\frac{3\,A\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\left(2\,B\,\mathrm{c}\right.}{12\,bd\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin
(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^2
+ 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c))/(b*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 0.26, size = 74, normalized size = 0.54

$$\frac{\left(\sqrt{\cos}\,\left(dx+c\right)\right)\left(2B\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)+3A\cos\left(dx+c\right)\sin\left(dx+c\right)+3A\left(dx+c\right)+4B\sin\left(dx+c\right)\right)}{6d\sqrt{b\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*cos(d*x+c)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(d*x+
c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(b*cos(d*x+c))^(1/2)
```

maxima [A] time = 0.67, size = 68, normalized size = 0.50

$$\frac{\frac{3 (2 dx+2 c+\sin(2 dx+2 c))A}{\sqrt{b}} + \frac{B\left(\sin(3 dx+3 c)+9 \sin\left(\frac{1}{3} \arctan(\sin(3 dx+3 c),\cos(3 dx+3 c))\right)\right)}{\sqrt{b}}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + B*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b))/d
```

mupad [B] time = 1.82, size = 95, normalized size = 0.70

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\left(3A\sin(c+dx)+3A\sin(3c+3dx)+10B\sin(2c+2dx)+B\sin(4c+4d\right.}{12\,b\,d\,\left(\cos(2c+2dx)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c
+ 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*
x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.867 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+c)
^(1/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c}{2d\sqrt{b\cos(c+dx}}$$

Mathematica [A] time = 0.09, size = 57, normalized size = 0.58

$$\frac{\sqrt{\cos(c+dx)}\,(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*d*Sqrt[b*Cos[c + d*x]])
```

fricas [A] time = 0.94, size = 210, normalized size = 2.14

$$\left[-\frac{B\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,(B\cos(dx}{4\,bd\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)),
 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x +
 c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 0.23, size = 55, normalized size = 0.56

$$\frac{\left(\sqrt{\cos}\ (dx+c)\right)(B\cos(dx+c)\sin(dx+c)+2A\sin(dx+c)+B(dx+c))}{2d\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*cos(d*x+c)^(1/2)*(B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c))/(
b*cos(d*x+c))^(1/2)
```

maxima [A] time = 0.66, size = 40, normalized size = 0.41

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{\sqrt{b}}+\frac{4\,A\sin(dx+c)}{\sqrt{b}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/sqrt(b) + 4*A*sin(d*x + c)/sqrt(b))
/d
```

mupad [B] time = 1.37, size = 82, normalized size = 0.84

$$\frac{\sqrt{\cos(c+d\,x)}\ \sqrt{b\ \cos(c+d\,x)}\ \left(B\ \sin(c+d\,x)+4\,A\ \sin(2\,c+2\,d\,x)+B\ \sin(3\,c+3\,d\,x)+4\,B\,d\,x\ \cos(c+d\right.}{4\,b\,d\ (\cos(2\,c+2\,d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c +
2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b*d*(cos(2*c + 2*d*
x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.868 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=59

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] A*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(
b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (B*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int (A+B\cos(c+dx))\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.71

$$\frac{\sqrt{\cos(c+dx)}\,(A(c+dx)+B\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]]
)
```

fricas [A] time = 0.93, size = 187, normalized size = 3.17

$$\left[-\frac{A\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,\sqrt{b\cos(d}}{2\,bd\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*
B*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)), (A*sqrt(b)*arctan(sq
rt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
+ sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)
```

maple [A] time = 0.18, size = 39, normalized size = 0.66

$$\frac{\left(\sqrt{\cos}\ (dx+c)\right)(A(dx+c)+B\sin(dx+c))}{d\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/(b*cos(d*x+c))^(1/2)
```

maxima [A] time = 0.59, size = 40, normalized size = 0.68

$$\frac{\frac{2\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{B\sin(dx+c)}{\sqrt{b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + B*sin(d*x + c)/sqrt(
b))/d
```

mupad [B] time = 0.54, size = 61, normalized size = 1.03

$$\frac{\sqrt{\cos(c+d\,x)}\ \sqrt{b\ \cos(c+d\,x)}\ \left(B\ \sin(2\,c+2\,d\,x)+2\,A\,d\,x\ \cos(c+d\,x)\right)}{b\,d\ \left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*co
s(c + d*x)))/(b*d*(cos(2*c + 2*d*x) + 1))
```

sympy [A] time = 12.68, size = 46, normalized size = 0.78

$$\begin{cases} \frac{Ax}{\sqrt{b}}+\frac{B\sin(c+dx)}{\sqrt{b}\,d} & \text{for}\ d\neq 0 \\ \frac{x(A+B\cos(c))\sqrt{\cos(c)}}{\sqrt{b\cos(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((A*x/sqrt(b) + B*sin(c + d*x)/(sqrt(b)*d), Ne(d, 0)), (x*(A + B*c
os(c))*sqrt(cos(c))/sqrt(b*cos(c)), True))
```

3.869 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c)^
(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {18, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int(A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.67

$$\frac{\sqrt{\cos(c+dx)}\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*
x]])
```

fricas [B] time = 1.11, size = 215, normalized size = 3.58

$$\left[-\frac{2A\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)+B\sqrt{-b}\,\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\right.}{2bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/2*(2*B*sqrt(b)
*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A
*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d
*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))), x
)
```

maple [A] time = 0.18, size = 54, normalized size = 0.90

$$-\frac{\left(2A\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\left(dx+c\right)\right)\left(\sqrt{\cos}\ (dx+c)\right)}{d\sqrt{b\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c))*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(1/2)
```

maxima [A] time = 0.59, size = 92, normalized size = 1.53

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/sqrt(b))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\ \cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\ \sqrt{b\ \cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\sqrt{b\cos(c + dx)}\,\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.870 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*
cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 8, 3770}

$$\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (A*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -}{d\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.74

$$\frac{A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
[Out] (B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]])
```

fricas [A] time = 1.06, size = 211, normalized size = 3.10

$$\left[\frac{B\sqrt{b}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx}{2bd\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d
*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*
x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)), x
)
```

maple [A] time = 0.17, size = 59, normalized size = 0.87

$$\frac{-2B\cos(dx+c)\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)}{d\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))/(b*c
os(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

maxima [B] time = 0.67, size = 125, normalized size = 1.84

$$\frac{\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4\,A\sqrt{b}\,\sin(2\,dx+2\,c)}{b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*A*sqrt(b)*s
```

```
in(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*
x + 2*c) + b))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cos(c + d\,x)}{\cos(c + d\,x)^{3/2}\,\sqrt{b\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\sqrt{b\cos(c + dx)}\,\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)), x
)
```

3.871 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/d/cos
(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {18, 2748, 3768, 3770, 3767, 8}

$$\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (
A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c +
d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}} - \frac{(B\cdots}{\cdots}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\cdots}{d\cdots}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.61

$$\frac{\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*
x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

fricas [A] time = 0.88, size = 231, normalized size = 2.16

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2B\cos\left(dx+c\right)+A\right)\sqrt{b}\cdots}{4bd\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c
))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*cos(
d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*co
s(d*x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)), x
)
```

maple [A] time = 0.20, size = 120, normalized size = 1.12

$$\frac{-A\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A\left(\cos^{2}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+2B\cos\left(dx+c\right)\sin\cdots}{2d\sqrt{b\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+2*B*cos(d*x+c)*sin(d*x+c)+A*
sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
```

maxima [B] time = 0.71, size = 722, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c
)^2 + 2*b*cos(2*d*x + 2*c) + b) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c
) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4
*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)
+ 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((2*(2*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.872 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=145

$$\frac{A\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}}{2d\sqrt{}}$$

```
[Out] 1/2*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*A*sin(d*x+c)^3
/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b
*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+
c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 3768, 3770}

$$\frac{A\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}}{2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (
B*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c +
d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3)/(3*d
*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}} - \frac{(A}{}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} +$$

Mathematica [A] time = 0.09, size = 76, normalized size = 0.52

$$\frac{2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{6d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos
[2*(c + d*x)])*Tan[c + d*x])/(6*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

fricas [A] time = 0.85, size = 259, normalized size = 1.79

$$\left[\frac{3\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,A\cos\left(dx+c\right)^{2}+3\,B\cos\right.}{12\,bd\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-
b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)))
)*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)), x
)
```

maple [A] time = 0.22, size = 139, normalized size = 0.96

$$\frac{-3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+4A\left(\cos^{2}\left(dx+c\right)\right)\right.}{6d\sqrt{b\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*(-3*B*cos(d*x+c)^3*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*B*cos
(d*x+c)^3*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*A*cos(d*x+c)^2*sin(d*x
+c)+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+
c)^(5/2)
```

maxima [B] time = 0.71, size = 957, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1
)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4
*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x
 + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6
*cos(2*d*x + 2*c) + 1)*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*
c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4
*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4
*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.873 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{Ax\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*A*x*cos(d*x+
c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*
x+c))^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ax\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(2*b*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(
3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos^3(c+dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2b\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+a\right.}{}\\ &= \frac{Ax\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(}{2bd\sqrt{b\cos(c+}\end{aligned}$$

Mathematica [A] time = 0.09, size = 69, normalized size = 0.47

$$\frac{\cos^{\frac{3}{2}}(c+dx)(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*
x)] + B*Sin[3*(c + d*x)]))/(12*d*(b*Cos[c + d*x])^(3/2))
```

fricas [A] time = 1.18, size = 236, normalized size = 1.59

$$\left[-\frac{3\,A\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\left(2\,B\,c\right.}{12\,b^{2}d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b^2*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*s
in(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b^2*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2), x
)
```

maple [A] time = 0.22, size = 74, normalized size = 0.50

$$\frac{\left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\left(2B\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)+3A\cos\left(dx+c\right)\sin\left(dx+c\right)+3A\left(dx+c\right)+4B\sin\left(dx+c\right)\right)}{6d\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/6/d*cos(d*x+c)^(3/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(d*x+
c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(b*cos(d*x+c))^(3/2)
```

maxima [A] time = 0.69, size = 68, normalized size = 0.46

$$\frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{3}{2}}}+\frac{B\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{3}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + B*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(3/2))/d
```

mupad [B] time = 1.57, size = 95, normalized size = 0.64

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,A\,\sin\left(c+d\,x\right)+3\,A\,\sin\left(3\,c+3\,d\,x\right)+10\,B\,\sin\left(2\,c+2\,d\,x\right)+B\,\sin\left(4\,c+4\,d\,x\right)\right.}{12\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c
+ 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*
x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.874 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+
c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*
x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int\cos(c+dx)(A+B\cos(c+dx))\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(}{2bd\sqrt{b\cos(c+}$$

Mathematica [A] time = 0.10, size = 57, normalized size = 0.53

$$\frac{\cos^{\frac{3}{2}}(c+dx)(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*d*(b*Cos[c + d*x])^(3/2))
```

fricas [A] time = 1.16, size = 210, normalized size = 1.96

$$\left[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2(B\cos(d}{4b^2d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)
), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x
 + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2), x
)
```

maple [A] time = 0.17, size = 55, normalized size = 0.51

$$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)(B\cos(dx+c)\sin(dx+c)+2A\sin(dx+c)+B(dx+c))}{2d\,(b\cos(dx+c))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*cos(d*x+c)^(3/2)*(B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c))/(
b*cos(d*x+c))^(3/2)
```

maxima [A] time = 0.68, size = 40, normalized size = 0.37

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{3}{2}}}+\frac{4\,A\sin(dx+c)}{b^{\frac{3}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(3/2) + 4*A*sin(d*x + c)/b^(3/2))
/d
```

mupad [B] time = 0.69, size = 82, normalized size = 0.77

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(B\,\sin(c+d\,x)+4\,A\,\sin(2\,c+2\,d\,x)+B\,\sin(3\,c+3\,d\,x)+4\,B\,d\,x\,\cos(c+d\right.}{4\,b^2\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c +
2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b^2*d*(cos(2*c + 2*
d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.875 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] A*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b
/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int (A+B\cos(c+dx))\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.65

$$\frac{\cos^{\frac{3}{2}}(c+dx)(A(c+dx)+B\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/
2))
```

fricas [A] time = 1.10, size = 187, normalized size = 2.88

$$\left[-\frac{A\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2 + 2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c) - b\right) - 2\,\sqrt{b\cos(d}}{2\,b^2 d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*
B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)), (A*sqrt(b)*arctan(
sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c
) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x
+ c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2), x
)
```

maple [A] time = 0.17, size = 39, normalized size = 0.60

$$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)(A(dx+c)+B\sin(dx+c))}{d(b\cos(dx+c))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(3/2)*(A*(d*x+c)+B*sin(d*x+c))/(b*cos(d*x+c))^(3/2)
```

maxima [A] time = 0.60, size = 40, normalized size = 0.62

$$\frac{\frac{2\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}+\frac{B\sin(dx+c)}{b^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2) + B*sin(d*x + c)/b^(3/
2))/d
```

mupad [B] time = 1.02, size = 61, normalized size = 0.94

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(B\,\sin(2\,c+2\,d\,x)+2\,A\,d\,x\,\cos(c+d\,x)\right)}{b^2\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*co
s(c + d*x)))/(b^2*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.876 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c
)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]
]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int(A+B\cos(c+dx))\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Mathematica [**A**] time = 0.05, size = 40, normalized size = 0.61

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Cos[c + d*x]^(3/2))/(d*(b*Cos[c + d*x])^
(3/2))
```

fricas [**A**] time = 1.19, size = 215, normalized size = 3.26

$$\left[-\frac{2\,A\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)+B\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\right.}{2\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(2*B*sqrt(
b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) +
 A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos
(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2), x
)
```

maple [A] time = 0.16, size = 54, normalized size = 0.82

$$-\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\,(dx+c)\right)}{d\,(b\cos(dx+c))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(3/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c))/(
b*cos(d*x+c))^(3/2)
```

maxima [A] time = 0.60, size = 92, normalized size = 1.39

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{b^{\frac{3}{2}}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/b^(3/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\cos(c + dx)}}{(b\cos(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(b*cos(c + d*x))**(3/2), x
)
```

3.877 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c)
)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 8, 3770}

$$\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (
A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{\cos(c+dx)}\right)\mathrm{Subst}(\int 1\,dx, x}{bd\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.68

$$\frac{\sqrt{\cos(c+dx)}\,\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])
)/(d*(b*Cos[c + d*x])^(3/2))
```

fricas [A] time = 0.73, size = 211, normalized size = 2.85

$$\left[\frac{B\sqrt{b}\,\cos\left(dx+c\right)^2\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx\right.}}{2\,b^2d\cos\left(dx+c\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos
(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(
d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))),
 x)
```

maple [A] time = 0.19, size = 59, normalized size = 0.80

$$\frac{\left(-2B\cos\left(dx+c\right)\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin\left(dx+c\right)\right)\left(\sqrt{\cos}\,\left(dx+c\right)\right)}{d\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*cos(
d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2)
```

maxima [B] time = 0.67, size = 133, normalized size = 1.80

$$\frac{\frac{4A\sqrt{b}\sin(2dx+2c)}{b^2\cos(2dx+2c)^2+b^2\sin(2dx+2c)^2+2b^2\cos(2dx+2c)+b^2}+\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(\right.\right.}{b^{\frac{3}{2}}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + B*(log(cos(d*x + c)^2 + sin(d*x +
 c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
*x + c) + 1))/b^(3/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))),
 x)
```

3.878 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b/d
/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*A*arctanh(sin(d*x+c))*cos(d*x+c)
^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {18, 2748, 3768, 3770, 3767, 8}

$$\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) +
 (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[
c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}}-\\ &= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.56

$$\frac{\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*
x])/(2*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
```

fricas [A] time = 0.88, size = 231, normalized size = 1.99

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2B\cos\left(dx+c\right)+A\right)\sqrt{b}}{4b^{2}d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b^2*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x +
 c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*co
s(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*
d*cos(d*x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2)),
 x)
```

maple [A] time = 0.18, size = 121, normalized size = 1.04

$$-\frac{A\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-A\left(\cos^{2}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-2B\cos\left(dx+c\right)\sin}{2d\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-2*B*cos(d*x+c)*sin(d*x+c)-A*
sin(d*x+c))/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)
```

maxima [B] time = 0.70, size = 739, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b*co
s(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x +
 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.879 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$\frac{A\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c-}{2b}$$

```
[Out] 1/2*B*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*A*sin(d*x+c)
^3/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/b/d/cos(d*x+c)^(1
/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*
cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 3768, 3770}

$$\frac{A\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c-}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) +
 (B*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[
c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3
)/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int(A+B\cos(c+dx))\sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^3(c+dx)\,d}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}} -$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.48

$$\frac{2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{6d\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos
[2*(c + d*x)])*Tan[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)
)
```

fricas [A] time = 0.90, size = 259, normalized size = 1.65

$$\left[\frac{3\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,A\cos\left(dx+c\right)^{2}+3\,B\cos\right.}{12\,b^{2}d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4), -1/6*(3*B*sqrt
(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)),
 x)
```

maple [A] time = 0.20, size = 139, normalized size = 0.89

$$\frac{-3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+4A\left(\cos^{2}\left(dx+c\right)\right)\right.}{6d\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/6/d*(-3*B*cos(d*x+c)^3*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*B*cos
(d*x+c)^3*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*A*cos(d*x+c)^2*sin(d*x
+c)+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))/(b*cos(d*x+c))^(3/2)/cos(d*x+
c)^(3/2)
```

maxima [B] time = 0.72, size = 983, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2
+ 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 +
18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*co
s(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*
d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x +
4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) - 3*(4*(sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
 + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x
+ 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*B/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x
 + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2
+ 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)
*sqrt(b)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.880 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=148

$$\frac{Ax\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*A*x*cos(d*
x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*
cos(d*x+c))^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^
(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ax\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x
]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos^3(c+dx)$$

$$= \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2b^2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+\right.}{}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin}{2b^2d\sqrt{b\cos(c+}}$$

Mathematica [A] time = 0.07, size = 72, normalized size = 0.49

$$\frac{\sqrt{\cos(c+dx)}\,(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*
x)] + B*Sin[3*(c + d*x)]))/(12*b^2*d*Sqrt[b*Cos[c + d*x]])
```

fricas [A] time = 0.81, size = 236, normalized size = 1.59

$$\left[-\frac{3\,A\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\left(2\,B\,c\right.}{12\,b^{3}d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b^3*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*s
in(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b^3*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x
)
```

maple [A] time = 0.21, size = 74, normalized size = 0.50

$$\frac{\left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\left(2B\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)+3A\cos\left(dx+c\right)\sin\left(dx+c\right)+3A\left(dx+c\right)+4B\sin\left(dx+c\right)\right)}{6d\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/6/d*cos(d*x+c)^(5/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(d*x+
c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(b*cos(d*x+c))^(5/2)
```

maxima [A] time = 0.67, size = 68, normalized size = 0.46

$$\frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{5}{2}}}+\frac{B\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{5}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + B*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2))/d
```

mupad [B] time = 1.56, size = 95, normalized size = 0.64

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,A\,\sin\left(c+d\,x\right)+3\,A\,\sin\left(3\,c+3\,d\,x\right)+10\,B\,\sin\left(2\,c+2\,d\,x\right)+B\,\sin\left(4\,c+4\right.\right.}{12\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c
+ 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*
x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.881 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*
x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*
cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin}{2b^2d\sqrt{b\cos(c+}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.56

$$\frac{\sqrt{\cos(c+dx)}\,(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

fricas [A] time = 1.02, size = 210, normalized size = 1.96

$$\left[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2\,(B\cos(d}{4\,b^3d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)
), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x
 + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2), x
)
```

maple [A] time = 0.18, size = 55, normalized size = 0.51

$$\frac{\left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\left(B\cos\left(dx+c\right)\sin\left(dx+c\right)+2A\sin\left(dx+c\right)+B\left(dx+c\right)\right)}{2d\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*cos(d*x+c)^(5/2)*(B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c))/(
b*cos(d*x+c))^(5/2)
```

maxima [A] time = 0.68, size = 40, normalized size = 0.37

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{5}{2}}}+\frac{4\,A\sin(dx+c)}{b^{\frac{5}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(5/2) + 4*A*sin(d*x + c)/b^(5/2))
/d
```

mupad [B] time = 0.70, size = 82, normalized size = 0.77

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(B\,\sin\left(c+d\,x\right)+4\,A\,\sin\left(2\,c+2\,d\,x\right)+B\,\sin\left(3\,c+3\,d\,x\right)+4\,B\,d\,x\,\cos\left(c+d\right.\right.}{4\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c +
2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b^3*d*(cos(2*c + 2*
d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.882 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] A*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)
/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int(A+B\cos(c+dx))\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\cos(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\,(A(c+dx)+B\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d
*x]])
```

fricas [A] time = 1.53, size = 187, normalized size = 2.88

$$\left[-\frac{A\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,\sqrt{b\cos(d}}{2\,b^3d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*
B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), (A*sqrt(b)*arctan(
sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c
) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x
+ c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2), x
)
```

maple [A] time = 0.15, size = 39, normalized size = 0.60

$$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(A(dx+c)+B\sin(dx+c))}{d(b\cos(dx+c))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(5/2)*(A*(d*x+c)+B*sin(d*x+c))/(b*cos(d*x+c))^(5/2)
```

maxima [A] time = 0.60, size = 40, normalized size = 0.62

$$\frac{\frac{2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}+\frac{B\sin(dx+c)}{b^{\frac{5}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2) + B*sin(d*x + c)/b^(5/
2))/d
```

mupad [B] time = 0.48, size = 61, normalized size = 0.94

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(B\,\sin(2\,c+2\,d\,x)+2\,A\,d\,x\,\cos(c+d\,x)\right)}{b^3\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*co
s(c + d*x)))/(b^3*d*(cos(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.883 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=66

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x
+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*
x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int(A+B\cos(c+dx))\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.65

$$\frac{\sqrt{\cos(c+dx)}\,\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c
+ d*x]])
```

fricas [A] time = 0.87, size = 215, normalized size = 3.26

$$\left[-\frac{2\,A\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) + B\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^2 + 2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\right.}{2\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/2*(2*B*sqrt(
b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) +
 A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos
(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2), x
)
```

maple [A] time = 0.14, size = 54, normalized size = 0.82

$$-\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)\right)}{d\,(b\cos(dx+c))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(5/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c))/(
b*cos(d*x+c))^(5/2)
```

maxima [A] time = 0.60, size = 92, normalized size = 1.39

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{b^{\frac{5}{2}}}+\frac{4B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/b^(5/2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.884 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+
c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 8, 3770}

$$\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{\cos(c+dx)}\right)\mathrm{Subst}(\int 1\,dx}{b^2d\sqrt{b\cos(c+d}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.68

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])
)/(d*(b*Cos[c + d*x])^(5/2))
```

fricas [A] time = 0.92, size = 211, normalized size = 2.85

$$\left[\frac{B\sqrt{b}\,\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}\,A\sqrt{\cos(dx}{2\,b^3d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos
(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(
d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2), x
)
```

maple [A] time = 0.17, size = 59, normalized size = 0.80

$$\frac{\left(-2B\cos(dx+c)\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)\left(\cos^{\frac{3}{2}}(dx+c)\right)}{d\,(b\cos(dx+c))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*cos(
d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2)
```

maxima [B] time = 0.66, size = 133, normalized size = 1.80

$$\frac{\frac{4A\sqrt{b}\sin(2dx+2c)}{b^3\cos(2dx+2c)^2+b^3\sin(2dx+2c)^2+2b^3\cos(2dx+2c)+b^3}+\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(\right.\right.}{b^{\frac{5}{2}}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + B*(log(cos(d*x + c)^2 + sin(d*x +
 c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
*x + c) + 1))/b^(5/2))/d
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.885 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b
^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*A*arctanh(sin(d*x+c))*cos(d*
x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {18, 2748, 3768, 3770, 3767, 8}

$$\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
 + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*
Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,d
\\
&= \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 0.56

$$\frac{\sqrt{\cos(c+dx)}\,\left(\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[
c + d*x])*Sin[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))
```

fricas [A] time = 0.89, size = 231, normalized size = 1.99

$$\left[\frac{A\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(2B\cos(dx+c)+A)\sqrt{b}}{4b^3d\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b^3*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x +
 c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*co
s(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*
d*cos(d*x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c))),
 x)
```

maple [A] time = 0.20, size = 120, normalized size = 1.03

$$\frac{\left(-A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A\left(\cos^2(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+2B\cos(dx+c)\sin\right.}{2d\,(b\cos(dx+c))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+2*B*cos(d*x+c)*sin(d*x+c)+A*
sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2)
```

maxima [B] time = 0.69, size = 757, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*
cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*
b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*co
s(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*s
qrt(b)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out
```

3.886 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{A\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos}}{}$$

```
[Out] 1/2*B*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*A*sin(d*x+
c)^3/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/b^2/d/cos(d*x
+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b
^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 3768, 3770}

$$\frac{A\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
 + (B*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*
Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c +
d*x]^3)/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int(A+B\cos(c+dx))\sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^3(c+dx)\,d}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.48

$$\frac{\sqrt{\cos(c+dx)}\left(2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{6d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c +
 d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*(b*Cos[c + d*x])^(5/
2))
```

fricas [A] time = 0.94, size = 259, normalized size = 1.65

$$\left[\frac{3\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,A\cos\left(dx+c\right)^{2}+3\,B\cos\right.}{12\,b^{3}d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4), -1/6*(3*B*sqrt
(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)),
 x)
```

maple [A] time = 0.20, size = 139, normalized size = 0.89

$$\frac{-3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3B\left(\cos^{3}\left(dx+c\right)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+4A\left(\cos^{2}\left(dx+c\right)\right)}{6d\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/6/d*(-3*B*cos(d*x+c)^3*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*B*cos
(d*x+c)^3*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*A*cos(d*x+c)^2*sin(d*x
+c)+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))/(b*cos(d*x+c))^(5/2)/cos(d*x+
c)^(1/2)
```

maxima [B] time = 0.71, size = 1033, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c
)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x +
 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)
^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2
*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(
4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x +
6*c))*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*
c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d
*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(
4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((b^2*cos(4*d*x + 4*c)^2 + 4*
b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 +
2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.887 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/10*A*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(
d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/13*B*(b*cos(d*x+c))^(13/3)*hypergeom([1
/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]
^2]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*
b^4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{7/3}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{7/3}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{10/3}\,dx}{b^3}$$
$$= -\frac{3A(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+}{10b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.18, size = 94, normalized size = 0.79

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cos^2(c+dx)\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\left(13A\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)+10B\cos(c+dx)\,{}_2F_1\left(\right.\right.}{130d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(13*A*Hypergeometric
2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] + 10*B*Cos[c + d*x]*Hypergeometric2F1[1/
2, 13/6, 19/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(130*d)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^3 + A\cos\left(dx+c\right)^2\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\left(A+B\cos\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.888 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*A*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d
*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([1/
2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1
0/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^3
*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\cos(c+dx))^{4/3}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{7/3}\,dx}{b^2}$$
$$= -\frac{3A(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c}{7b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.15, size = 89, normalized size = 0.75

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cot(c+dx)(b\cos(c+dx))^{4/3}\left(10A\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};co\right.\right.}{70bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6, 1
3/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(70*b*d)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^2 + A\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \cos \left(dx+c\right) \left(b \cos \left(dx+c\right)\right)^{\frac{1}{3}} \left(A+B \cos \left(dx+c\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B \cos \left(dx+c\right)+A\right) \left(b \cos \left(dx+c\right)\right)^{\frac{1}{3}} \cos \left(dx+c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.889 $\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2\right.}{7b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*A*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*
x+c)/b/d/(sin(d*x+c)^2)^(1/2)-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/
6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2\right.}{7b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)
*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^2*d*S
qrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = A\int \sqrt[3]{b\cos(c+dx)}\,dx + \frac{B\int(b\cos(c+dx))^{4/3}\,dx}{b}$$
$$= -\frac{3A(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\,c}{}$$

Mathematica [A] time = 0.09, size = 86, normalized size = 0.72

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\left(7A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c\right.\right.}{28d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3, 5/
3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos
[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*d)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{1}{3}}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.890 $\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=114

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\ {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+\right.}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*A*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+
c)/d/(sin(d*x+c)^2)^(1/2)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[
5/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\ {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+\right.}{4bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hyp
ergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= (Ab)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx + B\int \sqrt[3]{b\cos(c+dx)}\,dx$$
$$= -\frac{3A\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+d}{d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.10, size = 86, normalized size = 0.75

$$-\frac{3b\sqrt{\sin^2(c+dx)}\,\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] +
B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{1}{3}}(A+B\cos(dx+c))\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{1/3}(A+B\cos(c+dx))}{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

3.891 $\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*b*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c
))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2]
,[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (3*A*b*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1/3)
*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx + (bB)\int \frac{1}{(b\cos(c+dx))^{2}}$$
$$= \frac{3Ab\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3B\sqrt[3]{b\cos}}{}$$

Mathematica [A] time = 0.13, size = 86, normalized size = 0.77

$$\frac{3b\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\left(A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)-2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\right)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2*
B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{1}{3}} (A+B\cos(dx+c)) \left(\sec^2(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{1/3}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.892 $\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{5/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

```
[Out] 3/5*A*b^2*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x
+c))^(5/3)/(sin(d*x+c)^2)^(1/2)+3/2*b*B*hypergeom([-1/3, 1/2],[2/3],cos(d*x
+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{5/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5
*d*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[
-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*S
qrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx$$
$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx}$$
$$= \frac{3Ab^2\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,{}_2F}{2d}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 0.80

$$\frac{3\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\sec^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,\left(2A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(\right.\right.}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(2*A*Hypergeometric2F1[-5/6, 1/2, 1/
6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos
[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(10*d)
```

fricas [F] time = 1.67, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (b\cos (dx+c))^{\frac{1}{3}} (A+B\cos (dx+c)) \left(\sec^{3} (dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos (dx+c)+A) (b\cos (dx+c))^{\frac{1}{3}} \sec (dx+c)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos (c+d\,x))^{1/3}\,\left(A+B\,\cos (c+d\,x)\right)}{\cos (c+d\,x)^3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.893 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{16/3}\,{}_2F_1\left(\frac{1}{2},\frac{8}{3};\frac{11}{3};\right.}{16b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/13*A*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*si
n(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/16*B*(b*cos(d*x+c))^(16/3)*hypergeom(
[1/2, 8/3],[11/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{16/3}\,{}_2F_1\left(\frac{1}{2},\frac{8}{3};\frac{11}{3};c\right.}{16b^4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*
x]^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])
^(16/3)*Hypergeometric2F1[1/2, 8/3, 11/3, Cos[c + d*x]^2]*Sin[c + d*x])/(16
*b^4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{10/3}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{10/3}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{13/3}\,dx}{b^3}$$
$$= -\frac{3A(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\sin}{13b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.21, size = 94, normalized size = 0.79

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cos^2(c+dx)\cot(c+dx)(b\cos(c+dx))^{4/3}\left(16A\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)+13B\cos(c+dx)\,{}_2\right.}{208d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(16*A*Hypergeometric
2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2] + 13*B*Cos[c + d*x]*Hypergeometric2F1[
1/2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(208*d)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^4 + Ab\cos\left(dx+c\right)^3\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^3)*(b*cos(d*x + c))^(1/3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\left(A+B\cos\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.894 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos\right.}{13b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/10*A*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(
d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/13*B*(b*cos(d*x+c))^(13/3)*hypergeom([1
/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos\right.}{13b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]
^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*
b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{7/3}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\cos(c+dx))^{7/3}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{10/3}\,dx}{b^2}$$
$$= -\frac{3A(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c$$
$$10b^2d\sqrt{\sin^2(c+dx)}$$

Mathematica [A] time = 0.19, size = 89, normalized size = 0.75

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cot(c+dx)(b\cos(c+dx))^{7/3}\left(13A\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)+10B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\right.\right.}{130bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 5/3, 8
/3, Cos[c + d*x]^2] + 10*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 13/6, 19/6,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(130*b*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^3 + Ab\cos\left(dx+c\right)^2\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \cos \left(d x+c\right) \left(b \cos \left(d x+c\right)\right)^{\frac{4}{3}} \left(A+B \cos \left(d x+c\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(B \cos \left(d x + c\right) + A\right)} \left(b \cos \left(d x + c\right)\right)^{\frac{4}{3}} \cos \left(d x + c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.895 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos\right.}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*A*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([1/2,
 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos\right.}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(10/
3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d
*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = A\int (b\cos(c+dx))^{4/3}\,dx + \frac{B\int (b\cos(c+dx))^{7/3}\,dx}{b}$$
$$= -\frac{3A(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}} - \frac{3B(}{}$$

Mathematica [A] time = 0.05, size = 86, normalized size = 0.72

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{4/3}\left(10A\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos\right.\right.}{70d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6, 1
3/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(70*d)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos(dx+c)^2 + Ab\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (b\,\cos(c+d\,x))^{4/3}\,(A+B\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.896 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c\right.}{7bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*A*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*
x+c)/d/(sin(d*x+c)^2)^(1/2)-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6]
,[13/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c\right.}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*H
ypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$$
$$= (Ab)\int \sqrt[3]{b\cos(c+dx)}\,dx + B\int (b\cos(c+dx))^{4/3}\,dx$$
$$= -\frac{3A(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c}{4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.01, size = 87, normalized size = 0.75

$$-\frac{3b\sqrt{\sin^2(c+dx)}\,\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\,\left(7A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2\right.\right.}{28d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3,
5/3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*d)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \left(b \cos \left(d x +c \right)\right)^{\frac{4}{3}} \left(A +B \cos \left(d x +c \right)\right) \sec \left(d x +c \right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

3.897 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=112

$$-\frac{3Ab\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\ {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*A*b*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*
x+c)/d/(sin(d*x+c)^2)^(1/2)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3]
,[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3Ab\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\ {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (-3*A*b*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x
]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*H
ypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx + (bB)\int \sqrt[3]{b\cos(c+dx}$$
$$= -\frac{3Ab\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+}{d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.79

$$-\frac{3b^2\sqrt{\sin^2(c+dx)}\,\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
+ B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(b \cos \left(dx+c\right)\right)^{\frac{4}{3}} \left(A+B \cos \left(dx+c\right)\right) \left(\sec^{2}\left(dx+c\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B \cos \left(dx+c\right)+A\right) \left(b \cos \left(dx+c\right)\right)^{\frac{4}{3}} \sec \left(dx+c\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.898 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{3Ab^2\sin(c+dx)\,_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*b^2*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x
+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*b*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6,
1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab^2\sin(c+dx)\,_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2
*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*(b*Cos[c + d*x])^(
1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt
[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+d$$
$$= \frac{3Ab^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3bB\sqrt[3]{}}{}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.77

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\left(A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)-2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\right)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] -
2*B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2+Ab\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}} (A+B\cos(dx+c)) \left(\sec^3(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}} \sec(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{4/3}\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.899 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos\right.}{10b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*A*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d
*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([1/
2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos\right.}{10b^4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1
0/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^4
*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{4/3}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{7/3}\,dx}{b^3}$$
$$= -\frac{3A(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\text{co}}{}$$

Mathematica [A] time = 0.18, size = 94, normalized size = 0.79

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cos^2(c+dx)\cot(c+dx)\left(10A\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+\right.\right.}{70d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6, 13/6, Cos
[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(70*d*(b*Cos[c + d*x])^(2/3))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3)/b, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(dx+c\right)\right)\left(A+B\cos\left(dx+c\right)\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

3.900 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*A*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*
x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2,
7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/
3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^3*d
*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int \sqrt[3]{b\cos(c+dx)}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{4/3}\,dx}{b^2}$$
$$= -\frac{3A(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos}{}$$

Mathematica [A] time = 0.02, size = 89, normalized size = 0.75

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\,\left(7A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(\right.\right.}{28bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3, 5/
3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos
[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*b*d)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)/b, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(dx + c\right) \left(A + B \cos \left(dx + c\right)\right)}{\left(b \cos \left(dx + c\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B \cos \left(dx + c\right) + A\right) \cos \left(dx + c\right)}{\left(b \cos \left(dx + c\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(c + d\,x\right)\,\left(A + B\,\cos \left(c + d\,x\right)\right)}{{\left(b\,\cos \left(c + d\,x\right)\right)}^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

3.901 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=117

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c\right.}{4b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*A*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3]
,[5/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2748, 2643}

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c\right.}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*H
ypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b^2*d*Sqrt
[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = A\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx + \frac{B\int \sqrt[3]{b\cos(c+dx)}\,dx}{b}$$
$$= -\frac{3A\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{4/3}\;{}_2F_1}{4b^2d}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.73

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + B*
Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c +
 d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(2/3), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(dx + c)}{(b\cos(dx + c))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
[Out] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{(b\,\cos(c + d\,x))^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(2/3),x)
[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{(b\cos(c + dx))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
[Out] Integral((A + B*cos(c + d*x))/(b*cos(c + d*x))**(2/3), x)
```

3.902 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))
^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[
7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1/3)*H
ypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= (Ab)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx + B\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3B\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6}\right.}{bd\sqrt{s}}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.75

$$\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\left(A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)-2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\right)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2*B*
Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin[c +
 d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(d*
x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(dx+c))\sec(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

3.903 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{5/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

```
[Out] 3/5*A*b*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c
))^(5/3)/(sin(d*x+c)^2)^(1/2)+3/2*B*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^
2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{5/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*b*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d
*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3
, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx\\ &= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx+(bB)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3Ab\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(\right.}{2d(b\cos(c+dx))^{2/}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 89, normalized size = 0.78

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(2A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\right)}{10d(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(2*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]
+ 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2])*Sqrt[
Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(8/3))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*cos(
d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(dx + c))\left(\sec^2(dx + c)\right)}{(b\cos(dx + c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx + c) + A)\sec(dx + c)^2}{(b\cos(dx + c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos(c + d\,x)}{\cos(c + d\,x)^2\,(b\,\cos(c + d\,x))^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(b*cos(c + d*x))**(2/3), x)
```

3.904 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{8/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{5/3}}$$

```
[Out] 3/8*A*b^2*hypergeom([-4/3, 1/2],[-1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*
x+c))^(8/3)/(sin(d*x+c)^2)^(1/2)+3/5*b*B*hypergeom([-5/6, 1/2],[1/6],cos(d*
x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{8/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*b^2*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
8*d*(b*Cos[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1
[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)*
Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{11/3}}\,dx$$
$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{11/3}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx$$
$$= \frac{3Ab^2\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\text{co}\right.}{5d(b\cos(c+dx}$$

Mathematica [A] time = 0.15, size = 89, normalized size = 0.76

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\csc(c+dx)\left(5A\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)+8B\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\right)}{40d(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(5*A*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]
 + 8*B*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(40*d*(b*Cos[c + d*x])^(8/3))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos(
d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(dx+c))\left(\sec^{3}(dx+c)\right)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{3}}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3}\,(b\,\cos(c+d\,x))^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^{3}(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(b*cos(c + d*x))**(2/3), x)
```

3.905 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/5*A*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d
*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2,
 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8
/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^4*d
*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\int (b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{2/3}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{5/3}\,dx}{b^3}$$
$$= -\frac{3A(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\,c}{}$$

Mathematica [A] time = 0.18, size = 94, normalized size = 0.79

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cos^2(c+dx)\cot(c+dx)\left(8A\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c\right.\right.}{40d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*(8*A*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[
c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(40*d*(b*Cos[c + d*x])^(4/3))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{2}{3}}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(dx+c\right)\right)\left(A+B\cos\left(dx+c\right)\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

3.906 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2\right.}{5b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*A*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*
x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2,
5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2\right.}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/
3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d
*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+B\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}$$
$$= \frac{A\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{2/3}\,dx}{b^2}$$
$$= -\frac{3A(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(}{}$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.75

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\cot(c+dx)\left(5A\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\right)}{10bd\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*
B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[Sin[
c + d*x]^2])/(10*b*d*(b*Cos[c + d*x])^(1/3))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}}{b^2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)\left(A+B\cos\left(dx+c\right)\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

3.907 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))
^(1/3)/(sin(d*x+c)^2)^(1/2)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2]
,[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2748, 2643}

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(2/3)*H
ypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*Sqrt
[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = A\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx + \frac{B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(\right.}{2b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.13, size = 85, normalized size = 0.73

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)-2A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\right)}{2d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(-2*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
B*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(2*d*(b*Cos[c + d*x])^(4/3))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
[Out] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(4/3),x)
[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
[Out] Timed out
```

3.908 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{4/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}$$

```
[Out] 3/4*A*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))
^(4/3)/(sin(d*x+c)^2)^(1/2)+3*B*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*s
in(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{4/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Si
n[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx\\ &= (Ab)\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx + B\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx\\ &= \frac{3A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+\right.}{bd\sqrt[3]{b\cos(c+dx)}\,\sqrt{\mathrm{si}}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.75

$$\frac{3b\sqrt{\sin^2(c+dx)}\,\cot(c+dx)\left(A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\right)}{4d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Cot[c + d*x]*(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*
B*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2])*Sqrt[Sin[
c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(
d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(dx+c))\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

3.909 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{7/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{4/3}}$$

```
[Out] 3/7*A*b*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+
c))^(7/3)/(sin(d*x+c)^2)^(1/2)+3/4*B*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)
^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{7/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*b*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/
3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt
[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{10/3}}\,dx + (bB)\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx$$
$$= \frac{3Ab\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2\right.}{4d(b\cos(c+dx))}$$

Mathematica [A] time = 0.20, size = 89, normalized size = 0.78

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(4A\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\right)}{28d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]
 + 7*B*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(10/3))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^2}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*co
s(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(dx + c))\left(\sec^2(dx + c)\right)}{(b\cos(dx + c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx + c) + A)\sec(dx + c)^2}{(b\cos(dx + c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cos(c + d\,x)}{\cos(c + d\,x)^2\,(b\cos(c + d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

3.910 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{10/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{7/3}}$$

```
[Out] 3/10*A*b^2*hypergeom([-5/3, 1/2],[-2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d
*x+c))^(10/3)/(sin(d*x+c)^2)^(1/2)+3/7*b*B*hypergeom([-7/6, 1/2],[-1/6],cos
(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{10/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*b^2*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
10*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2
F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/
3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{13/3}}\,dx$$
$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{13/3}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{10/3}}\,dx$$
$$= \frac{3Ab^2\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos\ldots\right.}{7d(b\cos(c+dx)\ldots}$$

Mathematica [A] time = 0.20, size = 89, normalized size = 0.76

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\left(7A\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)+10B\cos(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\right)}{70d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(7*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]
 + 10*B*Cos[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(70*d*(b*Cos[c + d*x])^(10/3))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^3}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*co
s(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(dx+c))\left(\sec^{3}(dx+c)\right)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{3}}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3}\,(b\,\cos(c+d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

3.911 $\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=157

$$-\frac{A\sin(c+dx)\cos^{m+1}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(c+dx)\right)}{d(m+n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)\,c}{}$$

```
[Out] -A*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2+1/2*m+1/2*n],[3/2+
1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1+m+n)/(sin(d*x+c)^2)^(1/2)-B*cos(
d*x+c)^(2+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1+1/2*m+1/2*n],[2+1/2*m+1/2*n
],cos(d*x+c)^2)*sin(d*x+c)/d/(2+m+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {20, 2748, 2643}

$$-\frac{A\sin(c+dx)\cos^{m+1}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(c+dx)\right)}{d(m+n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + m
+ n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m + n)*Sqrt[Si
n[c + d*x]^2])) - (B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric
2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2
+ m + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{m+n}(c+dx)(A+$$

$$= \left(A\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{m+n}(c+dx)\,d$$

$$= -\frac{A\cos^{1+m}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+\right.}{d(1+m+n)\sqrt{\sin^2}}$$

Mathematica [A] time = 0.27, size = 130, normalized size = 0.83

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)(b\cos(c+dx))^n\left(A(m+n+2)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);c\right.\right.}{d(m+n+1)(m+n+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(2 + m + n)*Hype
rgeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2] + B*(1 + m
 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos
[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m + n)*(2 + m + n)))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

maple [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \left(\cos^{m}\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\left(A+B\cos\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(c+dx\right)\right)^{n}\left(A+B\cos\left(c+dx\right)\right)\cos^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)
```

3.912 $\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=141

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,{}_2F_1\left(\frac{1}{2},\frac{n+4}{2};\frac{n+}{2}\right.}{b^4d(n+4)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -A*(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(4+n)*hyperg
eom([1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(4+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,{}_2F_1\left(\frac{1}{2},\frac{n+4}{2};\frac{n+}{2}\right.}{b^4d(n+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b
*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{2+n}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{2+n}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{3+n}\,dx}{b^3}$$
$$= -\frac{A(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\sin}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.31, size = 120, normalized size = 0.85

$$-\frac{\sqrt{\sin^2(c+dx)}\,\cos^2(c+dx)\cot(c+dx)(b\cos(c+dx))^n\left(A(n+4)\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)+B(n+3)\cos\right.}{d(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(4 + n)*Hypergeometric
2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + B*(3 + n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(3 + n)*(4 + n)))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^3 + A\cos\left(dx+c\right)^2\right)\left(b\cos\left(dx+c\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

maple [F] time = 1.64, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\left(A+B\cos\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(A+B\,\cos\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.913 $\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=141

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2}\right.}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -A*(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(3+n)*hypergeom(
[1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {16, 2748, 2643}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2}\right.}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b
*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{1+n}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\cos(c+dx))^{1+n}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{2+n}\,dx}{b^2}$$
$$= -\frac{A(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\sin}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.25, size = 118, normalized size = 0.84

$$-\frac{\sqrt{\sin^2(c+dx)}\,\cos(c+dx)\cot(c+dx)(b\cos(c+dx))^n\left(A(n+3)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)+B(n+2)\cos\right.}{d(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + n)*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + B*(2 + n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^
2])/(d*(2 + n)*(3 + n)))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos(dx+c)^2+A\cos(dx+c)\right)(b\cos(dx+c))^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

maple [F] time = 1.18, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(b\cos\left(dx+c\right)\right)^{n}\left(A+B\cos\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(A+B\,\cos\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \cos{\left(c + dx \right)}\right)^{n} \left(A + B \cos{\left(c + dx \right)}\right) \cos{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*cos(c + d*x), x)
```

3.914 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=141

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+}{2}\right.}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -A*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(2+n)*hypergeo
m([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)
^2)^(1/2)
```

Rubi [**A**] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2748, 2643}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+}{2}\right.}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b*C
os[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d
*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = A\int (b\cos(c+dx))^n\,dx + \frac{B\int (b\cos(c+dx))^{1+n}\,dx}{b}$$
$$= -\frac{A(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}} - \frac{B(}{}$$

Mathematica [A] time = 0.18, size = 112, normalized size = 0.79

$$-\frac{\sqrt{\sin^2(c+dx)}\,\cot(c+dx)(b\cos(c+dx))^n\left(A(n+2)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)+B(n+1)\cos(c+dx)\,{}_2F_1\left(\right.\right.}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(2 + n)*Hypergeometric2F1[1/2, (1 + n
)/2, (3 + n)/2, Cos[c + d*x]^2] + B*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[
1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + n
)*(2 + n)))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (b\,\cos(c+d\,x))^n\,\left(A+B\,\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int((b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n\,(A+B\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x)), x)
```

3.915 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=132

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos\right.}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -A*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+
c)/d/n/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*
n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {16, 2748, 2643}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos\right.}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] -((A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]
^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{-1+n}(A+B\cos(c+dx))\,dx$$
$$= (Ab)\int (b\cos(c+dx))^{-1+n}\,dx + B\int (b\cos(c+dx))^n\,dx$$
$$= -\frac{A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\sin(c+}{dn\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.20, size = 109, normalized size = 0.83

$$-\frac{b\sqrt{\sin^2(c+dx)}\,\cot(c+dx)(b\cos(c+dx))^{n-1}\left(A(n+1)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)+Bn\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\right.\right.}{dn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(A*(1 + n)*Hypergeometric2F1[1/
2, n/2, (2 + n)/2, Cos[c + d*x]^2] + B*n*Cos[c + d*x]*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(1 + n)
))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas
")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \left(b\cos \left(dx+c\right)\right)^{n}\left(A+B\cos \left(dx+c\right)\right)\sec \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x)
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos \left(dx+c\right)+A\right)\left(b\cos \left(dx+c\right)\right)^{n}\sec \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima
")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos \left(c+d\,x\right)\right)}^{n}\,\left(A+B\,\cos \left(c+d\,x\right)\right)}{\cos \left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x),x)
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos \left(c+dx\right)\right)^{n}\left(A+B\cos \left(c+dx\right)\right)\sec \left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c),x)
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

3.916 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{Ab\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\text{cc}\right.}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] A*b*(b*cos(d*x+c))^(-1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c
)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^n*hypergeom([
1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{Ab\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\text{cc}\right.}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (A*b*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Co
s[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int (b\cos(c+dx))^{-2+n}(A+B\cos(c+dx))\,dx$$
$$= \left(Ab^2\right)\int (b\cos(c+dx))^{-2+n}\,dx + (bB)\int (b\cos(c+dx))^{-}$$
$$= \frac{Ab(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+d\right.}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.19, size = 109, normalized size = 0.83

$$-\frac{b\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^{n-1}\left(An\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)+B(n-1)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\right.\right.}{d(n-1)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(A*n*Hypergeometric2F1[1/2, (-1
 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + B*(-1 + n)*Cos[c + d*x]*Hypergeometri
c2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n
)*n))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n (A+B\cos(dx+c)) \left(\sec^2(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^n\ (A+B\cos(c+d\,x))}{\cos(c+d\,x)^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n (A+B\cos(c+dx)) \sec^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.917 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=139

$$\frac{Ab^2\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}+\frac{bB\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\right.}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] A*b^2*(b*cos(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)+b*B*(b*cos(d*x+c))^(-1+n)*hypergeo
m([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{Ab^2\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}+\frac{bB\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\right.}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (A*b^2*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (b*B*(b*Cos[
c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int (b\cos(c+dx))^{-3+n}(A+B\cos(c+dx))\,dx$$
$$= \left(Ab^3\right)\int (b\cos(c+dx))^{-3+n}\,dx + \left(b^2B\right)\int (b\cos(c+dx$$
$$= \frac{Ab^2(b\cos(c+dx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+d}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.18, size = 118, normalized size = 0.85

$$-\frac{\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\sec^2(c+dx)(b\cos(c+dx))^n\left(A(n-1)\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)+B(n-2)\cos(}{d(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + n)*Hypergeometric2F1[1/2, (-2 +
 n)/2, n/2, Cos[c + d*x]^2] + B*(-2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2
, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^
2])/(d*(-2 + n)*(-1 + n)))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

maple [F] time = 1.37, size = 0, normalized size = 0.00

$$\int \left(b \cos \left(d x+c\right)\right)^{n} \left(A+B \cos \left(d x+c\right)\right) \left(\sec^{3}\left(d x+c\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(B \cos \left(d x + c\right) + A\right)} \left(b \cos \left(d x + c\right)\right)^{n} \sec \left(d x + c\right)^{3}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.918 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{Ab^3\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}+\frac{b^2B\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\right.}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] A*b^3*(b*cos(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*
x+c)^2)*sin(d*x+c)/d/(3-n)/(sin(d*x+c)^2)^(1/2)+b^2*B*(b*cos(d*x+c))^(-2+n)
*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(2-n)/(sin(d*
x+c)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{Ab^3\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}+\frac{b^2B\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\right.}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (A*b^3*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2]) + (b^2*
B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= b^4\int (b\cos(c+dx))^{-4+n}(A+B\cos(c+dx))\,dx\\ &= \left(Ab^4\right)\int (b\cos(c+dx))^{-4+n}\,dx+\left(b^3B\right)\int (b\cos(c+dx)\\ &= \frac{Ab^3(b\cos(c+dx))^{-3+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1+n);\cos\right.}{d(3-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.18, size = 118, normalized size = 0.84

$$-\frac{\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\sec^3(c+dx)(b\cos(c+dx))^n\left(A(n-2)\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)+B(n-3)\cos(\right.}{d(n-3)(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-2 + n)*Hypergeometric2F1[1/2, (-3 +
 n)/2, (-1 + n)/2, Cos[c + d*x]^2] + B*(-3 + n)*Cos[c + d*x]*Hypergeometric
2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2])*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]
^2])/(d*(-3 + n)*(-2 + n)))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n (A+B\cos(dx+c)) \left(\sec^4(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n \sec(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.919 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{9}{2}}(c$$

```
[Out] -2*A*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*cos(d*x+c)^
(9/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 9/4+1/2*n],[13/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(9+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{2A\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{9}{2}}(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*A*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n
)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c +
d*x]^2]) - (2*B*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2
, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{5}{2}+n}(c+dx)(A+$$

$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{5}{2}+n}(c+dx)\,dx$$

$$= -\frac{2A\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}\right.}{d(7+2n)\sqrt{\sin^2(c+}}$$

Mathematica [A] time = 0.45, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+9)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx\right.\right.}{d(2n+7)(2n+9)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(9 + 2*n)*Hyperge
ometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + B*(7 + 2*n)*Co
s[c + d*x]*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2
])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(9 + 2*n))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^3+A\cos\left(dx+c\right)^2\right)\left(b\cos\left(dx+c\right)\right)^n\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{5}{2}}(dx+c)\right)(b\cos(dx+c))^n(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{5/2}\,(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.920 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{7}{2}}(c+\cdots}{}$$

```
[Out] -2*A*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*cos(d*x+c)^(
7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{7}{2}}(c+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*A*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n
)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)(A+$$

$$= \left(A\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\,dx$$

$$= -\frac{2A\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}\right.}{d(5+2n)\sqrt{\sin^2(c+}}$$

Mathematica [A] time = 0.36, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+7)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right.\right.}{d(2n+5)(2n+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(7 + 2*n)*Hyperge
ometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + B*(5 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]
)*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(7 + 2*n))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(B\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*
x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{3}{2}}(dx+c)\right)(b\cos(dx+c))^n(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{3/2}\,(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.921 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+$$

```
[Out] -2*A*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*cos(d*x+c)^(
5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*A*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n
)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{1}{2}+n}(c+dx)(A$$

$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{1}{2}+n}(c+dx)\,d$$

$$= -\frac{2A\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\right.}{d(3+2n)\sqrt{\sin^2(c+}}$$

Mathematica [A] time = 0.26, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+5)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx\right)\right.}{d(2n+3)(2n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(5 + 2*n)*Hyperge
ometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + B*(3 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2])
*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(5 + 2*n))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n (A+B\cos(dx+c))\left(\sqrt{\cos}\,(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+d\,x)}\,(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.922 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c$$

```
[Out] -2*B*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*A*(b*cos(d*x+c
))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n
)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)(A+B\cos(c+$$

$$= \left(A\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\,dx + \left(B\cos^{-}\right.$$

$$= -\frac{2A\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos\right.}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.24, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+3)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx\right.\right.}{d(2n+1)(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(3 + 2*n)*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + B*(1 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])
*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(3 + 2*n))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n (A+B\cos(dx+c))}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(c+dx))^n (A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.923 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\,\sqrt{\cos(c+dx)}}-\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+$$

[Out] 2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*B*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\,\sqrt{\cos(c+dx)}}-\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]) - (2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)(A+B\cos(c+$$

$$= \left(A\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx + (B\cos^{-}$$

$$= \frac{2A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)\sin}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.27, size = 133, normalized size = 0.82

$$-\frac{2\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+1)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)+B(2n-1)}{d\left(4n^2-1\right)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-1
 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + B*(-1 + 2*n)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n (A+B\cos(dx+c))}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n\,\left(A+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(c+dx))^n (A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.924 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\right.}{d(1-2n)\sqrt{\sin^2(c+}}$$

```
[Out] 2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*x+c
))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-
2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\right.}{d(1-2n)\sqrt{\sin^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 +
 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)(A+B\cos(c+$$
$$= \left(A\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\,dx + (B\cos^{-}$$
$$= \frac{2A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)\sin}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.23, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\,\csc(c+dx)(b\cos(c+dx))^n\left(A(2n-1)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)+B(2n-3)\right.}{d(2n-3)(2n-1)\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + B*(-3 + 2*n)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n (A+B\cos(dx+c))}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n\,(A+B\,\cos(c+d\,x))}{\cos(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.925 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\right.}{d(3-2n)\sqrt{\sin^2(c+}}$$

```
[Out] 2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*x+
c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3
-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\right.}{d(3-2n)\sqrt{\sin^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c +
d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1
+ 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{7}{2}+n}(c+dx)(A+B\cos(c+$$

$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{7}{2}+n}(c+dx)\,dx + (B\cos^{-}$$

$$= \frac{2A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n);\cos^2(c+dx)\right)\text{s}}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.24, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\,\csc(c+dx)(b\cos(c+dx))^n\left(A(2n-3)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)+B(2n-5)\right.}{d(2n-5)(2n-3)\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-3 + 2*n)*Hypergeometric2F1[1/2, (-
5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + B*(-5 + 2*n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\,(A+B\cos(dx+c))}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n\,\left(A+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

3.926 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{7}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\right.}{d(5-2n)\sqrt{\sin^2(c+}}$$

```
[Out] 2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -7/4+1/2*n],[-3/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(7-2*n)/cos(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*x+
c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(
5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{7}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\right.}{d(5-2n)\sqrt{\sin^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Cos[c + d*x]^(7/2)*Sqrt[Sin[c +
d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1
 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sq
rt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{9}{2}+n}(c+dx)(A+B\cos(c+$$
$$= \left(A\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{9}{2}+n}(c+dx)\,dx + \left(B\cos^{-}\right.$$
$$= \frac{2A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-7+2n);\frac{1}{4}(-3+2n);\cos^2(c+dx)\right)\text{s}}{d(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.24, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\,\csc(c+dx)(b\cos(c+dx))^n\left(A(2n-5)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)+B(2n-7)\right.}{d(2n-7)(2n-5)\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-5 + 2*n)*Hypergeometric2F1[1/2, (-
7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2] + B*(-7 + 2*n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c +
 d*x]^2])/(d*(-7 + 2*n)*(-5 + 2*n)*Cos[c + d*x]^(7/2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n (A+B\cos(dx+c))}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n\,\left(A+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

3.927 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=169

$$-\frac{3Ab\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}-\frac{3bB\sin(c+dx)\sqrt[3]{}}{}$$

```
[Out] -3*A*b*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13
/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)-3*b*B*cos
(d*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1/2*m],[8/3+1/2*m],c
os(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3Ab\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}-\frac{3bB\sin(c+dx)\sqrt[3]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*b*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2,
(7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[
Sin[c + d*x]^2]) - (3*b*B*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hyper
geometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)(A+B\cos(c+}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{\left(Ab\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}} + \frac{\left(bB\sqrt[3]{b}\right.}{}$$

$$= -\frac{3Ab\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3\right.}{d(7+3m)\sqrt{\sin^2(}}$$

Mathematica [A] time = 0.51, size = 140, normalized size = 0.83

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\csc(c+dx)(b\cos(c+dx))^{4/3}\cos^{m+1}(c+dx)\left(A(3m+10)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2\right.\right.}{d(3m+7)(3m+10)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(B*(7 + 3*m)*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] +
A*(10 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2 + Ab\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos
(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))(b\cos(dx+c))^{\frac{4}{3}}(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{4/3}\,(A+B\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.928 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=167

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b}{}$$

```
[Out] -3*A*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2*m],[11/6
+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(5+3*m)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*x
+c)^(2+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 4/3+1/2*m],[7/3+1/2*m],cos(d
*x+c)^2)*sin(d*x+c)/d/(8+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5
 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeom
etric2F1[1/2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
8 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c+dx)(A+B\cos(c+d$$

$$= \frac{\left(A(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{2}{3}+m}(c+dx)\,dx}{\cos^{\frac{2}{3}}(c+dx)} + \frac{\left(B(b\cos\right.}{}$$

$$= -\frac{3A\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3\right.}{d(5+3m)\sqrt{\sin^2(}}$$

Mathematica [A] time = 0.32, size = 140, normalized size = 0.84

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\left(A(3m+8)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(\right.\right.}{d(3m+5)(3m+8)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(A*(8 + 3*m)*H
ypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + B*(5 + 3
*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 3*m)*(8 + 3*m))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))(b\cos(dx+c))^{\frac{2}{3}}(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{2/3}\,(A+B\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.929 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=167

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\sqrt[3]{b\,\mathrm{c}}}{}$$

```
[Out] -3*A*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2*m],[5/3+
1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(4+3*m)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*x+
c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13/6+1/2*m],cos(d
*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\sqrt[3]{b\,\mathrm{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4
 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeom
etric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{\frac{1}{3}+m}(c+dx)(A+B\cos(c+dx))\,d}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}} + \frac{\left(B\sqrt[3]{b\cos(\right.}{}$$

$$= -\frac{3A\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\right.}{d(4+3m)\sqrt{\sin^2(c+}}$$

Mathematica [A] time = 0.31, size = 140, normalized size = 0.84

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+1}(c+dx)\left(A(3m+7)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{m}{2}+\frac{5}{3};\cos^2(c+dx)\right)\right.}{d(3m+4)(3m+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(A*(7 + 3*m)*H
ypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + B*(4 + 3*m)
*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(d*(4 + 3*m)*(7 + 3*m))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))\,(b\cos(dx+c))^{\frac{1}{3}}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{1/3}\,(A+B\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx))\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)
```

3.930 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=167

$$-\frac{3A\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1(}{d(3m+5)\sqrt{\sin^2(}}$$

```
[Out] -3*A*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*
sin(d*x+c)/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*x+
c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d
/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3A\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1(}{d(3m+5)\sqrt{\sin^2(}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m)/
6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x]
)^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int \cos^{\frac{2}{3}+}}{\sqrt[3]{b\cos(c+dx)}}$$

$$= -\frac{3A\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)\sin(}{d(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.32, size = 140, normalized size = 0.84

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(A(3m+5)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)+B(3m+\right.}{d(3m+2)(3m+5)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(5 + 3*m)*Hypergeometric2F1[1/2, (
2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + B*(2 + 3*m)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos(
d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(\cos^m(dx+c))(A+B\cos(dx+c))}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^m\,\left(A+B\,\cos(c+d\,x)\right)}{(b\,\cos(c+d\,x))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\cos^m(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/3),x)
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)
```

3.931 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=167

$$-\frac{3A\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\right.}{d(3m+4)\sqrt{\sin^2(c}}$$

```
[Out] -3*A*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x+c)^2)*
sin(d*x+c)/d/(1+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*x+
c)^(2+m)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/
(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3A\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\right.}{d(3m+4)\sqrt{\sin^2(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin
[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/
6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x]
)^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{(b\cos(c+dx))^{2/3}}$$

$$= \frac{\left(A\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{-\frac{2}{3}+m}(c+dx)\,dx}{(b\cos(c+dx))^{2/3}} + \frac{\left(B\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{\frac{1}{3}+m}}{(b\cos(c+dx))^{2/}}$$

$$= -\frac{3A\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\cos^2(c+dx)\right)\sin(c}{d(1+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.31, size = 140, normalized size = 0.84

$$-\frac{3\sqrt{\sin^2(c+dx)}\,\csc(c+dx)\cos^{m+1}(c+dx)\left(A(3m+4)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)+B(3m+\right.}{d(3m+1)(3m+4)(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(4 + 3*m)*Hypergeometric2F1[1/2, (
1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + B*(1 + 3*m)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^
2])/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos(
d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(\cos^m(dx+c))\,(A+B\cos(dx+c))}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^m\,\left(A+B\,\cos(c+d\,x)\right)}{(b\,\cos(c+d\,x))^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\cos^{m}{(c + dx)}}{(b\cos{(c + dx)})^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)
```

3.932 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=171

$$\frac{3A\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}\ldots\right.}{bd(3m+2)\sqrt{\sin^2(c+\ldots}}$$

```
[Out] 3*A*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)^2)*sin(
d*x+c)/b/d/(1-3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*x+c)
^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/b/d/
(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{3A\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}\ldots\right.}{bd(3m+2)\sqrt{\sin^2(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +
 d*x]^2]) - (3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (
8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(
1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{4}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{4}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{1}{3}}}{b\sqrt[3]{b\cos(c+d}}$$

$$= \frac{3A\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\cos^2(c+dx)\right)\sin(c+}{bd(1-3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.34, size = 140, normalized size = 0.82

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(A(3m+2)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)+B(3m-}{d(3m-1)(3m+2)(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(2 + 3*m)*Hypergeometric2F1[1/2, (
-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + B*(-1 + 3*m)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-1 + 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*co
s(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(\cos^m(dx+c))(A+B\cos(dx+c))}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^m\,\left(A+B\,\cos(c+d\,x)\right)}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\cos^m(c + dx)}{(b\cos(c + dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(4/3), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```